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IMPERFECT COMPETITION IN INPUT GOOD MARKET: A DYNAMIC ANALYSIS

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Abstract

A competitive industry uses an input good in its production. This input good is produced by constant returns to scale technology from renewable resources. An increase in input good demand leads to an increase in resources and in resource-owner's wealth but also in her indebtedness. Non-competitiveness in upward industry generates transitional dynamics, that is, it is advantageous to divide the increase in reserves over time.

Keywords: dynamics of imperfect competition, input markets, renewable resources, forestry

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1. INTRODUCTION

This paper analyses a situation where a final good industry (which I call paper industry) uses input good (which I call wood) produced by an upstream industry (which I call forestry). Wood flow production is a constant share of forest reserves. There is no depreciation of these reserves. Investment increases the effective size of reserves. In forestry there is congestion. This is presented as increasing investment cost with respect to reserves. A representative forest owner- wood producer maximizes her intertemporal utility subject to an intertemporal budget constraint. Paper industry is competitive while forestry may be competitive or non- competitive. The model applied is chosen so that special consequences of non-competitiveness can be brought forth. In a competitive situation there is no transitional dynamics. If there is an increase in wood demand the forest-owner adjusts instantaneously reserves to the desired long-run level. In contrast, if the forest-owner's supply affects wood price and she perceives it, this adjustment is gradual. For this, I will deal with two cases. In the first, the forest-owner has monopoly in wood selling while in the second I use a more general formulation.

The model is presented in Sec. 2. Basically, it may be seen as a special case of a more general situation with upward and downward industries with emphasis on the former. Analytically, my model has similarities to those presented, e.g., by Blanchard and Fischer (1989, Ch.2.4.) or Sen and Turnovsky (1991). In their models transitional dynamics is generated by investment installation costs which increase with respect to the size of investment flow or to relative size of investment to capital in competitive circumstances with decreasing returns to scale technology. For comparison, in my model there is constant returns to scale in production and transitional dynamics is generated by imperfect competition.

After presenting the model, long-run consequences are discussed in Section 3 while transitional dynamics is presented in Section 4. Based on these, indebtedness and wealth effects are analyzed in Section 5. Concluding remarks are in Section 6.

2. THE MODEL

Wood is produced in forests owned by a representative forest owner-wood producer. The effective size of these forests is denoted by R which will be called (forest) reserves. These reserves yield wood at a constant rate σ, that is, there is a continuous flow of wood $\sigma R(t)$ where t denotes time. Variable wood production costs (paid to outsiders) are assumed equal to zero. Investment, I, increases reserves and is financed by borrowing and/or from retained earnings. There is no depreciation of reserves. Installation costs of investments increase with the size of R. A simple formulation of this is to assume that the total costs of investing I is $(1+h(R))I$, $h' > 0$. The forest owner maximizes her intertemporal utility depending on her consumption c. She may take loans from a fully functioning credit market at a constant interest rate i*. Her subjective discount rate is equal to this. Her indebtedness at time t is B(t). All in all, her optimization may be presented as

$$\max \int_0^{\infty} u(c(t))e^{-i^*t}dt \qquad (1)$$

subject to

$$dB/dt = i^*B + c + (1+h(R))I - w\sigma R \qquad (2a)$$

$$dR/dt = I \qquad (2b)$$

where u denotes instantaneous utility, $u' > 0$, $u'' < 0$ and w denotes wood price. In a competitive situation wood price is exogenous. In a non-competitive situation I analyze two cases, a monopoly in selling wood and a more general formulation. For monopoly, we need demand for wood. A simple formulation is to assume that a representative final-good firm produces according to a variant of Cobb-Douglas production function:

$$F = A(\sigma R)^{\beta} \quad , 0 < \beta < 1 \qquad (3)$$

The constant A can be understood as a measure of paper production efficiency. For simplicity, we set final good's other production costs than those caused by buying wood equal to zero. In this situation the firm's profit is $pA(\sigma R)^{\beta} - w\sigma R$ where p denotes paper price. p is assumed exogenous. It may, for example, be equal to paper's world market price. Maximizing this profit with respect to wood input σR yields the inverse demand function for wood as

$$w = pA\beta(\sigma R)^{-(1-\beta)} \qquad (4)$$

The more general non- competitive situation may be presented so that wood price is a decreasing function of wood supply which is, for its part, a function of reserves, that is, $w = f(\sigma R) = w(R)$ where $w' < 0$, $w'' < 0$.

