

LLULL'S WRITINGS ON ELECTORAL SYSTEMS*

1. Introduction

We present three of Llull's works dealing with electoral systems, to be cited with the abbreviations given in the parentheses:

Artifitium electionis personarum (AEP)
En qual manera Natana fo eleta a abadessa (B24)
De arte electionis (DAE)

B24 is the twenty-fourth chapter of the novel *Blaquerna*. For a long time this was the only source on Llull's electoral systems until, in 1937, *Honecker* [10] rediscovered DAE. In his commentary, *Honecker* [10, p. 307, n. 11a] doubted whether the available bibliographical evidence would imply the existence of yet another work of similar title and content. In 1959, finally, *Pérez Martínez* [28] announced the discovery of the manuscript of AEP, and later included a description of it in his catalogue of Roman libraries [29, p. 39]. While *Pérez Martínez* was referenced by *Platzeck* [30, vol. II, p. 7] and *Llinarès* [12, p. 177, n. 25], neither of the three authors mentioned *Honecker's* discovery of DAE. Conversely, in vol. 17 (1989) of the authoritative *Raimundi Lulli Opera Latina* [20, p.

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X] the editors claimed that the *Ars electionis* is preserved only in a manuscript in Kues, that is DAE, thus neglecting AEP.

The oldest list of Llull's works is that in his autobiographical *Vita coetanea* [19, p. 307], reproduced with annotations in *Hillgarth* [9, p. 341]. The items of interest to us are:

- 86. *Ars notandi*
- 87. *Ars eleccionis*
- 88. *Alia ars eleccionis*

We believe AEP to be identical with number 87 and DAE to coincide with number 88, on the grounds that in Cod. Vat. lat. 9332 the text of AEP is immediately preceded by that of the *Ars notandi*, and that AEP is longer and more detailed than DAE. DAE comprises about 650 words, while AEP is about 100 words longer. Furthermore, *Honecker* [10, p. 304, n. 2a] quotes two titles, *De arte eligendorum prelatorum magna* and *De arte electionis compendiosa*, from a Renaissance bibliography drawn up by Arias de Loyola. Honecker identifies DAE with the second title. This assignment seems persuasive to us, in that it leaves the first title for AEP, which indeed mentions repeatedly that its electoral system is intended for the election of prelates.

The recent literature, surveyed by *Lohr/Bonner* [22], concentrates on Llull's philosophy and does not embrace his electoral systems. Moreover, the electoral systems proposed both by Llull and by Cusanus have long been ignored in the political science literature. This is beginning to change, thanks chiefly to the research of *McLean/London* [24, 25], *Meuthen* [27], and *Felsenthal/Machover* [5], as evidenced in the recent monographs of *McLean/Urken* [26] and *Colomer* [3].

Our paper is organized as follows. In Section 2 we comment on the chronological sequence of AEP, B24, and DAE. For a particular work, we refer to the number and the date of composition given in *A Chronological Catalogue of Ramon Llull's Works of Bonner* [1, vol. II, pp. 1257-1304], quoted as «Bo». Sections 3-5 contain transcriptions and translations of the three texts. In Section 6 we describe and compare the electoral systems in these writings. The tract AEP, edited here for the first time, contributes at least three details leading beyond B24 and DAE:

- (1) multiple records are kept during the election session,
- (2) *unus punctus* is recorded for the winner of each pairwise comparison,
- (3) lots are thrown if candidates are tied with the same number of winning points.

In Section 7 we discuss alternative interpretations. Section 8 concludes the paper with a final résumé.

The present paper is accompanied by a web edition at www.uni-augsburg.de/llull/ which offers a linked triple text consisting of facsimile, transcription and translation, for each of the three works. For details of this web edition, see *Drton et al.* [4]. A narrative on the somewhat accidental circumstances that led to the present research can be found in *Pukelsheim* [34].

2. The sources

The dating of the first text, AEP (Bo II.A.10: 1274-83), is least certain. *Platzek* [30, vol. II, p. 7] places it in the period 1273-5, without providing any detailed evidence. The recent study of *Santanach i Suñol* [39] reconfirms the period 1274-83 from Bonner's catalogue. The expert opinion is that, in all probability, Llull completed the novel, *Blaquerna*, in 1283 in Montpellier; see *Soler i Llopart* [42]. We shall argue in Section 6 that the electoral systems in AEP and in B24 are distinct from each other. In fact, they are so different that it would seem to us most unlikely that Llull wrote the two works back to back. Therefore we would tend to believe that AEP was written quite some time before 1283.

The AEP is preserved in a single manuscript, Codex Vaticanus latinus 9332 of the Biblioteca Apostolica Vaticana, as reported by *Pérez Martínez* [28]. The call number 9322 quoted in *Pérez Martínez* [29, p. 38] is a misprint. The bibliographic information given by *Glorieux* [8, p. 301], that AEP appears in codex Clm. 9332 in Munich, confuses libraries; at no time did the Bayerische Staatsbibliothek own a codex with this call number.

Cod. Vat. lat. 9332 is an undecorated collection of fascicles in quarto (22.5 x 15 cm), consisting of several distinct parts and totaling 326 folios (paper). It begins with Llull's *Ars notatoria* (Bo II.A.2: 1274-6), immediately followed by AEP on folios 11r-12v. Both texts date from the second half of the fifteenth century, and are written by the same hand. The first text starts with a heading giving the title. The second text, AEP, has no heading; instead the title has to be inferred from the colophon: *Finis artificii electionis personarum*. The appearance of AEP arouses sympathetic feelings as to how difficult life was for a medieval copyist: the pen blunted, the ink blotted, the writing blurred. *Gayà* [13, p. 23] tactfully calls the impression that AEP conveys as being *de caligrafía más oscura*.

AEP appears to have been written by a skilled hand, as a quick, cursory excerpt. The small cursive, with many abbreviations, is in parts hard to read. The number of lines per page changes repeatedly, the rather complicated figure on folio 11r is drawn free-hand. There are no references to the author of the text, nor to the copyist of the manuscript. However, on folios 13r-15v of the codex appears a third Llull text, *Liber de accidente et substantia* (Bo IV.75: 1313). This text, though by a different hand, says that it is *a Raimundo compilatus*.

Formerly, Cod. Vat. lat. 9332 was owned by Pier Leoni (d. 1492), court physician of Lorenzo de' Medici. It is most likely that it was Leoni himself who copied the *Ars notatoria* and the *Artifitium electionis personarum*. This is the conclusion arrived at by *Ruysschaert* [38, p. 57], on the basis of a comparison of the hands in Cod. Vat. lat. 11585, 9414, 9425 and in *Reginensis* 1773, all from Leoni's library. Note that *Ruysschaert's* singular, *le premier texte*, comprises a plural, the *Ars notatoria* and the *Artifitium electionis personarum*. For more on Leoni and his relations to Lull's work see *Rotzoll* [37].

For B24, Chapter 24 of the novel *Blaquerna* (Bo II.A.17: 1283), we rely on the Catalan manuscript, Codex Hispanicus 67 of the Bayerische Staatsbibliothek, Munich. The volume consists of 268 folios (paper), and dates from the end of the fourteenth or the beginning of the fifteenth century. Authors occasionally quote the obsolete call number Cod. Hisp. 610, from p. 91 in volume 7 of the 1858 (reprinted in 1971) *Catalogus codicum manu scriptorum Bibliothecae Regiae Monacensis*. The transcription in Section 4 below was kindly provided by *Soler i Llopart* [15]. We also consulted the transcription of *Galmés/Ferrà* [17], the English translations of *Peers* [14] and *McLean/London* [24, 25], and the French translation of *Llinarès* [18].

DAE (Bo III.38: 1299) is dated rather exactly by Lull himself: it was written on 1 July 1299, in Paris. The *codex unicus* preserving this text is the Codex Cusanus 83 in the Library of the Sankt Nikolaus-Hospital/Cusanusstift in Bernkastel-Kues. *Honecker* [11, p. 571, n. 23] considers it almost certain that it was written by Nicholas Cusanus. This view is reconfirmed by the more recent findings that Cusanus spent the spring of 1428 in Paris, copying parts of Lull's works available in the Paris libraries. *Roth* [36, Sections A.I and A.V] reviews the current state of research on the relation of Cusanus to Lull, and also gives a description of Cod. Cus. 83. For the transcription and the translations we consulted *Honecker* [10, pp. 308-309] and *McLean/London* [24, 25].

In our transcriptions, numbers in small print are the line counts in the manuscripts; most of the paragraph breaks are ours. In the translations, we follow the transcriptions rather closely except for adding punctuation. A few additions and omissions that we considered helpful are indicated by brackets []. In the main text, bracketed numbers refer to reference items at the end of the paper. Lull's electoral systems rely on his Third Figure, the quaternary 16 candidate version being part of AEP and the ternary 9 candidate version heading DAE. These tables are built up from entities which we call *cells*, following today's statistical terminology. While Lull's notion of *ars* carries a rather broad meaning, see *Bonner* [1, vol. I, p. 62], we translate *ars electionis* simply by *electoral system*, for the purposes of the present paper.

