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Mergers, trade and industrial policy

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MERGERS, TRADE AND INDUSTRIAL POLICY

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Abstract

The aim of this paper is to show that the *ex post* incentives for governments to tax and subsidize affect firms' *ex ante* decisions about merging. I analyze situations in which a merger of two national firms in different countries would always take place in the absence of an endogenous (or any) government intervention, but the endogenous response of government intervention may make the merger unprofitable. Examples are given where the merger is never profitable with optimal policies. At the same time, such unprofitable mergers could increase welfare. Examples are also given where mergers are profitable with optimal policies.

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1. INTRODUCTION

In 1992, the European Union and the United States reached an agreement that would limit subsidies given to the aircraft industry, that is, to Airbus and Boeing. However, this agreement still allows the respective governments to give, as a subsidy, one third of the development costs and various forms of variable costs (see Irwin and Pavcnik (2004)). On the other hand, both companies would undoubtedly save both development and variable costs by co-operating, the extreme of which would be the merging of these firms. In the case of a merger, there would also be the added benefit of obtaining a potential monopoly position (practically) for aircraft production. Such a merger would increase profit. The explanation is straightforward: the merged enterprise could always keep the initial production pattern and so any change in this pattern must lead to an increase in profit. This contrasts with the results to be obtained in relation to strategic trade and industrial policies because, as will be shown, typically, while the optimal policy in the case of national firms is subsidy, it is tax in the case of an international merger. A result, in this paper, is that with this optimal taxation the merger may not be profitable. It is noteworthy that this may be so even if there are no subsidies for national firms, that is, optimal taxation may, in itself, be sufficient to prevent international mergers.

I analyse duopolistic situations where two firms in different countries produce either a homogenous good or two heterogenous goods. These firms compete either by adjusting quantities (Cournot duopoly) or by adjusting prices (Bertrand duopoly). The owners of these firms bargain over a horizontal merger, that is, over the equity shares of the merged enterprise and the division of the merger profit. This is modelled as a Nash bargaining game with equal bargaining strengths. If there were no policies, the solution is the familiar split-the-difference division: each owner-group obtains its initial pre-merger profit plus half of the merger surplus. The introduction of production subsidies/taxes complicates the situation in many ways. As an example, consider the basic case to be presented in Sec.3 where the two merging firms produce a homogenous good with constant unit-costs under Cournot competition and export this good to a third country. Here, the pre-merger optimal policy is subsidization and the post-merger optimal policy is taxation. It will be shown that in this basic case the merger is never profitable with optimal policies.

Subsequently, I modify this basic case. For simplicity, I assume in all these modifications that unit costs are similar. First, the good's domestic consumption does not change the conclusion that the merger is not profitable with optimal policies. An additional element is that the larger country with larger consumption of the good has a lower tax (or even a subsidy). Second, I introduce two heterogenous goods which can be produced in the respective

countries only. Under Cournot competition it can be shown, again, that with policies the merger is not profitable. On the other hand, if the firms compete in Bertrand fashion, optimal policies, both pre-merger and post-merger, are taxes. We may expect that because the merger-ownership is shared the taxes are then larger. As will be shown, the merger is now profitable if the goods' substitutability is even modest. The monopoly advantages are then larger than the disadvantages caused by higher taxes.

In the relevant literature, the well-known result in Brander and Spencer (1985) is that in a Cournot duopoly with national firms in different countries, a production subsidy is an optimal policy. This was subsequently modified in several ways. Eaton and Grossman (1986) showed that in a Bertrand-duopoly a production tax is optimal in both countries if all production is exported to a third country. Dixit (1984) showed that if there are several Cournot-competitive firms in the policy country, a tax may well be optimal. This is so because a subsidy to one domestic firm is, in itself, harmful to other domestic firms. In all of these, the firms are owned by nationals. A further modification concerns cross-ownership of the imperfectly competitive firms. Lee (1990) and Dick (1993) showed that if these firms are partially owned by foreigners, subsidy recommendation may change to tax because a part of the benefit caused by a subsidy goes to foreigners. In both Lee (1990) and Dick (1993), the firms continue to make independent production decisions from which ownership is separated and the cross-ownership shares are modelled as exogenous. An extension concerns the subsidization or taxation of research and development (R&D), instead of final goods. As shown by Brander and Spencer (1983), subsidization is still optimal. However, as presented by Qiu and Tao (1998), if there are considerable exogenous spill-overs from the R&D activity to foreign firms, subsidy recommendation may be reversed. Zhou *et al.* (2001) discuss investment taxes and subsidies in a duopoly situation that entails a low product quality country (firm) and a high quality country (firm), both of which export to a third country. They discovered that under Cournot competition a tax is optimal in the low-quality country while a subsidy is optimal in the high-quality country. Joint welfare is increased by taxes in both countries.

Section 2 presents the general framework of the models to be applied. Section 3 deals with the basic case. The influence of domestic consumption is discussed in Section 4. Heterogenous goods are introduced in Sections 5 and 6. In the former, the firms adjust quantities while in the latter they adjust prices. Concluding remarks and suggestions for a wider framework are offered in Section 7.

2. THE MODEL

There are two countries, denoted by the letters h (h-country) and f (f-country). A standard model is applied (see, e.g., Markusen and Venables (1988)). Each country has a single factor of production, labor, the endowment of which is L^h in h-country and L^f in f-country. These shares are normalized so that $L^h + L^f = 1$. Labor is used in two production sectors in both countries. The first is a competitive sector where a tradable, composite good is produced in both countries under constant returns to scale. The second sector is imperfectly competitive, producing a single homogenous good or two differentiated goods. In the production of this good (these goods) there are two national firms, firm 1 in h-country, owned by h-country residents, and firm 2 in f-country, owned by f-country residents. These firms consider merging. Marginal production costs are constant. Unless otherwise stated, these costs are assumed similar to the firms, pre-merger and post-merger and, by choosing the units suitably, we may set them equal to zero. The governments' policy instruments are a production tax or subsidy. For h-country it is denoted by s_1 per production unit. If $s_1 > 0$ we speak of a subsidy, but if $s_1 < 0$ we speak of a tax. The corresponding denotation for f-country is s_2 . There are no trading costs and the markets are integrated. The demand for this good (these goods) takes a linear form. I analyse a dynamic game with complete information for the following moves. First, two national firms decide whether to merge or not. Second, given a merger, they bargain over the ownership shares. This is formulated as a Nash bargaining game with equal bargaining strengths. Third, the governments set optimal subsidies or taxes separately. Fourth, firms choose outputs or prices. This dynamic game is solved by backward induction and it is, accordingly, subgame perfect.

Let us start with one homogenous good and Cournot competition. The overall demand for this good is, in inverse form,

$$p = A - q \quad (1)$$

where p denotes the price of the good, A is a constant and q is, in market equilibrium, the combined supply of both countries. With national firms $q = q_1 + q_2$, where q_1 is the production in h-country and q_2 in f-country. Firms adjust their production quantities so that their profits are maximized at all levels of subsidies/taxes. The optimal subsidy/tax for h-country is obtained by maximizing the indirect utility function $w^h(p, I^h)$ with respect to s_1 , applying Roy's identity and assuming constant marginal utility of income, normalized to

unity. The income in h-country, I^h , consists of labor income plus the profit of the national firm (or the share in the merger profit) minus subsidy costs (or plus the tax revenue). For example, in h-country, with a non-merged firm the income I^h is equal to $L^h + \pi_1 - s_1 q_1$, where wages are normalized to unity and π_1 denotes the profit of the national firm 1. In later sections, I will extend this model to situations where there are two differentiated goods under Cournot competition and Bertrand-competition in the pre-merger situation.

3. THE BASIC CASE

Here a single homogenous good is produced and all of it is exported to a third country. To begin with, we assume similar unit costs, normalized to zero. The zero cost also applies to the merged enterprise, which implies that there are no production synergies from the merger. In this section and in the next section, it is assumed that the merged enterprise can shift production of the good from one country to the other at no cost (except the potential costs via policy changes such shifts may cause). National firms compete in Cournot fashion. Familiar results ensue: if national firms are domestically owned, subsidization is the optimal policy in both countries. If subsidies are given to the firms their profits are $\pi_1 = \pi_2 = (A/2.5)^2 = 0.16A^2$. Here, these subsidies for national firms are not decisive regarding my conclusions. To show this later on, I calculated the profits without policies which are equal to $(A/3)^2$. Next, firms 1 and 2 merge (or consider merging at least). Obviously, without policies it does not matter where the merged firm produces. Optimal policies change this. Assume first that the merged enterprise produces a fraction k of the overall production of the good, q , in h-country and $1-k$ in f-country, $0 < k < 1$. These production shares are assumed exogenous. This implies that they are policy-independent. The merged enterprise chooses q so that its profit $\pi = (p+s_1)kq + (p+s_2)(1-k)q$ is maximized. In that which follows, I continue to use the letter π without subscripts to refer to the profit of the merged enterprise. Subsequently, the governments choose optimal policies to maximize welfare. These policies are

$$s_1 = -2(1-z)q/k = -(1-z)A/2k \quad (2a)$$

$$s_2 = -2zq/(1-k) = -zA/[2(1-k)] \quad (2b)$$

where z denotes the share of the merged enterprise (and its profits) belonging to h-country residents (owners of firm 1) while $1-z$ is the share which belongs to f-country residents (owners of firm 2), $0 \leq z \leq 1$. As Eqs. (2a) and (2b)

show, the optimal policy is taxation in both countries and the larger the ownership share, z or $1-z$, is in a country, the smaller is the unit tax. This will be discussed later on. Additionally, the larger the production share, k or $1-k$, is in a country, the lower is the unit tax there. And vice versa: with a very small production share in one country the tax would be very large there. The intuitive explanation is that a higher tax on a smaller production share does not affect the overall profit so much while it increases tax revenue. With production in both countries and optimal taxes, the overall profit of the merged firm can be shown to be $\pi = (A/4)^2$.

On the other hand, if all production is concentrated in h-country, its optimal tax, denoted by s , can be shown as

$$s = -2(1-z)q = -(1-z)A/(4-2z) \quad (3a)$$

If all production takes place in f-country, the optimal tax there is

$$s = -2zq = -zA/(2+2z) \quad (3b)$$

In (3a) s is an increasing function of z and in (3b) an increasing function of $1-z$. A higher tax decreases profit. With a small ownership share, a large part of this loss is borne by foreigners. As may be seen, if the producing country inhabitants own the whole merged enterprise the optimal tax there is zero: because the firm has a monopoly there is no strategic motivation for taxation (or subsidization).