For forest owner's optimization, the Hamiltonian is:

$$H = \{u(c) - \mu\,[i^*B + c + (1+h(R))I - w\sigma R] + \mu qI\}\, e^{-i^*t} \qquad (5)$$

The control variables are c and I while B and R are state variables. The co- state variables are $-\mu(t)e^{-i^*t}$ for debt accumulation and $\mu(t)q(t)e^{-i^*t}$ for reserve accumulation. Three first degree optimum conditions are the same for non-competitive and competitive situations:

$$u'(c(t)) = \mu(t) \qquad \text{(from } dH/dc = 0\text{)} \qquad (6a)$$

$$1+h(R) = q \qquad \text{(from } dH/dI = 0\text{)} \qquad (6b)$$

$$d(-\mu e^{-i^*t})/dt = -(\partial H/\partial B) = \mu i^* e^{-i^*t} \qquad (6c)$$

The fourth condition for a non- competitive situation where forest owner has monopoly in selling wood is, by inserting Eq.(4) into (5),

$$d\,[\mu q e^{-i^*t}]\,/dt = -(\partial H/\partial R) = \mu\,[\,-\,pA\beta^2\sigma^\beta R^{\beta-1} + h'(R)I\,]\,e^{-i^*t} \qquad (6d)$$

For the more general formulation, inserting w = w(R) into (5) yields as the fourth condition the following:

$$d\,[\mu q e^{-i^*t}]\,/dt = \mu\,[\,-\,w\sigma - w'\,\sigma R + h'(R)I\,]\,e^{-i^*t} \qquad (6d')$$

In a competitive situation, the condition is:

$$d\,[\mu q e^{-i^*t}]\,/dt = \mu\,[\,-\,w\sigma + h'(R)I\,]\,e^{-i^*t} \qquad (6d'')$$

where w is exogenous. Additionally, it can be shown that transversality conditions are fulfilled in all cases.

From Eq. (6c), $d\mu/dt = 0$ or μ is constant. Because the utility function u is decreasingly increasing, we may conclude from Eq. (6a) that consumption is the same for all t, that is, there is a complete consumption smoothing. Denoting the steady state values by asterisks, noticing from Eq. (2b) that $I^* = 0$ and applying Eq. (2a) consumption may be presented as

$$c(t) = c^* = w^*\sigma R^* - i^*B^* \qquad (7)$$

Differentiating Eq. (6b) with respect to q we may see that $R_q > 0$ and from this that $I_q > 0$. Using dμ/dt = 0 and Eq. (6c) we obtain from Eq. (6d), in the monopoly case,

$$dq/dt = i^*q - pA\beta^2\sigma^\beta R^{\beta-1} + h'(R)I \qquad (8)$$

In the more general formulation of non-competitive market, from Eq. (6d'),

$$dq/dt = i^*q - w\sigma - w'\sigma R + h'(R)I \qquad (8')$$

In a competitive situation this is, by setting w' = 0,

$$dq/dt = i^*q - w\sigma + h'(R)I \qquad (8'')$$

3. LONG RUN EFFECTS

In the monopoly case, Eqs. (8) and (6b) yield in the steady state

$$i^*q^* = pA\beta^2\sigma^\beta R^{*\beta-1} = \beta w^*\sigma = i^*(1+h(R^*)) \qquad (9)$$

As may be concluded from Eq.(9) in the steady state marginal revenue from resources is equal to their marginal interest cost. For the more general formulation, from Eqs. (8') and (6b),

$$i^*q^* = w^*\sigma + w'(R^*)\sigma R^* = i^*(1+h(R^*)) \qquad (9')$$

The competitive case is straightforward because here w' = 0. From (9'),

$$i^*q^* = w\sigma = i^*(1+h(R^*)) \qquad (9'')$$

Notice that the same condition (9'') would be obtained by choosing resources so that the discounted net profits would be maximized, that is, from maximizing the following with respect to R*:

$$\int_0^\infty w\sigma R^* e^{-i^*t}\,dt \quad - \quad \int_{R_0}^{R^*} (1 + h(R))dR$$

where the last term presents the costs of the instantaneous increase in reserves by $R^* - R_0$.

