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Abstract: We discuss the nonparametric approach to profit efficiency analysis at the firm and industry levels in the absence of complete price information, and propose two new insights. First, choosing one commodity (whose price is known) as a numeraire good enables us to measure profit inefficiency in absolute monetary terms. Second, imposing a 'Law of One Price' (LoOP) constraint that all firms should be evaluated in terms of the same input-output prices allows us to aggregate firm-level profit inefficiencies to the overall industry inefficiency. Moreover, the LoOP restrictions increase the discriminatory power of the method by better capturing firm-level allocative inefficiencies. Besides the measurement of profit losses, the presented approach enables one to recover absolute price information from quantity data. We conduct a series of Monte Carlo simulations to study the performance of the proposed approach in controlled production environments.

Keywords: Profit Efficiency; Industry Inefficiency; Data Envelopment Analysis; Absolute Prices; Law of One Price; Weight Restrictions; Simulation

JEL classification: C14, C61, D21, D24, D61

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1. INTRODUCTION

According to the standard neo-classical microeconomic theory, the firm behavior can usually be characterized by profit maximization. Therefore, it is generally interesting to test if the empirical production data are consistent with the profit maximization hypothesis (see Afriat, 1972; Hanoch and Rothschild, 1972; Diewert and Parkan, 1983; Varian, 1984). If profit maximization fails empirically, one may proceed to estimate the resulting loss. The notion of profit efficiency was first introduced by Nerlove (1965) in the context of parametric estimation of production functions. The nonparametric estimation of profit inefficiency can rely on the same well-established theoretical principles and axioms; see e.g. Banker and Maindiratta (1988) and Färe and Grosskopf (1995).

While profit maximization is technically one of the simplest firm objectives considered in the literature,[1] the construction of a well-defined index of profit efficiency presents two fundamental obstacles. First, both observed and maximal profit can be positive, negative or zero, and thus the usual ratio-form efficiency measures are generally ill-defined. Second, the price data are often unobserved, unreliable or inconsistent with a competitive equilibrium; hence it is not self-evident which prices should be used in the calculation of profit inefficiencies. Chambers et al. (1996, 1998) have recently addressed both these challenges by introducing a normalized profit efficiency measure, which has an appealing dual interpretation as the directional distance function.[2] Their efficiency measure has desirable index number properties (e.g., it is homogenous of degree zero in prices, homogenous of degree one in quantities). Moreover, the approach is directly applicable in the nonparametric data envelopment analysis (DEA) framework (Farrell, 1957; Charnes et al., 1978), which enables one to estimate profit inefficiencies when prices are unknown, as it merely employs quantity data.

While the approach of Chambers et al. comes a long way in addressing the fundamental challenges in profit efficiency measurement, two important problems still remain.

1) The price normalization (i.e., the direction vector) must be specified *a priori*. In practice, the results of the efficiency analysis depend on the choice of the normalization (see e.g. Färe and Grosskopf, 2004). Yet, there is usually no particular reason to prefer certain normalization to another. Therefore, more detailed guidelines regarding the appropriate selection of the direction vector are needed.

2) The price estimates provided by DEA typically exhibit large variation across firms. This is at odds with the competitive market equilibrium, where the Law of One Price (henceforth LoOP) should apply. While the DEA prices have a compelling interpretation at the level of an individual firm, price variation across firms imply arbitrage opportunities. Thus, DEA prices do not convey value information required for the coordination and reallocation of resources at the industry level. Related to this point, if the LoOP holds, the Koopmans theorem (Koopmans, 1957) implies that the industry profit inefficiency is simply the sum of the firm-level profit inefficiencies. If the LoOP fails, the aggregation of efficiency measures will require some rather restrictive conditions (see Blackorby and Russell, 1999, and Briec et al., 2003, for some general results).

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[1] Other common firm objectives such as cost minimization of budget constrained revenue maximization can typically be seen as special cases of constrained profit maximization; see Kuosmanen (2003) for discussion.

[2] The directional distance function can be seen as the adaptation of Luenberger's (1992) benefit function to the production context.

The purpose of this paper is to address these two challenges. Firstly, we propose an alternative price normalization scheme, which offers a compelling interpretation of profit inefficiency as an absolute, monetary scale measure (in dollars, euros, or any other desired currency).[3] More specifically, we choose one of the input or output commodities as a numeraire good, and consequently express all other (unknown) prices in terms of this numeraire.[4] Indeed, even if complete price information is typically not available, the price of at least one input or output is known in most applications. Secondly, we propose to impose a set of price restrictions to ensure a common set of prices across all firms, such that the DEA prices satisfy the LoOP. The resulting price system minimizes the inefficiency of the industry as a whole.[5] Taken together, we obtain a method that yields absolute and uniform shadow prices giving a direct economic interpretation for profit inefficiency as a monetary loss (both at the firm and at the industry level).

Apart from the interpretation, our approach can considerably enhance the power of DEA: the suggested price normalization and LoOP constraints both represent more stringent performance criteria than those usually applied in the DEA context. Consequently, the proposed approach will typically identify greater inefficiencies than the usual DEA methods. To some extent, the extra power is obtained by imposing more stringent assumptions.[6] In this respect, it is worth noting that the proposed tools can be applied in more relaxed form, and independent of each other. The LoOP constraints apply equally well in the traditional, relative efficiency framework. On the other hand, it is possible to apply the absolute price normalizations without the LoOP constraints (see Kortelainen and Kuosmanen, 2005).

The main aim of the paper is to show how absolute and uniform prices can be exploited in profit efficiency analysis at the firm and industry level. However, we also demonstrate how the presented approach can be applied to recover price information. If firm behavior is generally guided by profit maximization in a competitive market environment, but price information is unavailable for some inputs or outputs, then the observed input-output mix can reveal the underlying prices. If individual firms fail to maximize profit due to some market distortions, identifying the underlying prices is challenging. By identifying the price system that minimizes the overall inefficiency of the industry, we may come up with reasonable price estimates. Using Monte Carlo simulations, we investigate the effective ability of the proposed model to recover the 'true' profit losses and price levels in controlled production environments.

The rest of the paper is organized as follows. Section 2 concentrates on profit efficiency analysis at the firm level and presents the measures of profit efficiency in four different cases (respectively characterized by the availability or unavailability of price and/or technology information). In addition, we present the absolute price

[3] Kortelainen and Kuosmanen (2005) apply a similar approach to ours in the context of eco-efficiency analysis of consumer durables. Their approach differs substantially from the first proposal to measure efficiency in absolute rather than relative scale presented by Womer et al. (2003).

[4] Färe et al. (1990) presented the idea of absolute shadow price in the context of non-parametric efficiency measurement, but did not use it for efficiency measurement.

[5] Kuosmanen et al. (2006) have earlier applied similar LoOP constraints in the context of cost efficiency analysis. The present paper extends their approach to the context of profit efficiency measurement.

[6] However, cost and return data are frequently used as proxies for output and input quantities in empirical applications. This approach is only valid if the underlying input/output prices are the same across all firms in the sample (i.e., the LoOP holds).

normalization scheme. Section 3 subsequently considers industry-level analysis and introduces the LoOP constraints. Section 4 presents the Monte Carlo simulation results of our performance assessment of the presented method. Section 5 draws some concluding remarks.

2. PROFIT EFFICIENCY AT THE FIRM LEVEL

2.1 KNOWN PRICES AND TECHNOLOGY

This section starts by introducing the notion of profit efficiency in the theoretical case of perfect price and technology information. In the subsequent sub-sections, we gradually move towards its estimation under incomplete knowledge about prices and/or technology.

Adopting the standard notation, $\mathrm{y} = (y_1 \ldots y_s)' \in \mathbb{R}^s_+$ represents an output vector and $\mathrm{p} = (p_1 \ldots p_s)' \in \mathbb{R}^s_{++}$ the associated price vector. Similarly, $\mathrm{x} = (x_1 \cdots x_r)' \in \mathbb{R}^r_+$ denotes an input quantity vector and $\mathrm{w} = (w_1 \ldots w_r)' \in \mathbb{R}^r_{++}$ the associated price vector. Suppose an observed sample of N firms, indexed by $n \in \nu \equiv \{1,\ldots,N\}$. For each firm observation $n \in \nu$, $\mathrm{X}_n = (X_{n1} \ldots X_{nr})'$ and $\mathrm{Y}_n = (Y_{n1} \ldots Y_{ns})'$ represent the corresponding input and output vectors and $\mathrm{W}_n = (\mathrm{W}_{n1} \ldots \mathrm{W}_{nr})'$ and $\mathrm{P}_n = (\mathrm{P}_{n1} \ldots \mathrm{P}_{ns})'$ and the respective price vectors.

The technology is defined by the production possibility set

(1) $$T = \left\{ (\mathrm{x},\mathrm{y}) \in \mathbb{R}^{r+s}_+ \middle| \mathrm{x} \text{ can produce } \mathrm{y} \right\}.$$

We assume that this set satisfies the following well-known regularity properties: (1) closedness, (2) non-emptiness, (3) scarcity, and (4) no free lunch. See e.g. Färe and Primont (1995) for a detailed discussion. Using (1), we next define the profit function as

(2) $$\pi(\mathrm{w},\mathrm{p}) \equiv \max_{x,y} \left\{ \mathrm{p}\cdot\mathrm{y} - \mathrm{w}\cdot\mathrm{x} \middle| (\mathrm{x},\mathrm{y}) \in T \right\}.$$

Nerlove (1965) suggests two alternative measures of profit efficiency: the ratio measure and the difference measure. For an evaluated firm $n \in \nu$, the ratio measure can be written as:

(3) $$PE^R_n \equiv \frac{\mathrm{P}_n\mathrm{Y}_n - \mathrm{W}_n\mathrm{X}_n}{\pi(\mathrm{W}_n,\mathrm{P}_n)},$$

and the difference measure as

(4) $$PE^D_n \equiv \pi(\mathrm{W}_n,\mathrm{P}_n) - (\mathrm{P}_n\mathrm{Y}_n - \mathrm{W}_n\mathrm{X}_n).$$

An attractive property of the ratio measure (3) is that it is homogeneous of degree zero in prices and quantities, which makes it invariant to the units of measurement (such as the currency unit for the input and output prices). Still, it is generally ill-defined (i.e., it equals infinity) if the maximum profit equals zero. In addition, the measure is difficult to interpret in case of negative (maximum and/or actual) profit levels. The opposite holds for the difference measure (4). This measure is homogenous of degree one in prices and quantities, and thus not invariant to the units of measurement. Nevertheless, it is capable of handling negative or zero profits: its value is always a nonnegative and finite real number interpretable as the absolute profit loss (in money terms).