7Hec quidem figura signat artem per quam semper poterunt eligi homines 8conuenientes ad dignitatem prelati- cam et per quam quod in electione ipsa non 9fiat fraus nec simonia poterit evitari. [folio 11v] 1Littere enim xvi ex quibus composita est ipsa figura sig- nant 16m personas habentes 2uocem ad eligendum prelatum. Si uero in ca- pitulo fuerint plures 3aut pauciores persone habentes uocem ad faciendum ipsam electionem 4multiplicari debent uel diminui littere ac etiam figura pro- portionaliter secundum quod 5persone plures uel pauciores existent. Si autem essent tot persone 6quot littere alpha- beti non sufficerent loco ipsarum po- nantur alia signa 7uelud numerus vt vnus duo tres uel ut primus secundus tertius 8quartus et sic multiplicando quousque quelibet persona habet sig- num 9suum. Et postea ex dictionibus ipsis debet componi figura 10sicut ex litteris consistit. Composita enim pre- dicta figura non sufficit 11nisi pro per- sonis xvim.

Prius quidem ordinari oportet 12quod secundum quod aliquae persone fuerint in capitulo primus⁴ alteris in sedibus 13preponatur ac quod eis attribuantur priores littere uel priora signa 14ipsius figure atque quod primo uocentur et prius de ipsis inuestigentur.

15Post autem hoc oportet statui quod in electione considerentur tria quorum

This figure signifies the system by which one can always elect men worthy of the dignity of a prelate, and by which [it is guaranteed] that in the election fraud does not occur and by which simony can be avoided. Now the 16 letters of which this figure is composed signify 16 persons having a vote in the election of the prelate. If in the chapter there are more or fewer persons having a vote to conduct this election, the letters and also the figure must be augmented or diminished in accordance with the greater or lesser number of persons. If, however, there are so many persons that the letters of the alphabet do not suffice, other signs are introduced, such as one, two, three, or first, second, third, fourth, and so on, continuing until each person has his own sign. And thereafter the figure must be composed from these [signs] as if it consisted of letters. The figure composed as described above, however, suffices only for 16 persons.

Initially it is necessary to establish that, since several persons are in the chapter, the first is seated before the others, and that the prior letters or the prior signs of the figure are assigned to them, and that they should be the first to be called upon and examined in these [letters].

After this it is necessary to ascertain that in the election three things should

⁴ (AEP f.11v, l.12): We are uncertain whether *primus* means the most senior person, the person highest in rank, or the foremost person.

16Primum est honestas et sanctitas u-
te. Secundum est scientia et sapientia.
17Tertium est conueniens dispositio
cordis. Que quidem tria quelibet per-
sona 18habens uocem in capitulo ad
sacra dei euangelia[#] iuret considerare
ac 19semper preeligere personam in
qua ipsa tria melius fuerint.

20Post uero ordinacionem predictam
oportet quod habeantur tres figure
21similes figure predictae que teneantur
in locis diuersis. Et 22postmodum^c
imponantur nomina uidelicet quod
vna personarum uocetur *b* 23alia *c* alia
d et sic de singulis quousque quelibet
persona habet litteram [folio 12r] 1sibi
appropriatam. Si autem figura com-
posita fuerit ex aliis signis 2attribuan-
tur ipsa signa personis ipsis secun-
dum quod dixi de litteris. 3Quo qui-
dem ordinato ponant se in domo et
incipiant facere electionem 4suam tali
modo.

Primo enim oportet quod exeant do-
mum ille due 5persone quibus littere
uel signa prime camere attribute fue-
rint 6et postea querat omnibus aliis per
sacramentum que ipsarum 7duarum
melius conueniens et digna fuerit se-
cundum tria predicta ad 8dignitatem
ipsam habendam et etc. Et omnes res-
ponderint et eligerint 9prout eis uidebitur
fiat vnus punctus in littera attri-
buata illi 10persone que plures uoces

be considered, of which the first is ho-
nesty and holiness of life, the second is
knowledge and wisdom, and the third
is a suitable disposition of the heart.
Each person having a vote in the chap-
ter should take an oath by the holy
gospels of God to consider these three
things and to always elect the person in
whom they are best [embodied].

After this ruling it is necessary that
three figures identical to the figure
described above be held ready at dis-
tinct locations. And thereafter the
names should be inserted in such a
way that one of the persons is denoted
by *b*, another by *c*, another by *d*, and
so on for each, until every person has
an appropriate letter. If, however, the
figure is composed of other signs,
these signs are assigned to the persons
according to what I described for the
letters. With this matter taken care of,
[the electors] should betake them-
selves into a hall and begin to conduct
the election in the following way.

Firstly, the two persons whose letters
or signs appear in the first cell should
leave the hall. And afterwards [some-
body] inquires of all the others on oath
which of the two is better suited and
worthier according to the three things
described above to be bestowed this
dignity, etc. And all shall respond, and
shall elect as it appears [fit] to them.
[Then] a dot is placed by the letter
assigned to the person who has the

[#] (AEP f.11v, l.18): The ms. has *evagnelia*, a Tuscan variant of *euangelia*.

^c (AEP f.11v, l.22): *postmodum* corrected from *posmodum*.

habuerit. Qui punctus fiat ipsi¹¹ littere in qualibet figurarum existentium in locis diuersis. Si uero vna¹² habuerit tot uoces ut altera fiat in qualibet littera ipsius camere¹³ punctus vnus et hoc in qualibet figurarum.

Cum autem inuestigatum¹⁴ fuerit taliter de camera prima videlicet quod de camera ipsius *b c* fiat¹⁵ illud idem de camera 2a videlicet ipsorum *b d* et *c* debet ad locum¹⁶ suum redire et facere illud quod altere persone facient quousque¹⁷ uocabuntur camere in quibus ipse^d erit. Et hec talis inuestigatio¹⁸ semper fiat per ordinem videlicet quod primo fiat inuestigatio de omnibus ca¹⁹meris in quibus erit *b* et postea de illis in quibus erit *c* postremo²⁰ uero de illis in quibus erit *d* et sic de omnibus per ordinem.

Facta autem²¹ inuestigatione camerarum omnium debent numerari puncta²² cuiuslibet littere et si in aliqua litterarum sive signiorum ex quibus composita²³ fuit figura ipsa reperiantur plura puncta quam in aliqua litterarum persona²⁴ illa pro qua illa littera seu signum posita fuerit eligatur ad²⁵ dignitatem propter quam facta fuerit inuestigatio supradicta [folio 12v] quecumque dignitas sit.

Si autem contigerit duas personas aut²⁶ plures habuisse numerum uocum equalem oportet ipsas exire domum

most votes. Such a dot is marked in each of the figures existing at the distinct locations. If now one [person] has as many votes as another, then a dot is placed by both letters of this cell, and this [is done] in each of the figures.

After in this way [the pairwise comparison] has been examined in the first cell, namely in the cell with *b* and *c*, the same is done in the 2nd cell, namely that with *b* and *d*; and *c* must go back to his place and do what the other persons do, until again cells will be called in which his letter occurs. And this examination continues in the following order, namely that the examination is first extended over all cells in which *b* appears, and afterwards over those in which *c* appears, and then over those in which *d* appears, and this way over all [cells], in order.

Once the examination of all cells has been completed, the dots of each letter must be counted. And if in any of the letters or signs from which this figure is composed more dots are determined than in any of the other letters, the person for whom this letter or sign stands is elected to the dignity for which the examination that is described above has been conducted, whatever the dignity.

If, however, it happens that two persons or more have an equal number of votes, it is necessary that they leave

^d (AEP f.12r, l.17): *ipse* corrected from *ipsum*.

³et quod relique persone remanentes
 qui nequaquam tot uoces habuerint
⁴item de nouo iurent obseruando tria
 superius posita preheligere ⁵digniorem
 et magis conuenientem ad ipsam dig-
 nitatem habendam ⁶et quod illa in qua
 plures uoces erunt concordēs eligatur.
 Si autem contingat ⁷tot uoces concor-
 dari in una sicut in altera miscentur^e
 sortes super ⁸illas que in ista electione
 última equalem numerum uocum ha-
 bebunt ⁹et illa cui sors euenerit eli-
 gatur.

Si autem in principio inuestigationis
¹⁰predicte absens fuerit aliqua persona
 habens uocem in ipso capitulo ¹¹opor-
 tet quod attribuitur ei vna predictarum
 litterarum ipsius figure uel signiorum
¹²Si fuerint alia signia quam littere. Si
 uero plures absentes fuerint ¹³tribu-
 atur cuilibet littera vna uel signum. Et
 de ipsa uel de ipsis ¹⁴quemadmodum
 de presentibus inuestigetur secundum
 ordinationem predictam.

¹⁵Finis artificii electionis personarum.

the hall and that the other remaining
 persons, no matter how many votes
 they have, again take an oath to select,
 while observing the above-mentioned
 three things, the one who is more wor-
 thy and suitable for this dignity, and
 that the one for whom more votes are
 counted will be elected. If now it hap-
 pens [yet again] that as many votes
 are counted for one [person] as for
 another, lots are thrown over those
 who had an equal number of votes in
 the last election, and the one whose
 lot wins is elected.

If now at the beginning of the above
 examination a person who has a vote
 in this chapter is absent, it is neces-
 sary that he be assigned one of the
 said letters of this figure, or one of the
 signs if signs other than letters are
 used. If more are absent, each is assig-
 ned one letter or sign. And with the
 one or more [absent] as well as with
 those present the examination is con-
 ducted according to the rules describ-
 ed above.