In all three cases the steady state value of q, the shadow price of resources, is equal to cost of the last unit of new reserves, that is, $q^* = 1+h(R^*)$. Comparing Eq. (9'') with Eq. (9) or (9') we see that with imperfect competition optimal long-run resources are smaller than in the competitive case. Additionally, the right hand side of Eq. (9'') shows that the interest cost of the last additional unit of forest is equal to the revenue obtained from it.

In the monopoly case we obtain from Eq. (9) the effects of two potential sources of an increase in wood demand, an improvement in paper production efficiency and an increase in paper price, as

$$\partial R^*/\partial A = p\beta^2\sigma^\beta R^{*\beta-1}/[i^*h'(R^*) + pA\beta^2(1-\beta)\sigma^\beta R^{*-(2-\beta)}] > 0 \quad (10a)$$

$$\partial R^*/\partial p = A\beta^2\sigma^\beta R^{*\beta-1}/[i^*h'(R^*)+ pA\beta^2(1-\beta)\sigma^\beta R^{*-(2-\beta)}] > 0 \quad (10b)$$

Both lead to an increase in long-run reserves. Correspondingly, with the more general formulation, the effect of an autonomous upward shift in wood price, for whatever reason, is obtained by differentiating (9') as

$$\partial R^*/\partial w = \sigma / [-w''\sigma R^*- w'\sigma +i^*h'(R^*)] \quad (11)$$

which is positive because w', w'' < 0 and h' > 0. An autonomous increase in wood price leads to an increase in long-run reserves. Finally, as may be seen from (11), in the competitive case,

$$\partial R^*/\partial w = \sigma / [i^*h'(R^*)] > 0 \quad (12)$$

An increase in wood price leads to an increase in long-run reserves of forests. Comparing Eqs. (12) and (11) we may conclude that the reserves' increase is larger in the competitive situation than in the non-competitive situation.

4. SYSTEM DYNAMICS

The system may be seen as functions of two variables, R and q. It is solved in a standard way by starting from Equations (2b) and (8) or (8') or (8''). Linearizing the non-linear functions around the steady state values R* and q* yields

$$dR/dt = I_q(q^*)(q-q^*) \qquad (13a)$$

$$dq/dt = i^*q - pA\beta^2\sigma^\beta R^{*-(1-\beta)} + pA(1-\beta)\beta^2\sigma^\beta R^{*-(2-\beta)}[R-R^*] + I_q(q^*)h'(R^*)[q-q^*] \qquad (13b)$$

or

$$dq/dt = i^*q - w^*\sigma - w'(R^*)\sigma R^* - [2w'(R^*)\sigma + w''(R^*)\sigma R^*][R-R^*] + I_q(q^*)h'(R^*)[q-q^*] \qquad (13b')$$

or

$$dq/dt = i^*q - w\sigma + I_q(q^*)h'(R^*)[q-q^*] \qquad (13b'')$$

(13b) refers to the monopoly case, (13b') to the more general case and Eq. (13b'') to the competitive case. These equations may be presented in matrix form as:

$$\begin{vmatrix} dR/dt \\ dq/dt \end{vmatrix} = \begin{vmatrix} 0 & I_q(q^*) \\ E & G \end{vmatrix} \begin{vmatrix} R \\ q \end{vmatrix} + \begin{vmatrix} \text{constant} \\ \text{constant} \end{vmatrix} \quad (14)$$

where, in the monopoly situation, $E = pA(1-ß)ß^2 \sigma^ß R^{*-(2-ß)} > 0$ and $G = i^*+I_q h'(R^*) > 0$. In the more general case, $E = -[2w'(R^*)\sigma + w''(R^*)\sigma R^*] > 0$. Notice that these constants may differ between the cases. When solving these two simultaneous differential equations, one of the characteristic roots is negative, equal to $v = [G - (G^2 + 4I_qE)^{1/2}]/2$, and the other is positive and equal to $[G + (G^2+4I_qE)^{1/2}]/2$. Accordingly, we have saddle-path dynamics in the non-competitive cases. Applying Eq. (9'') Eq. (13b'') shows that in a competitive situation E = 0 which implies that the characteristic root v is zero and that there is no transitional dynamics. Reserves jump instantaneously to the steady state level.