Chambers et al. (1998) recently proposed a normalized profit inefficiency measure, which combines the virtues of the measures (3)-(4):

$$(5) \qquad PE_n^{DD}(g^x, g^y) \equiv \frac{\pi(W_n, P_n) - (P_n \cdot Y_n - W_n \cdot X_n)}{P_n \cdot g^y + W_n \cdot g^x}.$$

This measure defines a normalized profit difference, with the specific normalization defined by the so-called 'direction vectors' $(g^x, g^y) \geq 0, (g^x, g^y) \neq 0$, which will be discussed in more detail below (especially in Section 2.5).[7] This general construction encompasses a multitude of profit efficiency measures, depending on the specification of the direction vectors; see e.g. Färe and Grosskopf (2004). Clearly, the measure is homogenous of degree zero in prices and quantities, and it is well-defined also for non-positive profits. Given these attractive properties, our following discussion will specifically focus on this profit inefficiency measure.

2.2 KNOWN PRICES, UNKNOWN TECHNOLOGY

In empirical studies, the production technology is usually unknown, which means that the profit function π must be estimated. Various estimation techniques are available for estimating profit function. This paper focuses on the nonparametric approach (e.g. Afriat, 1972; Varian, 1984; Färe and Grosskopf, 1995), which does not impose particular structure on the technology, but uses the minimalistic prior that only technically feasible input-output vectors are observable, implying $(X_n, Y_n) \in T$ for all $n \in \nu$. Replacing the profit function of (5) with the sample estimate $\max_{i \in \nu}(P_n \cdot Y_i - W_n \cdot X_i)$ gives the following empirical measure for profit inefficiency:

$$(6) \qquad \widehat{PE}_n^{DD} = \frac{\max_{i \in \nu}(P_n \cdot Y_i - W_n \cdot X_i) - (P_n \cdot Y_n - W_n \cdot X_n)}{P_n \cdot g^y + W_n \cdot g^x}.$$

If the quantity data are error-free, the empirical profit function gives a lower bound for the true but unknown profit function, which further implies that $\widehat{PE}_n^{DD} \leq PE_n^{DD}$.

2.3 UNKNOWN PRICES, KNOWN TECHNOLOGY

In practical efficiency analysis, these prices are often not observed or the observed prices do not constitute reliable proxies for the implicit costs and revenues faced by the evaluated firm. For clarity, we next consider the situation where prices are unknown, but the technology (characterized by profit function π) is known. In that case, profit inefficiency can be measured by resorting to the 'most favorable' or 'shadow' prices (i.e. we apply 'benefit-of-the-doubt' pricing in the absence of full price information). This yields the directional distance function measure

[7] To simplify the notation, we indicate the evaluated firm by the sub-script, and suppress any uninformative arguments of the efficiency indices / distance functions.

(7) $$DD_n \equiv \min_{(\mathrm{w},\mathrm{p})\in\mathbb{R}_+^{r+s}} \frac{\pi(\mathrm{w},\mathrm{p})-(\mathrm{p}\cdot \mathrm{Y}_n - \mathrm{w}\cdot \mathrm{X}_n)}{\mathrm{p}\cdot \mathrm{g}^y + \mathrm{w}\cdot \mathrm{g}^x}.$$

Notice that in contrast to previous cases, the prices enter this inefficiency measure as decision variables of the optimization problem. Comparison of (7) with (5) immediately reveals that $DD_n \le PE_n^{DD}$. Thus, the directional distance function (7) can be interpreted as a lower bound for the true but unknown profit inefficiency; this is directly analogous with the well-known interpretation of the Farrell input efficiency measure as the upper bound for cost efficiency. If production possibility set *T* is convex, then the difference between the profit inefficiency (5) and the directional distance function (7) is solely due to allocative inefficiencies, which are accounted in (5) but not in (7). Thus, for convex technologies we could measure allocative inefficiency by $AE_n^{DD} = PE_n^{DD} - DD_n$.

The notion "directional distance function" refers to the dual expression of (7) as

(8) $$DD_n = \min\left\{\delta \left| (\mathrm{X}_n - \delta \mathrm{g}^x, \mathrm{Y}_n + \delta \mathrm{g}^y) \in cm(T)\right.\right\},$$

with $cm(T)$ denoting the convex monotonic hull of *T* (see Chambers et al., 1998). In this expression, the vectors g^x and g^y determine the direction of projection of the evaluated input-output vector onto the boundary of the reference technology, which elicits their interpretation as direction vectors. For compactness, our following discussion will abstract from dual representations such as (8), but it is worth indicating that such dual results are easily obtained (starting from (8)).

2.4 UNKNOWN PRICES AND TECHNOLOGY

In many empirical studies, both prices and technology are unknown and must be estimated from data. Combining the insights of Sections 2.2 and 2.3, we can formulate a measure of profit inefficiency that utilizes minimal technology and price information. Specifically, we can plug in the empirical profit function $\max_{i\in\nu}(\mathrm{p}\cdot \mathrm{Y}_i - \mathrm{w}\cdot \mathrm{X}_i)$ into the directional distance function measure (7). This yields a minimax problem where we select the prices that minimize profit inefficiency, while the (endogenously selected) reference firm *m* maximizes the corresponding empirical profit function. Conveniently, this minimax problem can be transformed into the following linear programming problem:

(9) $$\begin{aligned} &\widehat{DD}_n = \min_{(\mathrm{w},\mathrm{p})\in\mathbb{R}_+^{r+s}} \rho \\ &s.t. \\ &\rho \ge (\mathrm{p}\cdot \mathrm{Y}_m - \mathrm{w}\cdot \mathrm{X}_m) - (\mathrm{p}\cdot \mathrm{Y}_n - \mathrm{w}\cdot \mathrm{X}_n) \quad \forall m \in \nu \\ &\mathrm{p}\cdot \mathrm{g}^y + \mathrm{w}\cdot \mathrm{g}^x = 1 \\ &\mathrm{p},\mathrm{w} \ge 0 \end{aligned}$$

The empirical profit function at shadow prices (w,p) (to be optimized) will be implicitly represented by expression $(\mathrm{p}\cdot \mathrm{Y}_m - \mathrm{w}\cdot \mathrm{X}_m)$ for firm(s) *m* for which the first (profit difference) constraint of (9) is binding. The second constraint in (9) presents the price normalization.

As noted in the Introduction, the specification of the direction vector $(\mathbf{g}^x, \mathbf{g}^y)$ remains somewhat arbitrary. Different specifications of the direction vector may yield a dramatically different picture regarding the profit inefficiency, and its decomposition to technical and allocative inefficiency components. To resolve this ongoing issue, we next consider price normalizations (direction vectors) that facilitate the interpretation of profit inefficiency in absolute money terms.

2.5 ABSOLUTE PRICE NORMALIZATIONS

We start by noting that linear programming problem (9) obtains profit inefficiency measures and shadow prices that merely bear a relative meaning. The inefficiency measure has to be proportioned with the value of the direction vector, and its interpretation is not always clear. The estimated input-output prices convey information about the relative value of one commodity in comparison to another, but we cannot express these values in money. In many instances, it may be useful to express the prices and profit inefficiency in absolute, monetary terms. Such absolute values can be obtained when the price of at least one input or output commodity is known a priori.

In most empirical applications, well-defined and meaningful prices are observed for some input-output goods, but not for all of them; this involves a setting that may be considered as a hybrid form of those considered in Sections 2.2 and 2.4. In such a setting, it is sensible to make use of the available price information in the profit inefficiency measurement, and apply shadow prices only for those inputs and outputs for which direct price observations are not available. Our argument is that these price observations can be meaningfully incorporated in the analysis through an appropriate specification of the direction vector.

Our argument is best illustrated by a simple example. Suppose the unit price of output 1 is known to equal \$2. Specifying the direction vector as $\mathbf{g}^x = \mathbf{0}$ and $\mathbf{g}^y = (\frac{1}{2}, 0, \ldots, 0)$, we effectively capture this price information in the normalization constraint; it is easily verified that this enforces the price of output 1 equal to 2 without restricting the prices of other outputs or inputs.[8] With this specific normalization, all other prices are measured relative to the price of output 1, and may thus be interpreted in absolute dollar terms. In this case, problem (9) boils down to

$$\widehat{DD}_n(\mathbf{0}, (\tfrac{1}{2}, 0, \ldots, 0)) = \min_{\substack{(\mathbf{w},\mathbf{p}) \in \mathbb{R}_+^{r+s} \\ \rho \in \mathbb{R}_+}} \rho$$

$$s.t.$$

(10) $$\rho \geq (\mathbf{p} \cdot \mathbf{Y}_m - \mathbf{w} \cdot \mathbf{X}_m) - (\mathbf{p} \cdot \mathbf{Y}_n - \mathbf{w} \cdot \mathbf{X}_n) \quad \forall m \in \nu$$

$$p_1 = 2$$

$$\mathbf{p}, \mathbf{w} \geq \mathbf{0}$$

Like before, the resulting value of the directional profit efficiency measure captures the difference between the maximum and actual profit, computed at shadow prices. However, given that the specific price normalization

[8] Referring to the dual representation (8), this price normalization obviously entails a very particular direction for projecting the evaluated input-output vector onto the boundary of *cm*(*T*).

captures the true price for the first output (i.e. $2), this profit difference is now expressed in absolute dollar terms. In fact, this formulation of the directional profit efficiency measure recovers the original Nerlovian difference measure (4), but is now expressed in (absolute) shadow prices.

