Modelling forest stock effects of forest investments in Finland 1960-2004

Mikael Linden

ISBN 952-458-823-4
ISSN 1795-7885
no 35

MODELLING FOREST STOCK EFFECTS OF FOREST INVESTMENTS IN FINLAND 1960-2004 *)

Mikael Linden **)
Department of Business and Economics
University of Joensuu

MAY 2006

Abstract

The national forest growth process and forest policy experienced in Finland in past 50 years are analyzed in terms of modern dynamic investment theory. Optimal forest investments and forest stock are derived in dynamic optimization framework. The private investments are subsidized by the government to stimulate forest growth. The optimal level of investments and growth effects depend on harvesting rate, on marginal productivity of forest stock, and on marginal benefits of investments. Under reasonable conditions government investment aid induces forest growth and supports less rigid adjustment path to higher optimal level of forest stock than without aid. Some regression results with Finnish regional data promote considered positive investment effects on forest stock.

Keywords: Forest policy in Finland, forest stock volume, forest investments, public aid, dynamic analysis

---

*) Paper presented at Scandinavian Society of Forest Economics (SSFE) conference in Uppsala, 8 -11 May 2006.

**) Author's address: Department of Business and Economics, University of Joensuu, Yliopistokatu 7, PO Box 111, FIN-80101. E-mail: mika.linden@joensuu.fi

I. Introduction

In Finland, the framework of public intervention in non-industrial private forestry was created during the 20th century, initially with legislation, and since 1928 with extension and funding for selected forestry activities. In the 1950s, cuttings were exceeding annual growth, and the sustainable cutting budget was of increasing concern to public decision makers. A major change in forestry policy took place during the 1960s with increased public intervention in forest management and financing. This was carried out via several large-scale forestry programmes and additional budget expenditures. The aim was to increase the long-term cutting potential of the forests (Uusitalo 1978, Palosuo 1979). The target of the new forest policy was to increase forest investments and, consequently, growing forest stock, and commercial fellings. Increasing investments into forestry were also seen as a growth factor for Finnish economy (Juurola et al. 1999).

Figure 1 shows the volume of Finnish growing forest stock based on extrapolation of Nation Forest Inventories (NFI) during years 1955 – 2004. The stock was not increasing before year 1972 but after it a steady growth process has taken place. The underlying process behind the Figure 1 is also seen in Figure 2 where forest stock increment, drain and removals (in million m^3) in Finland 1955-2004 are shown. After the beginning of 1970s the stock increment has been in every year larger than total drain, i.e. natural drain added with cuttings.

Forestry intensification was achieved especially in Northern Finland by increasing the share of clear cuttings in final fellings in 1950s and 1960s. This led to an expansion of artificial regeneration and consequent need for tending seedling and young stands. In addition, many peat lands were drained, fertilization was increased and a dense network of forest roads was built. All these (silvicultural) investment measures were made feasible by directions and substantial financial assistance from government. The change in forest policy was implemented by the Forest Financing Programmes (MERA) during the period 1965-1975, and these forestry programmes had successors well into the 1980s. In all these programmes the basic target was increase the extension of wood production in

Figure1. Forest stock volume (in million m^3) in Finland 1955 - 2004

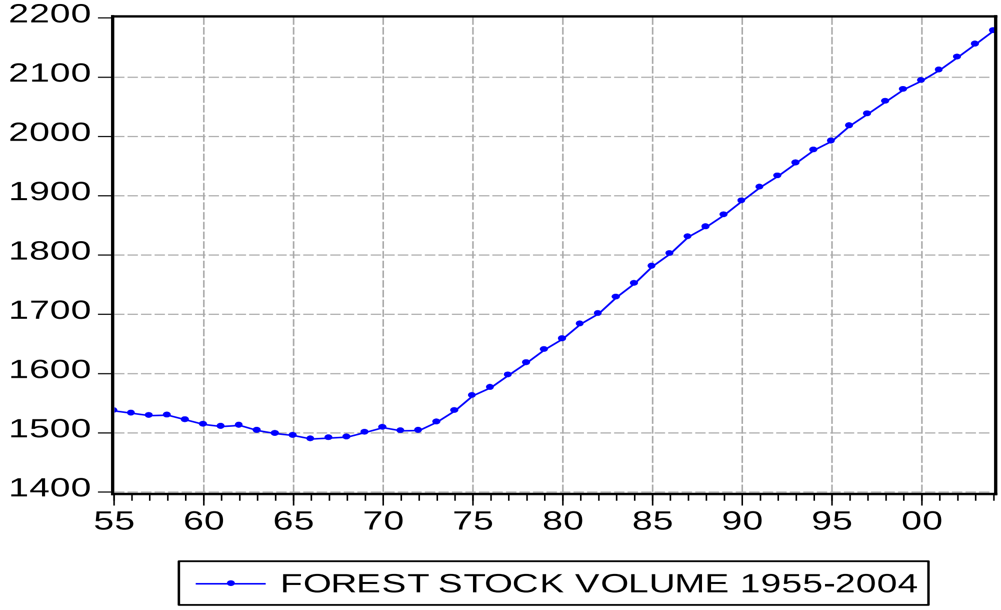
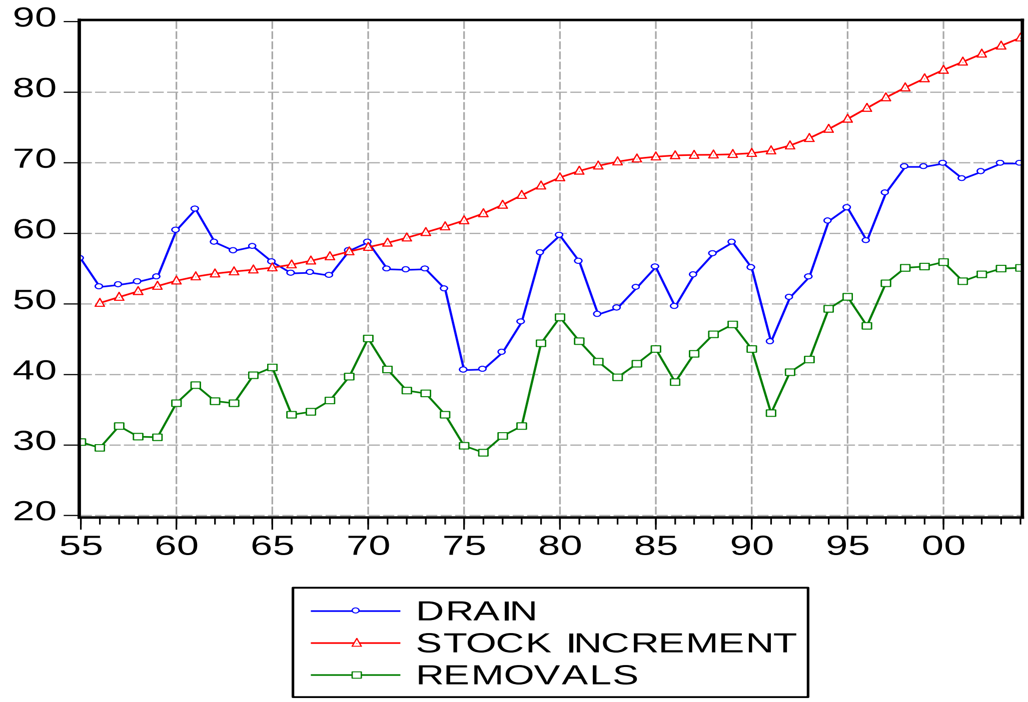


Figure 2. Forest stock increment, drain and removals (in million m^3) in Finland 1955-2004



both measures of areas and wood volume per hectare. The forest investment outlays and their effects can be measured in many ways. Figure 3 shows the affected hectares by different types of investments (drainage, fertilization, regeneration, and tending). For all cases a major increase took place during the years of 1965 -1978 starting with drainage and artificial regeneration.