In the non-competitive cases, the optimal path is the stable branch of the saddle-paths. Applying Eqs. (9) or (9') into Eqs. (13) or (13') a particular solution of these simultaneous differential equations turns out to be $q(t) = q^*$, $R(t) = R^*$ and so we obtain the time path of R as:

$$R(t) = R^* + (R_0 - R^*)e^{vt} \quad (15)$$

where R_0 is the initial reserves. Eq. (15) shows the gradual growth of reserves to their new steady state size caused, for example, by paper price increase. Differentiating Eq. (15) and inserting it into Eq. (14a) yields the corresponding optimal path for q, the shadow price of reserves, as

$$q(t) = q^* + (v/I_q)(R_0 - R^*)e^{vt} = q^* + (v/I_q)(R(t) - R^*) \quad (16)$$

5. DEBT AND WEALTH

In the non-competitive cases, using Eqs. (4), (6b), (7), (15) and (16) in Eq. (2a) and linearizing we obtain the following first-order differential equation for monopoly and for the more general case (see Appendix):

$$dB/dt = i^*B - i^*B^* + [q^*v - \beta p^*\sigma](R_0 - R^*)e^{vt} \qquad (17)$$

$$dB/dt = i^*B - i^*B^* + [q^*v - w^*\sigma - w'(R^*)\sigma R^*](R_0 - R^*)e^{vt} \qquad (17')$$

Solving these yields, applying Eqs. (9) or (9')

$$B(t) = B^* + \Omega[R_0 - R^*]\,e^{vt} = B^* + \Omega\,[R(t) - R^*] \qquad (18)$$

where $\Omega = q^* = 1+h(R^*) > 1$. We may see that forest owner's debt increases in the same way as the reserves increase. Differentiating Eq. (18) yields, for the monopoly case,

$$\partial B^*/\partial p = \Omega\,[\partial R^*/\partial p] \quad > 0 \qquad (19a)$$

$$\partial B^*/\partial A = \Omega\,[\partial R^*/\partial A] \quad > 0 \qquad (19b)$$

For the more general formulation and for the competitive case we obtain, for an autonomous increase in wood price,

$$\partial B^*/\partial w = \Omega\,[\partial R^*/\partial w] \qquad (19c)$$

As can be seen, in all cases the long-run indebtedness of the forest owner increases proportionately to the long-run reserves increase. For the competitive case, the adjustment is straightforward: reserves increase instantaneously to the long-run level. This is entirely financed by increased debt.

From Eq. (18) $B^* - B(0) = \Omega\,[\,R^* - R_0]$. This should not interpreted so that because $\Omega > 1$ the wealth of the forest-owner decreases in the long run as a consequence of a positive shock. Instead, we should compare debt with the market value of reserves. This value V is the present (discounted) value of sales revenue at time 0 or

$$V = \int_0^\infty w(t)\sigma R(t)e^{-i^*t}dt \qquad (20)$$

As an example consider a competitive situation where wood price increases from w_1 to w_2. This leads to an instantaneous increase in reserves from R_1^* to R_2^*. The value of reserves R_2^* at wood price w_2 is, from Eq.(20) and applying Eq. (9''),

$$V_2 = \int_0^\infty w_2\,\sigma R_2^* e^{-i^*t}dt$$

$$= w_2\,\sigma R_2^*/i^* = (1+h(R_2^*))R_2^*$$

Correspondingly, the value of reserves R_1^* at price w_1 is

$$V_1 = \int_0^{\infty} w_1 \sigma R_1^* e^{-i^*t} dt = w_1 \sigma R_1^*/i^* = (1+h(R_1^*))R_1^*$$