This is the end [of our exposition] of a
 system for the election of persons.

^e (AEP f.12v, l.7): *miscentur* corrected from *mictantur*.

4. En qual manera Natana fo eleta a abadesa (Bayerische Staatsbibliothek, Cod. Hisp. 67, f. 32v-34r)

[folio 32v] ¹Nathana e totes les dones qui ha₂vien veu a elecció foren en lo capitol per ele₃ger abadesa. Natana dix a totes les dones que necessari₄tat es molt gran haver bo major, car en la bonitat ₅del major dona Deus virtut al menor. —On, con nostre ₆major sia passat d'esta vida en l'altra segons que ₇natura e rahó requer, neçessaria cosa es a nosaltres ₈que encerquem enfre nos qual de nos es major en ₉santa vida e en la amor de Deu; cor, aquel es digne ₁₀de esser nostre pastor, segons l'ordenament e la vo₁₁lentat de Deu.

Totes les dones volgueren eleger ₁₂abadesa segons la manera en la qual havien acustu₁₃mat a eleger, mas Nathana dix que ella havia en₁₄tesa novella manera de elecció, la qual está en art ₁₅e en figures, la qual art segueix les condicions del *Li₁₆bre del gentil e dels iij savis*, lo qual segueix la *Art ₁₇de atrobar veritat*. —Per aquella manera— dix Natana, —[folio 33r] ¹es atrobada veritat per la qual veritat podem atrobar ₂aquella dona qui es pus cuvinent e mellor a esser ₃abadesa.

Pregada fo Natana per totes les dones que ₄digués la manera segons la qual per art poguesen ₅atrobar e eleger la dona qui es millor a abadesa. ₆Nathana respós dient estes paraules: —De

Natana and all the sisters who had a vote in the election gathered in a chapter to elect the abbess. Natana said to all the sisters that there was a very great necessity to have a good superior, for through the goodness of the superior God gives virtue to the subordinate. «Now, since our superior has passed from this life to the other as is required by nature and reason, it is needful for us all that we seek from among ourselves the one who is greatest in holiness of life and the love of God; for she is worthy to be our pastor according to the ordinance and the will of God.»

All the sisters wanted to elect the abbess according to the method to which they had been accustomed to elect. But Natana said that she had heard of a new electoral method based on a system and figures; this system follows the conditions of the *The Book of the Gentile and the Three Wise Men*, which follows *The Art of Finding the Truth*. «By this method,» said Natana, «is found the truth; by this truth we will be able to find the sister who is most suitable and best to be our abbess.»

Natana was asked by all the sisters to describe the method according to which, on the basis of the system, one can find and elect the sister who is best suited to be abbess. Natana replied

la art de elec₇ció vos diré breument los començamens. Aquella art ₈es departida en dues parts: la primera part es de e₉leger los elegendors qui elegen lur pastor; segona ₁₀part es en qual manera dejen eleger lur major; e, per ₁₁açó, primerament vos vull recomptar de la primera ₁₂part e puy de la segona.

Natana dix: –Nos som ₁₃xx dones en est capitol qui havem veu a eleger pas₁₄tor. Segons art se cové que elejam, d'estes xx dones, ₁₅nombre senar qui sia en v o en vij, cor aquest nom₁₆bre es pus cuvinent a elecció que altre; e lo vij nom₁₇bre es pus cuvinent quel v. On, primerament, sia ₁₈fet sagrament per totes les dones a dir veritat e sia ₁₉demanat secretament a la primera dona qual de les ₂₀xix dones son pus cuvinents a esser vij qui elegen ₂₁major; e, après, sia demanada la segona dona, he ₂₂puxes la terça e axí per orde tro a la derrera. E cas₂₃cuna vegada, scriva hom ço que diu cascuna de les ₂₄dones; a la fi, sia vist quals son aquelles dones qui han ₂₅haudes mes veus; e aquelles que hauran haudes ₂₆mes de veus, aquelles sien les vij dones qui degen ₂₇eleger abadessa.

«Segona part de elecció es con los ₂₈vij elegendors elegen pastor. On, primerament, [folio 33v] ₁cové que los vij elegendors se covenguen al eleger de ₂çert nombre e de çertes persones, segons quel[s] será vijares, ₃e que cas-

with these words: «I will tell you briefly the principles of the electoral system. The system is divided into two parts. The first part is to elect the electors who then elect their pastor; the second part concerns the method by which they should elect their superior. So I will first tell you about the first part, and then about the second.»

Natana said: «In this chapter we are 20 sisters who have a vote in electing our pastor. According to the system we must elect from these 20 sisters an odd number, say 5 or 7; for such a [odd] number is more convenient for the election than another [even number], and the number 7 is more convenient than 5. First let an oath be taken by all the sisters to tell the truth. Then let the first sister be asked in secrecy which of the other 19 sisters are most suitable to be among the 7 who elect the superior. Afterwards, let the second be asked, and then the third, and so on, in order, until the last. And on each occasion, let it be written down what each sister says. In the end, let it be seen who are the sisters who have received the most votes. And those who have the most votes shall be the 7 sisters to elect the abbess.»

«The second part of the electoral system is how the 7 electors elect the pastor. Firstly, it is necessary that the 7 electors agree upon a certain number and upon certain persons, according to what seems to them good, and that

cuna persona comparen ab l'altra, se-
gons quatre 4condicions, ço es, a sa-
ber: qual ama e coneix mes Deu, ni
5qual ama e coneix mes virtuts, ni
qual coneix e desa6ma pus fortment
vicis; quarta es qual ha pus cu7vinent
persona.

Cascú dels *vij* elegedors pot eleger
8una persona a esser en lo nombre del
qual deu esser elet 9major; e cascú
dels *vij* elegedors cové esser en aquel
10nombre on pusca esser elet major. E,
per ço que pus 11planament puschats
entendre la art, sotsposem que 12lo
nombre sert sia en *ix* persones de les
quals sia triat 13e elet nostre pastor;
on, primerament, cové que los *vij*
14sien devesits en dues parts: *ij* a la
una part e *v* a l'al15tra e cové que los *v*
encerquen quals dels *ij* deu 16esser elet
e scrivas secretament aquell qui ha
mes 17de veus. Aprés cové que ab la *i*
qui ha mes de veus 18haudes sia com-
parat altre d'aquells *v* e que sia mes en
lo 19loch de aquell qui es estat vençut
per menors veus; 20e aquell vençut sia
mes en lo loch d'aquell qui es com-
21parat ab lo primer o ab lo segon. E
açó mateix, per 22orde, sia en tots los
altres, e sien en est nombre 23meses
lo[s] *vij* el *ix* qui no son dels elege-
dors. On 24siguent aquest nombre, se-
ran multiplicades *xxx*25*vj* cambres en
les quals aparran les veus de cas26cú, e
sia elet aquell qui haurá mes veus en
mes 27cambres.

they compare each person with every
other according to four conditions,
namely: which of them best loves and
knows God, which of them best loves
and knows the virtues, which of them
knows and most strongly hates the
vices and, fourthly, which is the most
suitable person.»

«Each of the 7 electors can select a
person to be among those from whom
the superior will be elected, and each
of the 7 electors necessarily is among
this number from whom the superior
is elected. And so that you can under-
stand the system more plainly, let us
suppose that this number consists of 9
persons from whom our pastor is to be
elected. Firstly, it is necessary that the
7 are divided into two groups: 2 on the
one side and 5 on the other; and then
the 5 must decide which of the 2
should be elected; and in secrecy let it
be written down who has the most
votes. Afterwards, the 1 who has the
most votes should be compared to
another one of the 5, and that this sis-
ter be set in the place of the one who
is defeated by fewer votes; and the
one who is defeated is put in the place
of the sister who is now compared
with the first or with the second. And
let this be done, in order, with all the
others; and to this number are added
the 8th and 9th persons who are not
among the electors. Following this
numbering, 36 cells are generated in
which appear the votes of each; and
let her be elected who has the most
votes in the most cells.»

Con Nathana hac mostrada la art de e₂₈lecció, una dona demaná a Nathana: –Si s'esdevé que ₂₉en les cambres haja qui hagen eguals veus, quen [folio 34r] ₁mana fer la art? Respós Nathana: –La art mana que ₂sia sobre aquells *ij* o *iiij* o mes encerchat per art ₃tant solament; e sia encerchat qual d'aquells se cové ₄mils ab les *iiij* condicions demunt dites; e aquel qui ₅mills se cové ab les condicions, aquell es digne de ₆esser elet.

Molt plach la art e la manera de elec₇ció a totes les dones, e totes digueren que segons ₈aquella art no podia hom errar en elecció; e totes ₉feeren establiment que per tots temps elegisen per aquella art ₁₀e manera que Natana recomptá; e trameteren encerchar ₁₁la art e aprengheren aquella. Aprés poch de dies ₁₂feeren elecció segons la art e atrobaren per art que Na₁₃thana devia esser abadesa.