If h-country is the producer the merger profit is

$$\pi = q^2 = [A/(4-2z)]^2 \quad (4a)$$

while if f-country is the producer the profit is

$$\pi = q^2 = [A/(2+2z)]^2 \quad (4b)$$

The country with the largest profit is chosen. As Eqs. (4a)–(4b) show, it is h-country if $z > 1/2$ and f-country if $z < 1/2$. In other words, the chosen country is the one whose residents own a majority share of the merged enterprise. We

may see that the profit is larger when the good is produced in one country instead of in both countries because in the latter case the merger profit is $(A/4)^2$. In conclusion, the merged enterprise concentrates the production into one country.

How are the ownership-shares determined? These merger shares are bargained for between the native owners of the two national firms and this bargaining concerns the division of the profit of the merged enterprise. Here, I apply a standard Nash bargaining model with equal bargaining strengths. Apart from satisfying certain plausible axioms, as presented by Nash (1953), this game has an additional strength. As was shown by Binmore *et al.* (1986), the solution of this Nash bargaining game is the same as in a dynamic game of alternating offers when the length of a single bargaining period approaches zero. This solution is reached instantaneously. In a Nash bargaining game, the solution is obtained by maximizing the Nash product. For our purposes, the Nash product is $[z\pi - \pi_1][(1-z)\pi - \pi_2]$, which is maximized with respect to z . In game theoretical terms, the national firms' profits π_1 and π_2 represent disagreement points that ensue if the bargaining does not come to an agreeable sharing of the merger profit π . The feasibility requires that $z\pi > \pi_1$ and that $(1-z)\pi > \pi_2$. Obviously, a sufficient condition for non-feasibility is that $\pi < \pi_1 + \pi_2$ because at least one owner-group loses from merging. Notice that in the Nash-product the profits also present utilities because we have already assumed the marginal utility of income to be equal to one. The first order condition which applies to all our cases can be shown to be

$$\frac{\pi + z(\partial\pi/\partial z)}{z\pi - \pi_1} = \frac{\pi + (1-z)(\partial\pi/\partial(1-z))}{(1-z)\pi - \pi_2} \quad (5)$$

From the profit maximization by the merged enterprise producing a single homogenous good, we obtain the production quantity q , price p and, consequently, the profit π as functions of policy s . If production takes place in h -country, we differentiate Eq. (3a) with respect to s , invert the derivative and insert it into the differentiated form of the profit function. Because above the national firms profits were similar, we may set them equal to zero. Eq. (5) becomes

$$\frac{\pi + 2zq^2/(2-z)}{z\pi} = \frac{\pi - 2(1-z)q^2/(2-z)}{(1-z)\pi} \quad (6a)$$

If production takes place in f-country we differentiate Eq. (3b) and proceed as above. This yields Eq. (5) as

$$\frac{\pi - 2zq^2/(1+z)}{z\pi} = \frac{\pi + 2(1-z)q^2/(1+z)}{(1-z)\pi} \quad (6b)$$

It may now be inferred from Eqs. (6a) and (6b) that the residents in the producing country obtain a larger share of the merged enterprise because a larger numerator must correspond to a larger denominator. This outcome may seem counter-intuitive. After all, the national firms were quite similar. However, a larger ownership share in the producing country means lower tax there. A part of the consequent increase in the merger profit goes to the owners in the non-producing country. Another question is whether this is large enough to compensate for the loss caused by the lower ownership-share. I leave this question open. Without policies the sharing is equal, that is, $z = (1-z) = \frac{1}{2}$ as may be seen from Eq. (5).

Other games yield other results. If side-payments (ex ante lump-sum transfers) are allowed, the division of the merger profit changes once again (for side-payments, see, e.g., Weber (1994)). Let T denote the side-payment of firm 1's owners to firm 2's owners. Net merger profit for firm 1's owners is $z\pi - T$. Correspondingly, it is $(1-z)\pi + T$ for firm 2's owners. Again, by choosing the units suitably, we may set $\pi_1 = \pi_2 = 0$. The Nash product is now $[z\pi - T][(1-z)\pi + T]$. By differentiating this with respect to T , the new first order condition becomes

$$T = -(1/2)(1 - 2z)\pi \quad (7)$$

From Eq. (7), the income for h-country residents is $z\pi - T = \pi/2$ and so this bargaining game yields equal sharing.

A central question is how do optimal policies affect the merger feasibility. Without policies, the merger profit is always larger than the sum of the profits of the national firms because the merged enterprise could always produce the earlier overall quantity. Consequently, any changes must increase profit and there is an incentive for merging. With policies this may not be so. After all, the policy changes from subsidies to taxation. The sum of national firms' profits is $0.32A^2$ which is always larger than the merger profit because the latter is always smaller than $A^2/4 = 0.25A^2$, as can be seen from Eqs. (4a) and (4b). Accordingly, at least one of the owner-groups of the two national firms loses from the merger. In other words, optimal trade and industrial policies make the merger unprofitable and infeasible. This does not decisively depend on the fact that the national firms were subsidized. By comparing the merger profit with the sum of national firms profit without subsidies ($=2(A/3)^2$) we may see, from Eqs. (4a) and (4b), that the merger is feasible only if the producing country residents own, approximately, more than 93 per cent of the merged enterprise,

The question emerges whether the merger is good from a welfare point of view. In the basic case, the relevant (normalized) utility concepts in a situation of national firms is the profit of the respective firm minus subsidy costs. These may be shown to be equal to $(1/2)(A/2.5)^2$ for each country. In a merger situation, the utility of the producing country is its share of the merger profit plus tax income. In the other country, it is simply the share of the merger profit going there. If the production takes place in h-country, its utility can be shown to be equal to $(1/2)A^2/(4-2z)$. We may directly see that it is, for all values of z , larger than h-country's utility with national firms. f-country's utility is $(1-z)[A/(4-2z)]^2$, which is a decreasing function of z . As was shown previously, the bargaining results for the producing h-country in a share z , which is equal or larger than $1/2$. With equal sharing, f-country's welfare is smaller with a merger than with national firms. Consequently, so it is with larger h-country shares. Analogous reasoning applies if f-country is the producing country. The conclusion is that in the basic case the merger raises the welfare of the producing country and decreases the welfare of the non-producing country. This depends on the tax income in the producing country and the corresponding lack of this revenue in the other country.

So far, it has been assumed that the governments did not cooperate with each other. Brander and Spencer (1985) showed for a homogenous goods duopoly with national firms that if both countries maximize separately the combined welfare of both countries and take the other country's policy as exogenous it is optimal to pay lower subsidies than in a non-cooperative situation. In particular,

if both countries export all their production to a third country, the optimal policy reverses to taxation. In the merger case, nearly the opposite ensues. Maximizing the combined welfare of the two countries with respect to the policy variable yields the outcome that laissez-fair is the optimal policy, both with shared production and with concentrated production. In itself, this is not surprising. We are dealing with a monopoly, so there are no strategic motives for taxation or subsidizing, nor are there policy motives caused by foreign ownership if all merger ownership is in h- and f-countries, as was assumed. Obviously, with co-operative pre- and post-merger policies, as presented above, the merger is always feasible.

Above, we have assumed equal costs. Let us consider a situation where these costs differ. By choosing the units suitably, assume the unit production cost in h-country to be c , $c > 0$, while in f-country it is equal to zero. Proceeding as above, it can be shown that the two conclusions still hold. First, it is always profitable to concentrate the production in one country. Second, the merger is never feasible with optimal policies. As can be shown, the sum of the profits of the national firms is always larger than $(1/3.88)A^2$, while the merger profit is always smaller than $(1/4)A^2$.

4. DOMESTIC CONSUMPTION

If there is domestic consumption of the good, it may be expected that pre-merger subsidies would be higher and post-merger taxes lower. I analyze a case where the homogenous good is fully consumed in h- and f-countries. The size of the relative consumption in the two countries can be measured by (normalized) labour endowments, L^h and L^f , $L^h + L^f = 1$. Similar constant production costs (normalized to zero) are assumed. In the initial situation of national firms, by applying standard methods, we obtain, as optimal policies, subsidies which are equal to $s_1 = L^h A$ and $s_2 = L^f A$. In the larger country, the subsidy is larger because there are more consumers who benefit from the subsidy-induced price fall. If all is consumed in one country, laissez fair is the optimal policy in the other country. Furthermore, at least one of the subsidies is larger than in the basic case, in which the subsidies are both equal to $A/5$. The profits of the national firms can be shown to be $\pi_1 = (L^h A)^2$ and $\pi_2 = (L^f A)^2$. The sum of these profits is $(2L^{h2} - 2L^h + 1)A^2$, which obtains its minimum value at $L^h = 1/2$. Inserting this, we may see that the sum of the national firms' profits is larger than $A^2/2$. In an extreme situation where all is consumed in one of the two countries, this country pays so large a subsidy to its firm that the

other firm's profit is nil. This firm closes and the merger question becomes irrelevant. In the following, I assume that there is consumption in both countries.

If the two firms merge and the k :th part is produced in h -country and the $(1-k)$ th part in f -country, $0 < k < 1$, the policies are $s_1 = -2(1-z-L^h/2)q/k$ and $s_2 = -2(z-L^f/2)q/(1-k)$. The merger profit is $\pi = (A/3)^2$. Unlike in the basic case, here the optimal policy may be subsidy if a country is large and its inhabitants own a large share of the merged enterprise. On the other hand, if all production is concentrated in one country, the optimal policy is, if h -country is the producing country,

$$s = -2(1-z-L^h/2)q \quad (8a)$$

or, if f -country is the producer,

$$s = -2(z-L^f/2)q \quad (8b)$$

The country with the smaller tax, hence the larger profit, is chosen. This depends on the ownership-shares and country sizes. If all production takes place in h -country, the merger profit is

$$\pi = q^2 = [A/(4-2z-L^h)]^2 \quad (9a)$$

and if all production is in f -country,

$$\pi = q^2 = [A/(2+2z-L^f)]^2 \quad (9b)$$

The merger profit is always larger than $(A/2.5)^2$. For example, if h -country is chosen it must be that the denominator of (9a) is smaller than the denominator of (9b). In other words, $4-2z-L^h < 2+2z-L^f = 2+2z-(1-L^h) =$

$1+2z+L^h$ or $2z+L^h > 3/2$. We can see the result by inserting this into Eq. (9a). This also applies to f-country if it is the producing country. Because with shared production the merger profit was $(A/3)^2$, here also it is profitable for the merged enterprise to concentrate production in one country only.

The other question concerns the feasibility of the merger. By comparing profits, we may see that a sufficient condition for non-feasibility is that $4-2z-L^h > 1.42$ or $2+2z-L^f > 1.42$. Unless the countries and, at the same time, the ownership shares are very dissimilar, it is not profitable for the national firms to merge.