The cost of the instantaneous reserves increase from R_1^* to R_2^* may be shown to be

$$C = \int_{R_1^*}^{R_2^*} (1 + h(R))dR = (1+h(\xi))(R_2^* - R_1^*)$$

where $R_1^* < \xi < R_2^*$. Because there is no transitional dynamics this is equal to (instantaneous) debt increase, that is, $C = B_2^* - B_1^*$. The forest owner spends the increase in income flow, $w_2 \sigma R_2^* - w_1 \sigma R_1^*$, on interest payments an consumption flow. The increase in net wealth is $V_2 - V_1 - (B_2^* - B_1^*)$. Inserting the corresponding values we obtain the wealth increase as

$$(1+h(R_2^*))R_2^* - (1+h(R_1^*))R_1^* - (1+h(\xi))(R_2^* - R_1^*)$$

$$=[h(R_2^*) - h(\xi)] R_2^* + [h(\xi) - h(R_1^*)]R_1^*$$

This is positive because $R_1^* < \xi < R_2^*$ and $h(R)$ is an increasing function.

6. CONCLUDING REMARKS

If the forest-owner has some monopoly power in selling wood she finds it advantageous in case of an increase in wood demand to divide her investments in new resources over time. Her indebtedness follows the same pattern, that is, it increases over time. Her wood sales increase over time while their unit price decreases. Her income increases which she spends on higher consumption and higher interest payments. In contrast, if wood market is competitive, within the model I have applied an increase in exogenous wood price leads to an instantaneous increase in reserves which is financed by borrowing.

As mentioned in Introduction a standard way to introduce transitional dynamics into a Ramsey- type model in a competitive framework is to assume installation costs which increase with the size of investment flow. Here it is non-competitiveness which causes transitional dynamics. Why is it so? For illustration, let us consider a situation where investment is done as a consequence of an upward shift in exogenous wood price. Assume that here there are two alternatives only: either all new (profitable) investment is done at once (time 0) or it is divided in two parts, the first part at time 0 and the second part at time 1. The nominal investment costs are equal in both cases as was shown. Also, all investment at time 0 is financed by borrowing. From this it follows that in the second alternative there are smaller interest cost between time 0 and time 1 of that part of loan which is investment-induced. Notice, however, that because consumption jumps up at time 0 there may be borrowing to finance higher consumption between times 0 and 1 in this second alternative where the revenue flow is smaller than in the first alternative. So the effect of interest costs may be ambiguous but these costs are likely to be larger in the first alternative. What is unambiguous is that, in the second alternative, the producer must give up a part of sales revenue between time 0 and time 1 which would have been made possible by the additional production capacity created if all investment would have been made at time 0. As was shown in the competitive situation it is advantageous to do all investment at once as compared with any situation where investment is divided over time. If we introduce non-competitiveness the decisive factor is obviously sales revenue. Now in the first alternative sales revenue would grow proportionally less than sales quantity because of the price fall. This may well make the second alternative more advantageous. What was shown in my analysis is that dividing investment over time really is more advantageous.

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APPENDIX

SEC.2: TRANSVERSALITY CONDITIONS are fulfilled:

Optimality requires the following transversality conditions:

$$\lim_{t \to \infty} H = 0$$

$$\lim_{t \to \infty} -\mu e^{-i^* t} = 0$$

$$\lim_{t \to \infty} \mu q \, e^{-i^* t} = 0$$

$$H = \{ u(c) - \mu \, [\, i^*B + c + (1+h(R))I \; - pA\beta(\sigma R)^{\beta} \,] + \mu q \, I \} \, e^{-i^*t}$$
(monopoly)

$$H = \{ u(c) - \mu \, [\, i^*B + c + (1+h(R))I \; - w(R)\sigma R \,] + \mu q \, I \} \, e^{-i^*t}$$
(general)

$$H = \{ u(c) - \mu \, [\, i^*B + c + (1+h(R))I \; - w\sigma R \,] + \mu q \, I \} \, e^{-i^*t}$$
(competitive)

Because μ and c are constant and I* = 0, the conditions are fulfilled if

$$\lim_{t \to \infty} R e^{-i^* t} = 0$$

$$\lim_{t \to \infty} B \, e^{-i^* t} = 0$$

$$\lim_{t \to \infty} q \, e^{-i^* t} = 0$$

Because R*, B* and q* are finite the conditions are fulfilled.