Eleta fo Nathana a a₁₄badesa; gran desplaer hac Natana de son honrament. ₁₅Deus beney qui la volch honrar sobre totes les altres. ₁₆Duptá que les dones haguessen errat en la art e ₁₇volch veer les *xxvij* cambres en que está la art, ₁₈per tal que si havien errada la art e ella no devia esser ₁₉abadesa, que elegisen aquella ab qui l'endrechament ₂₀de la art se covengués. Natana e les altres dones ₂₁qui no eren stades de les *vij* dones qui havien ele₂₂git veeren la manera que havien

When Natana had explained the electoral system, one of the sisters asked her: «If it turns out that in the cells there are some who have equal votes, in which manner does the system proceed?» Natana replied: «The system demands that among these 2 or 3 or more one decides solely by means of the system; and that [in this way] one decides who of them combines best the 4 conditions mentioned above. And she who combines the conditions best is worthy to be elected.»

All the sisters were very pleased with the system and the electoral method; and all said that according to this system an electoral error could not occur. They all established a rule that at all times they would elect using the system and the method which Natana had described; and they sent away to find out about the system and they learned it. After a few days they held an election according to the system, and by the system it was found that Natana was to be abbess.

Natana was elected abbess; [but] Natana felt great displeasure at being thus honored. She blessed God for intending to honor her above all the others, [yet] feared that the sisters had erred in the system. And she wanted to see the 36 cells of which the system consists so that, if they had made an error in the system and she should not be abbess, they should elect another one in keeping with the proper working of the system. Natana and the other sisters who had not been among the 7 sisters who

hauda, segons art, ²³en la elecció, e atrobaren que la art havien seguit²⁴da segons ques convenia.

En gran pensament entrá Na²⁵thana con pogués e sabés regir si mateixa e les do²⁶nes e tots jorns cogitava com pogués ordenar lo ²⁷monestir a bones custumes.

had participated in the election, reviewed the method which had been followed in the election, according to the system, and found that the system had been followed as stipulated.

Natana began to think deeply on how she could and should rule herself and the sisters, and contemplated every day how she could direct the convent so as to encourage good ways.

5. De arte electionis

(Sankt Nikolaus-Hospital/Cusanusstift, Cod. Cus. 83, f. 47v-48r)

<i>bc</i>	<i>cd</i>	<i>de</i>	<i>ef</i>	<i>fg</i>	<i>gh</i>	<i>hi</i>	<i>ik</i>
<i>bd</i>	<i>ce</i>	<i>df</i>	<i>eg</i>	<i>fh</i>	<i>gi</i>	<i>hk</i>	
<i>be</i>	<i>cf</i>	<i>dg</i>	<i>eh</i>	<i>fi</i>	<i>gk</i>		
<i>bf</i>	<i>cg</i>	<i>dh</i>	<i>el</i>	<i>fk</i>			
<i>bg</i>	<i>ch</i>	<i>di</i>	<i>ek</i>				
<i>bh</i>	<i>ci</i>	<i>dk</i>					
<i>bi</i>	<i>ck</i>						
<i>bk</i>							

[folio 47v] ¹ [Q]uoniam bona electio in sancta ecclesia est ²valde necessaria ad eligendum personas ³communes cum per ipsas ecclesia gubernetur ⁴et ipse pugnent contra ecclesie inimicos qui faciunt peccata ut^f ⁵infideles et scismatici. Et hoc persone communes

Evidently a good electoral procedure in the Holy Church is very necessary for electing office holders, since by them the church is governed and they fight against the church's enemies who commit sins like the infidels and schismatics. And the elected office holders

^f (DAE f.47v, l.4): Obscured by correction. Honecker [10, p. 308, n. 3] reads: *qui sunt peccatores*.

electe⁶ facere non possunt nisi bone⁶ sint et bene proporcionate matri⁷ earum que bona est et alta. Quarum mater est sacrosancta roma⁸na ecclesia que magnas habet passiones per illos qui se fingunt⁹ bonos esse filios qui boni non sunt sed mali matri derogantes¹⁰ et bona eius iniuste occupantes. Unde propter hoc doctrinam¹¹ dare vo¹¹lumus ad elygendum communem personam secundum modum tercię figure artis¹² generalis ita quod per artem suam sequendo processum poterunt si velint eligentes¹³ meliorem eligere personam et hoc palam et si meliorem non eligant erit¹⁴ omnibus in capitulo existentibus manifestum quod peyorem eligunt et ipsi sunt periuri sine¹⁵ aliquo colore excusacionis. Modus autem eligendi est iste.

Primo ut dispositio¹⁶ne apparet¹ *b* autem primam personam significat que in ecclesia vacante prius fuit¹⁷ recepta *c'* significat secundam personam et sic deinceps usque ad *k* ita quod prima¹⁸ persona vacantis ecclesie vocetur *b* et secunda persona vocetur *c^k* et sic de aliis¹⁹ suo modo. Si vero in ecclesia plures sint quam *ix* persone multiplicentur camere²⁰ in figura suprascripta addendo *l*. Et si sint undecim addatur *m*. Si autem²¹ plures sint quam littere sint in alphabeto apponatur alius nu-

cannot do this unless they are good and in good relations with their mother who is good and noble. Their mother is the Holy Roman Church who endures great pains from those who pretend to be good sons, yet who are not good but evil, and who disparage their mother and unjustly take her goods. Therefore we want to give this explanation on how to elect an office holder, based on the Third Figure of the *Ars generalis*, so that in following this system the electors can, if they wish, elect the better person, and do so in public. And if they do not elect the better one, it will be manifest to all who exist in the chapter that they are electing an inferior one and they are perjuring themselves without any shadow of an excuse. The electoral method is the following.

Firstly, as appears in the [above] display, let *b* signify the person who was received first into the vacant church, *c* signifies the second person and so on to *k*, so that the first person of the vacant church is called *b*, the second person is called *c*, and so on with the others in the same way. If, however, there are more than 9 persons in the church, the cells in the figure shown above are increased by adding *l*. And if there are eleven, *m* is added. Now if there are more [persons] than there are

⁶ (DAE f.47v, l.6): *bone* corrected from *boni*.

¹¹ (DAE f.47v, l.10): *doctrinam* duplicated.

¹ (DAE f.47v, l.16): Honecker [10, p. 308, n. 5] reads: *Primo ut discretioni vestre apparet ...*

² (DAE f.47v, l.17): *c* corrected from *s*.

^k (DAE f.47v, l.18): *c* corrected from *s*.

merus scilicet unus vocetur ²²primus frater alius secundus etcetera et omnes sequuntur modum predictum.

In principio ²³omnes facient iuramentum quod personam eligent meliorem et magis ydoneam.^l ²⁴Deinde qui eligere debeant sedeant et *b c* sint stantes ad partem ita prope ²⁵quod audiant verba eligencium et ut omnes ipsos videant. Postmodum *d* petat ²⁶ab unoquoque sedente quem de *b c* plus voluerit pro abbate priore aut ²⁷episcopo et sic de aliis et tunc computentur voces singulorum eligencium^m et si *b* ²⁸plures voces habeat quam *c*ⁿ sedeat *c*^o in locum suum et *b* pedes remaneat vel ²⁹converso. Et mittatur *d* ad *b* et tunc surgat *c* qui ab unoquoque petat ³⁰quem de *b d* voluerit pro prelato et si *b* plures voces habeat sedeat *d* et ³¹vadat *e* ad *b* et tunc petat *c* vel *d* aut alius ab unoquoque coram omnibus ³²quem plus voluerit pro domino scilicet *b* aut *e* et si *e* vincat *b* tunc vadat ³³*f* ad *e* aut *e* contrario et si *b* sit devictus *c* petat ab unoquoque quem de *e f* [folio 48r] ¹plus voluerit pro domino et sic per ordinem usque ad *k* ita quod []^p in []^o *k* fiat ²determinacio electionis sive *k* devincat vel devincatur et sic per ordinem predictum

letters in the alphabet, some other number is taken, that is, someone is called the first brother, another the second, and so on, and all follow in the manner described above.

Initially all take an oath that they will elect the better and more suitable person. Next, those who must elect sit down, and *b* and *c* stand on the side near enough so they can hear the words of the electors, and that all see them. Then *d* inquires of everyone who is sitting whom of *b* or *c* he prefers for abbot, prior, or bishop, and likewise of the others. And afterwards the votes of all electors are added. And if *b* has more votes than *c*, then *c* sits down in his place and *b* remains standing, or vice versa. Then *d* is set against *b* and afterwards *c* stands up and inquires of everybody whom of *b* or *d* he prefers for prelate. And if *b* has more votes, then *d* sits down, and now *e* goes up to *b*, and afterwards *c* or *d* or somebody else inquires of everybody and in the presence of all whom he prefers for master, that is, *b* or *e*. And if *e* wins over *b*, then *f* goes up to *e*, or vice versa. And if *b* is defeated, then *c* inquires of everybody whom of *e* or *f* he prefers for master, and so on, in order, up to *k*, so that with *k* the election comes to an end whether *k* wins

^l (DAE f.47v, l.23f): *deinde* duplicated.

^m (DAE f.47v, l.27): *eligencium* corrected from *elegencium*.

ⁿ (DAE f.47v, l.28): *c* corrected from *s*.

^o (DAE f.47v, l.28): *c* corrected from *s*.

^p (DAE f.48r, l.1): Erasure [].

^q (DAE f.48r, l.1): Erasure [].

³scilicet si in capitulo sint decem fiat determinacio in *l* et sic per ordinem ut iam ⁴dictum est.