Figure 3. Forest investments (in 1000 hectares) in Finland 1958-2004

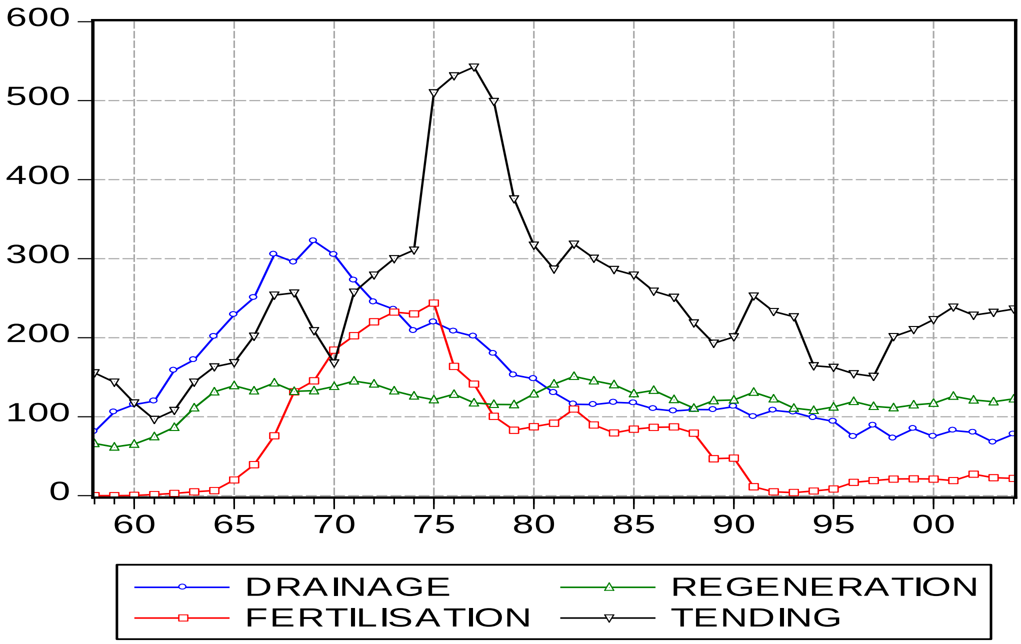
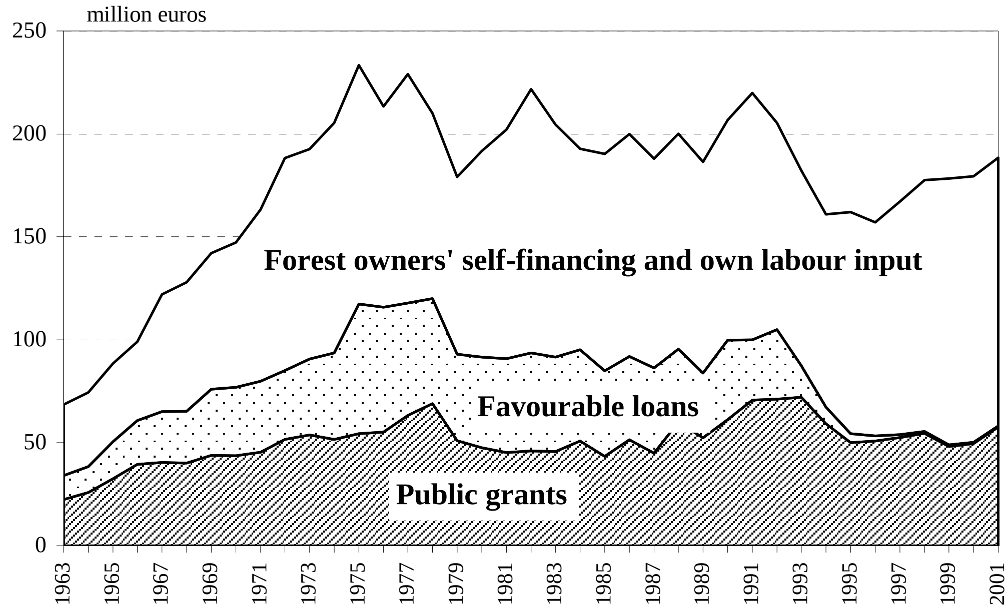


Figure 4 summarizes the forest investments in monetary terms. In years 1963 – 1978 both private and government aid to forest investments increased in real terms to maximum levels.

The question of forest investment effect on growing forest stock is seldom *directly* analyzed theoretically or empirically in forest economic papers. Typically the stock growth analysis is conducted with forest production function or with rotation models

Figure 4. Real forest investments (in million euros, 2001 prices) in Finland 1963-2001



without any explicit investment function (e.g. Chang 1981, Nautiyal & Couto 1983, Williams & Nautiyal 1990). Dynamic analyses are few. Lyon & Sedjo (1983) developed an economic maximizing optimal control model of long term timber supply potential. However the main focus in their paper is in optimal harvest of different types of forest. Vehkamäki (1986) derives a target forest stock for government's forest policy in aggregate neo-classical growth and consumption model with biological stock production function. Optimal conditions of allocation of capital (investments) between forest stock and non-forest capital in gross national product depending linearly on the supply of raw wood are derived. However he uses linear target investment functions without any adjustment cost process. A more general approach that uses neo-classical investment theory directly added with biological production function for forest stocks are few (Anderson 1976, Omwani 1988). Note however that any papers connecting renewable resources in general and investments are also relevant in his context (e.g. Clarke et al. 1979, Clarke 1990, Nyarko & Olson 1991, Jorgenson & Kort 1997).

Empirical papers relating factors to timber harvesting and forest investments are many but only few connect forest stocks and investment to each other. In their review paper on forest investments and harvesting Beach et al. (2005) do not document any papers on forest stock effects of investments. Some papers analyze effects of plot and resource conditions on investments (see Beach et al. 2005, Linden & Leppänen 2006 and 2003b). The results confirm the positive relation between stock measures and investments but the proposed causality runs from stock variables to investments, not vice versa which is the focus of this study.

The lack of interest in dynamic investment models of renewable resources like forest can be understood easily. In theory renewable resources are self-sustaining processes where investments to keep (forest) capital level at desired level are not necessary needed. As long as harvesting is sustainable investments are redundant. Investments are only considered if a higher level of forest capital must be obtained. This was the relevant case in Finland in mid 1960s when the extensive forest investment policy started. On the empirical side lack of detailed measurements of aggregate forest stock and very slowly maturing forest investment effects have hindered the empirical analyses.

Next we build an optimal control model of forest investments targeted to maximize the value of forest stock. Model incorporates together convex adjustment cost of investments, stock dynamics depending on stock level, investments and parametric harvesting rate, and government investment subsidy. It is shown that optimal control levels of forest investments and stock exist and they can be obtained with investment subsidies speeding up the stiff adjustment process. Some forest growth regression results with aggregate data from Finnish forestry board districts in years 1965-2003 are also presented to complete the analysis.

II. Optimal investments and forest stock

The theoretical standpoint for deriving optimal forest investments is based on the neo-classical capital theory (see Precious 1987, Heijdra & van der Ploeg 2002, Ch. 4).

Assume that net value of forest capital of the representative private forest owner is given by what is left over of stock value after the investment outlays have been paid

(1) $$V(t) = P_1(t)S(t) - P_2(t)[1 - s_I(t)]\Phi(I(t))$$

where $V(t)$ is net value of the forest in period t,

$S(t)$ is the forest stock in m^3,

$P_1(t)$ is the (stumpage) price of forest unit,

$P_2(t)$ is the price of investment goods unit,

s_I is the government investment subsidy, and

$\Phi(I(t))$ is the stock adjustment cost function, with $\Phi' > 0$ and $\Phi'' > 0$.

The real net value is obtained with dividing Eq. 1) by $P_1(t)$

$$v(t) = S(t) - p_2(t)[1 - s_I(t)]\Phi(I(t))$$

where $p_2(t) = P_2(t)/P_1(t)$ is the relative price of investment goods.

Assume that $p_2(t)$ is constant over time (i.e. we can assume that $p_2(t) = 1$).