More generally, we can fix the price of any input or output, or a combination thereof. If only a single price is fixed, then it can be modeled as a special case of the directional distance function; see the above example. If multiple prices are known then we simultaneously need multiple normalization constraints; the practical implementation follows the same lines as the above example.

Even if all prices are unknown (which is sometimes the case), it may be useful to select one output or input as a numeraire commodity, and specify the direction vector such that the element corresponding to the numeraire good equals unity, and all other elements are equal to zero. The absolute interpretation of the estimated profit difference is then conditional upon the specific price value for that selected commodity.

Another possibility to formulate absolute price normalizations is to use information about the total cost, revenue, or profit of the firm. For example, total operating cost (or budget) data are often available for public sector firms (such as schools, university departments, health care units, and tax offices), even if input-specific prices are unknown. If the total cost for firm *n* is known to be C_n, we may normalize the prices such that the "shadow cost" of firm *n* (i.e., total cost calculated using shadow prices) equals the observed costs. It is easily verified that specifying the direction vectors as $\mathbf{g}^x = (X_{n1}/C_n, X_{n2}/C_n, ..., X_{nr}/C_n)$ and $\mathbf{g}^y = 0$ effectively enforces such a constraint. This normalization facilitates the absolute monetary interpretation for the shadow prices, which are measured in currency units. If the total revenue (turnover) or profit of the firm is known, we may apply directly analogous price normalizations: if total revenue of firm *n* is R_n, we may specify the direction vector as $\mathbf{g}^x = 0$ and $\mathbf{g}^y = (Y_{n1}/R_n, Y_{n2}/R_n, ..., Y_{ns}/R_n)$; if total profit of firm *n* is $PR_n > 0$, we may specify the direction vector as $\mathbf{g}^x = (-X_{n1}/PR_n, -X_{n2}/PR_n, ..., -X_{nr}/PR_n)$ and $\mathbf{g}^y = (Y_{n1}/PR_n, Y_{n2}/PR_n, ..., Y_{ns}/PR_n)$.

Finally, if the prices or cost/revenue/profit aggregates are not known precisely, but some meaningful upper and lower bounds can be established, one may impose these bounds explicitly in the measurement problem to narrow down the possible price range. In the context of cost efficiency analysis, Kuosmanen and Post (2001) proposed to impose relative price restrictions of the form $A \leq \frac{w_i}{w_j} \leq B$, which restrict the price of input *i* relative to the price of input *j* to the closed interval [*A*,*B*]; utilizing this information, they subsequently computed upper and lower bounds for cost efficiency. Similar restrictions are directly applicable in the present context as well; the formulation of relative constraints for output prices or cost/revenue/profit aggregates is directly obvious. It is worth emphasizing that the absolute price normalizations suggested above also enable us to impose absolute price restrictions of the form $C \leq w_i \leq D$, which restrict the price of input *i* to the closed interval [*C*,*D*]. We note that this contrasts with the usual DEA practice, which typically do not employ absolute weight restrictions (see Dyson et al., 2001, for discussion).

3. INDUSTRY LEVEL ANALYSIS

3.1 GENERAL

Aggregation of efficiency indices from the firm level to the industry level has attracted some deserved attention in the recent literature; the Introduction of Briec et al. (2003) offers an excellent review. This literature has predominantly focused on the aggregation of technical efficiency indices. Notable contributions include Blackorby and Russell (1999) who consider the radial Farrell measures and the main non-radial contenders, and Briec et al. (2003) who assess the directional distance function discussed above. Both these studies conclude that the aggregation of technical efficiency measures requires rather restrictive assumptions such as linearity of the production technology (i.e., T is a half-space). By contrast, in the present context of profit efficiency analysis we can proceed with the very general properties stated in Section 2.1, because the profit function (2) is linear by definition. In particular, we may resort to the Koopmans (1957) theorem, which established that the profit function of the industry is the sum of the firms' profit functions: if π_n is the profit function of firm $n \in \nu$ and Π is the industry profit function, then

(11) $$\Pi(\mathrm{w,p}) = \sum_{n \in \nu} \pi_n(\mathrm{w,p}) .$$

Two observations are worth noting regarding the Koopmans theorem. First, summing the firm-level profit functions to the aggregate profit function of the industry requires that the input-output prices are the same across all firms. Second, this result allows for firm-specific production technologies, characterized by firm specific profit functions. If all firms have access to the same technology, as usually assumed in the empirical studies, then the profit functions are also the same for all firms: $\pi_n = \pi \;\; \forall n \in \nu$. In this case, the Koopmans theorem implies

(12) $$\Pi(\mathrm{w,p}) = N \cdot \pi(\mathrm{w,p}) .$$

Denote the aggregate industry input-output vectors by $(\mathfrak{X},\mathfrak{Y}) \equiv \left(\sum_{n \in \nu} \mathrm{X}_n , \sum_{n \in \nu} \mathrm{Y}_n \right)$. Assume the LoOP holds so that all firms take the same input-output prices. Denoting the observed price vectors by (W,P), we may measure the normalized profit inefficiency of the industry analogous to (5) as

(13) $$IPE^{DD} \equiv \frac{\Pi(\mathrm{W,P}) - (\mathrm{P} \cdot \mathfrak{Y} - \mathrm{W} \cdot \mathfrak{X})}{\mathrm{P} \cdot \mathrm{g}^y + \mathrm{W} \cdot \mathrm{g}^x} .$$

The Koopmans theorem directly implies that this industry profit inefficiency measure is obtained as the sum of the firms' profit inefficiencies:

(14) $$IPE^{DD} = \frac{N \cdot \pi(\mathrm{W,P}) - \sum_{n \in \nu} (\mathrm{P} \cdot \mathrm{Y}_n - \mathrm{W} \cdot \mathrm{X}_n)}{\mathrm{P} \cdot \mathrm{g}^y + \mathrm{W} \cdot \mathrm{g}^x}$$

(15) $$= \sum_{n \in \nu} \frac{\pi(\mathrm{W,P}) - (\mathrm{P} \cdot \mathrm{Y}_n - \mathrm{W} \cdot \mathrm{X}_n)}{\mathrm{P} \cdot \mathrm{g}^y + \mathrm{W} \cdot \mathrm{g}^x} = \sum_{n \in \nu} PE_n^{DD} .$$

Under the LoOP, the industry profit inefficiency measure can be interpreted as a total profit loss resulting from technical and allocative inefficiencies of the industry. However, it is difficult to draw a sharp distinction between technical and allocative inefficiencies here, as the division crucially depends on the level of aggregation.

This point is best illustrated by a numerical example. Suppose the firms and the industry share the same the Cobb-Douglas technology $T=\{(x_1,x_2,y)|y\le x_1^{0.5}\cdot x_2^{0.5}\}$ that exhibits constant returns to scale.[9] Let $P=2$, $W_1=W_2=1$, and $g^y=1$, $g_1^x=g_2^x=0$. Suppose our data consist of two firms, with input output vectors $(x_{11},x_{12},y_1)=(1,4,2)$ and $(x_{21},x_{22},y_1)=(4,1,2)$. Note first that both firms are profit inefficient: the aggregate profit inefficiency may be calculated as $PE_1^{DD}(1,1,1)+PE_2^{DD}(1,1,1)=\frac{0-(4-1-4)}{2}+\frac{0-(4-4-1)}{2}=1$. Clearly, both firms are technically efficient in the sense that the input-output vectors lie on the boundary of *T* (moreover, $DD_1=DD_2=0$). Therefore, the firms' profit inefficiencies are entirely due to the allocative inefficiencies. However, the picture looks completely different when examined from the aggregate perspective. Clearly, the aggregate input-output vector $(\mathcal{X}_1,\mathcal{X}_2,\mathcal{Y})=(5,5,4)$ lies in the interior of *T*, implying technical inefficiency at the aggregate level. In fact, calculating the directional distance function for the aggregate input-output vector gives $DD_{ind}=1$, which reveals that the profit loss at the industry level is entirely due to technical inefficiency. In conclusion, what is allocative inefficiency at the firm level may be technical inefficiency at the industry level, and *vice versa*.

From an empirical perspective, the LoOP condition assumed above may seem too restrictive. In reality, input-output prices often differ across firms due to imperfect competition or transaction costs. If the input prices exhibit substantial variations across firms, one might be tempted to gauge profit inefficiency of the industry as the sum of firms' inefficiencies at the observed input-output prices (i.e., $\sum_{n\in\nu}\frac{\pi(\mathrm{W}_n,\mathrm{P}_n)-(\mathrm{P}_n\cdot\mathrm{Y}_n-\mathrm{W}_n\cdot\mathrm{X}_n)}{\mathrm{P}_n\cdot\mathrm{g}^y+\mathrm{W}_n\cdot\mathrm{g}^x}$). While this sum captures the technical and allocative inefficiencies at the firm level, its interpretation as the aggregate industry inefficiency is dubious. The price variations can be seen as a signal of arbitrage opportunities, or a lack of coordination; such price observations cannot represent the competitive market equilibrium. Therefore, this sum does not adequately measure the industry-level profit inefficiency in the Pareto-Koopmans sense. In fact, even if this sum is equal to zero, the aggregate input-output vector may be technically inefficient. We may modify the previous numerical example by introducing firm-specific price vectors $(W_{11},W_{12},P_1)=(4,1,4)$ and $(W_{21},W_{22},P_1)=(1,4,4)$. One can verify that the sum of firm-specific profit inefficiencies is equal to zero, even though we earlier noted that the aggregate input-output vector is technically inefficient. Therefore, we reject the idea of measuring industry-level profit inefficiency at firm-specific prices.