The outcome of the merger game can be obtained by differentiating Eq. (8a) or (8b) and proceeding in a similar way as in the previous section. If production takes place in h-country, Eq. (5) becomes

$$\frac{\pi + 2zq^2/(2-z-L^h/2)}{z\pi - \pi_1} = \frac{\pi - 2(1-z)q^2/(2-z-L^h/2)}{(1-z)\pi - \pi_2} \quad (10a)$$

If production is in f-country, Eq. (5) is

$$\frac{\pi - 2zq^2/(1+z-L^f/2)}{z\pi - \pi_1} = \frac{\pi + 2(1-z)q^2/(1+z-L^f/2)}{(1-z)\pi - \pi_2} \quad (10b)$$

For various reasons, the production is likely to take place in the larger country. As Eqs. (8a) and (8b) show, a larger country size leads to a lower post-merger tax due to consumer effects. Besides, with national firms the subsidy is larger in the larger country and so the profit, π_1 or π_2 , is larger there. As shown by Eqs. (10a) or (10b), this contributes to a larger share. This, in itself, further reduces the tax. Both of these reasons yield a larger merger profit. In relation to co-operative policies, it can be shown that a post-merger subsidy is the optimal policy.

5. HETEROGENOUS GOODS: COURNOT-COMPETITION

In the following two sections I extend the discussion into differentiated goods, good 1 and good 2. Unlike in the previous sections, the production sites are now inflexible: good 1 may be produced in h-country only and good 2 in f-country only, pre- and post-merger. This may be justified, e.g., by assuming sunk costs. The production costs of the two goods are constant, similar and normalized to zero. All is exported to a third country. There is a quadratic sub-utility function for these goods which yields linear demands. In the present section, the firms adjust quantities and the inverted demand functions are $p_1 = C - q_1 - \beta q_2$ and $p_2 = C - q_2 - \beta q_1$, where C is a constant, p_1 and p_2 are the prices of respective goods, and q_1 and q_2 are their production quantities. Goods 1 and 2 are imperfect substitutes, that is, $0 < \beta < 1$. In the initial situation, there are two national firms, firm 1 in h-country producing good 1 and firm 2 in f-country producing good 2. Similar results ensue as with the homogenous good: it is optimal for both governments to pay subsidies. The profits of the firms with these subsidies are $\pi_1 = \pi_2 = [C/(2+\beta-0.5\beta^2)]^2$.

Let us now turn to a situation in which firms 1 and 2 merge. Good 1 is produced in h-country in plant 1 of the merged enterprise and good 2 in f-country in plant 2. The plants use separate accounting so that the government may tax or subsidize the production of the domestic plants. The merger situation is formulated as a joint maximization of the combined profit $\pi = (p_1+s_1)q_1 + (p_2+s_2)q_2$. Maximizing this yields the production quantities (and the prices) as functions of policies s_1 and s_2 . Welfare maximization by the governments yields the key expressions for optimal policies as:

$$s_1 = -2(1-z)(1-\beta^2)q_1 \quad (11a)$$

in h-country. Correspondingly, in f-country, as

$$s_2 = -2z(1-\beta^2)q_2 \quad (11b)$$

Taxing is the optimal policy in both countries if ownership is shared. There is a motive for taxation because a part of it is paid by foreign shareholders. On the other hand, suppose the merged enterprise is fully h-country owned ($z=1$).

Then the h-country optimal tax is zero as Eq. (11a) shows: obviously there is no strategic motive for taxation (or subsidization). Now, however, there is a strong motive for taxation in f-country, as shown by Eq. (11b), because all profits go to h-country.

The merger game is similar to that presented in Sec. 3: The merger is bargained over between the native owners of the two national firms. This bargaining concerns the sharing of the merged enterprise and its profit and the bargaining solution is obtained by maximizing the Nash-product. The first order condition is as presented in Eq. (5). Its terms can be obtained by differentiating Eq. (11a) with respect to s_1 and Eq. (11b) with respect to s_2 . The inverted terms are then inserted into the differentiated form of the profit function $\pi = \pi(s_1, s_2)$, yielding the terms in Eq. (5) as

$$\begin{aligned} z(\partial\pi/\partial z) \\ = 2z(q_1)^2(1-\beta^2)/(2-z) - 2z(q_2)^2(1-\beta^2)/(1+z) \end{aligned} \quad (12a)$$

and

$$\begin{aligned} (1-z)(\partial\pi/\partial(1-z)) = \\ - 2(1-z)(q_1)^2(1-\beta^2)/(2-z) + 2(1-z)(q_2)^2(1-\beta^2)/(1+z) \end{aligned} \quad (12b)$$

The solution of the whole dynamic game is such that the production quantities, taxes and ownership shares are similar in the two countries and for the two goods, as may be inferred from profit maximization and Eqs. (5), (11a), (11b), (12a) and (12b). The merger profit for each owner-group is $\pi/2 = (1+\beta)C^2/(3+2\beta-\beta^2)^2$. It can be shown, e.g. by simulating, that this is always smaller than the profit of a national firm. Consequently, the merger is not feasible for any values of β .

In relation to welfare in the merger situation, it is, for each country, half of the merger profit plus tax income. This can be shown to be equal to $(2+\beta-\beta^2)C^2/(3+2\beta-\beta^2)^2$. In the initial situation with national firms, each country's welfare is the national firm's profit minus subsidy costs, which is equal to $(1-\beta^2/2)C^2/(2+\beta-\beta^2/2)^2$. This is larger than the welfare in the merger situation only if $\beta < 0.42$ (approximately). Otherwise, the merger increases

welfare. Obviously, this depends on tax revenues because the merger was shown to be non-profitable to the owners. This is clarified by considering a situation where there are no subsidies for national firms. Here the merger increases welfare if $\beta > 0.5$ (approximately).

6. HETEROGENOUS GOODS: BERTRAND COMPETITION

I continue to consider a situation with two differentiated goods. Good 1 is produced, pre- and post- merger, in h-country and good 2 in f- country. The difference here is that firms adjust prices instead of quantities. The demands for the two goods are linear: $q_1 = B - p_1 + bp_2$ and $q_2 = B - p_2 + bp_1$, where B is constant and b describes the substitutability between goods, $0 < b < 1$. The other denotations are the same as in the previous section. I continue to assume zero costs, inflexibility of production sites and that all is exported to a third country. In the initial duopoly situation of national firms, the optimal policy in both countries is taxation (as is well-known, see Eaton and Grossman (1986)). The firms' profits are equal to $\pi_1 = \pi_2 = B^2 (2-b^2)^2 / (4-2b - b^2)^2$.

In the merger situation, the merged enterprise chooses the prices so as to maximize the combined profit. Optimal policies here are taxes in both countries. In h-country it is, for good 1,

$$s_1 = -2(1-z)q_1 \quad (13a)$$

and in f-country for good 2,

$$s_2 = -2zq_2 \quad (13b)$$

The merger game is still presented by Eq. (5). By differentiating Eq. (13a) with respect to s_1 and Eq.(13b) with respect to s_2 and proceeding as explained in the previous section, the terms in Eq. (5) become

$$z(\partial\pi/\partial z) = 2z(q_1)^2/(2-z) - 2z(q_2)^2/(1+z) \quad (14a)$$

and

$$(1-z)(\partial\pi/\partial(1-z)) = -2(1-z)(q_1)^2/(2-z) + 2(1-z)(q_2)^2/(1+z) \quad (14b)$$

The solution of the dynamic game leads to equal production quantities and prices for the two goods, equal policies in the two countries and equal ownership sharing of the merged enterprise. Half of the merger profit is $\pi/2 = B^2/[(3-b)^2(1-b)]$. With reasonable degrees of substitutability ($b > 0.6$ approximately), this can be shown to be larger than a national firm's profit, that is, the merger is feasible. To recall, in the corresponding Cournot case the merger was never feasible. An obvious explanation is that now there are both pre-merger and post-merger taxes. On the other hand, shared ownership of the merged enterprise leads to higher tax. However, if the substitutability of goods is reasonable, the monopoly advantage annuls this.

7. CONCLUDING REMARKS

We have discussed cases where two firms in different countries plan to merge to form a monopoly. Optimal policies may make this merger infeasible. The situation becomes more complicated if there would be more firms. As an illustration, consider a situation where there are initially two firms in h-country and one firm in f-country. They produce (and trade) a homogenous good at zero cost under Cournot competition and export this good to a third country. It can be shown that in the initial situation with three national firms the optimal policy is laissez fair in h-country and subsidy in f-country. (If there would be three or more firms in h-country its policy would be tax, for reasons see Sec.1). Assume that any two of these three firms can merge while the third stays separate. The negotiation about merger ownership shares is modelled as a two-players Nash bargaining game, presented in Sec.3. (For corresponding situations with coalitional games see Horn and Persson (2001)). Without

policies, all the merger combinations are equally profitable but with optimal policies there are several features which favour a domestic merger between the two h-country firms. First, in a domestic merger both owner-groups obtain half of the merger shares while in an international merger between the f-country firm and one of h-country firms, the share of this h-country firm's owners is decreased by the relative profits of the national firms, as shown in the denominators of Eq. (5). Initially, f-country firm's profit is larger than the profit of a h-country firm because of f-country's subsidy. Second, as was shown in the beginning of Sec. 3, when there is one domestic merged enterprise and one foreign firm, this merger receives a subsidy while in an international merger the shared ownership in itself leads to decreased subsidies in both countries or even to a tax. Finally, notice that there is an additional effect from policies. As was shown in Sec. 3, if h-country firms merge and the f-country firm stays separate, optimal policies make forming a further merged monopoly infeasible.

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DERIVATIONS AND EXPLANATIONS

SEC. 3. HOMOGENOUS GOOD. ALL EXPORTED TO A THIRD COUNTRY. SIMILAR PRODUCTION COSTS (=0).