Iste autem modus eleccionis utilis est valde et securus quia remotus ⁵est a scrutinio secreto et speciali compromissione per que plures fieri possent ⁶fraudes quam in modo predicto. Qui sic palam eligunt dispositi sunt quod ab eorum ⁷sociis magnam habeant verecundiam si male eligant illi vero qui secrete eligunt ⁸non.

Est etiam autem⁸ novus modus eleccionis bonus in quantum est tantum ⁹generalis^s esse non potest. Et hoc est quia de omnibus personis in capitulo existentibus ¹⁰in eleccione fit mensuracio quarum quelibet ad suam eligit voluntatem in qualibet camera ¹¹supradicte figure et sic de eleccione magis est contentus.

Item per hunc modum ¹²eleccionis quelibet persona capituli forte proponeret esse bona et honesta ¹³si in eleccionibus prelatorum modus iste esset vsitatus et procuraret amicos in [essen¹⁴cia]^t ecclesia et pacem acquireret et inimiciciam evitaret ut in tempore elec¹⁵cionis esset electus et socii se invicem []^u diligent ut in elec-

or is defeated, and this in the order described above. That is, if there are ten in the chapter then the end is reached with *l*, and in the order that has just been described.

This electoral method is very useful and secure, since it is far from the secret *scrutinium* and the election *per compromissum* wherein more fraud can be committed than in the above method. Those who elect publicly face great disgrace by their colleagues if they elect badly; those who elect secretly do not.

This new electoral method is also good in that it is as general as [no other] can be. And this is so because in the election a comparison is made between all persons who exist in the chapter. Each of them elects according to his will in each cell of the above figure, and thus is more content with the election.

Moreover, with this electoral method each person in the chapter presumably would strive to be good and honorable, if this method were used in the election of prelates; and [each would] procure friends in the church, bring about peace, and avoid animosity, so that at the time of the [next] election he might be elected. And the brethren would

⁸ (DAE f.48r, l.8): *etiam autem* corrected from *ecciam ante*.

^s (DAE f.48r, l.9): Honecker [10, p. 309, n. 15] inserts *quantum alius* before *esse non potest*.

^t (DAE f.48r, l.13f): *essencia* expunged.

^u (DAE f.48r, l.15): Erasure [].

cione unus^v staret ¹⁶pro alio et sic exaltatum esset capitulum scilicet per exaltacionem quam fratres haberent ¹⁷per caritatem iusticiam prudenciam et per alias virtutes. Si autem electio ¹⁸sit facienda de personis absentibus fiat electio secundum predictum etc.

Factus est iste ¹⁹modus electionis parvysius anno incarnationis domini nostri Jhesu Christi *Mo* ducentesimo ²⁰*xcix*o primo die Julii. Deo^w gracias.

show mutual esteem, so that in an election one would stand for the other. And so the chapter would be enhanced, that is, [it would gain] through the enhancement which the brethren would enjoy through charity, justice, prudence, and other virtues. If now an election of absent persons is to be made, the election should be carried out as described above, etc.

This electoral method was devised in Paris, on the first day of July in the year 1299 of the Incarnation of our Lord Jesus Christ. Thanks be to God.

6. Llull's electoral systems

6.1. *The trail to truth*

Llull's electoral systems in AEP, B24 and DAE have in common that they are intended for the election of church officials in clerical communities. In AEP there are 16 persons or a few more who elect a prelate, without any further indication as to who they are or how they are selected. In B24 it is 20 nuns of a convent who elect a new abbess. DAE has 9 persons or a few more electing an abbot, prior, or bishop.

In each of the three writings Llull quotes some of his prior work. AEP and B24 both mention the *Ars compendiosa inveniendi veritatem* (Bo II.A.1: c. 1274), see [32]. In addition, AEP refers in a somewhat cursory fashion to the *Liber principiorum theologiae* (Bo II.A.6: 1274-8), the *Liber principiorum philosophiae* (Bo II.A.7: 1274-8), and the *Liber principiorum juris* (Bo II.A.8: 1274-8). For an edition of these along with the *Liber principiorum medicinae* (Bo II.A.9: 1274-8) see [21]. B24 quotes the famous *Llibre del gentil e dels tres savis* (Bo II.A.4: 1274-6), see [16].

^v (DAE f.48r, l.15): *unus* corrected from *unius*.

^w (DAE f.48r, l.20): *Deo* corrected from *De*.

In DAE, Llull says that the Third Figure is taken from the *Ars generalis*. Platzeck [30, vol. II, p. 34, n. 85] identifies this reference with the *Ars compendiosa* (Bo III.36: 1299). Bonner [1, vol. I, p. 22, n. 88] points out, however, that from 1294 on Llull used the term *Ars generalis* as a generic reference to his system, the final version of which appeared in the *Ars brevis* (Bo III.77: 1308) and the *Ars generalis ultima* (Bo III.80: 1305-8). A version of the Third Figure is reproduced on plate XVII in Bonner [1, vol. I, after p. 582].

Self-references of authors to their own works have been popular from the middle ages till today and, besides advertising the author's oeuvre, serve to claim an advanced level of competence. It would seem to us that, in the present case, Llull may want to send a more pointed message to his readers, emphasizing the ultimate function that he attributes to an electoral system. In our time we view an election as a fundamental building block of mass democracies, a means to collect the opinions of individual voters and to aggregate them across the entire society. This integrating function of popular elections in democratic societies would have appeared totally alien to Llull.

As a devoted Christian in his time, Llull's concern was not the aggregation of many individual truths, but the discovery of the one and only truth existing, the truth of God. We understand Llull's electoral systems as manifestations of his *Ars generalis*, and as such they are means to set mankind on a trail leading to the unique, divine truth: «By this method,» said Natana, «is found the truth; by this truth we will be able to find the sister who is most suitable and best to be our abbess.» (B24, f. 32v, l. 17 - f. 33r, l. 3).

6.2. The bookkeeping of sincerity

The Third Figure of the *Ars generalis* is displayed right at the beginning of DAE. It is an arrangement of the 36 pairs that can be formed from 9 candidates named *b, c, ..., k*. A larger version of the figure, for the 120 pairs from 16 candidates *b, c, ..., r*, is included on the first folio of AEP. The use of letters to indicate candidates serves a double purpose, of exhibiting the universal applicability of the system, and of producing a figure small enough to fit the limited space available.

The question arises, however, whether the sequence of letters is meant to indicate a ranking of the candidates. If so, the candidate with the first letter would hold the highest rank, and the candidate who is assigned the last letter would be ranked lowest. The other candidates would be ranked in-between, as determined by the order of the letters. If the letters were to indicate a ranking, then it is conceivable that Llull might have wished to design a biased electoral

system favoring candidates of higher merits at the expense of candidates of lower merits. We maintain that such hypothesis, that the assignment of letters presupposes a ranking by merits, is inconsistent with Llull's way of thinking.

Llull felt himself called upon to devote his life to spreading the Christian truth to the Muslim world. *Pindl* [16, p. 264] recalls a tragic incident from Llull's autobiography, the *Vita coetanea*, as an episode of lasting effect, see [19, p. 279] or [1, vol. I, p. 21]: Llull and his Saracen teacher of Arabic enter into a fierce argument. The teacher draws the sword against his master. Llull successfully defends himself, and imprisons the teacher. While Llull contemplates what to do next, the discouraged Muslim hangs himself. Thus powerful demonstration of authority has led to a double defeat: the infidel is lost to everlasting damnation, and Llull failed to convert the disbelieving soul to his Christian God. Henceforth Llull builds on the force of arguments, not on the power of authority. He wishes to convert his opponents to Christianity through dialogue, not to dictate.

To this end the combinatorial aids of the *Ars generalis* serve as a warranty of fairness, so that no contraposition of opinions is omitted. For the electoral systems, the Third Figure is used to implement a bookkeeping of sincerity, that is, to secure a complete comparison of all arguments. Furthermore, Llull uses combinatorial structures quite generally to organize his philosophical view. But nowhere does Llull imply that the assignment of letters presupposes a rank order of arguments, see *Platzeck* [30], *Pring-Mill* [33], *Hillgarth* [9], or *Bonner* [1]. Such a clandestine outmaneuvering of opponents is simply incompatible with a *protagonist of the cross-cultural dialogue*, as Llull is hailed by *Pindl* [16, p. 259]. We believe that it was entirely outside Llull's way of thinking that the assignment of the letters might bias his electoral systems in one way or the other.

Not surprisingly, the instructions of how to assign letters to persons turn out to be cursory and incidental. In addition, the relevant section in AEP (f. 11v, l. 11-14) is somewhat hard to decipher. We understand that somebody, a *primus* of whatever sort, takes the lead to randomly attribute letters to persons. A brief examination then makes sure that all electors remember their letters. B24 makes mention neither of the letters nor of the assignment process. We take this as an indication of how unimportant this step is, in Llull's view. In DAE (f. 48r, l. 9-10) Llull praises his electoral system *because in the election a comparison is made between all persons who exist in the chapter*; that is, because all chapter members are treated equally.

In the sequel our analysis is based on the assumption that all pairwise comparisons are of equal value, so that the decision between the first and the second candidates, *b* and *c*, enters the final evaluation with exactly the same weight as the decision between the first and the last candidates, *b* and *k* (or *b* and *r*).