Following Kuosmanen et al. (2006), we prefer to interpret the LoOP condition not as a *descriptive* (empirical) hypothesis, but rather as a *normative* requirement for Pareto efficiency. Koopmans (1951a,b) showed that the existence of a set of "efficiency prices" that are uniform across all producers is both a necessary and sufficient condition for Pareto efficiency. Koopmans emphasized that the "one price" efficiency condition is by no

[9] Li and Ng (1995) have shown that, under constant returns to scale, if all firms have the same technology $T_n=S$, then the industry technology is $T\equiv\sum_{n\in\nu}T_n=S$.

means limited to competitive market settings, but concerns all production activities, including production of non-for-profit and public sector firms, even centrally planned economies: "*the price concept established does not in any way presuppose the existence of a market or of exchange of commodities between owners*" (Koopmans 1951b, p. 462).

A problem with the Koopmans' efficiency prices is that they are usually not empirically observable. The fact that empirical price data frequently show large variations across firms simply means that these price observations do not represent valid efficiency prices. Therefore, even if we do observe prices, these price data may be useless for estimating the overall profit loss due to inefficiency. We therefore propose to operationalize the efficiency prices by resorting to the shadow prices, applying the insights of Sections 2.3-2.5 to the industry level profit efficiency analysis.

3.2 Shadow price approach

Recent papers by Li and Ng (1995), Ylvinger (2000), and Kuosmanen et al. (2006) show how cost efficiency of the industry can be measured by means of shadow prices. Briec et al. (2003) consider a similar approach to construct a lower bound for industry's allocative inefficiency based on quantity data. In the following, we extend these ideas to the measurement of profit inefficiency of the industry in the absence of relevant (efficiency) price data.

Following Kuosmanen et al. (2006), we may consider two distinct approaches: the *Top-Down* and the *Bottom-Up* approaches. The Top-Down approach evaluates the aggregate input-output vector directly based on (13), whereas the Bottom-Up approach constructs the industry level inefficiency as the sum of firms' inefficiencies utilizing (15). The Koopmans theorem implies that both approaches are equivalent. We next discuss the two approaches in more detail.

3.2.1 Top-Down Approach

The Top-Down approach starts directly from the aggregate input-output vector $(\mathfrak{X},\mathfrak{Y})$ and the definition of IPE^{DD} in (13). We maintain the assumption that all firms have access to the same technology *T*, but in contrast to Kuosmanen et al. (2006), we do not need to impose constant returns to scale. Obviously, under variable returns to scale, the aggregate input-output vector $(\mathfrak{X},\mathfrak{Y})$ is not necessarily contained within set *T*. Nevertheless, the average input-output vector $(\mathfrak{X}/N,\mathfrak{Y}/N)$, which may be interpreted as the "representative firm" of the industry, will always be contained in the convex monotonic hull *cm*(*T*); the average input-output vector is a particular convex combination of the observed firms, and *cm*(*T*) satisfies convexity by construction. Moreover, the profit function defined with respect to *T* is equivalent to the profit function defined with respect to *cm*(*T*): $\pi^T(\mathrm{w},\mathrm{p}) = \pi^{cm(T)}(\mathrm{w},\mathrm{p})$ (Kuosmanen, 2003, Theorem 3.2). Therefore, we can harmlessly express the industry profit inefficiency measure in terms of the representative firm as

$$(16) \qquad IPE^{DD} = N \cdot \left(\frac{\pi(\mathrm{W,P}) - (\mathrm{P} \cdot \mathfrak{Y}/N - \mathrm{W} \cdot \mathfrak{X}/N)}{\mathrm{P} \cdot \mathrm{g}^y + \mathrm{W} \cdot \mathrm{g}^x} \right).$$

If the price vector (W,P) is known but the technology is unknown, we may resort to the same non-parametric approach as in Section 2.2 and estimate the unknown profit function by the maximum observed profit. This gives the empirical industry profit inefficiency measure

$$(17) \qquad \widehat{IPE}^{DD} = N \cdot \left(\frac{\max_{i \in \nu}(\mathrm{P} \cdot \mathrm{Y}_i - \mathrm{W} \cdot \mathrm{X}_i) - (\mathrm{P} \cdot \mathfrak{Y}/N - \mathrm{W} \cdot \mathfrak{X}/N)}{\mathrm{P} \cdot \mathrm{g}^y + \mathrm{W} \cdot \mathrm{g}^x} \right).$$

On the other hand, if the prices are not observed but the technology is known, then we can resort to the shadow prices as in Section 2.3. This motivates the following 'industry' directional distance function as an indicator for industry profit inefficiency:

$$(18) \qquad IDD = \min_{(\mathrm{w,p}) \in \mathbb{R}_+^{r+s}} N \cdot \left(\frac{\pi(\mathrm{w,p}) - (\mathrm{p} \cdot \mathfrak{Y}/N - \mathrm{w} \cdot \mathfrak{X}/N)}{\mathrm{p} \cdot \mathrm{g}^y + \mathrm{w} \cdot \mathrm{g}^x} \right).$$

Analogous to the firm level distance functions, this industry directional distance function gives the lower bound for the true but unknown industry profit inefficiency: $IDD \le IPE^{DD}$.

Finally, if both prices and technology are unknown, we can combine from the results in (17) and (18) (compare with Section 2.4), and measure efficiency of the representative firm relative to the empirical profit function at the most favorable shadow prices. This industry profit inefficiency measure is obtained as the optimal solution to the linear programming problem (compare with (9))

$$\widehat{IDD} = \min_{\substack{(\mathrm{w,p}) \in \mathbb{R}_+^{r+s} \\ \rho \in \mathbb{R}_+}} N \cdot \rho$$

$$s.t.$$

$$(19) \qquad \rho \ge (\mathrm{p} \cdot \mathrm{Y}_m - \mathrm{w} \cdot \mathrm{X}_m) - (\mathrm{p} \cdot \mathfrak{Y}/N - \mathrm{w} \cdot \mathfrak{X}/N) \quad \forall m \in \nu$$

$$\mathrm{p} \cdot \mathrm{g}^y + \mathrm{w} \cdot \mathrm{g}^x = 1$$

$$\mathrm{p,w} \ge 0$$

The optimal solution to (19) provides an estimate of the industry profit inefficiency, but it also reveals the optimal shadow prices $(\mathrm{w}^*, \mathrm{p}^*)$ that minimize the aggregate inefficiency. We may subsequently use these industry shadow prices within the firm level profit efficiency analysis. Specifically, we can plug in the industry shadow prices $(\mathrm{w}^*, \mathrm{p}^*)$ to the firm level problem (6) to assess profit inefficiency of each specific firm. However, this would require that the industry shadow prices are unique. While we would expect the shadow prices to be unique in most non-trivial applications, it is possible to construct numerical examples where shadow prices are non-unique. Fortunately, we may test for the uniqueness before moving top-down from the industry level. A detailed procedure is presented by Kuosmanen et al. (2006, Section 4.1).

As a concluding remark, it is worth to note that the representative firm approach has been widely used in the DEA literature for assessing the structural efficiency of the industry, stemming from the pioneering work of Førsund and Hjalmarsson (1979). The recent paper by Ylvinger (2000) sharply criticizes this approach, concluding that: "…*the bad news is that all these problems suggest that many misleading results may have been published using the average-unit measures; the good news is that these studies are at least not focused on something of vital importance, such as nuclear reactors (thank god).*" We must emphasize here that Ylvinger's critique concerns the aggregation of technical efficiency indices, and it does not carry over to the present context of profit efficiency measurement. Indeed, our Top-Down model re-institutes the representative firm approach with a sound theoretical foundation in the nonparametric profit efficiency analysis.

3.2.2 Bottom-Up Approach

The Bottom-Up approach starts from the firm-specific input-output data and constructs the industry profit inefficiency measure as the sum of the firm inefficiencies using (15). Things are particularly simple if the efficiency prices (W,P) are directly observed. If both prices and technology are known, the left-hand side of (15) can be directly used for calculating the industry profit efficiency. If the observed prices satisfy the LoOP but the technology is unknown, we may first calculate the profit inefficiencies at the firm level using (6), and subsequently sum over all firms to obtain the industry profit inefficiency measure.

As argued above, the efficiency prices are usually unobservable. If the prices are unknown but the technology is known, we can resort to the shadow prices as in Section 2.3. However, we cannot solve problem (7) for each firm separately, because this would likely result as shadow prices that violate the LoOP and thus do not allow for aggregation. Therefore, we need to identify the shadow prices simultaneously for all firms by minimizing the sum of firms' profit inefficiencies. Thus, problem (7) can be adapted to the industry level as

$$IDD = \min_{(\mathbf{w},\mathbf{p}) \in \mathbb{R}_+^{r+s}} \sum_{n \in \nu} \left(\frac{\pi(\mathbf{w},\mathbf{p}) - (\mathbf{p} \cdot \mathbf{Y}_n - \mathbf{w} \cdot \mathbf{X}_n)}{\mathbf{p} \cdot \mathbf{g}^y + \mathbf{w} \cdot \mathbf{g}^x} \right). \tag{20}$$

Comparing problem (20) with (18), we can verify that both approaches yield the same result.

Finally, if both prices and technology are unknown, we can plug in the empirical profit function in problem (20). As a result, we essentially solve problem (9) for all firms simultaneously, minimizing the sum of firms' profit inefficiencies. This minimization problem can be formally expressed as

$$\begin{aligned} \widehat{IDD} = & \min_{\substack{(\mathbf{w},\mathbf{p}) \in \mathbb{R}_+^{r+s} \\ (\rho_1 \dots \rho_N) \in \mathbb{R}_+^N}} \sum_{n \in \nu} \rho_n \\ & s.t. \\ & \rho_n \geq (\mathbf{p} \cdot \mathbf{Y}_m - \mathbf{w} \cdot \mathbf{X}_m) - (\mathbf{p} \cdot \mathbf{Y}_n - \mathbf{w} \cdot \mathbf{X}_n) \quad \forall m,n \in \nu \\ & \mathbf{p} \cdot \mathbf{g}^y + \mathbf{w} \cdot \mathbf{g}^x = 1 \\ & \mathbf{p},\mathbf{w} \geq 0 \end{aligned} \tag{21}$$

It is illustrative to compare this problem with problem (9) in Section 2.4 and problem (19) in Section 3.2.1. Problem (21) is consistent with the LoOP because it applies the same input and output prices to all firms. By contrast, solving problem (9) for each firm separately will typically yield different shadow prices for each firm. We also note that problems (19) and (21) will always yield the same results regarding both the industry inefficiency as well as the shadow prices.

