Asset Price Dynamics by Economic Forces

Estola, Matti
Hokkanen, Veli-Matti

ISBN 952-458-706-8
ISSN 1795-7885
no 26

Asset Price Dynamics by Economic Forces

Matti Estola
University of Joensuu, Dept of Business and Economics
P.O. Box 111, FIN-80101 Joensuu, Finland
Tel: +358-13-2514587; E-mail: matti.estola@joensuu.fi

Veli-Matti Hokkanen
University of Jyväskylä, Department of Mathematics
P.O. Box 35 FIN-40351 Jyväskylä, Finland
Tel:+358-14-602700; E-mail: vmho@math.jyu.fi

9th June 2005

Abstract

We introduce a model describing the evolution of an asset price. Individual investors in the stock market are assumed to use a subjective set of information in making their expectations of the future asset price. Security trading takes place according to investors' expectations of the rate of return of investing in this asset, and the asset price changes in the perfectly competed stock market according to excess demand or supply. We define the 'force' which acts upon the asset price and show that the adjustment may be stable or unstable. The possible equilibrium asset price is conditional on the distribution of expectations of individual investors.[1] (JEL G12).

Keywords: Investor, asset price, stochastic dynamics.[2]

[1]We are grateful to Pentti Suomela and two anonymous referees for helpful comments. Remaining errors are naturally on our responsibility.

[2]The corresponding author is: Matti Estola, University of Joensuu, Dept of Business and Economics, P.O. Box 111, FIN-80101 Joensuu, Finland. Tel: +358-13-2514587; E-mail: matti.estola@joensuu.fi

1 Introduction

The concept of equilibrium used in economics was borrowed from physics and introduced in economics by Canard in 1801 (Mirowski 1989). Although equilibrium means a balance of forces situation, in economics the balancing 'forces' in various situations have not been defined. In order to efficiently exploit the concept of equilibrium, however, we should be able to distinguish whether the equilibrium is stable or not, and to understand the adjustment process, we should define the forces which either'push' economic quantities toward their equilibrium values or cause their evolution with time.

The use of the term 'force' as the cause of a change in an economy is old. For example, the 'invisible hand' of Adam Smith can be seen as a force field; a reason which 'pushes' an economy toward its equilibrium state. In physics, the existence of the (invisible) gravitational force field can be demonstrated by letting an object fall; without the force field the object would not move. We can similarly demonstrate the existence of an (invisible) economic force field by observing that an economic quantity changes, and call the reason for this a 'force'. What remains is to give a meaningful expression for this force.

Static neoclassical theory as a whole is an application of equilibrium analysis. In the theory only equilibrium states are studied, and the adjustment from one equilibrium to another is not modelled even though it is understood in the framework that in a non-equilibrium situation economic agents change their behavior with time to reach their optimum. The equilibrium thinking in economics is so deeply rooted that it is also applied in the modelling of financial phenomena, such as asset prices and exchange rates, which show high fluctuations and where an equilibrium state seldom exists. This kind of evolutionary behavior requires a dynamic framework for modelling. In physics, too, the equilibrium states of various dynamic systems were known much before Newton defined his dynamic laws of mechanics. Inspired by this we introduce a dynamic extension for the equilibrium analysis of asset prices analogous to that Newton gave for classical mechanics, namely, we define the forces which create the motion of asset prices. Cox et al. (1985), too, use this terminology in their introduction: 'Our framework is general enough to include many of the fundamental forces affecting asset markets...'.

Jeremy Bentham (1963) writes: "Nature has placed mankind under the governance of two sovereign masters, pain and pleasure. It is for them to point out what we ought to do, as well as to determine what we shall do. ... They govern us in all we do, in all we say, in all we think." "Although Bentham explicitly states that the pleasure-pain calculus is applicable to what we 'shall' do as well as to what we 'ought' to do, he was primarily interested in 'ought' and did not develop a theory of actual human behavior with many testable implications" (Becker 1976 p. 8). "I am saying that the economic approach provides a valuable unified framework for understanding all human behavior. ... If this argument is correct, the economic approach provides a unified framework for understanding behavior that has long been sought by and eluded Bentham, Comte, Marx, and others" (ibid. p. 14). "Only after long reflection on this work and the rapidly growing body of related work by others did I conclude that the economic approach was applicable to all human behavior" (ibid. p. 8).