The forest stock accumulation is given by

(2) $$\dot{S}(t) = F(S(t)) + I(t) - hS(t),$$

where $F(S(t))$ is the "forest" production function with $F'(S(t)) > 0$, i.e. forest stock effects on growth of forest, and h is the constant share of stock harvested every period .

Under these assumptions the net present value of forest is

(3) $$v(0)=\int_0^{\infty} v(t)e^{-rt}dt=\int_0^{\infty} e^{-rt}[S(t)-(1-s_I(t))\Phi(I(t))]dt.$$

The forest owner maximizes the value of forest (3) under the restriction (2). The current value Hamiltonian can be written as

(4) $$H(t)=e^{-rt}[S(t)-(1-s_I(t))\Phi(I((t))+q(t)[F(S(t))+I(t)-hS(t)]].$$

$q(t)$ is the Lagrange multiplier for forest stock accumulation restriction, e.g. the shadow price of existing forest stock. It measures how much the value of forest stock would change ($dv(t)$) if initial forest capital stock were slightly increased ($dS(t)$), that is $q(t))=dv(t)/dS(t)$.

The first order conditions of optimization of Eq. 4) are

5a) $$\frac{\partial H(t)}{\partial I(t)}=e^{-rt}[q(t)-(1-s_I(t))\Phi'(I(t))]=0$$

5b) $$-\frac{\partial H(t)}{\partial S(t)}=e^{-rt}[\dot{q}(t)-rq(t)]=-e^{-rt}[1+q(t)(F'(S(t))-h)].$$

Eq. 5a) implies investment function (see Appendix I) like

6) $$q(t)=(1-s_I(t))\Phi'(I(t)) \quad \Rightarrow$$
$$I(t)=I(q(t),s_I(t)), \quad \text{with } I_q>0 \text{ and } I_{s_I}>0.$$

The interpretation of optimality condition 5a) for investment is simple: the shadow price of forest stock, i.e. the marginal benefit of investment $q(t)$, is equal to marginal cost of investment $(1-s_I(t))\Phi'(I(t))$. Lower is the marginal cost due the high investment subsidy less the marginal benefit of investment is allowed (i.e. higher is the level of investments).

Eq. 5b) gives the intertemporal efficiency condition. It implies that

$$\dot{q}(t) = q(t)[r - F'(S(t) + h] - 1$$

or

$$\frac{\dot{q}(t)}{q(t)} = [r - F'(S(t))] - \frac{1}{q(t)} + h\ .$$

The shadow capital gain rate $\dot{q}(t)/q(t)$ is increasing when (subjective) interest rate, harvesting rate, and level of shadow price are large but stock yield, $F'(S)$, is small. If the forest owner is impatient the shadow value of his (lost) forest capital is increasing. Now the shadow or the opportunity value of forest investment is high. If the forest yield must equal the market rate of return on other (financial) assets, $r - F'(S(t)) = 0$, then capital gain rate is still positive when harvesting rate is high $\dot{q}(t)/q(t) = h - 1/q(t) > 0$ (i.e. the foregone gains of investments are high as they are harvested away).

For constant shadow value $\frac{\dot{q}(t)}{q(t)} = 0$: $F'(S(t)) = r + h - \frac{1}{q(t)}$. The forest yield is larger than interest rate but the small shadow value (gains from forest stock investments) depresses it. Thus marginal gain of investments is *positively* related to stock yield: forest marginal productivity (i.e. forest yield) has to cover both interest and harvesting rates *minus* the inverse of shadow value of forest capital. Note, if $r = 1/q(t)$ then $F'(S(t)) = h$, corresponding to *MSY* harvesting rule.

Finally, $\frac{\dot{q}(t)}{q(t)} = [r - F'(S(t))] - \frac{1}{q(t)} + h < 0$, can happen for a positive, albeit low, shadow value of forest capital and high forest yield. Thus if forest yield is already high (i.e. young forest) then the capital gain rate can decrease in time. Investments are less useful in this case.

Differential system of our model consists of

7) $$\begin{cases} \dot{S} = F(S) + I(q, s_I) - hS \\ \dot{q} = q[r - (F'(S) - h)] - 1 \end{cases}$$

State phase results depend on properties of

8) $$d\dot{S} = (F'(S) - h)dS + I_q dq + I_{s_I} ds_I - Sdh$$

9) $$d\dot{q} = dq[r - (F'(S) - h)] + qdr - qF''(S)dS + qdh \, .$$

The slopes of $\dot{S} = 0$ and $\dot{q} = 0$ curves in (q, S)-space are determined by

$$(\frac{dq}{dS})_{|\dot{S}=0} = -\frac{F'(S) - h}{I_q} \quad (F'(S) > 0,\ I_q > 0)$$

$$(\frac{dq}{dS})_{|\dot{q}=0} = \frac{qF''(S)}{[r - (F'(S) - h)]} \quad (F''(S) < 0)$$

leading to three cases of variable forest stock yield with given harvesting rate and interest rate:

	I	**II**	**III**
	$F' - h > 0,\ r - (F' - h) > 0$	$F' - h > 0,\ r - (F' - h) < 0$	$F' - h < 0,\ r - (F' - h) > 0$
$(\frac{dq}{dS})_{\lvert\dot{S}=0}$	$-$	$-$	$+$
$(\frac{dq}{dS})_{\lvert\dot{q}=0}$	$-$	$+$	$-$

Case I: $F'(S)-h>0,\ r-(F'(S)-h)>0$ corresponds to high forest yield compared to harvesting rate. This is called *an almost mature forest* case, where

$$(\frac{dq}{dS})_{|\dot{S}=0} = -\frac{F'(S)-h}{I_q} < 0,$$

$$\frac{\partial \dot{S}}{\partial S} = F'(S)-h > 0 \quad \text{and} \quad \dot{S}=0: \quad \text{S} = \frac{1}{h}[F(S)+I(q,s_I)],$$

$$(\frac{dq}{dh})_{|\dot{S}=0} = \frac{S}{I_q} > 0, \quad \text{and} \quad (\frac{dS}{dh})_{|\dot{S}=0} = \frac{S}{F'(S)-h} > 0.$$

$\dot{S}=0$ curve is decreasing in (q,S)-space and $\dot{S}>0$ with larger S. Increased harvesting rate shifts $\dot{S}=0$ curve outwards since larger harvest does not mean less forest stock (in long run) since stock yield is bigger than harvesting rate. Marginal investments are now more valuable than earlier $(\frac{dq}{dh})_{|\dot{S}=0} > 0$.

$$(\frac{dq}{dS})_{|\dot{q}=0} = \frac{qF''(S)}{[r-(F'(S)-h)]} < 0,$$

$$\frac{\partial \dot{q}}{\partial q} = [r-(F'(S)-h)] > 0, \quad \text{and} \quad \dot{q}=0: \quad q = \frac{1}{[r-(F'(S)-h)]} > 0,$$

$$(\frac{dq}{dr})_{|\dot{q}=0} = -\frac{q}{[r-(F'(S)-h)]} < 0, \quad \text{and} \quad (\frac{dq}{dh})_{|\dot{q}=0} = -\frac{q}{[r-(F'(S)-h)]} < 0.$$

$\dot{q}=0$ curve is decreasing in (q,S)-space and $\dot{q}>0$ with larger q. Increasing interest rate and harvesting rate shift $\dot{q}=0$ curve inwards as future gains investment reduce.

As both $\dot{S}=0$ and $\dot{q}=0$ curves are decreasing in (q,S)-space their relative steepness can be solved with following arguments:

$$(\frac{dq}{dS})_{|\dot{S}=0} = -\frac{F'(S)-h}{I_q} \to 0^-, \quad \text{when } [F'(S)-h]^+ \to 0, \quad \text{and}$$

$$(\frac{dq}{dS})_{|\dot{q}=0} = \frac{qF''(S)}{[r-(F'(S)-h)]} \to -\infty, \quad \text{when } [r-(F'(S)-h)]^+ \to 0.$$

Thus a point $S^* = S$ (or alternatively $h^* = h$) with $F'(S) - h = 0$ exists where

$$(\frac{dq}{dS})_{|\dot{S}=0} = 0 \quad \text{but} \quad (\frac{dq}{dS})_{|\dot{q}=0} = \frac{qF''(S)}{r} < 0$$

showing that $\dot{S} = 0$ is less steeper than $\dot{q} = 0$ (see Figure 5, next page).