Both the Top-Down and Bottom-Up approaches have their own merits. The key advantage of the Top-Down model (19) is its computational efficiency. While problem (19) has $r+s+1$ variables and $N+1$ constraints, problem (21) involves $r+s+N$ variables and N^2+1 constraints. In applications that use large samples or bootstrap techniques, the computational advantage of the Top-Down model can be considerable. On the other hand, the Bottom-Up model is more flexible. For example, the Bottom-Up model allows one to "weaken" the strict LoOP condition adopted above, so as to allow for minor differences between the (shadow) prices associated with different firms (see Kuosmanen et al., 2006, for a more detailed discussion). Such weaker versions of the uniformity constraints may in particular be useful for investigating the sensitivity of the firm-specific efficiency results with respect to the LoOP constraint *stricto sensu*. Again, we abstract from a formal treatment in this study, but the extension of these methodological tools towards the current profit efficiency setting is easy.

3.3 PRICE NORMALIZATIONS AT THE AGGREGATE LEVEL

As noted in the Introduction, the specification of the direction vector is a topic of ongoing debate. In Section 2.5, we discussed the absolute price normalizations and the corresponding specifications of direction vectors, arguing that the absolute price normalizations offer an economically meaningful approach to specifying the direction vectors. Obviously, the absolute price normalizations are equally well applicable at both firm and industry levels. At the industry level, the absolute price normalizations could be a way to estimate a lower bound for the inefficiency loss (in money terms) due to imperfect competition even when observed prices do not represent a competitive market equilibrium. Thus, the absolute price normalizations can be a useful tool for production oriented welfare analysis.

In Section 2.5, we also noted the possibility of normalizing the prices based on cost, revenue, or profit aggregates. At the aggregate level, we may use information about the aggregate cost, revenue, or profit of the industry, even if firm-specific cost, revenue, or profit data are unavailable. For example, if the total turnover of the industry is known to be $\mathcal{R}$, we may specify the direction vector as $\mathrm{g}^x = 0$ and $\mathrm{g}^y = (\mathfrak{Y}_1/\mathcal{R}, \mathfrak{Y}_2/\mathcal{R}, \ldots, \mathfrak{Y}_s/\mathcal{R})$. The resulting price normalization implies that the profit inefficiency and the shadow price estimates are expressed in the same currency units as the industry turnover.

4. MONTE CARLO SIMULATIONS

4.1 BASE SCENARIO

To assess the goodness of the proposed approaches in estimating total profit loss and the absolute shadow prices, we conducted a series of simulations in a controlled production environment. To make the simulation setting as "realistic" as possible, the data generating process was specified to conform with the most standard assumptions found in the microeconomic theory and econometrics literature. We first introduce a base scenario, which serves as a benchmark case for comparisons with 14 alternative scenarios where we change some of the key parameters of the data generating process.

Our base scenario involves a sample of 100 profit maximizing firms transforming three inputs to a single output using a decreasing returns to scale Cobb-Douglas technology of the form $y = x_1^{0.25} \cdot x_2^{0.25} \cdot x_3^{0.25}$. The true input prices are equal to one, and the output price is equal to three. Allocative inefficiency is modeled in the spirit of Lau and Yotopoulos (1971) by introducing firm-specific, "subjective" price perceptions: firms maximize profit at random prices drawn from a uniform distribution. Input prices were drawn from Uni[0.8, 1.2] and output prices from Uni[2.4, 3.6], with expected values 1 and 3 respectively. The expected prices are interpreted as the "true" prices. Subsequently, the profit maximizing input-output combination was calculated for each firm, using the subjective price perceptions. Finally, the resulting production vector was perturbed by output-oriented technical inefficiency. Random inefficiency terms were drawn from the half-normal distribution |N(0,0.09)|, and the exponent of their negative values were subsequently used to scale down the profit maximizing output level. This procedure generates a sample of firms operating under the assumed economic hypotheses and standard technology, subject to both technical and allocative inefficiency.

Recall that the presented approaches necessitate that the price of at least one input or output is given ex ante (e.g. taken as a numeraire). The more correct information we have about the prices, the more accurate the result. We therefore examine how the information about the true prices influences the estimated levels of unknown prices and the industry profit inefficiency. In the base scenario, we consider all possible price normalizations and their combinations when 1, 2, 3, or 4 prices are known, 15 combinations in total. In addition, we also consider a price normalization that is based on the knowledge of the total costs of the industry (see Sections 2.5 and 3.3 for discussion). Instead of fixing any specific price, this normalization requires that the total industry costs at the estimated shadow prices must be equal to the known total industry costs. In the simulations, the total cost of the industry varies randomly from one simulation to another. To implement the total cost normalization, we imposed the following price constraint:

$$\sum_{i=1}^{3} \mathfrak{X}_i w_i = \sum_{i=1}^{3} \mathfrak{X}_i 1$$

In this equation, $\mathfrak{X}_i$ denotes the observed total use of input *i* in the industry and w_i are the prices to be estimated. Next, the number 1 on the right hand side refers to the 'true' price, which in this case is one for each input. Hence, the cost normalization constraint imposes that the sum on the left-hand side of the equality, which represents the total cost of the industry at the 'estimated' prices, must be equal to the total cost of the industry at the 'true' prices, which appears on the right-hand side.

One hundred replications were performed for all these different model variants. Inefficiency estimates were calculated by using the formulas given in Section 3.2.[10] The results are reported in Table 1 below. In that table, the first column indicates which prices are ex ante known in each model variant. The second column reports the average shadow price estimates, the third column the average estimated industry-level profit inefficiency (profit loss) and column four the percentage of the counted average of profit loss relative to the true

[10] For the estimation of profit losses and shadow prices, we could use either the Bottom-Up or the Top-Down approach presented in Section 3.2. However, the Top-Down approach is computationally less demanding.

profit loss. The estimated profit loss of the industry is calculated as in Equation (19) or (21). The true profit loss is calculated by subtracting the profit of simulated output and input combinations at 'true' prices from the maximal profit of optimal output and input combination at 'true' prices. The standard deviations of the estimates are reported between brackets.

Table 1: Results of the base scenario

Known prices		Shadow prices				Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2	w_3		
1 prices known	p=3		0.91	0.94	0.91	27.47	86%
			(0.40)	(0.42)	(0.43)	(2.72)	(4%)
	w_1=1	0.87		0.00	0.00	12.50	39%
		(0.11)		(0.00)	(0.00)	(1.43)	(3%)
	w_2=1	0.86	0.00		0.00	12.37	39%
		(0.11)	(0.00)		(0.00)	(1.26)	(3%)
	w_3=1	0.86	0.00	0.00		12.53	39%
		(0.12)	(0.00)	(0.00)		(1.30)	(3%)
2 prices known	p=3,w_1=1			0.88	0.89	28.07	88%
				(0.35)	(0.36)	(2.67)	(4%)
	p=3,w_2=1		0.88		0.91	28.18	88%
			(0.32)		(0.35)	(2.57)	(4%)
	p=3,w_3=1		0.88	0.90		28.31	89%
			(0.29)	(0.31)		(2.64)	(4%)
	w_1=w_2=1	1.87			0.00	21.63	68%
		(0.19)			(0.00)	(2.30)	(5%)
	w_1=w_3=1	1.88		0.00		21.89	69%
		(0.19)		(0.02)		(2.30)	(4%)
	w_2=w_3=1	1.84	0.00			21.74	68%
		(0.18)	(0.01)			(2.12)	(4%)
3 prices known	p=3,w_1=w_2=1				0.84	28.91	91%
					(0.14)	(2.67)	(4%)
	p=3,w_1=w_3=1			0.81		28.91	91%
				(0.17)		(2.74)	(4%)
	p=3,w_2=w_3=1		0.82			28.96	91%
			(0.16)			(2.67)	(4%)
	w_1=w_2=w_3=1	2.92				29.57	93%
		(0.18)				(2.83)	(3%)
4 prices known	p=3,w_1=w_2=w_3=1					30.20	95%
						(2.78)	(3%)
Industry costs known		2.97	0.95	1.06	0.99	28.56	89%
		(0.17)	(0.41)	(0.41)	(0.49)	(2.89)	(3%)

A main conclusion is that the results are heavily dependent on the amount of prior price information that is available. In particular, the knowledge of the output price proves critical in this base scenario. Note that every model variant where the output price is known yields a profit loss estimate that accounts for at least 86 percent of the true loss. The situation is very different when only one or two input prices are known. In all these model variants, the price estimates of the other inputs tend to zero and, as a result, the overall profit loss is under-estimated. When three of the prices are known, it does not make difference whether or not we know all the input prices or only two input prices and the output price. Finally, knowing the fourth price improves the result only slightly.

The bottom row of Table 1 reports the results obtained by normalizing the input prices to match with the total costs of the industry. We find that the average price estimates are fairly good both for the output and inputs, but the standard deviations are large. This model variant is capable of accounting for 89% percent of the total profit inefficiency on average.

Observe that the input prices were under-estimated in all model variants, except for the last one where the total industry costs are constrained to their true level. In general, the lower the input prices, the higher the firm profits. Thus, this downward bias in input prices seems to be a general feature of these model variants. Imposing constraints on the total industry costs could help to alleviate this bias. Even if the exact total costs are unknown, it may be possible to estimate a meaningful lower bound.