SEC. 3:

In the steady state of the general case

$$i^*q^* = - w^*\sigma - w'(R^*)\sigma R^* = i^*(1+h(R^*))$$

Differentiating this,

$$- \sigma - (\partial w'/\partial R^*)(\partial R^*/\partial w)\sigma R^* - w'(R^*)\sigma(\partial R^*/\partial w)$$

$$= i^*(\partial h(R^*)/\partial R^*)(\partial R^*/\partial w)$$ from which

$$\partial R^*/\partial w = \sigma / [-w''\sigma R^* - w'\sigma + i^*h'(R^*)]$$

SEC. 4:

From Eq. (6c)

$$d(-\mu e^{-i^*t})/dt = [d(-\mu)/dt]\, e^{-i^*t} + \mu i^* e^{i^*t} = - (\partial H/\partial B) = \mu i^* e^{i^*t}$$

From this, $d\mu/dt = 0$. The fourth condition is:

$$d\,[\mu q e^{-i^*t}]\,/dt = \mu e^{-i^*t}(dq/dt) + q\, d(\mu e^{-i^*t})/dt$$

$$= \mu e^{-i^*t}(dq/dt) - q i^* \mu e^{-i^*t} = - (\partial H/\partial R)$$ where

$$-(\partial H/\partial R) = \mu\, [\, - pA\beta^2\sigma^{\beta} R^{\beta-1} + h'(R)I\,]e^{-i^*t}$$ if monopoly (Eq.(6d)

$$-(\partial H/\partial R) = \mu\, [- w\sigma - w'\sigma R + h'(R)I]\, e^{-i^*t}$$ if general, $w = w(R)$ (6d')

$$-(\partial H/\partial R) = \mu\, [- w\sigma + h'(R)]e^{-i^*t}$$ if competitive (6d'')

Dividing by μe^{-i^*t} yields

$dq/dt = i^*q - pA\beta^2\sigma^\beta R^{\beta-1} + h'(R)I$ if monopoly (Eq. (8))

$dq/dt = i^*q - w\sigma - w'\sigma R + h'(R)I$ if general (Eq. (8'))

$dq/dt = i^*q - w\sigma R + h'(R)I$ if competitive (Eq. (8''))

Linearizing these,

Monopoly: $dq/dt = i^*q - pA\beta^2\sigma^\beta R^{*-(1-\beta)}$

$+ pA(1-\beta)\beta^2 \sigma^\beta R^{*-(2-\beta)} [R-R^*] + I^* h'(R^*) + I_q(q^*)h'(R^*) [q-q^*]$

$+ I^* h''(R^*)[R-R^*]$

General: $dq/dt = i^*q - w^*\sigma - w'(R^*)\sigma R^*$

$- [2w'(R^*)\sigma + w''(R^*)\sigma R^*][R-R^*]$

$+ I_q(q^*)h'(R^*) [q-q^*]$

$+ I^* h'(R^*) + I_q(q^*)h'(R^*) [q-q^*]$

$+ I^* h''(R^*)[R-R^*]$

Competitive: $dq/dt = i^*q - w^*\sigma + I_q(q^*)h'(R^*) [q-q^*]$

$+ I^* h'(R^*) + I_q(q^*)h'(R^*) [q-q^*]$

$+ I^* h''(R^*)[R-R^*]$

Because $I^* = 0$, these are as presented in text

Linearizing Eq. (2b) yields

$$dR/dt = I^* + I_q(q-q^*) = I_q(q^*)(q-q^*)$$

SEC.5: FOREIGN DEBT

In the monopoly case:

$dB/dt = c + i^*B - w\sigma R + I \cdot (1+h(R))$ (use Eqs. (6b) and (7))

$= w^*\sigma R^* - i^*B^* + i^*B - w\sigma R + I \cdot (1+h(R))$ (use Eq.(4))

$= i^*B - i^*B^* + pAß(\sigma R^*)^{ß} - pAß(\sigma R)^{ß} + qI$ (linearize)

$= i^*B - i^*B^* + pAß(\sigma R^*)^{ß} - pß(\sigma R^*)^{ß} - pAß^{2} \sigma^{ß} R^{*ß-1} (R(t) - R^*)$

$+ q^* I_q (q-q^*)$ (use Eq. (15) and Eq. (16))