(i) NATIONAL FIRMS:

Firms' profit maximization:

$$\max_{q_1} \pi_1 = (p+s_1)q_1 = (A -q_1 -q_2 +s_1)q_1, \text{ and}$$

$$\max_{q_2} \pi_2 = (p+s_2)q_2 = (A -q_1 -q_2 +s_2)q_2$$

The first order conditions are

$$\partial\pi_1/\partial q_1 = A - 2q_1 -q_2 +s_1 = 0$$

$$\partial\pi_2/\partial q_2 = A -q_1 - 2q_2 +s_2 = 0 \text{ from which}$$

$$q_1 = A -q_1 -q_2 +s_1 = p + s_1 \text{ and } q_2 = p + s_2 \text{ which yield}$$

$$\pi_1 = q_1^2 \text{ and } \pi_2 = q_2^2$$

$$q_1 = (A +2s_1 -s_2)/3 \text{ and } q_2 = (A +2s_2 -s_1)/3$$

$$\partial q_1/\partial s_1 = \partial q_2/\partial s_2 = 2/3$$

$$\partial p/\partial s_1 = \partial (q_1 -s_1)/\partial s_1 = -1/3$$

h-country's normalized welfare is $w^h = \pi_1 - s_1 q_1 = q_1^2 - s_1 q_1$. Maximizing:

$$\partial w^h/\partial s_1 = 2q_1(\partial q_1/\partial s_1) - q_1 - s_1(\partial q_1/\partial s_1)$$

$$= 2q_1(2/3) - q_1 - s_1(2/3) = 0. \text{ This yields}$$

$$s_1 = (1/2)q_1.$$

Maximizing f-country's welfare yields, by symmetry, from $\partial w^f / \partial s_2 = 0$:

$$s_2 = (1/2)q_2$$

Applying these yields $q_1 = q_2$ and $s_1 = s_2$. Consequently,

$$q_1 = (A + q_1/2)/3 \text{ from which}$$

$$q_1 = q_2 = 2A/5, s_1 = s_2 = A/5$$

$$\pi_1 = \pi_2 = q_1^2 = q_2^2 = (A/2.5)^2 = 0.16A^2$$

$$w^h = \pi_1 - s_1 q_1 = (1/2)q_1^2 = (1/2)(A/2.5)^2 = 0.08A^2.$$

(Without policies set $s_1 = s_2 = 0$. This yields

$$\pi_1 = \pi_2 = (A/3)^2.)$$

(ii) MERGER: k and 1-k PRODUCTION

Merger's profit maximization (notice $p = A - q$):

$$\max_q \pi = (p+s_1)kq + (p+s_2)(1-k)q = pq + [ks_1 + (1-k)s_2]q$$

$$= [A - q + ks_1 + (1-k)s_2]q$$

$$\partial \pi / \partial q = A - 2q + ks_1 + (1-k)s_2 = 0 \quad \text{from which}$$

$$q = A - q + ks_1 + (1-k)s_2 = p + ks_1 + (1-k)s_2 \quad \text{from which}$$

$$q = (1/2)[A + ks_1 + (1-k)s_2]$$

$$\partial q / \partial s_1 = k/2 \quad \text{and} \quad \partial q / \partial s_2 = (1-k)/2$$

$$\partial p / \partial s_1 = \partial(A - q) / \partial s_1 = -k/2$$

$$\pi = [p + ks_1 + (1-k)s_2]q = q^2$$

h-country's welfare is its share of the merger profit minus subsidy cost/plus tax revenue:

$$w^h = z\pi - s_1kq. \text{ Maximizing:}$$

$$\partial w^h / \partial s_1 = z2q(\partial q / \partial s_1) - kq - s_1k(\partial q / \partial s_1)$$

$$= zkq - kq - s_1k^2/2 = 0, \text{ from which}$$

$$s_1 = -2(1-z)q/k. \quad \text{By symmetry,}$$

$$s_2 = -2zq/(1-k). \text{ Note that } k \neq 0, 1$$

Inserting

$$\begin{aligned} q &= (1/2)[A + ks_1 + (1-k)s_2] = (1/2)[A - 2(1-z)q - 2zq] \\ &= (1/2)(A - 2q) \quad \text{from which } q = A/4 \end{aligned}$$

$$\text{and so } \pi = (A/4)^2 \text{ and } s_1 = -A(1-z)/2k \text{ and } s_2 = -Az/2(1-k).$$

(iii) MERGER: ALL PRODUCTION TO ONE COUNTRY

$$\max_q \pi = (p+s)q = (A-q+s)q \text{ where } s = \text{subsidy/tax in the producing country}$$

$$\partial \pi / \partial q = A - 2q + s = 0, \text{ from which}$$

$$q = A - q + s = p + s$$

$$\partial q / \partial s = 1/2 \quad \text{and} \quad \partial p / \partial s = \partial q / \partial s - 1 = -1/2$$

$$q = (1/2)(A + s)$$

$$\pi = (p+s)q = q^2 \quad \text{and} \quad \partial \pi / \partial s = 2q \partial q / \partial s = q$$

If production is in h-country its normalized welfare is

$w^h = z\pi - sq$. Maximizing:

$$\partial w^h / \partial s = z\partial\pi / \partial s - q - s\partial q / \partial s$$

$$= zq - q - s/2 = 0, \text{ from which}$$

$$s = -2(1-z)q < 0. \text{ Inserting}$$

$$q = (1/2)(A - 2(1-z)q) \quad \text{from which}$$

$$q(1 + (1-z)) = (1/2)A \quad \text{or}$$

$$q = A / (4-2z) \text{ and so } \pi = [A/(4-2z)]^2$$

$$\text{and } s = -A(1-z)/(2-z) \text{ from which } \partial s / \partial z > 0$$

$$\text{Size of welfare: } w^h = z\pi - sq = zq^2 + 2(1-z)q^2 = (2-z)q^2 = A^2 / [4(2-z)]$$

$$= (1/2)A^2 / (4-2z) > (1/2)A^2 / 4 > (1/2)A^2 / 2.5^2 \text{ (h-country's welfare with national firms)}$$

$$\text{f- country's welfare is } w^f = (1-z)\pi = (1-z)q^2 = (1-z)[A/(4-2z)]^2$$

$$\partial w^f / \partial z = [A^2 / 4](-2z / (2-z)^4) < 0$$

$$\text{so because here } z > 1/2, w^f < (1/2)[A/3]^2 < (1/2)A^2 / 2.5^2$$

If the production is in f-country

$$w^f = (1-z)\pi - sq = (1-z)q^2 - sq. \text{ By symmetry,}$$

$$s = -2zq$$

$$q = A / (2+2z).$$

The welfare size:

$$w^f = (1-z)\pi - sq = (1-z)q^2 + 2zq^2 = (1+z)q^2 = A^2 / [4(1+z)]$$

$$= (1/2)A^2 / (2+2z) > (1/2)A^2 / 4 > (1/2)A^2 / 2.5^2$$

$$w^h = z\pi = zq^2 = z[A/(2+z)]^2$$

$$\partial w^h / \partial z = A^2(4-z^2)/(2-z)^4 > 0$$

so because here $z < 1/2$, $w^h < (1/2)A^2/2.5^2$

Three conclusions

a) it is profitable for the merged enterprise to produce in one country only:

$$\pi = q^2 > [A/4]^2 \text{ (the merger profit with shared production)}$$

b) If $z \neq 0,1$, the merger profit is smaller than the sum of national firms' profits. That is, the merger is infeasible.

c) the merger would increase the welfare of the producing country and decrease the welfare of the non-producing country

Co-operative policies in the merger cases

(ii) k:th part in h-country and (1-k)th part in f-country

$$w^h = z\pi - s_1 kq \quad \text{and} \quad w^f = (1-z)\pi - s_2(1-k)q, \text{ so}$$

$$w^h + w^f = \pi - s_1 kq - s_2(1-k)q$$

A country takes the other country's policy as exogenous and we know that $\partial q / \partial s_1 = k/2$ and $\partial q / \partial s_2 = (1-k)/2$ and $\pi = q^2$.

$$\partial(w^h + w^f) / \partial s_1 = 2q(\partial q / \partial s_1) - kq - s_1(\partial kq / \partial s_2) = kq - kq - s_1(\partial kq / \partial s_2) = 0$$

From this, $s_1 = 0$.

In a similar way it can be shown that $s_2 = 0$.

(iii) all produced in one country

$$w^h + w^f = \pi - sq = q^2 - sq$$

$$\begin{aligned}\partial(w^h+w^f)/\partial s &= 2q(\partial q/\partial s) - q - s(\partial q/\partial s) \\ &= q - q - s/2 = 0 \text{ or } s = 0\end{aligned}$$

The merger game

If production takes place in h-country $s = -2(1-z)q$ and so

$$\partial s/\partial z = 2q(\partial z/\partial s) - 2(1-z)(\partial q/\partial s) \text{ or}$$

$$1 = 2q(\partial z/\partial s) - 2(1-z)(1/2) \text{ from which}$$

$$\partial z/\partial s = (2-z) / 2q \text{ from which } \partial s/\partial z = 2q/(2-z) = -\partial s/\partial(1-z)$$

If production takes place in f-country $s = -2zq$ and so

$$\partial s/\partial z = -2q(\partial z/\partial s) - 2z(\partial q/\partial s) \text{ or}$$

$$\partial z/\partial s = -(1+z) / 2q \text{ from which } \partial s/\partial z = -2q/(1+z) = -\partial s/\partial(1-z)$$

STANDARD NASH BARGAINING GAME

The Nash product is

$$\max [z\pi - \pi_1][(1-z)\pi - \pi_2]$$

When maximizing this with respect to z , the first order condition is

$$[\partial(z\pi)/\partial z][(1-z)\pi - \pi_2] + \{[\partial(1-z)\pi]/\partial z\}\{z\pi - \pi_1\} = 0 \text{ or}$$

$$[\partial(z\pi)/\partial z][(1-z)\pi - \pi_2] - \{[(1-z)\pi]/\partial(1-z)\}\{z\pi - \pi_1\} = 0 \text{ or}$$

$$\frac{\pi + z(\partial\pi/\partial z)}{z\pi - \pi_1} = \frac{\pi + (1-z)(\partial\pi/\partial(1-z))}{(1-z)\pi - \pi_2} \quad (5)$$

In the basic case we know that $\partial q/\partial s = 1/2$ and $\pi = (p+s)q = q^2$ and $\partial\pi/\partial s = 2q\partial q/\partial s$, so

$$\partial\pi/\partial z = (\partial\pi/\partial s)(\partial s/\partial z) = q(\partial s/\partial z) = -\partial\pi/\partial(1-z)$$

If production is in h-country, $\partial s/\partial z = 2q/(2-z) = -\partial s/\partial(1-z)$ and Eq, (5) becomes

$$\frac{\pi + 2zq^2/(2-z)}{z\pi - \pi_1} = \frac{\pi - 2(1-z)q^2/(2-z)}{(1-z)\pi - \pi_2}$$

If production is in f-country, $\partial s/\partial z = -2q/(1+z) = -\partial s/\partial(1-z)$, so (5) is

$$\frac{\pi - 2zq^2/(1+z)}{z\pi - \pi_1} = \frac{\pi + 2(1-z)q^2/(1+z)}{(1-z)\pi - \pi_2}$$

Because $\pi_1 = \pi_2$ we can set them equal to zero by choosing the units suitably.