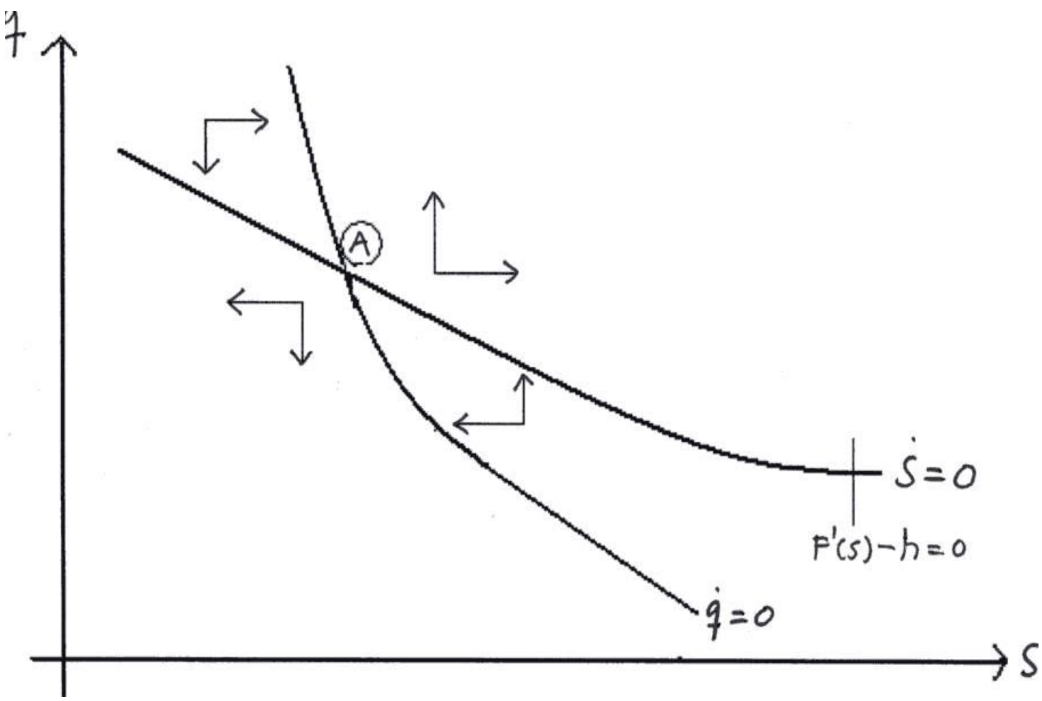
Case II: $F'(S) - h > 0,\ r - (F'(S) - h) < 0$ corresponds to case where forest yield is very high compared to given harvesting rate. This called an *young forest* case, where (like in Case I)

$$(\frac{dq}{dS})_{|\dot{S}=0} = -\frac{F'(S)-h}{I_q} < 0,$$

$$\frac{\partial \dot{S}}{\partial S} = F'(S) - h > 0 \quad \text{and} \quad \dot{S} = 0: \quad \mathrm{S} = \frac{1}{h}[F(S) + I(q, s_I)],$$

$$(\frac{dq}{dh})_{|\dot{S}=0} = \frac{S}{I_q} > 0, \quad \text{and} \quad (\frac{dS}{dh})_{|\dot{S}=0} = \frac{S}{F'(s) - h} > 0,$$

FIGURE 5. Saddle point stable *almost mature forest* stock and investment equilibrium (A).



but

$$(\frac{dq}{dS})_{|\dot{q}=0} = \frac{qF''(S)}{[r-(F'(S)-h)]} > 0\,,$$

$$\frac{\partial \dot{q}}{\partial q} = [r-(F'(S)-h)] < 0\,, \quad \text{and} \qquad \dot{q}=0: \quad q = \frac{1}{[r-(F'(S)-h)]} < 0,$$

$$(\frac{dq}{dr})_{|\dot{q}=0} = -\frac{q}{[r-(F'(S)-h)]} > 0\,,$$

$$(\frac{dq}{dh})_{|\dot{q}=0} = -\frac{q}{[r-(F'(S)-h)]} > 0\,.$$

Now $\dot{q}=0$ curve is increasing in (q,S)-space and $\dot{q}<0$ with larger but *negative* q. Thus investment gains for forest stock are harmful. Waste of capital takes place. The case is not relevant. Actually $\dot{S}=0$ and $\dot{q}=0$ curves do not intersect in positive (q,S) quadrant.

Case III: $F'(S)-h<0,\ r-(F'(S)-h)>0$ corresponds to case with low forest yield compared to given harvesting rate. This called a *mature forest* case, where

$$\left(\frac{dq}{dS}\right)_{|\dot{S}=0} = -\frac{F'(S)-h}{I_q} > 0\,,$$

$$\frac{\partial \dot{S}}{\partial S} = F'(S)-h<0 \qquad \text{and} \qquad \dot{S}=0: \quad \mathrm{S}=\frac{1}{h}[F(S)+I(q,s_I)]\,,$$

$$\left(\frac{dq}{dh}\right)_{|\dot{S}=0} = \frac{S}{I_q} > 0\,, \quad \text{and} \quad \left(\frac{dS}{dh}\right)_{|\dot{S}=0} = \frac{S}{F'(S)-h} < 0\,.$$

$\dot{S}=0$ curve is increasing in (q,S)-space but $\dot{S}<0$ with larger S due the low stock yield effects. Increased harvesting rate shifts $\dot{S}=0$ curve inwards since larger harvest means less forest stock, $\left(\frac{dS}{dh}\right)_{|\dot{S}=0}<0$, making the marginal investments more valuable than earlier, $\left(\frac{dq}{dh}\right)_{|\dot{S}=0}>0$.

$$\left(\frac{dq}{dS}\right)_{|\dot{q}=0} = \frac{qF''(S)}{[r-(F'(S)-h)]} < 0\,,$$

$$\frac{\partial \dot{q}}{\partial q} = [r-(F'(S)-h)]>0\,, \qquad \text{and} \qquad \dot{q}=0: \quad q=\frac{1}{[r-(F'(S)-h)]} > 0,$$

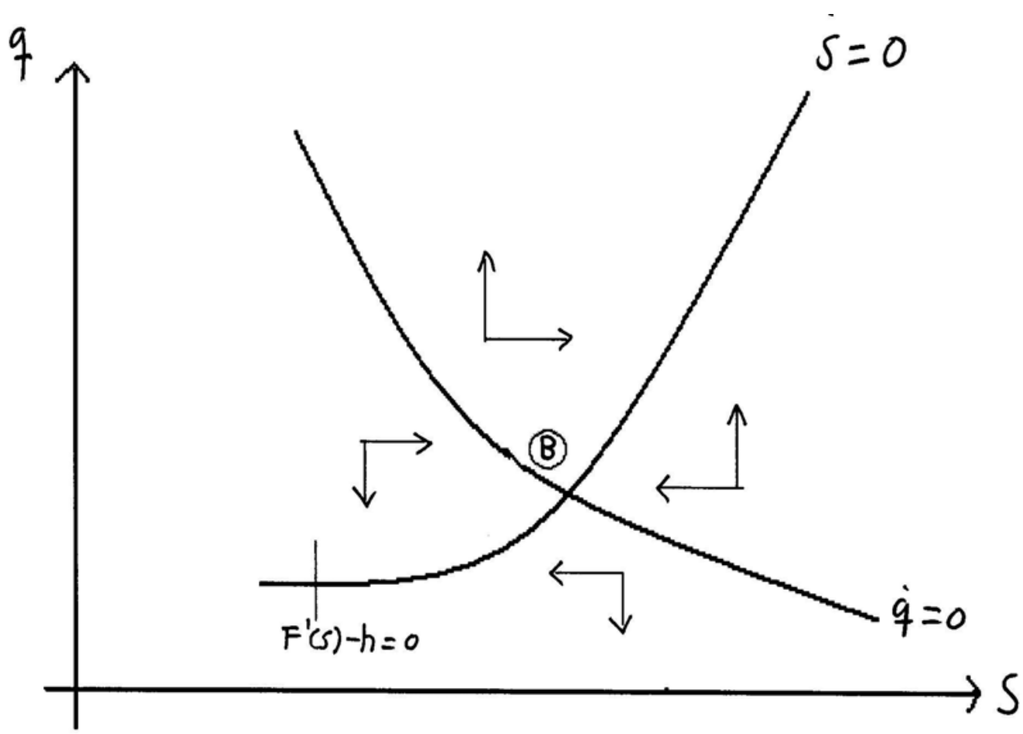
$$(\frac{dq}{dr})_{|\dot{q}=0} = -\frac{q}{[r-(F'(S)-h)]} < 0, \quad \text{and} \quad (\frac{dq}{dh})_{|\dot{q}=0} = -\frac{q}{[r-(F'(S)-h)]} < 0.$$