We continue in line with Bentham and Becker to search for a basis for modelling economic phenomena. We extend the neoclassical analysis as follows: *Economic agents like to improve their current situation, and in a decision-making situation, a rational economic agent compares the expected benefits and costs from the decision and makes the decision on the basis of which of these 'weighs' more.* This way we get an isomorphism between the mathematical model of the behavior of the lever, and the decision-making of a human being or an organization. The force acting upon a particular economic quantity — the change of which the decision-making concerns — is the difference between the benefits and costs for the decision-maker from the decision. With this assumption, we can model the observed evolution of economies and it gives the static neoclassical theory as a special (Pareto-optimal) case where the evolution ceases. The neoclassical assumption, that economic agents behave optimally, prohibits the modelling of dynamics in that framework, because none likes to change his optimal behavior. This is analogous as in dynamic games where each player is assumed to behave according an improvement algorithm, and in the Nash -equilibrium situation, none of the players likes to change his behavior. The proposed framework is applicable also for the modelling of economic growth, see Estola (2001).

2 Our Framework as Compared with Others

Our assumptions of the stock market behavior are: 1) Only a part of all security-owners and potential investors are active in the market at a particular moment in time with limited funds. 2) Transaction costs are abstracted away, and individual investors are assumed to plan to hold the assets they buy for a finite time. 3) Investors like to buy those securities whose rates of return they expect to exceed the others and the prevailing interest rate, and they like to sell other securities. 4) Asset prices change according to the investors' aggregate net demand. 5) The (rational) expectations of future asset prices of individual investors are conditional on information of factors investors consider relevant for their expectations. 6) The market expectation of a future asset price is a weighted average of those of all investors.

Our analysis deviates from Grossman (1976), Grossman & Stiglitz (1976) and Grossman & Stiglitz (1980) the following ways: 1) We simplify the common modelling of investors' risk-averseness with a concave utility function by introducing a parameter which determines the investor's attitude toward risk. 2) We assume that investors behave rationally both in buying and in selling assets, and both these components of net demand of assets affect their prices. 3) Grossman and Stiglitz (1980) assume that a correct rate of return exists for a risky asset which investors are able to find out in advance by investing enough in information. Our modelling, on the other hand, implies that asset prices change according to investors' buying and selling, which transactions are based on their beliefs of the rates of return of investing in them. The instantaneous 'equilibrium price of an asset' reflects the prevailing beliefs of all investors of the growth rate of this price, and it changes with these beliefs via investors' buying and selling. If some investors were able to know in advance the correct rates of return of risky assets, they should be able to forecast the average investment decisions of other investors in a given time unit and estimate the effects of these decisions on asset prices. John M. Keynes (1964 p. 156) was aware of this when he wrote that the investment behavior might be viewed as a beauty contest, where the judges are not concerned in finding out the prettiest girl, but in finding out who the other judges will vote as the prettiest one.

Merton (1973) assumes that the rates of return of risky assets follow a Wiener process, investors choose their optimal portfolios by maximizing their life-time expected von Neumann-Morgenstern utility functions, and the demands of assets are derived from the first order conditions of the investors' optimization problems. Merton shows that with these assumptions there exists a unique pair of efficient portfolios, one containing only risky assets and the other only the single riskless asset so that in the equilibrium all investors are indifferent between choosing their portfolios from the original n risky assets and the riskless asset or from these two 'funds'. The equilibrium defines unique returns for the risky assets. Cox et al. (1985) show that if the growth of real investments follows a continuous stochastic process and investors allocate their wealth by maximizing their expected lifetime utility, an equilibrium expected rate of return exists for any contingent claim together with the market clearing interest rate and total production and consumption plans. This general equilibrium results as a solution for the investors' dynamic stochastic control problems with budget restrictions. Duffie & Huang (1985) show that there exists dynamic trading strategies for investors which yield a Radner equilibrium allocation for long-lived securities and an equilibrium price process for every asset. Huang (1987) proves that in the equilibrium investors hold a fixed set of mutual funds rather than the market portfolio.

The studies referred have shown that if the returns from risky assets and real production are known to follow certain stochastic processes and investors behave optimally, an optimal investment strategy exists for every investor which yields to equilibrium asset prices. The methodology in these studies is that investors are assumed to maximize a target function with certain constraints, the demands of risky assets are derived from the first order conditions of investors' optimization problems, and an equilibrium in the asset market is formulated by setting the net demand for every asset equal to zero. We highly respect the works of these authors which have stated their rigorous proofs at a very general level. However, there are some aspects in this framework we like to discuss.