We next consider twelve alternative scenarios where we change some of the essential aspects of above base scenario so as to investigate the effects of the sample size, dimensionality, the levels of technical and allocative efficiency, curvature of the frontier, and stochastic noise on the goodness of the price and efficiency estimates. We resample random prices and inefficiencies for each scenario (holding all other model characteristics constant).

4.2 Effect of the sample size

Consider first the effect of sample size. It has been shown that DEA efficiency estimates converge to their true values when the sample size approaches infinity if the model assumptions are correct (see e.g., Kneip et al. 1998, Simar and Wilson 2000b). By imposing a more stringent assumption about the LoOP, the present models are less vulnerable to the curse of dimensionality than the standard DEA, and should thus exhibit faster rates of convergence. The purpose of this sub-section is to investigate how our simulation results change if the sample size is smaller or larger than the 100 observations assumed in the base scenario.

Table 2 reports the results of the small-sample scenario where the sample size is reduced by 50 percent (which obtains 50 observations in the new sample). Note that the total profit loss of the sample reduces accordingly. For brevity, we henceforth only report five representative model variants; the results of the other variants generally conform with those reported here.

As compared to the base scenario, the industry level inefficiency estimates become slightly more inaccurate in the small-sample scenario. The model variant where the output price is known accounts for about 82 percent of the total profit loss. Those model variants where only one input price is known perform almost equally well (or poorly) as in the base scenario. Note that even the model variant that contains all relevant price information leaves eight percent of the profit loss unaccounted for due to the small sample error. There are no clear differences between the shadow prices that apply to the base and small-sample scenarios. However, one should note that the estimates are more volatile for the small sample case, which is reflected by the higher standard deviations associated with all model variants.

Table 2: Results of the small-sample scenario (N=50)

Known prices		Shadow prices				Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2	w_3		
1 price known	p=3		0.95	0.98	0.82	12.95	82%
			(0.54)	(0.56)	(0.52)	(1.55)	(7%)
	w_1=1	0.89		0.00	0.00	5.82	37%
		(0.13)		(0.00)	(0.00)	(0.76)	(4%)
2 prices known	p=3,w_1=1			0.96	0.82	13.37	85%
				(0.44)	(0.47)	(1.58)	(7%)
4 prices known	p=3,w_1=w_2=w_3=1					14.59	92%
						(1.65)	(5%)
Industry costs known		2.92	0.99	1.05	0.96	13.36	84%
		(0.22)	(0.60)	(0.61)	(0.52)	(1.66)	(6%)

For comparison, we also consider a "large-sample" scenario, where the sample size is doubled from the base scenario to 200 observations. In this case, the true profit loss nearly doubles. The results of this scenario are reported in Table 3. We observe that the shadow prices become generally somewhat closer to the underlying true values than in the base scenario. Still, an even more important finding is that the associated standard deviations are considerably lower than for the original sample. This means that simulation replications become more consistent with each other, when the sample size in increased. The results for the profit inefficiency estimates are also slightly more accurate for the large-sample scenario. Model variants where the output price or the industry cost is known explain over 90 percent of the true profit loss and also other model variants produce better results when compared to the base scenario. However, note that even under perfect price information the sample estimate of the profit loss fails to account for 4 percent of the true profit loss. The main reason is the small-sample bias; DEA estimates tend to converge quite slowly to the true efficiency values.[11] In other words, findings are consistent with the general result that, although the sample bias is less problematic for larger samples, the bias generally disappears only asymptotically. Therefore, a very large data set is needed to get reliable and accurate estimates, which is usually impossible in empirical applications. However, if the available sample is small, one can always use the bootstrap methods suggested by Simar and Wilson (1998, 2000a) to calculate (small-sample) bias-corrected estimates and confidence intervals. These methods also allow for drawing statistical inference regarding both the firm and industry level inefficiency estimates. As a general conclusion, it seems recommendable to combine the presented framework with the bootstrap methods in empirical (especially small sample) applications.

[11] See Kneip et al. (1998) on the convergence of DEA estimators.

Table 3: Results of the large-sample scenario (N=200)

Known prices		Shadow prices				Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2	w_3		
1 price known	p=3		0.95	0.93	0.94	58.20	91%
			(0.25)	(0.24)	(0.25)	(3.61)	(2%)
	w_1=1	0.83		0.00	0.00	26.25	41%
		(0.09)		(0.01)	(0.00)	(1.97)	3%)
2 prices known	p=3,w_1=1			0.91	0.91	59.08	93%
				(0.20)	(0.21)	(3.60)	2%)
4 prices known	p=3,w_1=w_2=w_3=1					61.31	96%
						(3.59)	(2%)
Industry costs known		2.91	1.02	0.98	0.99	59.10	93%
		(0.10)	(0.26)	(0.23)	(0.23)	(3.67)	(2%)

4.3 EFFECT OF THE DIMENSIONALITY

Besides the sample size, another important factor influencing the rate of convergence is the number of input-output dimensions. We next consider a scenario where one of the inputs is excluded from the base scenario, and a scenario where another output is added to the base scenario. In the first scenario we simply drop out input 3 and the associated parameter, keeping the other parameters of the production function the same as in the base scenario. This also decreases the scale elasticity of the technology from 0.75 to 0.5. Table 4 reports the results of the five representative model variants. The construction of the three-input two-output model is more challenging, because it is difficult to generalize the Cobb-Douglas technology to a multi-input multi-output setting. For simplicity, we assume outputs to be perfect substitutes (marginal rate of transformation equals one), and apply the same prices for both outputs. This allows us to apply the base scenario and to partition the output in two distinct output vectors in a randomized fashion. The results of this multi-input multi-output scenario are reported in Table 5.

Table 4: Results of the two-input single-output scenario

Known prices		Shadow prices			Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2		
1 price known	p=3		0.95	0.98	44.47	93%
			(0.27)	(0.29)	(3.29)	(3%)
	w_1=1	0.90		0.00	22.25	46%
		(0.28)		(0.00)	(1.84)	(4%)
2 prices known	p=3,w_1=1			0.93	45.28	94%
				(0.16)	(3.28)	(3%)
3 prices known	p=3,w_1=w_2=1				46.03	96%
					(3.37)	(3%)
Industry costs known		2.10	1.02	0.98	39.15	82%
		(0.44)	(0.41)	(0.42)	(3.48)	(5%)

Table 5: Results of the three-input two-output scenario

Known prices		Shadow prices					Estimated industry profit inefficiency	% of true industry profit inefficiency
		p_1	p_2	w_1	w_2	w_3		
1 price known	p_1=3		2.91 (0.20)	0.92 (0.46)	0.95 (0.42)	0.85 (0.39)	26.65 (2.52)	84% (5%)
	w_1=1	0.87 (0.11)	0.87 (0.11)		0.00 (0.01)	0.00 (0.00)	12.07 (1.32)	38% (3%)
2 prices known	p_1=3,w_1=1		2.91 (0.16)		0.91 (0.34)	0.83 (0.35)	27.34 (2.59)	86% (5%)
5 prices known	p_1=p_2=3, w_1=w_2=w_3=1						30.07 (2.69)	94% (3%)
Industry costs known		2.95 (0.16)	2.94 (0.19)	0.98 (0.43)	1.03 (0.41)	0.98 (0.43)	27.81 (2.67)	87% (4%)

The results of the two-input single output scenario are virtually as expected. For the first four model variants, both shadow prices and profit loss estimates are on average closer to their true values than the corresponding estimates of the base scenario. The model variants where the output price is known capture over 93 percent of the true profit loss. However, for the last model variant the estimates are more inaccurate in the two-input single-output case than in the base scenario. The latter result can –at least partly- be explained by the low scale elasticity in the two-input model.

For the three-input two-output scenario the performance of the model variants is relatively good considering the multi-dimensionality of the setting. As before, the model variants where the price of output 1 or the total costs are known yield relatively good results, capturing at least 84 percent of the total inefficiency at the industry level. In fact, the simulation results for all model variants are almost as good as for the base scenario. Still, it should be noted that this can also be a consequence of our simplified assumption regarding the perfectly substitutable outputs.

A general conclusion drawn from the results of this section is that the method seems to perform better on average when the number of relevant inputs and outputs diminish. Therefore, a large number of inputs and outputs (as compared to the sample size) could provide yet another motivation for using bootstrap methods in practical applications.

4.4 Effect of technical efficiency level

We next consider the influence of the degree of technical efficiency of the different observations in the sample under evaluation. Recall that the base scenario assumes inefficiency levels that are drawn from the half-normal distribution |N(0,0.09)|. The following exercises modify this technical efficiency distribution by varying its variance term. We specifically consider two scenarios. The first involves low technical efficiency, which is implemented by doubling the standard deviation of the inefficiency distribution. Whereas the base scenario involved the average Farrell output efficiency of 0.787, the low efficiency scenario has average Farrell output efficiency of 0.619. The second involves high technical efficiency, which we incorporate by halving the standard deviation of the original inefficiency distribution. The resulting average Farrell output efficiency is 0.887.

Tables 6 and 7 report the results of the five representative model variants in the high and low technical efficiency scenarios, respectively. The percentage estimates of the true industry profit inefficiency in the low technical efficiency scenario (see the last column of Table 6) are similar to the corresponding figures of base scenario. However, the shadow prices are (on average) somewhat lower and standard deviations of shadow prices are notably higher than in the base scenario. As for the high technical efficiency scenario, the percentage estimates of the true industry profit inefficiency (see the last column of Table 7) are slightly lower as compared to base scenario and the low technical efficiency scenario. However, these differences are not very important, the result is even reversed for the second model variant. By contrast, when considering the shadow prices the model performs systemically better in the high technical efficiency scenario than in the low technical efficiency and base scenario, as (1) the shadow prices are closer to the true prices and (2) the associated standard deviations (of shadow prices) are substantially lower. Thus, these results suggest that absolute prices can be estimated more accurately when technical efficiency level is high. Instead, the technical efficiency level does not seem to affect the estimation accuracy of total inefficiency (when gauged as a percentage of the true profit inefficiency). From that perspective, it appears that the model is quite well capable of detecting technical inefficiencies. The next section investigates its vulnerability to allocative inefficiencies.