BARGAINING WITH SIDE PAYMENTS

$$c=0, \pi_1 = \pi_2 = 0.$$

$$\max_{T,z} [z\pi - T][(1-z)\pi + T]$$

When differentiating with respect to T, the first order condition is

$$-[(1-z)\pi + T] + z\pi - T = 0 \text{ or } T = -(1/2)(1-2z)\pi .$$

Accordingly the profit going to h-country residents is

$$z\pi - T = z\pi + (1/2)(1-2z)\pi = (1/2)\pi$$

END-PART OF SEC.3. OTHERWISE AS ABOVE EXCEPT THAT COSTS NOT SIMILAR, $c_1 = c > 0$, $c_2 = 0$

(i) NATIONAL FIRMS:

$$\max_{q_1} \pi_1 = (p+s_1-c)q_1 = (A -q_1 -q_2 +s_1 -c)q_1, \text{ and}$$

$$\max_{q_2} \pi_2 = (p-c+s_2)q_2 = (A -q_1 -q_2 +s_2)q_2$$

The first order conditions (foc) are

$$\partial\pi_1/\partial q_1 = A - 2q_1 -q_2 +s_1 -c = 0$$

$$\partial\pi_2/\partial q_2 = A -q_1 - 2q_2 +s_2 = 0 \text{ from which because } p = A -q_1 -q_2$$

$$q_1 = A -q_1 -q_2 +s_1 -c = p + s_1 -c$$

$$q_2 = A -q_1 -q_2 + s_2 = p + s_2 \text{ which yield}$$

$$\pi_1 = q_1^2 \text{ and } \pi_2 = q_2^2. \text{ Furthermore, solving the system,}$$

$$q_1 = (A +2s_1 -s_2 - 2c)/3 \text{ and } q_2 = (A +2s_2 -s_1 +c)/3$$

$$\partial q_1/\partial s_1 = \partial q_2/\partial s_2 = 2/3$$

$$\text{From foc, } \partial p/\partial s_1 = \partial (q_1 -s_1 +c)/\partial s_1 = -1/3$$

h-country's welfare is $w^h = \pi_1 - s_1 q_1 = q_1^2 - s_1 q_1$. Maximizing

$$\partial w^h/\partial s_1 = 2q_1(\partial q_1/\partial s_1) - q_1 - s_1(\partial q_1/\partial s_1)$$

$$= 2q_1(2/3) - q_1 - s_1(2/3) = 0 \text{ yielding}$$

$$s_1 = (1/2)q_1$$

Similarly, for f-country from $\partial w^f/\partial s_2 = 0$:

$$s_2 = (1/2)q_2$$

Applying these yields the system

$$3q_1 + 2q_2 - 2A + 2c = 0$$

$$2q_1 + 3q_2 - 2A = 0$$

Solving yields

$$q_1 = (6A - 18c)/15 = (2A - 6c)/5 = 2s_1 \quad (\text{this implies } c < A/3)$$

$$q_2 = (6A + 12c)/15 = (2A + 4c)/5 = 2s_2$$

$$\pi_1 + \pi_2 = q_1^2 + q_2^2 = (2/5)^2 [(A - 3c)^2 + (A + 2c)^2] = (2/5)^2 [2A^2 - 2Ac + 13c^2]$$

$$\partial(\pi_1 + \pi_2)/\partial c = (2/5)^2 (-2A + 13c)$$

Setting this equal to zero we obtain minimum at $c = 2A/13$.

$$\text{At this value } \pi_1 + \pi_2 > (2/5)^2 (2 - 4/13)A^2 = (84/325)A^2 \approx (1/3.87)A^2$$

This implies that, with all possible values of c ,

$$\pi_1 + \pi_2 > (1/3.88)A^2$$

(ii) MERGER: k VS. (1-k) PRODUCTION

$$\max_q \pi = (p + s_1 - c)kq + (p + s_2)(1 - k)q = pq + [(s_1 - c)k + (1 - k)s_2]q$$

$$= [A - q + ks_1 - kc + (1 - k)s_2]q$$

$$\partial\pi/\partial q = A - 2q + ks_1 - kc + (1 - k)s_2 = 0 \quad \text{from which}$$

$$q = A - q + ks_1 - kc + (1 - k)s_2 = p + ks_1 + (1 - k)s_2 - kc \quad \text{from which}$$

$$q = (1/2)[A + ks_1 + (1 - k)s_2 - kc]$$

$$\partial q/\partial s_1 = k/2 \quad \text{and} \quad \partial q/\partial s_2 = (1 - k)/2$$

$$\partial p/\partial s_1 = \partial(A - q)/\partial s_1 = -k/2 \quad \text{and} \quad \partial p/\partial s_2 = -(1 - k)/2$$

$$\pi = [p + ks_1 + (1-k)s_2 - kc]q = q^2$$

h-country's welfare is $w^h = z\pi - s_1kq$. Maximizing

$$\partial w^h / \partial s_1 = z2q(\partial q / \partial s_1) - kq - s_1k(\partial q / \partial s_1)$$

$$= zkq - kq - s_1k^2/2 = 0, \text{ from which}$$

$$s_1 = -2(1-z)q/k \quad \text{By symmetry,}$$

$$s_2 = -2zq/(1-k). \text{ Notice that } k \neq 0, 1$$

Interpretation (again): The larger the production share is, the smaller the tax

Inserting s_1 and s_2 :

$$q = (1/2)[A + ks_1 + (1-k)s_2 - kc] = (1/2)[A - 2(1-z)q - 2zq - kc]$$

$$= (1/2)(A - 2q - kc) \quad \text{from which } q = (A - kc)/4$$

and so $\pi = [(A - kc)/4]^2$ and $s_1 = -(A - kc)(1 - z)/k$ and $s_2 = -(A - kc)z/(1 - k)$.

Notice for later use $\pi = [(A - kc)/4]^2 < [A/(2 + 2z)]^2$ if $z \neq 1$

(iii) MERGER: ALL PRODUCTION TO ONE COUNTRY:

If all production is in h-country:

$$\max_q \pi = (p + s - c)q = (A - q + s - c)q$$

$$\partial \pi / \partial q = A - 2q + s - c = 0, \text{ from which}$$

$$q = A - q + s - c = p + s - c$$

$$\partial q / \partial s = 1/2 \quad \partial p / \partial s = \partial q / \partial s - 1 = -1/2$$

$$q = (1/2)(A + s - c)$$

$$\pi = (p + s - c)q = q^2 \quad \partial \pi / \partial s = 2q \partial q / \partial s$$

$$w^h = z\pi - sq$$

$$\begin{aligned} \partial w^h / \partial s &= z \partial \pi / \partial s - q - s \partial q / \partial s = 2zq(\partial q / \partial s) - q - s(\partial q / \partial s) \\ &= zq - q - s/2 = 0, \text{ from which} \end{aligned}$$

$$s = -2(1-z)q^* < 0. \text{ Inserting}$$

$$q = (1/2)(A - c - 2(1-z)q) \quad \text{from which}$$

$$q(1 + (1-z)) = (1/2)(A - c) \quad \text{or}$$

$$q = (A - c) / (4 - 2z) \text{ and so } \pi = [(A - c) / (4 - 2z)]^2$$

If all production is in f-country

$$\max_q \pi = (p + s)q = (A - q + s)q$$

$$\partial \pi / \partial q = A - 2q + s = 0, \text{ from which}$$

$$q = A - q + s = p + s$$

$$\partial q / \partial s = 1/2 \quad \partial p / \partial s = \partial q / \partial s - 1 = -1/2$$

$$q = (1/2)(A + s)$$

$$\pi = (p + s)q = q^2 \quad \partial \pi / \partial s = 2q \partial q / \partial s$$

$$w^f = (1-z)\pi - sq$$

$$\begin{aligned} \partial w^f / \partial s &= (1-z) \partial \pi / \partial s - q - s \partial q / \partial s = 2(1-z)q(\partial q / \partial s) - q - s(\partial q / \partial s) \\ &= (1-z)q - q - s/2 = 0, \text{ from which} \end{aligned}$$

$$s = -2zq, \quad q = [A/(2+2z)] , \quad \pi = [A/(2+2z)]^2$$

Two conclusions:

a) Because $[A/(2+2z)]^2 > [(A-kc)/4]^2$ it is profitable to concentrate production to one country only. A natural choice would be f-country. It does not need to be shown because if the merger profit would be larger in h-country (because of potential lower tax) the conclusion still holds.

b) The merger is never feasible. If production is in h-country, the merger profit is

$$\pi = [(A-c) / (4-2z)]^2 < A^2/4$$

and if production is in f-country

$$\pi = [A/(2+2z)]^2 < A^2/4$$

while the sum of national firms' profits was

$$\pi_1 + \pi_2 > (1/3.88)A^2$$

SEC.4. ALL CONSUMED IN h-COUNTRY AND f-COUNTRY, SIMILAR COSTS (=0)

(i) NATIONAL FIRMS

$$\max_{q_1} \pi_1 = (p+s_1)q_1 = (A -q_1 -q_2 +s_1)q_1, \text{ and}$$

$$\max_{q_2} \pi_2 = (p+s_2)q_2 = (A -q_1 -q_2 +s_2)q_2$$

The first order conditions are

$$\partial\pi_1/\partial q_1 = A - 2q_1 -q_2 +s_1 = 0 \text{ and}$$

$$\partial\pi_2/\partial q_2 = A -q_1 - 2q_2 +s_2 = 0 \text{ from which}$$

$$q_1 = A -q_1 -q_2 +s_1 = p + s_1 \text{ and } q_2 = p + s_2 \text{ which yield}$$

$$\pi_1 = q_1^2 \text{ and } \pi_2 = q_2^2. \text{ Furthermore,}$$

$$q_1 = (A +2s_1 -s_2)/3 \text{ and } q_2 = (A +2s_2 -s_1)/3$$

$$\partial q_1/\partial s_1 = \partial q_2/\partial s_2 = 2/3$$

$$\partial p/\partial s_1 = \partial(q_1 -s_1)/\partial s_1 = -1/3$$

h-country's welfare maximization:

$$\begin{aligned} \partial w^h/\partial s_1 &= -L^h(\partial p/\partial s_1)(q_1+q_2) + \partial(\pi_1 - s_1q_1)/\partial s_1 \\ &= L^h(1/3)(q_1+q_2) + 2q_1(\partial q_1/\partial s_1) - q_1 - s_1(\partial q_1/\partial s_1) \end{aligned}$$

$$= (L^h/3)(q_1+q_2) + 2q_1(2/3) - q_1 - s_1(2/3) = 0 \text{ or}$$

$$L^h(q_1+q_2) + 4q_1 - 3q_1 - 2s_1 = 0 \text{ or}$$

$$(L^h+1)q_1 + L^h q_2 = 2s_1 \text{ Correspondingly, for f-country from } \partial w^f/\partial s_2 = 0,$$

$$(L^f+1)q_2 + L^f q_1 = 2s_2$$

Inserting q_1 and q_2 :

$$(L^h+1)(A+2s_1-s_2)/3 + L^h(A+2s_2-s_1)/3 = 2s_1$$

$$(L^f+1)(A+2s_2-s_1)/3 + L^f(A+2s_1-s_2)/3 = 2s_2 \text{ or}$$

$$(4-L^h)s_1 + (1-L^h)s_2 = (1+2L^h)A$$

$$(4-L^f)s_2 + (1-L^f)s_1 = (1+2L^f)A$$

From these, (notice $L^f = 1-L^h$),

$$[(4-L^h)(4-L^f) - (1-L^h)(1-L^f)]s_1 = (4-L^f)(1+2L^h)A - (1-L^h)(1+2L^f)A$$

where

$$\begin{aligned} (4-L^f)(1+2L^h)A - (1-L^h)(1+2L^f)A &= (3+L^h)(1+2L^h)A - (1-L^h)(3-2L^h)A \\ &= 12L^hA \end{aligned}$$

and

$$(4-L^h)(4-L^f) - (1-L^h)(1-L^f) = (4-L^h)(3+L^h) - (1-L^h)L^h = 12$$

Accordingly $s_1 = L^hA$,

By symmetry, from f-country's welfare maximization:

$$s_2 = L^fA$$

Using these:

$$q_1 = (A+2s_1-s_2)/3 = [A+2L^hA-(1-L^h)A]/3 = L^hA$$

$$q_2 = L^fA$$

$$\pi_1 + \pi_2 = [(L^h)^2 + (L^f)^2]A^2 = [L^{h2} + (1-L^h)^2]A^2 = (2L^{h2} - 2L^h + 1)A^2$$

$\geq (1/2 - 1 + 1)A^2 = A^2/2$ because $L^h = 1/2$ gives the minimum value

Why? $d(2L^{h2} - 2L^h + 1)/dL^h = 4L^h - 2 = 0$. The extreme value $L^h = 1/2$ is obviously minimum.