6.3. *The evaluation of a single pairwise comparison*

All three variants of Llull's electoral system build on pairwise comparisons of two candidates at a time. They differ in whether an exhaustive, or a partial series of pairwise comparisons is stipulated. In the present subsection we discuss the evaluation of a single pairwise comparison.

The instruction in AEP (f. 12r, l. 9-10) is clearest: *[Then] a dot is placed by the letter assigned to the person who has the most votes*. Thus the victorious candidate scores one point, and what is recorded is this winning point and not the number of votes that led to the victory. We are reminded of an ordeal by judicial duel, in which the truth emerges on the side of the victor no matter how easy or how hard he had to fight. This way of arriving at a decision is concordant with Llull's belief in the uniqueness of the divine truth: among two candidates, one must be more worthy than the other.

The electors are humans tempted by sin, however, and it is not guaranteed that they recognize the truth in any case and at any time. Therefore Llull includes a rule for the situation when the electors fail to recognize the truth and two candidates are tied with the same number of votes (AEP, f. 12r, l. 11-12): *If now one [person] has as many votes as another, then a dot is placed by both letters of this cell*. By awarding a winning point to each of the two candidates, they remain equal between each other and yet gain an advantage over the other candidates.

The corresponding passage in B24 (f. 33v, l. 16-17) is briefer: *and in secrecy let it be written down who has the most votes*. We think that the meaning of the phrase is identical to the ruling in AEP. For at the beginning of B24 (f. 32v, l. 14-15), Llull claims to propose *a new electoral method based on a system and figures*. In the middle (B24, f. 33v, l. 3), he states that the electors *compare each person with every other*. Moreover (B24, f. 33v, l. 24-25), *a total of 36 cells are generated*. Clearly this points to the Third Figure, even more so since in the other two texts the figure is reproduced in its entirety. From this we conclude that 'written down' means 'marked': 'and in secrecy let the letter of the person be marked who has the most votes'. With this interpretation, B24 defines the same rule as does AEP. We cannot imagine that the quoted passage would support an extended meaning, to write down the number of votes of the victorious candidate, let alone two numbers, one for each candidate.

B24 does not contain a rule on how to break a tie. Llull may have hoped that his insistence on an odd number of electors (B24, f. 33r, l. 15) makes a tie-breaking rule superfluous. This is true for the majority of comparisons, when two of the seven electors are compared by the remaining five, or when the two additional candidates are compared by all seven electors. However, the other

cases are left dangling in the air. When one of the electors and one of the additional candidates are compared, then there are six electors and a tie with three votes for each candidate becomes possible. Such ties are not accounted for in B24.

Nor does the last text of the electoral trilogy, DAE, feature a tie-handling rule even though here it is needed most. DAE promotes a partial system of pairwise comparisons, based on successive eliminations. Therefore each pairwise comparison must leave a well-defined victor, who then proceeds to the next round. Other than in B24, no mention is made that an odd number of electors would be advantageous. Nor would the AEP rule, that tied candidates are treated equally, be of any help since just one candidate can proceed to the next round. In our view, the absence of a tie-breaking rule in DAE constitutes a fatal omission. If DAE was practiced as is, then the divine truth must have shone rather brightly in order to keep the electoral business on the road of decidability.

6.4. The overall evaluation of the election

The global evaluation of all pairwise comparisons is settled most clearly again in AEP (f. 12r, l. 20-21): *Once the examination of all cells has been completed, the dots of each letter must be counted.* The overall winner of the election is the one who scores the greatest number of winning points, that is, who wins most of the pairwise comparisons. Two extra rules take care of the possibility that two or more candidates are tied with the same number of winning points. Firstly, another election is to be conducted among those tied. Secondly, if yet again a tie emerges, lots are thrown.

The last rule sounds surprising since the Christian Church condemned the throwing of lots ever since the soldiers threw lots at the feet of the crucified Jesus to divide his garments among them (St John 19,24). *Maleczek* [23, p. 129] reports that the Church banned lots from electoral decisions in 1223. Did Llull realize that the inclusion of lots was detrimental to promoting his system to a clerical clientele? In any case, in B24 and DAE he does not follow up on the idea. Remarkably, in the political world of today, every electoral law we know of includes a final clause to break eventual ties - by drawing lots.

In B24 (f. 33v, l. 26-27) the winner of the election is the candidate *who has the most votes in the most cells.* We think that this ambiguous instruction is meant to set the same rule as in AEP, that the candidate with the largest number of winning points across all pairwise comparisons is the winner of the election. For the case of a tie between two or more candidates, only the first tie-handling rule of AEP is repeated, to conduct a further election among those tied.

In contrast to the exhaustive schemes in AEP and B24, of considering all pairwise comparisons, DAE advocates a partial system of successive eliminations. A candidate who loses in a pairwise comparison is eliminated from further consideration. Only the victorious candidate enters into the next round, and the last round's victor is the overall winner of the entire election. We are reminded of the devout Christian knight who, in Chapter 64 of *Blaquerna*, proves the Christian truth to the Saracen king by fighting duels with all the knights of the king's court, one after the other, see [31, p. 82] and [1, vol. II, p. 754, n. 44].

Today's understanding of electoral systems focuses on the freedom with which individuals cast their votes; then the exhaustive system of AEP and B24, and the DAE system of successive eliminations may lead to diverging results. However, Llull coped with devising a decision procedure that would unerringly converge to the one and only divine will. He may have hoped that his goal would not be hampered by changing some electoral details here and there. In fact, it is easy to propose some hypotheses under which he could rightly hope so.

6.5. A hypothetical convergence of systems

Formally, the exhaustive comparisons of AEP and B24 and the partial comparisons of DAE are different systems, possibly leading to different results. Under mild hypotheses, however, the differences vanish and the final conclusions coincide. For an illustration, let us agree on the following two hypotheses,

- (i) that the truth set by God is unique, and
- (ii) that, in any decision instance, it discloses itself to at least half of the electorate.

Llull would have emphatically supported our assumptions, or so we believe. Under hypotheses (i) and (ii), the electoral systems of AEP, B24, and DAE lead to the same result. We forgo a stringent proof of this claim, instead illustrating it by example.

Suppose there are nine electors $b, c, d, e, f, g, h, i, k$, each of whom also stands as a candidate. By hypothesis (i) we may assume that there is a unique *true ranking* (*) in which God sees the candidates. In this example, we suppose it is given as follows:

$$(*) \quad e < g < k < b < d < i < h < c < f$$

That is, e is the weakest candidate, g is the second weakest candidate, and so on up to c , the second most worthy candidate, and finally f , the worthiest candidate.

We assume that the six electors b, c, d, e, f, g wisely see the candidates in the same order that is determined by the true ranking (*). The other three electors, h, i, k , deviate from the divine will (*) and base their decisions on the following individual rankings:

Elector h : $d < i < h < k < e < f < g < b < c$

Elector i : $h < d < g < k < b < f < c < e < i$

Elector k : $h < e < k < d < i < f < c < g < b$

When two of the six wise electors are up for comparison, they are not permitted to vote. Therefore the least favorable decision instance still has four wise electors as opposed to three individualists, so that our assumptions satisfy hypothesis (ii).

Now the election begins. Round 1 features the pairwise comparison between b and c . The electors d, e, f, g check the ranking (*) to find that b ranks below c , and hence vote for c . So do electors h and i . Only elector k , upon checking his or her individual preference scheme, ranks b ahead of c , whence this vote goes to b . In total, c beats b by 6 to 1 votes. Candidate c records a winning point, and proceeds to the next round.

Following the rules of DAE, round 2 compares c and d . Everybody ranking c above d , candidate c wins with 7 to 0 votes. Thus d is eliminated. Candidate c enters round 3, together with e . It transpires that c beats e by 6 to 1 votes, whence e is out.

In round 4 the margin of victory turns out to be as narrow as possible. For, according to the electoral rules, candidates c and f are excluded from voting. This leaves just four electors blessed with the insight into the true ranking (*); they vote for f . When electors h, i, k check their individual preferences, each finds c ahead of f . Hence f beats c by 4 to 3 votes, and the victor of round 4 is f .

Moreover, f also wins rounds 5 through 8, against g, h, i, k . Thus eight pairwise comparisons, out of a total of 36 possible pairings, establish the end result: the overall winner is f , in agreement with the divine ranking (*).

Following the rules of AEP and B24, another 28 rounds are called for until all pairwise comparisons are exhausted. In each case the two candidates standing for comparison are banned from voting, and the remaining electors check the preferences peculiar to them before they cast their votes. Then *unus punctus* is marked near the letter of the winning candidate. In the end, the electoral record looks as follows:

$bc \bullet$	$\bullet cd$	$\bullet de$	$ef \bullet$	$\bullet fg$	$gh \bullet$	$\bullet hi$	$\bullet ik$
$bd \bullet$	$\bullet ce$	$df \bullet$	$eg \bullet$	$\bullet fh$	$gi \bullet$	$\bullet hk$	
$\bullet be$	$cf \bullet$	$\bullet dg$	$eh \bullet$	$\bullet fi$	$gk \bullet$		
$bf \bullet$	$\bullet cg$	$dh \bullet$	$ei \bullet$	$\bullet fk$			
$\bullet bg$	$\bullet ch$	$di \bullet$	$ek \bullet$				
$bh \bullet$	$\bullet ci$	$\bullet dk$					
$bi \bullet$	$\bullet ck$						
$\bullet bk$							

The final step is to count the number of winning points. Candidate f gains the largest number of winning points, 8, and hence is the winner of the election. Moreover, we may count the winning points for all other candidates, so as to arrange them in winning order: e is lowest with 0 winning points, g is second-lowest with 1 point, etc., thereby actually reproducing the true ranking (*). Thus Natana's promise is fulfilled that, by this method, is found the truth.