Table 6: Results of the low technical efficiency scenario (average output efficiency: 0.619)

Known prices		Shadow prices				Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2	w_3		
1 price known	$p=3$		0.87 (0.51)	0.94 (0.59)	0.89 (0.56)	45.05 (4.53)	87% (4%)
	$w_1=1$	0.80 (0.14)		0.00 (0.00)	0.00 (0.00)	17.24 (1.63)	33% (2%)
2 prices known	$p=3,w_1=1$			0.85 (0.53)	0.86 (0.53)	45.93 (4.67)	89% (4%)
4 prices known	$p=3,w_1=w_2=w_3=1$					49.13 (4.65)	95% (3%)
Industry costs known		2.74 (0.36)	0.99 (0.58)	1.17 (0.63)	0.85 (0.65)	45.79 (4.38)	89% (4%)

Table 7: Results of the high technical efficiency scenario (average output efficiency: 0.887)

Known prices		Shadow prices				Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2	w_3		
1 price known	p=3		0.92	0.96	0.92	16.31	84%
			(0.25)	(0.27)	(0.28)	(1.68)	(4%)
	w_1=1	0.99		0.05	0.04	9.46	49%
		(0.23)		(0.14)	(0.11)	(1.26)	(5%)
2 prices known	p=3,w_1=1			0.91	0.91	16.68	86%
				(0.22)	(0.23)	(1.72)	(4%)
4 prices known	p=3,w_1=w_2=w_3=1					18.14	94%
						(1.74)	(3%)
Industry costs known		3.07	0.99	1.05	0.96	17.07	88%
		(0.13)	(0.28)	(0.28)	(0.28)	(1.84)	(4%)

4.5 EFFECT OF ALLOCATIVE EFFICIENCY LEVEL

In light of the previous findings, it is interesting to test how the model behaves when we alter the allocative efficiency levels. Recall that allocative inefficiencies were modeled in the base scenario by drawing firms' subjective price perceptions from the uniform distributions: Uni[0.8, 1.2] for inputs and Uni[2.4, 3.6] for outputs. We next consider two alternative scenarios involving higher and lower levels of allocative efficiency. In the high allocative efficiency scenario, we draw the firms' subjective prices from Uni[0.95, 1.05] for inputs and Uni[2.85, 3.15] for outputs, enforcing the subjective price perceptions closer to the true prices. In the low allocative efficiency scenario we expand the price ranges by drawing subjective prices from Uni[0.5, 1.5] for inputs and Uni[1.5, 4.5] for outputs.

The results of the five representative model variants are reported in Tables 8 and 9 for the high and low allocative efficiency scenarios, respectively. When considering industry profit inefficiency, the high allocative efficiency scenario seems to yield more accurate estimates, although this is not true for all model variants. For the second and fifth model variants, the estimates corresponding to the low allocative efficiency scenario are more accurate. Still, model variants where only a single input price is known systematically under-estimate the profit loss in every scenario, and the level of allocative efficiency does not make such a radical difference for the fifth model variant. The results concerning the shadow prices are somewhat mixed. In many model variants the shadow prices are on average more accurately estimated for the high allocative efficiency scenario, except for the second and fifth model variants. In addition, the standard deviations associated with the shadow prices are substantially higher for the high allocative efficiency scenario.

In conclusion, the estimates seem to be relatively sensitive to the degree of allocative inefficiency. High allocative efficiency is generally desirable, but symmetric deviations from the true prices can counter-balance each other. In our simulations, large allocative inefficiencies combined with technical inefficiencies can create large asymmetric inefficiencies, which in turn distracts our model predictions.

Table 8: Results of the high allocative efficiency scenario

Known prices		Shadow prices				Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2	w_3		
1 price known	p=3		0.95	1.04	0.92	24.18	93%
			(0.81)	(0.81)	(0.76)	(1.82)	(3%)
	w_1=1	0.39		0.00	0.00	6.92	27%
		(0.24)		(0.00)	(0.00)	(0.63)	(3%)
2 prices known	p=3,w_1=1			1.01	0.89	24.52	95%
				(0.62)	(0.63)	(1.84)	(3%)
4 prices known	p=3,w_1=w_2=w_3=1					25.87	97%
						(1.85)	(2%)
Industry costs known		1.29	0.96	1.16	0.88	19.32	75%
		(0.63)	(1.09)	(1.05)	(1.03)	(1.88)	(7%)

Table 9: Results of the low allocative efficiency scenario

Known prices		Shadow prices				Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2	w_3		
1 price known	p=3		0.73	0.73	0.77	62.60	66%
			(0.22)	(0.26)	(0.24)	(10.24)	(5%)
	w_1=1	0.98		0.05	0.04	41.62	44%
		(0.26)		(0.11)	(0.08)	(8.11)	(4%)
2 prices known	p=3,w_1=1			0.65	0.67	66.27	70%
				(0.24)	(0.23)	(11.64)	(5%)
4 prices known	p=3,w_1=w_2=w_3=1					91.03	95%
						(16.71)	(2%)
Industry costs known		3.64	0.98	1.01	1.02	80.24	84%
		(0.26)	(0.30)	(0.33)	(0.30)	(15.05)	(4%)

4.6 EFFECT OF CURVATURE OF THE FRONTIER

Obviously, the shape of the production possibility frontier also influences the goodness of the model. For a linear technology, the allocative inefficiencies would not matter and the model would likely yield highly accurate results. In the next two scenarios we vary the parameters of the production function to investigate how the curvature of the frontier affects the results. Recall that in the base scenario the production function was of the form $y = x_1^{0.25} \cdot x_2^{0.25} \cdot x_3^{0.25}$, which implies a scale elasticity of 0.75. In the following scenarios we vary the technology parameters preserving the symmetry of inputs. The first, high scale elasticity scenario assumes the production function $y = x_1^{0.3} \cdot x_2^{0.3} \cdot x_3^{0.3}$, with a scale elasticity of 0.9. This technology comes closer to the constant returns to scale situation,[12] and the frontier becomes more linear compared to the base scenario. The second, low scale elasticity scenario halves the technology parameters by assuming the production function $y = x_1^{0.15} \cdot x_2^{0.15} \cdot x_3^{0.15}$,

[12] We remark that the Cobb-Douglas technology must exhibit decreasing returns to scale to guarantee a finite profit maximum
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with a scale elasticity of 0.45. This results as a more curved frontier. The results of the five representative model variants in these two scenarios are reported in Tables 10 and 11, respectively.

As a general feature, the standard deviations of the shadow price estimates increase when the scale elasticity decreases. For the model variants where all the input prices or the total industry costs are known, the high scale elasticity scenario with a less curved frontier yields average shadow prices closer to their true values than the base or the low scale elasticity scenarios. By contrast, for the model variants where the output price is known, the low scale elasticity scenario produces average results that deviate less from the true prices and the true profit loss of the industry.

Table 10: Results of the high scale elasticity (0.9) scenario

Known prices		Shadow prices				Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2	w_3		
1 price known	p=3		0.91 (0.38)	0.86 (0.41)	0.89 (0.42)	49.91 (9.33)	75% (5%)
	w_1=1	0.99 (0.09)		0.00 (0.02)	0.00 (0.02)	25.37 (5.03)	38% (45)
2 prices known	p=3,w_1=1			0.82 (0.40)	0.84 (0.39)	51.37 (9.62)	77% (5%)
4 prices known	p=3,w_1=w_2=w_3=1					64.31 (12.16)	97% (2%)
Industry costs known		3.32 (0.12)	1.03 (0.43)	0.97 (0.48)	1.00 (0.47)	56.04 (11.03)	84% (4%)

Table 11: Results of the low scale elasticity (0.45) scenario

Known prices		Shadow prices				Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2	w_3		
1 price known	p=3		0.94 (0.51)	0.92 (0.51)	0.92 (0.50)	30.04 (2.37)	90% (3%)
	w_1=1	0.37 (0.22)		0.00 (0.00)	0.00 (0.00)	9.39 (0.76)	28% (2%)
2 prices known	p=3,w_1=1			0.95 (0.45)	0.86 (0.43)	30.56 (2.41)	92% (3%)
4 prices known	p=3,w_1=w_2=w_3=1					31.70 (2.17)	96% (3%)
Industry costs known		1.33 (0.56)	0.93 (0.72)	1.17 (0.66)	0.91 (0.73)	24.16 (2.10)	73% (6%)

4.7 EFFECT OF ERRORS IN VARIABLES

We complete the simulation study by investigating robustness of the model results to stochastic noise in the data. In principle, the model is strictly deterministic and does not account for the possibility of errors in the input-output data. This could be seen as a disadvantage of the method – and the nonparametric approach as a whole – as empirical production data is usually subject to some noise. Therefore, it is also interesting to investigate how the model behaves if errors are introduced into the input-output data.

This section considers four different scenarios. In the first scenario, the output data is perturbed by adding a random noise term drawn from the normal distribution N(0,0.0025). The second scenario is characterized by errors in all variables, which is implemented by drawing a random noise term from this same distribution for all input and output data. We label these two scenarios as 'low noise-level' scenarios. We subsequently build up two 'high noise-level' scenarios, where the standard deviation of the noise term amounts to 0.15, i.e., three times the standard deviation that applies to the low noise-level scenarios. Analogously as for these low noise-level scenarios, we perturb (only) the output data in the third scenario while a random noise term is added to all input and output data in the fourth scenario.