$\dot{q}=0$ curve is decreasing in (q,S)-space and $\dot{q}>0$ with larger q. Increasing interest rate and harvesting rate shift $\dot{q}=0$ curve inwards as future gains investment reduce.

As $\dot{S}=0$ curve increases and $\dot{q}=0$ curve decreases in (q,S)-space (see Figure 6) we notice that :

$$(\frac{dq}{dS})_{|\dot{S}=0} = -\frac{F'(S)-h}{I_q} \to 0^+, \text{ when } [F'(S)-h]^- \to 0 \quad \text{and}$$

FIGURE 6. Saddle point stable *mature forest* stock and investment equilibrium (B).



$$(\frac{dq}{dS})_{|\dot{q}=0} = \frac{qF''(S)}{[r-(F'(S)-h)]} \to \infty, \text{ when } [r - (F'(S) - h)]^{+} \to 0.$$

Thus for point $S^* = S$ (or $h^* = h$) with $F'(S) - h \le 0$ we have

$$(\frac{dq}{dS})_{|\dot{S}=0} \ge 0 \quad \text{but} \quad (\frac{dq}{dS})_{|\dot{q}=0} = \frac{qF''(S)}{r-(F'(S)-h))} < 0.$$

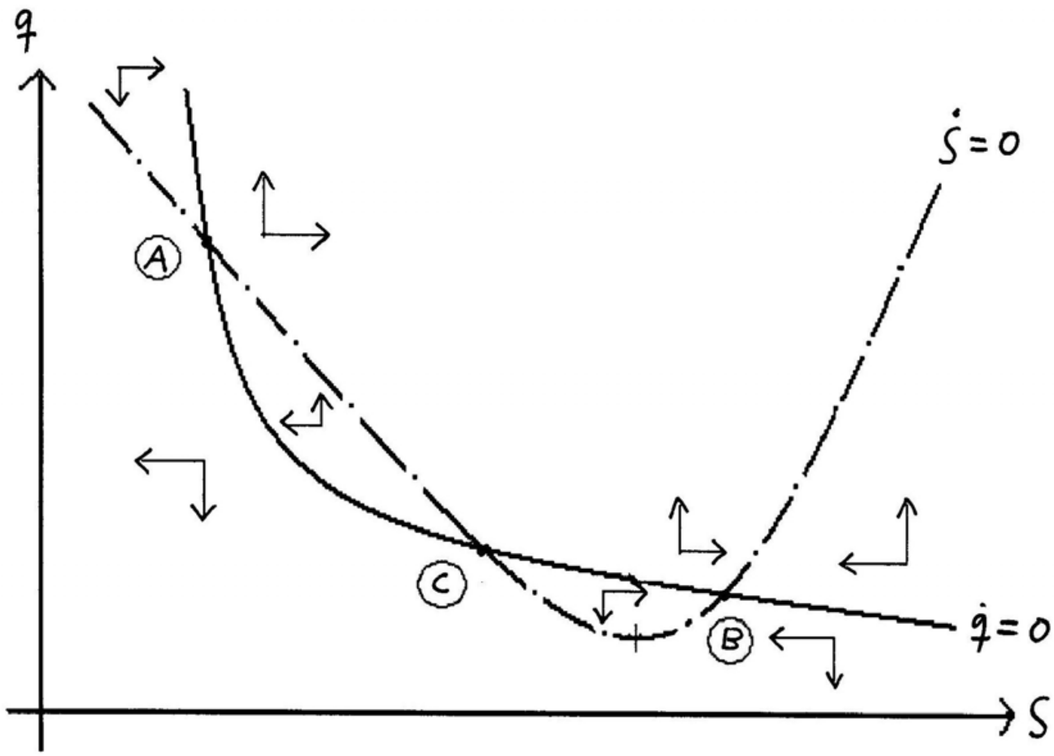
The analysis so far implies that cases I and III are relevant. The $\dot{S} = 0$ curve is U-shaped in (q, S) space, but $\dot{q} = 0$ curve is decreasing having slope of $-\infty$ when $(\frac{dq}{dS})_{|\dot{S}=0} < 0$ and $r-(F'(S)-h) \approx 0$. For big values of S $\dot{S} = 0$ curve is above $\dot{q} = 0$ curve with positive slope, $(\frac{dq}{dS})_{|\dot{S}=0} > 0$. The condition $F'(S) - h = 0$ divides the (q, S)-space in two sectors where dynamics are quite different. However in both sectors we have saddle point stability cases, i.e. cases I and III support equilibrium points, (A) and (B) for which optimal investment and forest stock level exist.

However the information concerning the case I can not rule out multiple intersection points in (q, S) space. $\dot{S} = 0$ and $\dot{q} = 0$ curves intersect three times if the slope of $\dot{S} = 0$ curve decreases steeper than slope of $\dot{q} = 0$ for some part in (q, S) space when $F'(S) - h > 0$. A condition for $|(\frac{dq}{dS})_{|\dot{S}=0}| \quad > \quad |\frac{dq}{dS})_{|\dot{q}=0}|$ is

$$\frac{F'(S)-h}{I_q} > \frac{q|F''(S)|}{[r-(F'(S)-h)]}.$$

This takes easily place if $I_q = \frac{1}{(1-s_I)\Phi''} \approx 0$ i.e. Φ'' is big: the adjustment costs are large. The result allows for Figure 7 where we have one unstable solution C), and two saddle point solutions A) and B).

FIGURE 7. Multiple equilibrium points in *(q,S)* space



Thus, in long run, if we start with young forest where investment gains are high but forest stock is low (equilibrium point A) the optimal investment policy with given (low) harvesting rate sustains a low steady state level of forest stock. A larger and more mature forest stock that allows also for higher harvesting rate is obtained at steady state point B. Between these points an unstable equilibrium point C may exist where from dynamics drive either toward point A or B. If equilibrium point B with large forest stock is considered to be socially more desirable than point A, then government investment subsidy program can help to obtain it effectively.