(ii) MERGER: k and (1-k) PRODUCTION

$$\max_q \pi = (p+s_1)kq + (p+s_2)(1-k)q = pq + [ks_1+(1-k)s_2]q$$

$$= [A-q + ks_1 + (1-k)s_2]q$$

$$\partial\pi/\partial q = A-2q + ks_1+(1-k)s_2 = 0 \quad \text{from which}$$

$$q = A-q + ks_1 + (1-k)s_2 = p + ks_1 + (1-k)s_2 \quad \text{from which}$$

$$q = (1/2)[A + ks_1 + (1-k)s_2]$$

$$\partial q/\partial s_1 = k/2 \quad \text{and} \quad \partial q/\partial s_2 = (1-k)/2$$

$$\partial p/\partial s_1 = \partial q/\partial s_1 - k = -k/2$$

$$\pi = [p+ks_1+(1-k)s_2]q = q^2$$

Optimal policies:

$$\partial w^h/\partial s_1 = -L^h(\partial p/\partial s_1) + \partial(z\pi - s_1kq)/\partial s_1$$

$$= kL^h/2 + z2q(\partial q/\partial s_1) - kq - s_1k(\partial q/\partial s_1)$$

$$= kL^h/2 + zkq - kq - s_1k^2/2 = 0, \text{ from which}$$

$$s_1 = -2(1-z-L^h/2)q/k. \quad \text{Correspondingly,}$$

$$s_2 = -2(z-L^f/2)q/(1-k). \quad \text{Inserting}$$

$$q = (1/2)[A + ks_1 + (1-k)s_2] = (1/2)[A - 2(1-z-L^h/2)q - 2(z-L^f/2)q]$$

$$= (1/2)(A - q) \quad (\text{note that } L^h + L^f = 1)$$

$$q = A/3 \quad \text{and so } \pi = (A/3)^2$$

(iii) MERGER: ALL PRODUCTION TO ONE COUNTRY

$$\max_q \pi = (p+s)q = (A-q+s)q$$

$$\partial\pi/\partial q = A-2q+s = 0 \text{ or } q = (1/2)(A+s) . \text{ From these,}$$

$$q = A-q+s = p+s$$

$$\partial q/\partial s = 1/2 \quad \partial p/\partial s = \partial q/\partial s - 1 = -1/2$$

$$\pi = (p+s)q = q^2 \quad \partial\pi/\partial s = 2q\partial q/\partial s = q$$

If production is in h-country, the foc for optimal policy s is

$$\begin{aligned} \partial w^h / \partial s &= -L^h q (\partial p / \partial s) + \partial z\pi / \partial s - q - s \partial q / \partial s \\ &= L^h q / 2 + 2zq (\partial q / \partial s) - q - s (\partial q / \partial s) \\ &= q L^h / 2 + zq - q - s / 2 = 0, \text{ from which} \end{aligned}$$

$$s = -2(1-z-L^h/2)q \text{ Inserting}$$

$$q = (1/2)(A - 2(1-z- L^h/2)q) \quad \text{from which}$$

$$q(1 + (1-z- L^h/2)) = (1/2)A \quad \text{or}$$

$$q = A / (4-2z-L^h) \text{ and so}$$

$$\pi = [A / (4-2z- L^h)]^2$$

Correspondingly, if production is in f- country

$$s = -2(z-L^f/2)q$$

$$\pi = [A / (2+2z- L^f)]^2$$

That country is chosen where the profit is larger, so

if h-country is producer, it must be that

$$4-2z- L^h < 2+2z- L^f = 2+2z-(1- L^h) = 1+2z+L^h \text{ or}$$

$$2z+L^h > 3/2 \text{ implying that } \pi = [A / (4-2z- L^h)]^2 > [A/2.5]^2$$

If f-country is producer, it must be that

$$4-2z-(1-L^f) > 2+2z-L^f \text{ or } 1/2 > 2z-L^f$$

$$\text{implying } \pi = [A/(2+2z-L^f)]^2 > [A/2.5]$$

THE MERGER GAME

If production takes place in h-country $s = -2(1-z-L^h/2)q$ So

$$\partial s / \partial s = 2q(\partial z / \partial s) - 2(1-z-L^h/2)(\partial q / \partial s) \text{ where } \partial q / \partial s = 1/2$$

From this,

$$\partial z / \partial s = (2-z-L^h/2) / 2q \text{ from which } \partial s / \partial z = 2q / (2-z-L^h/2) = -\partial s / \partial (1-z)$$

If production takes place in f-country $s = -2(z-L^f/2)q$. Differentiating:

$$1 = -2q(\partial z / \partial s) - 2(z-L^f/2)(\partial q / \partial s) \text{ or}$$

$$\partial z / \partial s = -(1+z-L^f/2) / 2q \text{ or}$$

$$\partial s / \partial z = -\partial s / \partial (1-z) = -2q / (1+z-L^f/2)$$

STANDARD NASH BARGAINING GAME

The Nash product is $[z\pi - \pi_1][(1-z)\pi - \pi_2]$

The first order condition is presented in Eq. (5) in the text:

$$\frac{\pi + zq(\partial s/\partial z)}{z\pi - \pi_1} = \frac{\pi + (1-z)q(\partial s/\partial(1-z))}{(1-z)\pi - \pi_2} \quad (5)$$

Inserting the derivatives we obtain if production is in h-country

$$\frac{\pi + 2zq^2/(2-z-L^h/2)}{z\pi - \pi_1} = \frac{\pi - 2(1-z)q^2/(2-z-L^h/2)}{(1-z)\pi - \pi_2}$$

If production is in f-country.

$$\frac{\pi - 2zq^2/(1+z-L^f/2)}{z\pi - \pi_1} = \frac{\pi + 2(1-z)q^2/(1+z-L^f/2)}{(1-z)\pi - \pi_2}$$

Co-operative policies

$$\begin{aligned} \partial(w^h+w^f)/\partial s &= -q(\partial p/\partial s) + \partial\pi/\partial s - q - s\partial q/\partial s \\ &= q/2 + 2q(\partial q/\partial s) - q - s(\partial q/\partial s) \\ &= q/2 + q - q - s/2 = 0 \quad \text{or } s = q \end{aligned}$$

SEC.5. HETEROGENOUS GOODS: COURNOT UNIT COSTS = 0, ALL EXPORTED TO A THIRD COUNTRY

NATIONAL FIRMS

Firms' profit maximization:

The profits are $\pi_1 = (p_1 + s_1)q_1 = (C - q_1 - \beta q_2 + s_1)q_1$

$$\pi_2 = (p_2 + s_2)q_2 = (C - q_2 - \beta q_1 + s_2)q_2$$

Profit maximization yields:

$$C - 2q_1 - \beta q_2 + s_1 = 0$$

$$C - 2q_2 - \beta q_1 + s_2 = 0$$

These can also be written as

$$p_1 = C - q_1 - \beta q_2 = q_1 - s_1$$

$$p_2 = C - q_2 - \beta q_1 = q_2 - s_2$$

From the latter pair

$$\pi_1 = q_1^2 \text{ and } \pi_2 = q_2^2$$

From the former pair

$$q_1 = [(2-\beta)C + 2s_1 - \beta s_2] / [4-\beta^2]$$

$$q_2 = [(2-\beta)C + 2s_2 - \beta s_1] / [4-\beta^2]$$

$$\partial q_1 / \partial s_1 = 2 / (4-\beta^2)$$

$$\partial q_2 / \partial s_1 = -\beta / (4-\beta^2)$$

$$\partial p_1 / \partial s_1 = \partial q_1 / \partial s_1 - 1 = -(2-\beta^2) / (4-\beta^2)$$

$$\partial p_2 / \partial s_1 = \partial q_2 / \partial s_1 = -\beta / (4 - \beta^2)$$

Governments' welfare maximization and policy choice:

Welfare is the national firm's profit minus subsidy costs. The f.o.c. for welfare maximization in h-country is:

$$\begin{aligned} \partial w^h / \partial s_1 &= \partial \pi_1 / \partial s_1 - q_1 - s_1 (\partial q_1 / \partial s_1) = 2q_1 (\partial q_1 / \partial s_1) - q_1 - s_1 (\partial q_1 / \partial s_1) \\ &= 0 \end{aligned}$$

Inserting the derivatives yields

$$s_1 = q_1 \beta^2 / 2$$

For f-country, symmetrically,

$$s_2 = q_2 \beta^2 / 2$$

The solution is $q_1 = q_2$, $p_1 = p_2$ and $s_1 = s_2 = q_1 \beta^2 / 2$

Inserting these yields $q_1 = [(2 - \beta)C + 2s_1 - \beta s_2] / [4 - \beta^2] = (C + s_1) / (2 + \beta)$ or

$$q_1 = q_2 = C / (2 + \beta - 0.5\beta^2)$$

$$\pi_1 = \pi_2 = q_1^2 = q_2^2$$

MERGER

The profit maximization of the merged enterprise is:

$$\max_{q_1, q_2} \pi = (p_1 + s_1)q_1 + (p_2 + s_2)q_2 = (C - q_1 - \beta q_2 + s_1)q_1 + (C - q_2 - \beta q_1 + s_2)q_2$$

The first order conditions are:

$$C - 2q_1 - \beta q_2 + s_1 = 0$$

$$C - 2q_2 - \beta q_1 + s_2 = 0$$

which can also be written as

$$p_1 = C - q_1 - \beta q_2 = q_1 + \beta q_2 - s_1$$

$$p_2 = C - q_2 - \beta q_1 = q_2 + \beta q_1 - s_2$$

From the former pair of focs:

$$q_1 = [C(1-\beta) + s_1 - \beta s_2] / [2(1-\beta^2)]$$

$$\partial q_1 / \partial s_1 = 1/(2-2\beta^2) \quad \text{and} \quad \partial q_2 / \partial s_1 = -\beta/(2-2\beta^2)$$

$$\partial p_1 / \partial s_1 = -\partial q_1 / \partial s_1 - \beta \partial q_2 / \partial s_1 = -1/2 \quad \text{and} \quad \partial p_1 / \partial s_2 = 0.$$

Analogous forms for good 2.