The results of the five representative model variants corresponding to these four scenarios are reported in Tables 12-15. The results offer a useful reminder that the nonparametric models should not necessarily underestimate the true efficiency if the data is subject to errors. In fact, all cases overestimate overall profit loss, except for the model where a single input price is known. For the low noise-level scenarios the estimates are yet relatively close to the true profit loss. However, the price estimates are volatile, as the standard deviations are high as compared to the average estimates. For the high noise-level scenario, the results are yet less encouraging; the different model variants substantially under- as well as over-estimate the true industry inefficiency. The systematic over-estimation is due to the fact that the noise terms randomly increase the profit for some firms while decreasing it for others; and our model wrongly qualifies these bogus profit differences as inefficiency. Observe, finally, that the model variant that assumes full knowledge of prices presents the greatest over-estimation of the profit loss in all scenarios.

Table 12: Results of the low noise-level scenario; noise in output

Known prices		Shadow prices				Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2	w_3		
1 price known	p=3		1.02 (0.61)	0.83 (0.62)	1.03 (0.71)	32.68 (3.34)	103% (8%)
	w_1=1	0.83 (0.12)		0.00 (0.00)	0.00 (0.00)	13.04 (1.49)	41% (4%)
2 prices known	p=3,w_1=1			1.03 (0.60)	0.86 (0.62)	34.13 (3.78)	107% (10%)
4 prices known	p=3,w_1=w_2=w_3=1					37.46 (4.70)	118% (13%)
Industry costs known		2.73 (0.22)	0.95 (0.60)	1.00 (0.60)	1.05 (0.60)	32.07 (2.98)	101% (7%)

Table 13: Results of the low noise-level scenario; noise in output and in all inputs

Known prices		Shadow prices				Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2	w_3		
1 price known	p=3		1.01	0.83	1.03		107%
			(0.71)	(0.67)	(0.74)	34.03 (3.97)	(10%)
	w_1=1	0.82		0.00	0.00		43%
		(0.13)		(0.01)	(0.01)	13.59 (1.52)	(5%)
2 prices known	p=3,w_1=1		1.00	0.88			112%
			(0.63)	(0.64)		35.57 (4.42)	(13%)
4 prices known	p=3,w_1=w_2=w_3=1						123%
						39.20 (5.46)	(16%)
Industry costs known		2.71	0.97	1.03	1.00		103%
		(0.25)	(0.63)	(0.67)	(0.60)	32.08 (2.97)	(7%)

Table 14: Results of the high noise-level scenario; noise in output

Known prices		Shadow prices				Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2	w_3		
1 price known	p=3		1.12	0.97	1.20		177%
			(1.20)	(1.05)	(1.24)	56.32 (8.75)	(29%)
	w_1=1	0.64		0.00	0.00		52%
		(0.17)		(0.00)	(0.00)	16.65 (1.85)	(6%)
2 prices known	p=3,w_1=1			1.34	1.00		186%
				(1.05)	(1.04)	58.97 (9.53)	(31%)
4 prices known	p=3,w_1=w_2=w_3=1					71.71	226%
						(17.32)	(57%)
Industry costs known		2.13	0.98	0.92	1.10		141%
		(0.38)	(0.99)	(0.96)	(1.01)	44.92 (4.74)	(15%)

Table 15: Results of the high noise-level scenario; noise in output and in all inputs

Known prices		Shadow prices				Estimated industry profit inefficiency	% of true industry profit inefficiency
		p	w_1	w_2	w_3		
1 price known	p=3		1.05	1.02	1.05	60.91	192%
			(0.98)	(1.02)	(1.02)	(11.58)	(37%)
	w_1=1	0.57		0.00	0.00		60%
		(0.17)		(0.00)	(0.01)	19.01 (1.95)	(7%)
2 prices known	p=3,w_1=1			1.14	0.99	63.89	201%
				(0.91)	(0.99)	(12.28)	(40%)
4 prices known	p=3,w_1=w_2=w_3=1					76.13	240%
						(19.32)	(64%)
Industry costs known		2.02	1.10	1.13	1.12		152%
		(0.43)	(0.86)	(0.86)	(1.01)	48.31 (5.14)	(15%)

As a final note, It is worth indicating that the error structure assumed in these two scenarios would be harsh for any efficiency assessment method, because the error variance is relatively high even for the so-called 'low-noise' scenario. This certainly applies to the scenarios where all input and output variables are perturbed by errors. For example, the parametric stochastic frontier models usually assume a single error term associated with the output. Naturally, even these models will likely fail to account for such a multi-dimensional error structure.

4.8 CONCLUSIONS

The simulation evidence suggests that the results can be sensitive to the *a priori* price information that is available. It seems that the model sometimes provides reasonably accurate results, while it may yield rather poor estimates in other situations. Fortunately, the simulations suggest a simple but effective diagnostic test for the goodness of the model. It appears that the industry-level inefficiency is estimated relatively accurately when all (or most) estimated prices are significantly greater than zero, whereas scenarios involving many zero-valued price estimates always yielded considerable underestimation of the industry profit loss. In this respect, we should note that the models that yield unrealistic price estimates (i.e., many zero prices) can still be useful if one is mainly interested in a conservative estimate for the industry profit loss.

In all scenarios, the results were particularly sensitive to the prior knowledge of the output price. In our simulations, the output variable plays a special role at least for four reasons: 1) the true level of the output price is higher than those of the input prices, 2) the range of the random shadow prices (and their variance) is higher, 3) technical inefficiency is modeled as output oriented, and 4) there are more inputs than outputs in all scenarios. All these features make the output price a more critical factor for the profit efficiency assessment than the prices of the input factors. This suggests that it is generally recommendable to identify which prices are most critical for the profit estimates in practical applications, and subsequently -if possible- estimate the associated prices prior to the actual efficiency analysis.

We also observed that model variants that fixed one or more prices directly tended to under-estimate the input prices (see e.g. the base scenario). It is therefore worthwhile to impose some further price constraints, which can be implemented at the firm level as well as at the aggregate level. For example, the fifth model variant that was considered in all scenarios only imposed the constraint that the total cost of the industry takes a specific (true) value. Alternatively, one could impose some upper bound for the total cost level using an inequality constraint. Finally, it may sometimes be meaningful to assume that every observed firm, or the industry as a whole, is making positive profit. Generally, it is advisable to apply this kind of constraints whenever possible, so as to guide price estimates further towards their correct levels.

A general observation, which applies to all scenarios that we have studied, is that the results showed potential vulnerability of our model to small-sample bias, which is a typical problem of the DEA approach. In fact, at least some of the results in our simulation experiment could probably be explained by the small-sample bias, because almost all scenarios were based on a relatively small sample (consisting of only 100 firms). Therefore, as the DEA estimates generally converge slowly to their true values, it seems advisable to use the sample-bias correcting bootstrap methods suggested by Simar and Wilson (1998, 2000a), at least in applications that involve a relatively small sample. By using these methods, one can calculate bias-corrected efficiency estimates (both at the firm and industry level) that take into account the suggested price normalizations and LoOP-constraints. In addition, bootstrap methods enable statistical inference regarding these (firm and industry level) efficiency estimates.

Finally, the simulations suggest that both the shadow prices and inefficiency estimates can be sensitive to errors in the input-output data. Errors can create bogus inefficiencies, and lead to severe over-estimation of the actual profit losses. Unfortunately, there are no simple, straightforward remedies for dealing with noisy data. If the error structure is unknown, also bootstrap methods are likely to fail. It therefore seems recommendable to refrain from using this method when sufficient reliability of the input-output data cannot be assured. However, if there is *a priori* information that suggests a particular error structure, and one can derive a specific probability distribution for the noise terms, then it could be possible to draw statistical inference by combining our method with a correspondingly defined parametric bootstrap approach. A more detailed treatment of such an extension falls beyond the scope of the current study, whence we leave the dealing with noisy data in this framework as an –admittedly interesting- avenue for future research.

5. Concluding Remarks

We have explored the nonparametric approach to profit efficiency analysis both at the firm and industry levels when information about production technology and economic prices is unavailable. We built on the directional distance function framework (Chambers et al., 1996, 1998) for the measurement of profit efficiency, and utilized traditional activity analysis (or data envelopment analysis) techniques (Koopmans, 1951a, Farrell, 1957, Afriat, 1972) for estimating the production technology and the associated shadow prices.

The novel contributions of this paper are two-fold:

1) We showed how the direction vector can be specified to facilitate absolute price normalizations that enable one to interpret the measured profit inefficiency as well as shadow prices directly in monetary units. Such a price normalization can be based on information about one or more input-output prices, or the cost, revenue, or profit aggregates. The suggested price normalizations constitute a useful practical addition to the directional distance function framework.

2) We showed how to ensure consistency between the firm and industry level profit efficiency measures by applying uniform shadow prices across all firms. Our contribution can be seen as an extension of some recent work (cited in Section 3) on the aggregation of technical efficiency indices and cost efficiency indices to the context of profit efficiency analysis. We believe the present framework is best suited for aggregation because the profit function exhibits the required linear structure by construction.

We assessed the goodness of the suggested methodology for estimating industry profit losses and absolute shadow prices in a controlled production environment by means of Monte Carlo simulations. More specifically, we have investigated how the sample size, the dimensionality of the input-output space, the degree of technical and allocative inefficiency, and the presence of stochastic noise influence the performance of the method. Our results underline the critical role of the direction vector specification (or price normalization). More specifically, the price normalizations based on the knowledge of the output price or the total cost of the industry provided reasonable profit inefficiency and price estimates throughout all scenarios, whereas the price normalizations based on a single input price yielded rather disappointingly poor estimates. We argued that this asymmetry in the 'usefulness' of input and output price information reflects the critical role of the output for the profit (efficiency) estimates in our specific simulation set-up (which included a single output and three inputs). More generally, our simulation results suggest it is advisable to use cost, revenue, or profit aggregates rather than individual commodity prices as a basis of absolute price normalizations.

The present paper has focused exclusively on profit efficiency analysis in a static setting. In future research, it would be interesting to extend our efficiency measurement framework to dynamic productivity and profitability analysis. In particular, future research could adapt the absolute price normalizations and the uniform price constraints proposed here to the context of Luenberger- or Bennet-Bowley type productivity indicators (Balk, 1998; Chambers, 2002).

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