III. Public subsidy effects

The analysis above indicated that Case III is relevant for active investment forest policy supporting optimal control and stable path to a equilibrium solution. We analyze next the effects of increase in public investment subsidy $(ds_I > 0)$. The change in s_I affects only the location of $\dot{S} = 0$ curve but have effects on shadow value forest stock since

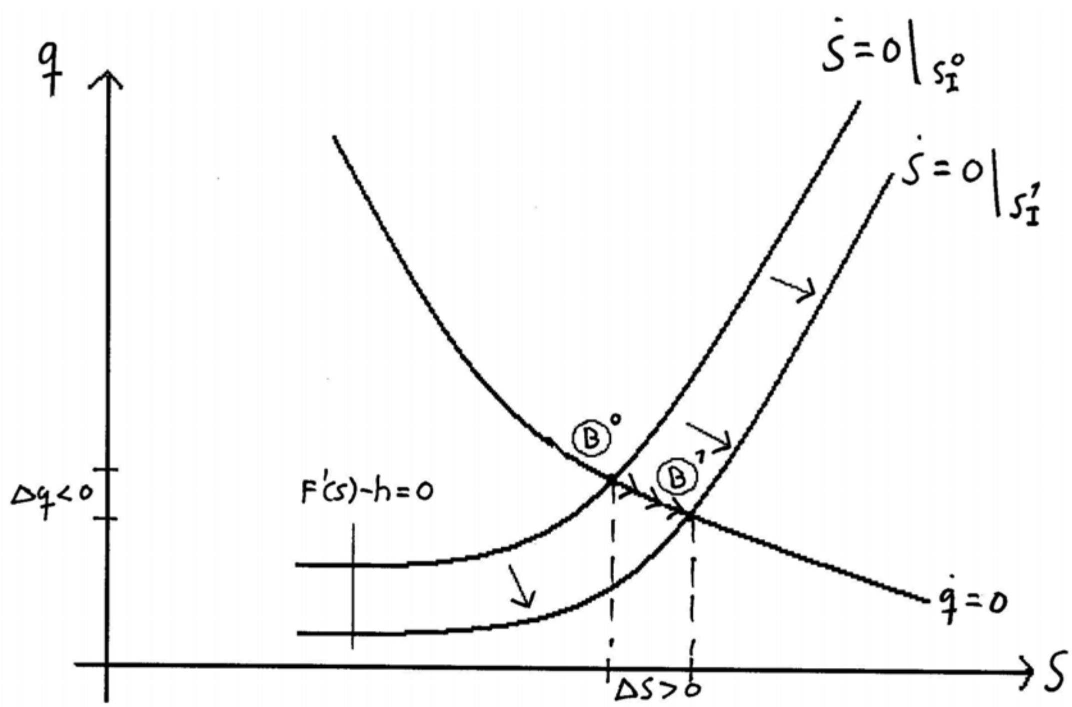
$$d\dot{S} = (F'(S) - h)dS + I_q dq + I_{s_I} ds_I - Sdh$$

and

$$(\frac{dq}{ds_I})_{|\dot{S}=0} = -\frac{I_{s_I}}{I_q} < 0, \quad \text{and} \quad (\frac{dS}{ds_I})_{|\dot{S}=0} = -\frac{I_{s_I}}{F'(S) - h} > 0.$$

Increasing public subsidy shifts $\dot{S} = 0$ curve down and right (Figure 8) since investment cost reduces and firms are willing to invest the same amount at lower value of q, $(\frac{dq}{ds_I})_{|\dot{s}=0} < 0$, i.e. larger investment subsidy increases forest stock, $(\frac{dS}{ds_I})_{|\dot{S}=0} > 0$.

FIGURE 8. Increase in public investment aid: Optimum level forest stock is larger than without aid.



Note that if subsidy increase is large enough points A and C will come closer to each other and finally join and disappear in Figure 7. Similarly in Figure 5 the point A will change to point B in Figure 6. Thus active state aided investment policy will in long run destroy the low forest stock equilibrium points. This means that state aided investments will help to find optimal equilibrium point B sooner and with less friction compared to pure private adjustment process to it.

Harvesting rate has also an interesting role in the determination of steady states. If the given harvest rate h is always larger than forest yield $F'(S)$ then we operate only with equilibrium point B (Case III: $F'(S) - h < 0$). Note that the level of steady state forest stock can be also quite small now as the increased harvesting rate shifts $\dot{S} = 0$ curve inwards (an opposite case to Figure 8 presentation). Anyway the gains of forest investments and public subsidy are always large as forest yield and marginal benefit of investments are positively related to each other at steady state level of marginal benefits (i.e. when $\dot{q} = 0$).

From practical and empirical point of view the equilibrium point B loses part of its relevance since under the extensive public forest investment subsidy program experienced in Finland since 1965 forest yield or forest stock increment has been larger than total drain (see Figure 2). Thus forest stock equilibrium points like A or C above has not only theoretical curiosity. Alternative we can argue that forest stock dynamics in Finland has not yet obtained any kind of steady state and the adjustment process is still going on. Some estimates concerning the process of national forest stock dynamics may help here. Thus next we present some regression results on Finnish forest stock dynamics.

IV. Empirical model

IV.1. Modeling forest growth

Assume that we have local (e.g. county or forest district based) observations of forest stocks in two, not necessarily successive, time periods S_{i,T_1} and S_{i,T_2} $(T_1 < T_2)$ with $i = 1, 2, ..., N$. The growth increment of stocks between these two periods in region i is defined as

$$\Delta S_{i,T_2-T_1} = S_{i,t=2} - S_{i,t=1} = F_i(S_{i,t\in(T_1,T_2)}) - \sum_{t=T_1}^{t=T_2} DRAIN_{i,t} \,,$$

where $F_i(S_{i,t\in(T_1,T_2)})$ gives the local stock effect on growth during the time period (T_1, T_2), and $\sum_{t=T_1}^{t=T_2} DRAIN_{i,t}$ is the local total drain consisting of fellings, felling waste and natural drain during (T_1, T_2).

This specification is problematic since $S_{i,t\in(T_1,T_2)})$ and $\sum_{t=T_1}^{t=T_2} DRAIN_{i,t}$ are dependent. The latter determines partly the level of the former. To avoid this dependency we consecrate on start level or first period stock effects on growth increment in a regression model setting in a following way

$$\Delta S_{i,T_2-T_1} = a_0 + a_1 S_{i,T_1} - a_2(\sum_{t=T_1}^{t=T_2} DRAIN_{i,t}) + \varepsilon_{i,T_2-T_1} \,.$$ [1)]

Next we introduce the forest investment effects in the model like

$$\Delta S_{i,T_2-T_1} = \alpha_0 + \alpha_1 S_{i,T_1} - \alpha_2(\sum_{t=T_1}^{t=T_2} DRAIN_{i,t}) + \alpha_3 INV_{i,T_1-D} + \varepsilon_{i,T_2-T_1} \,,$$

---

1) Note that we could estimate $G_i(S_{i,T_1})$ non-parametrically giving an interesting functional relationship between $\Delta S_{i,T_1-T_2}$ and $G_i(S_{i,t\in(T_1,T_2)})$ based on different local startup stock effects on local growth increment. This gives possibility to test if startup stock has scale or age effects on stock increment.

where INV_{i,T_1-D} are forest investments done D-period ago before period T_1 in local forests having delayed effects, say after 15-20 years later, on forest stock in period $T_2 - T_1$.

Naturally we could use in the model variables for other delayed periods since local forest investment programs last for many years. Alternative a cumulative measurement of delayed investments can be constructed. Thus we have

$$\Delta S_{i,T_2-T_1} = \alpha_0 + \alpha_1 S_{i,T_1} - \alpha_2 (\sum_{t=T_1}^{t=T_2} DRAIN_{i,t}) + \sum_{j=0}^{p} \alpha_{3,i} INV_{i,T_1-D-j} + \varepsilon_{i,T_2-T_1}$$

or

$$S_{i,T_2} = \alpha_0 + (1+\alpha_1) S_{i,T_1} - \alpha_2 (\sum_{t=T_1}^{t=T_2} DRAIN_{i,t}) + \alpha_3 (\sum_{j=0}^{p} INV_{i,T_1-D-j}) + \varepsilon_{i,T_2-T_1}$$

Finally, for testing government investment aid effects on stock growth, we can divide investment in two parts, to private and public investments, i.e. $\sum_{j=0}^{p} INV_{i,T_1-D-j}^{PRIV}$ and $\sum_{j=0}^{p} INV_{i,T_1-D-j}^{PUB}$,

$$S_{i,T_2} = \alpha_0 + (1+\alpha_1) S_{i,T_1} - \alpha_2 (\sum_{t=T_1}^{t=T_2} DRAIN_{i,t})$$

$$+ \alpha_3 (\sum_{j=0}^{p} INV_{i,T_1-D-j}^{PRIV}) + \alpha_4 (\sum_{j=0}^{p} INV_{i,T_1-D-j}^{PUB}) + \varepsilon_{i,T_2-T_1}$$

From viewpoint of practical regression model estimation the model is well defined but if the investment variable is not specified in same units as stocks and drains (m^3) the interpretation of coefficients α_3 and α_4 is difficult. Naturally we have to develop some measure of extension of investments that is convertible to m^3, e.g. if know the area affected by investments (in hectares) and know what is the volume of stand per hectares in different regions we can calculate a proper investment measure.