Using the latter pair of focs:

$$\pi = (p_1 + s_1)q_1 + (p_2 + s_2)q_2$$

$$= q_1(q_1 + \beta q_2) + q_2(q_2 + \beta q_1) = q_1^2 + 2\beta q_1 q_2 + q_2^2$$

Differentiating:

$$\partial \pi / \partial s_1 = 2q_1(\partial q_1 / \partial s_1) + 2\beta q_1(\partial q_2 / \partial s_1) + 2\beta q_2(\partial q_1 / \partial s_1) + 2q_2(\partial q_2 / \partial s_1) =$$

$$= 2q_1/(2-2\beta^2) - 2\beta q_1\beta/(2-2\beta^2) + 2\beta q_2/(2-2\beta^2) - 2q_2\beta/(2-2\beta^2) = q_1$$

Correspondingly

$$\partial \pi / \partial s_2 = q_2$$

h-country's welfare maximization:

$$\partial w^h / \partial s_1 = z[\partial \pi / \partial s_1] - \partial(s_1 q_1) / \partial s_1 = zq_1 - q_1 - s_1/(2-2\beta^2) = 0, \text{ from this}$$

$$s_1 = -2(1-z)(1-\beta^2)q_1 \quad (11a)$$

f-country's welfare maximization:

$\partial w^f / \partial s_2 = (1-z)[\partial \pi / \partial s_2] - \partial (s_2 q_2) / \partial s_2 = 0$, from which

$$s_2 = -2z(1-\beta^2)q_2 \quad (11b)$$

From merger game (see later) $z = 1/2$ and so $q_1 = q_2$ and $s_1 = s_2 = -(1-\beta^2)q_1 = -(1-\beta^2)q_2$ and so

$$q_1 = [C+s] / [2(1+\beta)]$$

$$(2+2\beta)q_1 = C - (1-\beta^2)q_1 \quad \text{or} \quad q_1 = C / (3+2\beta-\beta^2) = q_2$$

$$\pi = q_1^2 + 2\beta q_1 q_2 + q_2^2 = 2(1+\beta)q_1^2$$

$$\pi = 2(1+\beta)C^2 / (3+2\beta-\beta^2)^2$$

THE MERGER GAME

We have $s_1 = -2(1-z)(1-\beta^2)q_1$. Differentiating this implicitly with respect to s_1 yields:

$$\partial s_1 / \partial s_1 = 2(1-\beta^2)q_1(\partial z / \partial s_1) - 2(1-\beta^2)(1-z)(\partial q_1 / \partial s_1) \quad \text{or}$$

$$1 = 2(1-\beta^2)q_1(\partial z / \partial s_1) - 2(1-\beta^2)(1-z) / (2-2\beta^2) \quad \text{or}$$

$$\partial z / \partial s_1 = (2-z) / [2(1-\beta^2)q_1]. \quad \text{Inverting}$$

$$\partial s_1 / \partial z = 2(1-\beta^2)q_1 / (2-z).$$

We have $s_2 = -2z(1-\beta^2)q_2$. Differentiating this implicitly with respect to s_2 yields:

$$\partial s_2 / \partial s_2 = -2(1-\beta^2)q_2(\partial z / \partial s_2) - 2(1-\beta^2)z(\partial q_2 / \partial s_2) \quad \text{or}$$

$$1 = -2(1-\beta^2)q_2(\partial z / \partial s_2) - 2(1-\beta^2)z / (2-2\beta^2) \quad \text{or}$$

$\partial z / \partial s_2 = -(1+z) / [2(1-\beta^2)q_2]$. Inverting

$$\partial s_2 / \partial z = -2(1-\beta^2)q_2 / (1+z)$$

Because $\pi = \pi(s_1, s_2)$,

$$d\pi / dz = (\partial\pi / \partial s_1)(\partial s_1 / \partial z) + (\partial\pi / \partial s_2)(\partial s_2 / \partial z)$$

$$= q_1(\partial s_1 / \partial z) + q_2(\partial s_2 / \partial z)$$

Maximizing the Nash product yields Eq.(5) in the text:

$$\frac{\pi + z(\partial\pi / \partial z)}{z\pi - \pi_1} = \frac{\pi + (1-z)(\partial\pi / \partial(1-z))}{(1-z)\pi - \pi_2} \quad (5)$$

From the merged firm's profit maximization we obtained

$$\pi = \pi(s_1, s_2) \text{ where } \partial\pi / \partial s_1 = q_1 \text{ and } \partial\pi / \partial s_2 = q_2$$

From this and the derivatives above:

$$\begin{aligned} \partial\pi / \partial z &= (\partial\pi / \partial s_1)(\partial s_1 / \partial z) + (\partial\pi / \partial s_2)(\partial s_2 / \partial z) \\ &= 2(q_1)^2(1-\beta^2) / (2-z) - 2(q_2)^2(1-\beta^2) / (1+z) \end{aligned}$$

$$\begin{aligned} \partial\pi / \partial(1-z) &= (\partial\pi / \partial z)[\partial z / \partial(1-z)] = -\partial\pi / \partial z \\ &= -2(q_1)^2(1-\beta^2) / (2-z) + 2(q_2)^2(1-\beta^2) / (1+z) \end{aligned}$$

Using these we obtain Eqs. (12a) and (12b) in the text. We can now see that the solution of the dynamic game is such that $q_1 = q_2$ and $z = 1-z = \frac{1}{2}$ (from which $s_1 = s_2$ and $p_1 = p_2$).

Comparing the welfares

National firms:

$$w^h = \pi_1 - s_1 q_1 = q_1^2 - (\beta^2/2)q_1^2 = (1-\beta^2/2)q_1^2 = w^f$$

$$q_1 = (C+s_1)/(2+\beta) \text{ or } (2+\beta)q_1 = C + (\beta^2/2)q_1 \text{ from which}$$

$$q_1 = C/(2+\beta-\beta^2/2) = q_2$$

$$w^h = (1-\beta^2/2)C^2 / (2+\beta-\beta^2/2)^2$$

National firms without subsidies:

$$w^h = \pi_1 = q_1^2$$

$$q_1 = C / (2+\beta)$$

$$w^h = C^2 / (2+\beta)^2$$

Merger (notice the solution where $q_1 = q_2$ and $z = 1-z = \frac{1}{2}$)

$$w^h = \pi/2 - s_1 q_1 = (1/2) [(q_1 + \beta q_2)q_2 + [(q_2 + \beta q_1)q_1]] + (1-\beta^2)q_1^2$$

$$= (1+\beta)q_1^2 + (1-\beta^2)q_1^2 = (2+\beta-\beta^2)q_1^2 = w^f$$

$$q_1 = (C+s_1)/((2+2\beta) + [\beta-(1-\beta^2)q_1]) / [2+2\beta] \text{ or}$$

$$q_1 = C/(3+2\beta-\beta^2) = q_2$$

$$w^h = (2+\beta-\beta^2)C^2 / (3+2\beta-\beta^2)^2$$

SEC.6. BERTRAND COMPETITION , UNIT COSTS = 0, ALL EXPORTED TO A THIRD COUNTRY

The demand functions are

$$q_1 = B - p_1 + bp_2 \quad \text{and} \quad q_2 = B - p_2 + bp_1$$

NATIONAL FIRMS

The profit maximizations of national firms are

$$\max_{p_1} (p_1 + s_1)q_1 = (p_1 + s_1) (B - p_1 + bp_2)$$

$$\max_{p_2} (p_2 + s_2)q_2 = (p_2 + s_2) (B - p_2 + bp_1)$$

The first order conditions (focs) are

$$B - 2p_1 + bp_2 - s_1 = 0$$

$$B - 2p_2 + bp_1 - s_2 = 0$$

The focs can also be written as

$$q_1 = B - p_1 + bp_2 = p_1 + s_1$$

$$q_2 = B - p_2 + bp_1 = p_2 + s_2$$

From the former pair of focs

$$p_1 = [(2+b)B - 2s_1 - bs_2] / (4-b^2)$$

$$p_2 = [(2+b)B - 2s_2 - bs_1] / (4-b^2)$$

$$\partial p_1 / \partial s_1 = \partial p_2 / \partial s_2 = -2 / (4-b^2)$$

$$\partial p_2 / \partial s_1 = \partial p_1 / \partial s_2 = -b / (4-b^2)$$

and applying the latter pair of focs

$$\partial q_1 / \partial s_1 = \partial p_1 / \partial s_1 + 1 = (2-b^2)/(4-b^2) = \partial q_2 / \partial s_2$$

$$\partial q_2 / \partial s_1 = \partial p_2 / \partial s_1 = -b/(4-b^2)$$

Also from the latter pair, the profits of the national firms are

$$\pi_1 = (p_1 + s_1)q_1 = q_1^2 \text{ and } \pi_2 = q_2^2$$

From these

$$\partial \pi_1 / \partial s_1 = 2q_1(2-b^2)/(4-b^2)$$

$$\partial \pi_2 / \partial s_2 = 2q_2(2-b^2)/(4-b^2)$$

Optimal policy for h-country is obtained from the condition

$$\partial w^h / \partial s_1 = \partial \pi_1 / \partial s_1 - q_1 - s_1(\partial q_1 / \partial s_1) = 0 \text{ or}$$

$$-b^2 q_1 / (4-b^2) - s_1(2-b^2)/(4-b^2) = 0. \text{ From this,}$$

$$s_1 = -b^2 q_1 / (2-b^2) < 0. \text{ Correspondingly,}$$

$$s_2 = -b^2 q_2 / (2-b^2) < 0$$

Obviously, $p_1 = p_2$, $s_1 = s_2$, $q_1 = q_2$ and $\pi_1 = \pi_2$. Using foc

$$q_1 = p_1 + s_1 = [(2+b)B - 2s_1 - bs_2] / (4-b^2) + s_1$$

$$= (B - s_1)/(2-b) + s_1 = B/(2-b) + s_1(1-b)/(2-b) \text{ or}$$

$$(2-b)q_1 = B + s_1(1-b) = B - (1-b)b^2 q_1 / (2-b^2) \text{ or}$$

$$[(2-b)(2-b^2) + (1-b)b^2]q_1 = (2-b^2)B \text{ or}$$

$$(4 - 2b^2 - 2b + b^3 + b^2 - b^3)q_1 = (2-b^2)B \text{ or}$$

$$q_1 = (2-b^2)B / (4-2b - b^2) = q_2$$

$$\pi_1 = B^2 (2-b^2)^2 / (4-2b - b^2)^2 = \pi_2$$

MERGER

$$\max_{p_1, p_2} \pi = (p_1 + s_1) (B - p_1 + bp_2) + (p_2 + s_2) (B - p_2 + bp_1)$$