From our modern viewpoint it may seem overly simplistic to postulate hypotheses under which Llull's electoral systems cannot but find the truth. And evidently our two hypotheses are not exhaustive of Llull's thinking, because then he would not have added additional rules to handle ties. On the other hand it would seem anachronistic to accuse Llull, who lived seven centuries ago, of not providing a stringent argument that we find pleasing by today's standards. Llull may have well satisfied himself by checking a few sample situations, like the one we have discussed just now.

6.6. *The reduction of complexity*

For all of his life Llull endeavored to improve the structure of his *Ars generalis*, so as to make it more easily accessible to his audience. Llull's contemporaries were intrigued by the complexity of his philosophy, and presumably also felt that his electoral systems were fairly complicated and complex. We think that the changes from AEP over B24 to DAE may well have been motivated by Llull's desire to increase acceptability by decreasing complexity.

It is plainly evident that basing the electoral record on the Third Figure was overly demanding. None of its versions mentioned in the present paper is drawn

without error. In AEP the cells *bh* and *bi* are labeled incorrectly, in DAE the cell *ei* is misnamed, and so is the cell *EK* in plate XVII of *Bonner* [1, vol. I, after p. 582]. Perhaps this is why Llull requires multiple copies of the control figures (AEP, f. 11v, l. 20-21), and perhaps this is why Natana hopes that an electoral error spares her the burden of becoming abbess (B24, f. 34r, l. 16-24).

The first electoral system, from AEP, needs a considerable amount of time to be carried through. For every pairwise comparison

- (1) two candidates leave the electoral hall,
- (2) the remaining 14 electors submit their votes,
- (3) the votes are totaled,
- (4) a winning point is marked in the electoral records, and
- (5) the candidates are called in again.

Assuming that the electors declare their votes with appropriate deliberation, a single round may well have taken from 5 to 10 minutes. With 120 pairwise comparisons, the election then lasts from 10 to 20 hours. An adjournment overnight, however, counteracts Llull's concern to prevent fraud and simony, as expressed in AEP (f. 11r, l. 9) and DAE (f. 48r, l. 5-6).

B24 presents the accelerating idea of reducing the electoral community, by selecting a smaller electoral college. A college size of seven is taken to be optimum. In the least favorable case, two of the electors are up for comparison and hence cannot cast a vote themselves. This leaves five voters, and a winning majority must gather at least three votes. To claim legitimacy, a vote of three may have seemed sufficient to Llull, *for where there are two or three gathered together in my name, there am I in the midst of them* (Matt. 18,20), see also *Ganzer* [6, p. 76]. With an electoral college of seven, plus two additional candidates as in B24, the duration of the electoral business is shortened to last between 3 and 6 hours.

The third system, in DAE, does not decrease the number of electors. Instead it diminishes the number of pairwise comparisons. With a touch of ingenuity Llull turns away from an exhaustive scheme of pairwise comparisons, and replaces it by a partial system of successive eliminations. For n candidates, an exhaustive system needs $(n-1)n/2$ rounds, while a system of successive eliminations makes do with $n-1$ rounds. With 16 candidates, this is a spectacular reduction from 120 comparisons down to 15, with 9 candidates, from 36 down to 8. Since a system of successive eliminations does not require keeping a detailed electoral record, the election may need no more than one hour.

Curiously, though, Llull neglects the hazards of a successive eliminations system, which may suffer from a severe bias in favor of those who enter late at

the expense of those who enter early. In summary, we conclude that the changes from AEP via B24 to DAE entail a remarkable acceleration of the electoral business.

6.7. *The abandonment of saniority*

In medieval electoral systems it was neither time nor number that in the end constituted the binding effect of the electoral outcome. The crucial requirement is saniority, that is, the claim of the majority that in the face of God they are more sound of mind and more pleasing than the minority. *Maior et sanior pars* is the motto, see *Ganzer* [6] and *Schimmelpfennig* [40], [41]. The majority is well advised not to just celebrate their numerical victory, but to also claim a level of superior quality. If the minority succeeds in occupying this position, they may insist on having won the election on the grounds of better insight, even though they fall short numerically. With the result of the election contested, the way out is to take recourse to an arbitrator. Thus the notion of saniority necessitates an authority outside the electorate.

Llull never makes any mention of saniority, yet he shows a great concern that the more worthy and more suitable candidate be elected. Above and beyond the vow binding the electors as nuns (B24) or brethren (DAE), he makes them take an oath to rank the candidates according to pertinent properties. AEP (f. 11v, l. 15-19) stipulates that three things should be considered: honesty and holiness of life, knowledge and wisdom, and a suitable disposition of the heart. B24 (f. 33v, l. 4-6) defines four conditions: love and knowledge of God, love and knowledge of the virtues, knowledge of and opposition to vices, and greatest suitability. These attributes are similar to those that are characteristic of saniority, see *Maleczek* [23, p. 122]. We mention in passing that AEP, with three attributes, is from the quaternary phase of Llull's writings, while B24, with four attributes, is from the ternary phase.

In DAE (f. 47v, l. 23) the oath plainly demands to *elect the better and more suitable person* (personam meliorem et magis ydoneam). Thus Llull shifts the emphasis from worthiness of the electors (saniority), to worthiness of the candidates (idoneity). We take it that in Llull's writings the absence of the notion of saniority is a deliberate abandonment of the concept, not an accidental omission. Secret voting would have been another option to deal with the problem, and indeed was under consideration at that time. The 1283 Montpellier General Chapter of the Dominican Order introduced secrecy of the votes, see *Maleczek* [23, p. 129] and *Gaudemet et al.* [7, p. 332]. Secrecy does away with saniority, because a secret vote cannot be traced back to the voter and his or her state of

soundness. Llull remains somewhat undecided on whether he favors secret voting or not. In DAE (f. 48r, l. 6-8) he finally advocates open voting, welcoming the social control that comes with it.

We believe that Llull, designing his system so as to lead to the more worthy candidate, wants to shield it from saniority and outside intervention. The trail to truth envisioned by Llull needs no recourse to a higher worldly authority. When asked why in his electoral systems he mentions the notion of saniority not even once, Llull might have coined the phrase, half a millennium before Laplace: *Je n'ai pas besoin de cette hypothèse-là.*

7. Discussion

The credit for the discovery of DAE in Cusanus' library in Kues lies with Honecker. Regrettably, his comments on the electoral details are unreliable and have obscured the subject to date. We would like to correct his misjudgement of the overall evaluation of the election, his grossly misleading implication that all electoral systems lead to identical results, and his false statement on the transitivity of pairwise comparisons.

Honecker [11, p. 567] summarizes the global evaluation of a B24 election as follows: *Wer nun dabei in allen Wahlgängen zusammen die meisten Stimmen erhalten, das heißt in den meisten Einzelwahlen gesiegt hat, gilt als gewählt.* [«The one who among all pairwise comparisons has totaled the most votes, that is, has won most of the individual comparisons, wins the election.»] The conjunction *das heißt [that is]* is fatally misplaced, for the electoral system in the initial section of the sentence, building on vote totals, is different from the electoral system in the final section, counting victories. In our contemporary terminology the electoral system of adding votes is called the system of Borda, while the system of counting victories is named after Condorcet, see *McLean/London* [24, p. 106; 25, p. 34].

With his past as a courtier, Llull may have seen the electoral process as a voting tournament. His prescription that *unus punctus* is awarded to a duel's victor is most clearly formulated in the tract AEP edited here for the first time and not accessible to Honecker. Yet the texts that Honecker did have available should have prevented him from going astray. The evaluation of a single pairwise comparison is summarized correctly [11, p. 567]: *und es wird jedesmal geheim aufgeschrieben, wer bei jeder einzelnen Abstimmung die meisten Stimmen bekommt.* [«and each time it is recorded in secrecy who in each pairwise comparison gains the most votes.»] If in every round the name of the winner is recorded rather than the score of votes, then it is impossible to finally total those nonrecorded votes.

Honecker's misjudgment is presumably caused by the fact that in B24 the word *veus* stands for both the votes of the electors as well as the winning points of the candidates. Of course, it is irritating when one and the same word is used with distinct meanings. There are more examples, though, where Llull passes over a shift of meaning as he exposes his electoral systems in more and more detail: *persona* is a member of the convent as well as a member of the electoral college as well as a candidate, and *nombre* is the number of electors as well as the number of candidates as well as their numbering as well as the number of pairwise comparisons.