IV.2 Data

Our data consist of observation of forest stock, drain, and forest investments in 19 Finnish forestry board districts during years 1965-2001. The observations of following variables in different regions were obtained from Finnish Statistical Yearbook of Forestry:

$STOCK_{NFI9}$ = Stock volume (in m^3) of forest according to national forest inventory 1996-2003

$STOCK_{NFI8}$ = Stock volume (in m^3) of forest according to national forest inventory 1986-1994

$DRAIN_{1986\text{-}2001}$ = sum of yearly fellings, fellings waste, and natural drain (in m^3) in years 1986 – 2001 adjusted to regional years of NFI's.

$REGEN_{1965\text{-}1978}$ = sum of yearly hectares affected by artificial regeneration (seeding and planting) in years 1965-1978

$TEND_{1965\text{-}1978}$ = sum of hectares affected by tending of seeding stands and improvement of young stands (cleaning and thinning inc. pruning) in years 1965-1978

$FERTIL_{1965\text{-}1978}$ = sum of hectares affected by forest fertilization in years 1965-1978

$DRAINAGE_{1965\text{-}1978}$ = sum of hectares affected by forest drainage in years 1965-1978

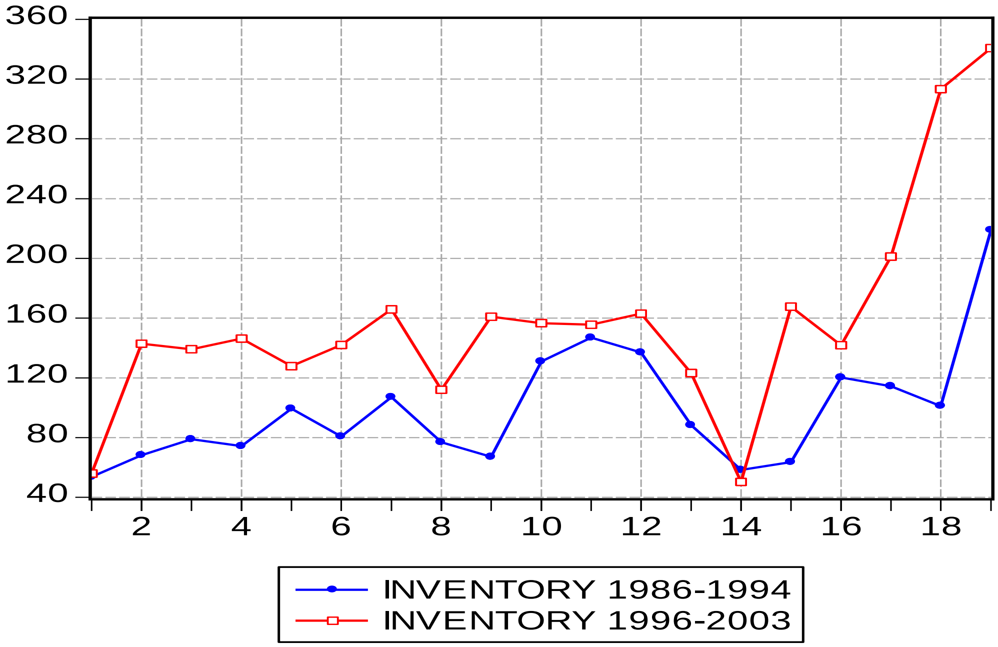
$PRIVc_{1965\text{-}1978}$ = sum of real private investment costs (in euros) in years 1965-1978

$PUBAIDc_{1965\text{-}1978}$ = sum of real financial aid and subsidy to private investments (in euros) in years 1965-1978

Figure 9. depicts the distribution of stock growth between the forestry board districts during the NFI's in 1986-1994 and in 1996-2003. In all regions, except in one (region 14), the forest stock has increased. In the northern part of Finland (regions with numbers of 17, 18, and 19) the forest growth has been unexpected fast during the past 15 years.

At this moment any proper solutions were not found to transform unit of investment observations (hectares or euros) to units of m^3. Thus some regression coefficient estimates lack direct physical interpretation. Thus the results are only indicative and qualitative in some parts. However we assume that used delayed investment effects stemming from years 1965-1978 are motivated and detect the growth effects of investment. Some preliminary analysis showed that results with model on change of forest stocks $\Delta STOCK_{NFI9\text{-}NFI8} = STOCK_{NFI9} - STOCK_{NFI8}$ were less satisfactory compared to model on level variable $STOCK_{NFI9}$.

Figure 9. Forest stock (in million m^3) in 19 Finnish forestry board districts according national forest inventories (NFI's) in 1986-1994 and 1996-2003



IV.3. Results

We first analyze the stock effects of forest investment measured in hectares (i.e. variables *REGEN, TEND, FERTIL* and *DRAINAGE*). Note that variable *TEND* includes investment actions (cleaning and thinning) that actually reduce the forest stock in short run. However the variable was included in regressions as it correlated strongly with other investment variables and preliminary results were poor without it. Typically all investment forms are closely connected to each other in forest management.

Table 1 gives the results of regression model on stock measurements from NFI9 in different regions. Exogenous variable includes different delayed investment actions measured in hectares and stock dynamic variables *($STOCK_{NFI8}$* and *$DRAIN_{1986-2001}$*). The results are expected but lack robustness in some parts. Stock effects on stock growth or increment span from 0.18 to 0.50 with average value of 0.35 corresponding to 3.5% yearly stock growth. Drain effects lack robustness over different specifications but effects are negative as expected. Stock effects from investments are all positive except for *TEND* that has surprisingly large negative effects on forest stock. However summing up all investment effects indicates strong positive stock growth effects. Results for tending, fertilization and drainage are robust but regeneration coefficient estimates are disperse. Model diagnostics support statistically significant results.

Table 1. OLS –regression model results of forest stock effects of forest investment in 19 Finnish forestry board districts during years 1965-2001. Investments measured in hectares. Endogenous variable: $STOCK_{NFI9}$ (N=19, HC t-values in parenthesis)

Constant	40.76 (1.23)	71.38 (2.17)*	35.05 (1.49)	90.02 (2.83)*	87.02 (3.85)*	16.68 (0.51)
$STOCK_{NFI8}$	1.18 (3.82*)	1.32 (4.69)*	1.40 (3.28)*			1.51 (3.34)*
$DRAIN_{1986-2001}$		–2.40 (–2.20)*		–0.26 (–0.13)*		1.25 (0.82)
$REGEN_{1965-1978}$			1.12 (1.79)*	2.15 (2.95)*	2.14 (3.60)*	0.99 (1.53)
$TEND_{1965-1978}$			–1.25 (–4.73)*	–1.05 (–2.13)*	–1.12 (–3.16)*	–1.49 (–3.76)*
$FERTIL_{1965-1978}$			0.83 (2.04)*	0.84 (1.35)	0.88 (1.66)	1.02 (2.16)*
$DRAINAGE_{1965-1978}$			0.84 (3.88)*	0.86 (2.04)*	0.98 (3.27)*	1.03 (3.20)*
R^2 *Normality*[1]	0.462 9.48*	0.587 3.05	0.812 3.31	0.656 0.531	0.654 3.84	0.822 5.52

*) statistically significant from zero at 10% level
[1)] B&J -test for model residual normality. H_0: residuals are normal

Table 2 gives corresponding results with monetary investments in two parts: private funding to forest investments and public financial investment aid and subsidy to private forest owners in years 1965-1978. The results are less satisfactory than in Table 1. Stock and drain effects are close to earlier ones but monetary investment effects are only in few cases statistically significant. The sign of private investment effects depend on the model specification. The negative private investment effects in most cases cast some doubts on earlier results concerning the non-substitution between private investments and public investment aid (see Linden & Leppänen 2006, 2003a). Regression results with excluding public aid produce a positive and significant estimate for the coefficient of private investment cost (last column in Table 2). As the correlation between *PUBc* and *PRIVc* is very high (0.89) we perhaps face here the problem of multicollinearity. Anyway, the public aid to private investments increases the forest stock clearly and supports our theoretical results.