The first order conditions are:

$$B - 2p_1 + 2bp_2 - s_1 + bs_2 = 0$$

$$B + 2bp_1 - 2p_2 - s_2 + bs_1 = 0$$

or

$$q_1 = B - p_1 + bp_2 = p_1 - bp_2 + s_1 - bs_2$$

$$q_2 = B - p_2 + bp_1 = p_2 - bp_1 + s_2 - bs_1$$

From the former pair,

$$(2-2b^2)p_1 = (1+b)B - (1-b^2)s_1$$

$$(2-2b^2)p_2 = (1+b)B - (1-b^2)s_2$$

From these,

$$\partial p_1 / \partial s_1 = \partial p_2 / \partial s_2 = -1/2$$

$$\partial p_1 / \partial s_2 = \partial p_2 / \partial s_1 = 0$$

$$\partial q_1 / \partial s_1 = \partial p_1 / \partial s_1 - b \partial p_2 / \partial s_1 + 1 = 1/2 = \partial q_2 / \partial s_2$$

$$\partial q_1 / \partial s_2 = \partial p_1 / \partial s_2 - b \partial p_2 / \partial s_2 - b = -b/2 = \partial q_2 / \partial s_1$$

$$\pi = (p_1 + s_1)q_1 + (p_2 + s_2)q_2. \text{ So,}$$

$$\begin{aligned}
\partial\pi/\partial s_1 &= \\
&= (\partial p_1/\partial s_1 + 1)q_1 + (p_1 + s_1)(\partial q_1/\partial s_1) + (\partial p_2/\partial s_1)q_2 + (p_2 + s_2)\partial q_2/\partial s_1 \\
&= (-1/2 + 1)(p_1 - bp_2 + s_1 - bs_2) + (p_1 + s_1)(1/2) + 0q_2 + (p_2 + s_2)(-b/2) \\
&= p_1 + s_1 - b(p_2 + s_2) = q_1 \text{ (the last form from focs)}
\end{aligned}$$

Correspondingly,

$$\partial\pi/\partial s_2 = q_2$$

The first order condition for determining optimal policy is, for h-country,

$$\partial w^h/\partial s_1 = z(\partial\pi/\partial s_1) - q_1 - s_1(\partial q_1/\partial s_1) = 0. \text{ Inserting the derivatives,}$$

$$s_1 = -2(1-z)q_1 \quad (13a)$$

Correspondingly,

$$s_2 = -zq_2 \quad (13b)$$

THE MERGER GAME

$$s_1 = -2(1-z)q_1 \quad \text{and} \quad s_2 = -2zq_2$$

$$\partial s_1/\partial z = 2q_1(\partial z/\partial s_1) - 2(1-\beta^2)(1-z)(\partial q_1/\partial s_1) \quad \text{or}$$

$$1 = 2q_1(\partial z/\partial s_1) - (1-z) \quad \text{or}$$

$$\partial z/\partial s_1 = (2-z)/[2q_1]. \text{ From this, } \partial s_1/\partial z = 2q_1/(2-z)$$

$$\partial s_2/\partial z = -2q_2(\partial z/\partial s_2) - 2z(\partial q_2/\partial s_2) \text{ or}$$

$$1 = -2q_2(\partial z/\partial s_2) - 2z/2 \text{ or}$$

$$\partial z / \partial s_2 = (1+z) / [2q_2] \text{ from which } \partial s_2 / \partial z = -2q_2 / (1+z)$$

Because $\pi = \pi(s_1, s_2)$,

$$\begin{aligned} d\pi/dz &= (\partial\pi/\partial s_1)(\partial s_1/\partial z) + (\partial\pi/\partial s_2)(\partial s_2/\partial z) \\ &= q_1(\partial s_1/\partial z) + q_2(\partial s_2/\partial z) \end{aligned}$$

where

$$\partial s_1/\partial z = 2q_1 / (2-z) = -\partial s_1/\partial(1-z)$$

$$\partial s_2/\partial z = -2q_2 / (1+z) = -\partial s_2/\partial(1-z)$$

As before, maximizing the Nash product yields Eq. (5) in the text:

$$\frac{\pi + z(\partial\pi/\partial z)}{z\pi - \pi_1} = \frac{\pi + (1-z)(\partial\pi/\partial(1-z))}{(1-z)\pi - \pi_2} \quad (5)$$

where

$$\partial\pi/\partial z = (\partial\pi/\partial s_1)(\partial s_1/\partial z) + (\partial\pi/\partial s_2)(\partial s_2/\partial z)$$

$$\partial\pi/\partial(1-z) = -\partial\pi/\partial z$$

Inserting the derivatives we obtain Eqs. (14a) and (14b) in the text:

$$z(\partial\pi/\partial z) \quad (14a)$$

$$= 2z(q_1)^2 / (2-z) - 2z(q_2)^2 / (1+z)$$

and

$$\begin{aligned} (1-z)(\partial\pi/\partial(1-z)) &= \\ -2(1-z)(q_1)^2 / (2-z) &+ 2(1-z)(q_2)^2 / (1+z) \end{aligned} \quad (14b)$$

The outcome in the overall dynamic game is $z = 1-z = 1/2$, $p_1 = p_2$, $s_1 = s_2$, $q_1 = q_2$. Accordingly, from the foc of profit maximization

$$q_1 = (p_1 + s_1)(1-b) = [B/(2-2b) - s_1/2 + s_1](1-b) =$$

$$= B/2 - (1-b)q_1/2 \quad \text{or}$$

$$(3-b)q_1 = B$$

$$(1/2)\pi = (1/2)(p_1+s_1)q_1 + (1/2)(p_2+s_2)q_2 = (p_1+s_1)q_1$$

$$= q_1^2/(1-b) = B^2 / [(3-b)^2(1-b)]$$

SEC.7. COURNOT, HOMOGENOUS GOOD, ALL EXPORTED TO A THIRD COUNTRY. n FIRMS IN h-COUNTRY, ONE FIRM IN f-COUNTRY. UNIT COSTS = 0

NATIONAL FIRMS

h-country firm i produces q_i , $i = 1, \dots, n$, subsidy/tax = s

f-country: firm f produces q_f , subsidy/tax s^*

$$\text{Price } p = A - \sum q_i - q_f$$

Profit maximization:

$$\max_{q_i} \pi_i = (p+s)q_i = (A - \sum q_i - q_f + s)q_i \quad i = 1, \dots, n$$

$$\max_{q_f} \pi_f = (p+s^*)q_f = (A - \sum q_i - q_f + s^*)q_f$$

First degree conditions are:

$$\partial \pi_i / \partial q_i = A - \sum q_j - 2q_i - q_f + s = 0, \quad i, j = 1, \dots, n, i \neq j$$

$$\partial \pi_f / \partial q_f = A - \sum q_i - 2q_f + s^* = 0$$

From the first n equations $q_1 = q_2 = \dots = q_n$. Using this, focs become

$$A - (n-1)q_i - 2q_i - q_f + s = 0$$

$$A - nq_i - 2q_f + s^* = 0 \quad \text{or}$$

$$A - (n+1)q_i - q_f + s = 0$$

$$A - nq_i - 2q_f + s^* = 0$$

Solving:

$$q_i = [A+2s-s^*]/(n+2)$$

$$q_f = A - (n+1)q_i + s = [A-ns + ((n+1)s^*)]/(n+2)$$

$$p = A - nq_i - q_f$$

$$= \{ (n+2) A - n[A+2s-s^*] - [A-ns + ((n+1)s^*] \} / (n+2)$$

$$= (A-ns-s^*)/(n+2)$$

Welfare maximization:

$$w^h = \sum \pi_i - \sum sq_i = n\pi_i - nsq_i = n(p+s)q_i - nsq_i = npq_i$$

$$= n\{ (A-ns-s^*)/(n+2) \} \{ [A+2s-s^*]/(n+2) \}$$

$$= [n/(n+2)^2] (A-ns-s^*)(A+2s-s^*) = \Omega (A-ns-s^*)(A+2s-s^*)$$

$$\partial w^h / \partial s = \Omega [-n(A+2s-s^*) + 2(A-ns-s^*)]$$

$$= \Omega [(2-n)A - 4ns - (2-n)s^*] = 0$$

$$w^f = \pi_f - s^*q_f = pq_f$$

$$= \{ (A-ns-s^*)/(n+2) \} \{ [A-ns + ((n+1)s^*)]/(n+2) \}$$

$$= [1/(n+2)^2] (A-ns-s^*)[A-ns + ((n+1)s^*)]$$

$$\partial w^f / \partial s^* = [1/(n+2)^2] [-A+ns-(n+1)s^* + (n+1)A-(n+1)ns-(n+1)s^*]$$

$$= [1/(n+2)^2] [nA - n^2s - 2(n+1)s^*] = 0$$

The equations system becomes:

$$(2-n)A - 4ns - (2-n)s^* = 0$$

$$nA - n^2s - 2(n+1)s^* = 0$$

From these,

$$s = (2-n)(3n+2)A / (n^3+6n^2+8n)$$

$$s^* = n(6-n)A / (n^2+6n+8)$$

AN ILLUSTRATION: $n = 2$.

National firms.

Setting $n = 2$ above:

$$s = 0, s^* = 2(6-2)A/(4+12+8) = A/3$$

$$q_1 = q_2 = (A+2s-s^*)/(n+2) = 2A/12 = A/6$$

$$q_f = [A-ns + ((n+1)s^*)]/(n+2) = (A+3A/3)/4 = A/2$$

$$p = A - nq_i - q_f = A - A/3 - A/2 = (6A-2A-3A)/6 = A/6$$

$$\pi_1 = \pi_2 = pq_1 = A^2/36$$

$$\pi_f = (p+s^*)q_f = (A/6+A/3)A/2 = A^2/4$$

Merger 12 between h-country firms 1 and 2.

The results are as presented in Sec. 3 in the text:

$$\pi_{12} = (A/2.5)^2 = A^2/6.25 > A^2/18 = \pi_1 + \pi_2$$

$$\pi_f = (A/2.5)^2 = A^2/6.25$$