Honecker [11, p. 568] also erroneously equates the results from the electoral system in B24 with those from the system in DAE: *Im Vergleich zur Wahl im Libre de Blancerna stellt der Wahlmodus des Traktats De arte electionis einen abgekürzten Prozeß dar. Die Zuverlässigkeit des Verfahrens wird davon aber nicht berührt.* [«Compared to the election in *Blaquerna*, the electoral system of the tract *De arte electionis* presents a shortened process. The reliability of the procedure, however, remains untouched.»] Everything is lumped together, including the contributions of Cusanus. Peculiar to Cusanus is [11, p. 574] *die 'Punktwertung', die das Verfahren, wenn man von der ars electionis ausgeht, nicht nur erheblich vereinfacht, sondern zugleich auch eine zuverlässige zahlenmäßige Auswertung der Abstimmung gestattet.* [«the 'scoring scheme' that not only simplifies the procedure considerably, as compared with the *ars electionis*, but at the same time permits a reliable numerical evaluation of the voting system.»] We cannot understand these quotations other than that, to Honecker, these systems always lead to the same result and differ only in how to get there. This is grossly misleading. As a matter of fact, the electoral systems of Llull/Condorcet and of Cusanus/Borda occasionally lead to different results, though, of course, not always.

Finally, Honecker [11, S. 574, n. 34] makes a false or, at best, confusing statement on the transitivity of pairwise comparisons. Transitivity means that the following logical conclusion holds: if *b* beats *c* and *c* beats *d*, then *b* beats *d*. For a system of exhaustive pairwise comparisons, as in AEP and B24, this conclusion is false, as may be illustrated with an example of three electors *X*, *Y*, *Z*, and three candidates *b*, *c*, *d*, see McLean/London [25, p. 23]:

Electors *X*: $d < c < b$

Electors *Y*: $b < d < c$

Electors *Z*: $c < b < d$

Here *b* beats *c* by 2 to 1 votes and, with the same score, *c* wins against *d*. Yet the voting duel between *b* and *d* sees *b* not winning, but losing by 1 to 2 votes.

Thus the Llull/Condorcet system suffers from occasional intransitivity. In contrast, the Cusanus/Borda system is free from this defect.

Blinded by the similarity of the various electoral systems, Honecker creates the impression - though he never explicitly says so - that the systems differ only in procedural details, but in the end lead to identical results. As we have pointed out, this is not true and, in particular, it inappropriately belittles the originality of Cusanus' contributions. A more detailed discussion of Cusanus' electoral system will be presented elsewhere [35].

McLean/London [25, p. 27], upon referring to the inauguration of an electoral college in B24, propose the following view: «Llull seems to wish to compromise between democracy and giving a more decisive voice to better qualified electors. This may represent Llull's attempt to compromise between the rival principles of *maior pars* and *sanior pars*.» We disagree and believe, as detailed in Section 6.7, that idoneity of the winning candidate is in Llull's eyes superior to seniority of the supportive electorate. Moreover, a *democracy-Llull* who devises an electoral system in order to mediate between the equal rights of the many and the specific talents of the few is, in our opinion, light years remote from *Llull's Llull* whom *Bonner* [1, vol. I, p. xii] tries to present.

One question remains open, namely whether Llull's electoral systems are as free of bias as depicted in our Section 6.1. After all, Llull lived in a time when hierarchies were omnipresent in all parts of society, and honoring them might have gone without saying. The relevant paragraph in AEP (f. 11v, l. 11-14) remains somewhat enigmatic to us.

Nor does DAE (f. 47v, l. 16-17) provide a definite answer. The key passage says that the letter *b* signifies *primam personam ... que in ecclesia vacante prius fuit recepta*. What does this mean? *Honecker* [11, p. 568], *McLean/London* [25, p. 29, n. 4], and *Meuthen* [27, p. 5] all consider, with varying degrees of confidence, that the phrase implies a ranking by seniority. They rely on the words *in ecclesia vacante*, a technical term from canon law, designating the church community who have lost their leader; note that the crucial word *vacante* is missing from n. 4 in *McLean/London* [25, p. 29]. Then *prius fuit recepta* might mean that the letters are assigned in the sequence in which the electors became members of that church community. Cusanus, in 1428 a canon lawyer recently graduated from university, may well have understood the phrase in this way. Llull, however, was neither a canon lawyer nor did he enjoy a university education.

What else could Llull have had in mind? *Honecker* [10, p. 308, n. 6] suspects that the DAE version copied by Cusanus stems from a Catalan original. His evidence consists of four substitutions of a long whip-*s* for the letter *c*, though it is not clear to us why this would point to the Catalan language. Nevertheless, the expunged *essencia* (DAE, f. 48r, l. 13-14) is more easily misread from a Catalan

esglesia, than from a Latin *ecclesia* on its sixth occurrence. Did indeed Llull draft a Catalan original, 1299 in Paris, which somehow or other mutated into the 1428 Latin copy of Cusanus?

Perhaps an original Catalan wording would be supportive of an alternative interpretation proposed by *McLean/London* [25, p. 29, n. 4]. Before starting the election, the electors celebrate of course a mass, see *Viollet* [43, p. 70]. Might the quote from DAE then be interpreted so that after the service the electors leave the church, and that the letters are assigned as they re-enter the empty building in random procession? The location where the election is to be held is never specified by Llull. Only in AEP (f. 12r, l. 3-4) is a *domus* mentioned although it remains open what exactly is meant by it. According to *Schimmel-pfennig* [41, p. 191], the election of a bishop normally took place in the canon chancel of the cathedral, or in the chapter house. A linguistic analysis of Llull's oeuvre may shed more light on the problem. For instance in *Felix*, Dame Reynard appeals to a church building [1, vol. II, p. 783]: «An election was once being held in a cathedral church, and in the chapter there was disagreement over the election of a bishop.» Later, though, the hermit refers to the church community [1, vol. II, p. 1067]: «Two clerics came before a bishop asking for a church position which was vacant.»

A more detailed semantic study may be also profitable in other ways. For example, apart from early years, Llull reserved the notion of *dignitates* for the divine virtues attributed to God, see *Bonner* [2]. The changing usage of the word is also noticeable in the electoral trilogy under discussion. When searching the transcriptions in www.uni-augsburg.de/llull/ for the occurrence of the string *dign*, AEP scores seven hits, B24 two, and DAE none. Clearly, AEP was written before *dignitas* was restricted to its narrower meaning.

8. Résumé

Llull's electoral systems have in common that they are exclusively based on pairwise comparisons between two candidates at a time, but they differ in detail. AEP and B24 describe systems of exhaustive pairwise comparisons, whereby AEP accepts everybody in the community as an elector, while B24 comprises a preliminary step of first selecting a smaller electoral college. In both systems the winner of the election is the candidate who is victorious in most of the pairwise comparisons. In contrast, DAE proposes a system of successive eliminations; the victor in the last pairwise contest is the winner of the whole election.

The newly edited manuscript AEP precedes B24 and DAE. It is longer than the two later texts, and contains more detailed instructions. All three writings

reflect Llull's fundamental view that every person will be rewarded with the divine truth provided he or she trusts in God and researches honestly. Llull developed his systems for clerical communities and, having fought vehemently throughout his life for founding new monasteries specializing in missionary work, he may have aimed at such institutions. However, whether the electoral systems have actually ever been used is not known.

Nor is it known what triggered Llull's interest in the subject, and which sources he built on. In 1283 the Dominican Order held a General Chapter in Montpellier, and electoral matters were on the agenda, see *Ganzer* [6, p. 76]. In 1285 Guilelmus de Mandagoto (d. 1321), archdeacon in Nîmes, published his influential *Tractatus de electionibus novorum praelatorum*; see *Schimmelpfenig* [40, p. 476]. *Viollet* [43, p. 88] points out that Mandagoto treats three systems, the elections *per scrutinium*, *per compromissum*, and *per inspirationem*, while the next generation canon Ioannis Andreae (1270-1348) recognizes only the first two methods. Llull fits in-between and anticipates Andreae, as DAE (f. 48r, l. 5) lists only *the secret scrutinium and the election per compromissum*.

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SUMMARY

We present three writings of Ramon Llull on electoral systems: *Artifitium electionis personarum* (before 1283), Chapter 24 *En qual manera Natana fo eleta a abadessa* from the novel *Blaquerna* (about 1283), and *De arte electionis* (1299). The tract *Artifitium electionis personarum*, edited here for the first time, proves to be an independent work shedding new light on the two later sources. In all three cases the electoral system is based exclusively on pairwise comparisons between two candidates at a time. The first two texts describe exhaustive schemes: the candidate winning most of the pairwise comparisons is the overall winner. The last text proposes a system of successive eliminations: the winner of the last comparison also wins the election.

ZUSAMMENFASSUNG

Wir präsentieren drei Schriften von Ramon Llull, in denen er sich zu Wahlsystemen äußert: *Artifitium electionis personarum* (vor 1283), das vierundzwanzigste Kapitel *En qual manera Natana fo eleta a abadessa* aus dem Roman *Blaquerna* (circa 1283) und *De arte eleccionis* (1299). Der hier erstmals edierte Traktat *Artifitium electionis personarum* erweist sich als ein eigenständiges Werk, das dem Verständnis der beiden jüngeren Quellen neue Impulse gibt. In allen drei Fällen basiert das Wahlsystem ausschließlich auf paarweisen Vergleichen je zweier Kandidaten. Die ersten beiden Texte beschreiben Systeme Jeder-gegen-jeden: wer in den meisten paarweisen Vergleichen siegt, ist der Wahlsieger. Der letzte Text enthält ein K.-o.-System: wer im letzten Vergleich siegt, hat die Wahl gewonnen.