Table 2. OLS –regression model results of forest stock effects of forest investment in 19 Finnish forestry board districts during years 1965-2001. Investments measured in euros. Endogenous variable: $STOCK_{NFI9}$ (N=19, HC t-values in parenthesis)

Constant	52.69 (1.76)*	117.96 (3.42)*	101.87 (2.84)*	62.25 (3.82)*	117.95 (3.53)*
$STOCK_{INV8}$	1.63 (3.71)*			1.47 (2.87)*	
$DRAIN_{1965-1978}$		−3.55 (−1.88)*		−1.09 (−0.62)	−3.53 (−2.48)*
$PRIVc_{1965-1978}$	−1.86 (−2.45)*	1.17 (1.03)	−0.29 (−0.39)	−1.26 (−1.77)	1.24 (3.23)*
$PUBAIDc_{1965-1978}$	2.77 (2.37)*	0.13 (0.07)	2.15 (1.39)	2.09 (1.88)*	
R^2 *Normality*	0.624 8.67*	0.417 0.337	0.279 6.25*	0.634 5.45	0.41 0.28

*) statistically significant from zero at 10% level
[1)] B&J -test for model residual normality. H_0: residuals are normal

V. Conclusions

An optimal control model where proposed to understand the forest policy actions made in Finland in years 1965-1978 to boost national wood production. The target of the new forest policy was to increase forest investments with public aid and to obtain larger national forest stock, faster forest growth, and larger potential commercial cuttings. The model results show that actions made in years 1965-1978 correspond to the model implications. As investment process in forestry is time demanding and rigid, implying large convex adjustment costs, the financial aid to private forest owners is cost reducing and incentive creating leading to less rigid investment adjustment process. Larger wood production is made possible with given marginal gain of investment when investment subsidies are distributed to private investors. Substantial public aid destroys the possible (unstable) equilibrium points stemming from large adjustment costs and make stable path to increased forest stock levels more feasible.

The optimal control level of forest stock is characterized by forest yield (marginal productivity of forest stock) that equals to the sum of interest and harvesting rates minus the inverse of gain of forest capital investments. This means that harvesting rate can be also larger than forest yield if interest rate is low and investment gains are low. The case corresponds to the mature or even to the old forest case.

Elementary empirical record of Finnish national forest dynamics since 1965 does not support steady state behaviour. Positive growth rate of forest stock has lately even been increasing. However these facts do not reject the possible path towards some steady state. Some supplementary empirical results were obtained with regression models which approximate the forest stock process analyzed theoretically. The data consisted of forest stock, drain, and investment observations from nineteen Finnish forestry board districts during years 1965-2001. The regression results confirmed generally the model predictions with economic and statistical significance. However some non-robustness of estimated coefficients was also obtained. A more detailed empirical study with more observation in needed next to reveal all relevant aspects of growth process of forest stocks in Finland in past 50 years.

Appendix I Derivation of investment function

Diffentiating the optimum condition 5a) gives

$$dq = (1 - s_I)\Phi'' dI - ds_I \Phi'$$

$$\Rightarrow$$

$$dI = \frac{dq}{(1 - s_I)\Phi''} + \frac{ds_I \Phi'}{(1 - s_I)\Phi''}$$

and $I(t) = I(q(t), s_I(t)),$ where

$$I_q = \frac{1}{(1 - s_I)\Phi''} > 0 \quad \text{and} \quad I_s = \frac{\Phi'}{(1 - s_I)\Phi''} > 0.$$

With the quadratic investment function $\Phi(I(t)) = I(t) + b[I(t)]^2$ with $b > 0$ investments takes a form

$$I(t) = \frac{1}{2b}[\frac{q(t)}{1 - s_I(t)} - 1].$$

References

Anderson, F.J (1976) "Control Theory and the Optimum Timber Rotation", *Forest Science* 22, 242-246.

Beach, R.H., Pattanayk, S.K., Yang, J.-C. , Murray, B.C. and Abt, R.C. (2005) "Econometric Studies of Non-industrial Private Forest Management: A Review and Synthesis", *Forest Policy and Economics* 7, 261-281.

Chang, S.J. (1981) "Determination of the Optimal Growing Stock and Cutting Cycle for an Uneven Aged Stand", *Forest Science* 27, 739-744.

Clark, C.W., Clarke, F.H. & Munro, G.R. (1979) "The Optimal Exploitation of Renewable Resource Stocks: Problems of Irreversible Investment", *Econometrica* 47, 25-47.

Clark, C. W. (1990) *Mathematical Bioeconomics. The Optimal Management of Renewable Resources*. 2nd ed. Wiley.

Heijdra, B.J & van der Ploeg, F. (2002), *Foundations of Modern Macroeconomics,* OUP.
Jorgenson, S. & Kort, P.M. (1997) "Optimal Investment and Finance in Renewable Resource Harvesting", *Journal of Economic Dynamics and Control* 21, 603-630.
Juurola, M., Ollonqvist, P., Pajuoja, H. & Toropainen, M. (1999) "Outcomes of Forest Improvement Work in Finland", *Silva Fennica* 33: 217-224.
Linden, M. & Leppänen, J. (2006) "Public Cost-Sharing in Private Forest Investments in Finland 1983-2000", *European Journal of Forest Research* (forthcoming).
________________(2003a) "Effects of Public Financed Aid on Private Forest Investments: Some Evidence from Finland, 1963-2000", *Scandinavian Journal of Forest Research* 18, 560-567.
_______________(2003b) "The Impact of Government Investment Assistance on Forest Investment and Timber Supply: A Theoretical and Empirical Analysis" in F.Helles, N. Strange & L. Wichman (eds*.), Recent Accomplishments in Applied Forest Economics Research,* 167-180, 2003. Kluwer Academic Pub. Dordrect
Lyon, K.S. & Sedjo, R.A. (1983) "An Optimal Control Theory Model to Estimate the Regional Long-Term Supply of Timber", *Forest Science* 29, 798-812.
Nautiyal, J.C. & Couto, L. (1984) "The Nature and Uses of the Timber Production Function; Eucalyptus grandis in Brazil", *Forest Science* 30, 761-773.
Nyarko, Y. & Olson, L.J. (1991) "Stochastic Dynamic Models with Stock-Dependent Rewards", *Journal of Economic Theory* 55, 161-168.
Omwami, R.K. (1988) *An Economic Model Underlying the Choice of Capital Intensity in Timber Production.* Acta Forestalia Fennica 204.
Precious, M. (1987), *Rational Expectations: Non-Market Clearing and Investment Theory*, OUP.
Palosuo, V.J. 1979. *MERA-programme in Finnish forestry.* Acta Forestalia Fennica 165. (In Finnish with English abstract).
Uusitalo, M. 1978. *Finnish Forestry During the MERA-Programme Period 1965-1975. A Review Based on Statistics.* Folio Forestalia 367. (in Finnish with English abstract).
Vehkamäki S. (1986) *The Economic Basis of Forest Policy: A Study on the Goals and Means of Forest Policy.* Acta Forestalia Fennica 194.
Williams, J.S. & Nautiyal, J.C. (1990) "The Long-Run Timber Supply Function", *Forest Science* 36, 77-86.