$= i^*B - i^*B^* - pAß^{2} \sigma^{ß} R^{*ß-1} (R_0 - R^*)e^{vt} + q^*v (R_0 - R^*)e^{vt}$

$= i^*B - i^*B^* - ßw^*\sigma(R_0 - R^*)e^{vt} + q^*v (R_0 - R^*)e^{vt}$

Solving this first-degree differential equation yields

$B(t) = Me^{i^*t} + e^{i^*t} \int [- i^*B^*] e^{-i^*t} dt$

$+ e^{i^*t} \int [[- ßw^*\sigma + q^*v][R_0 - R^*] e^{-i^*t + vt} dt$

$= Me^{i^*t} + B^* + [- ßw^*\sigma + q^*v][1/(v - i^*)] [R_0 - R^*] e^{vt}$

$= Me^{i^*t} + B^* + \Omega [R_0 - R^*] e^{vt}$

where $\Omega = [- ßw^*\sigma + q^*v][[1/(v - i^*)]$. From Eq. (9), $\Omega = q^* = (1+h(R^*)$. Setting t = 0 yields

$M = B(0) - B^* \;-\; \Omega\,[R_0 - R^*]$

Setting M into the equation above and dividing by e^{i^*t} yields

$$B(t)e^{-i^*t} = B(0) - B^* - \Omega\,[R_0 - R^*]$$

$$+\; B^*\, e^{-i^*t} + \Omega\,[R_0 - R^*]\, e^{vt-i^*t}$$

Letting t approach infinity and applying the transversality conditions yields

$$0 = B(0) - B^* - \Omega\,[R_0 - R^*] = M$$

Inserting this into the equation above yields

$$B(t) = B^* + \Omega\,[R_0 - R^*]\, e^{v\,t}$$

From this,

$$B(0) = B^* + \Omega\,[R_0 - R^*] \text{ and so}$$

$$\partial B^*/\partial A = \Omega\,[\partial R^*/\partial A] > 0 \text{ etc.}$$

<u>In the general case:</u>

$dB/dt = c + i^*B - w\sigma R + I \cdot (1+h(R))$ (use Eq. (7))

$= w^*\sigma R^* - i^*B^* + i^*B - w\sigma R + I \cdot (1+h(R))$ (use Eq.(6b))

$= i^*B - i^*B^* + w^*\sigma R^* - w\sigma R + qI$ (linearize)

$= i^*B - i^*B^* + w^*\sigma R^* - w^*\sigma R^* - [w^*\sigma + w'(R^*)\sigma R^*]\,(R(t) - R^*)$

$+\; q^*\, I_q\,(q-q^*)$ (use Eq. (15) and Eq. (16))

$= i^*B - i^*B^* - [w^*\sigma + w'(R^*)\sigma R^*](R_0 - R^*)e^{vt} + q^*v (R_0 - R^*)e^{vt}$

$= i^*B - i^*B^* - i^*q^*(R_0 - R^*)e^{vt} + q^*v (R_0 - R^*)e^{vt}$ (using (9'))

Solving this first-degree differential equation yields

$B(t) = Me^{i^*t} + e^{i^*t} \int [- i^*B^*] e^{-i^*t} dt$

$+ e^{i^*t} \int [[- i^*q^* + q^*v][R_0 - R^*] e^{-i^*t + vt} dt$

$= Me^{i^*t} + B^* + [- i^*q^* + q^*v][1/(v - i^*)] [R_0 - R^*] e^{vt}$

$= Me^{i^*t} + B^* + q^* [R_0 - R^*] e^{vt}$ (use (9'))

$= Me^{i^*t} + B^* + (1+h(R^*)) [R_0 - R^*] e^{vt}$

The rest is as with monopoly.

OVERALL INVESTMENT COSTS:

Costs of investing $R^* - R$ are

$(R_s - R_{s-1})(1+h_s) + (R_{s-1} - R_{s-2})(1+h_{s-1}) + \ldots..$

Letting the division approach zero this is equal to

$$\int_{R_0}^{R^*} (1 + h(R))dR$$

Which is equal to $(1+h(\xi))(R^*-R_0)$ by intermediate theorem where $R_0 < \xi < R^*$.