Firstly, the assumption that an equilibrium prevails all the time in the asset market requires that every investor continuously recalculates his expected life-time utility with new prices to find his demands for assets. In contrast to this, we assume that investors compare their expectations of growth rates of asset prices and adjust their portfolios when they believe this to be profitable. New information changes investors' expectations of future asset prices, and even with fixed expectations, a change in an asset price changes its expected growth rate. These factors force a revenue-seeking investor to adjust his wealth allocation. Our analysis is in line with Fisher (1983 pp. 9-12): "... I now briefly consider the features that a proper theory of disequilibrium adjustment should have ... if we are to show under what conditions the rational behavior of individual agents drives an economy to equilibrium. ... Such a theory must involve dynamics with adjustment to disequilibrium over time modeled. ...the most satisfactory situation would be one in which the equations of motion of the system permitted an explicit solution with the values of all the variables given as specific, known functions of time. ... Unfortunately, such a closed-form solution is far too much to hope for. ...the theory of the household and the firm must be reformulated and extended where necessary to allow agents to perceive that the economy is not in equilibrium and to act on that perception. ... A convergence theory that is to provide a satisfactory underpinning for equilibrium analysis must be a theory in which the adjustments to disequilibrium made by agents are made optimally."

Secondly, in the works referred the evolution of asset prices is not modelled in real time according to demand and supply as Samuelson (1941) proposed. Rather the equilibrium prices are either solved at every instant of time from a group of net demand equations set equal to zero, or they continue to follow an assumed stochastic process. In this we follow Samuelson.

Thirdly, the described general equilibrium models assume that the aim of investors is to maximize their life-time flow of consumption, and returns from financial assets are used to finance this consumption. However, nowadays the operations in asset markets great enough to affect asset prices are made by financial firms whose goal is to invest their clients' money in the most profitable way taking risks in account. This simplifies the definition for the target functions of investors so that we can omit consumption from them. We can thus study asset market in isolation with goods market by assuming that before their exact consumption plans, consumers fix the amount of money they will buy or sell assets at every time period.

Fourthly, in the models referred the uncertainty in asset prices results from the assumed uncertainties in real or financial returns, which are not modelled. In contrast to this, we model the uncertainty in asset prices by assuming that investors face a subjective stochastic flow of information concerning future asset prices, and they buy or sell assets on the basis of this information. A difference in demand and supply changes asset prices continuously, and in this way the stochastic information flows cause stochastic dynamics in asset prices.

An increase in any asset price decreases its expected growth rate and vice versa. This element in our model explains the observed short-term mean reverting behavior of asset prices; see for instance Lo & MacKinley (1988), Poterba & Summers (1988) or Risager (1998). According to Hull (1989 p. 216), Geometric Brownian Motion is the most widely used model of stock price behavior. We show that this model results from the rational buying and selling behavior of investors under uncertainty.

3 The Behavior of an Individual Investor

In order to keep our model as readable as possible, we assume only two assets in the market: a risky one and a riskless one. We assume that investor i has a fixed amount of money[3] S_i (\$) to be invested in a particular risky asset, or to be invested in another asset with risk-free interest rate[4] $r(t)$. For simplicity, the alternative cost of investing in the risky asset is supposed to be the lost interest returns[5]. Investor i makes his subjective forecast of the future asset price, calculates the expected rate of return from this investment

[3]Measurement units are denoted in braces after the quantities. A system of measurement units for economics is given in de Jong (1967).

[4]The interest rate (the rate of return of money, i.e. the flow of earned money divided by the invested capital) is a dimensional quantity measured in units $(\$/\Delta t)/\$ = 1/\Delta t$, where Δt is the time period the flow is measured, see de Jong (1967).

[5]This assumption is also made in Grossman and Stiglitz (1980). The alternative costs could also be the lost returns from investing into the next profitable expected asset.

and compares this with the risk-free interest rate. The planning function of investor i at time moment t is then

$$R_i(t) = Z_i(t)\left(\frac{\tilde{p}_i(t+\Delta_i t) - p(t)}{p(t)\Delta_i t} - b_i\right) + [S_i - Z_i(t)]r(t), \tag{1}$$

where R_i ($\$/\Delta_i t$) is the expected return flow from period $\Delta_i t$ (the investment period of investor i), Z_i ($\$$) the amount of money investor i invests in the risky asset, $S_i - Z_i$ the money investor i invests with interest rate $r(t)$ ($1/\Delta_i t$), $p(t)$ ($\$/unit$) the unit price of one asset at time moment t and $\tilde{p}_i(t+\Delta_i t)$ the investor's subjective expectation of the asset price at moment $t+\Delta_i t$. Dimensional constant b_i ($1/\Delta_i t$) measures the investor's attitude toward risk; situations $b_i > 0$, < 0, $= 0$, respectively, correspond to a risk-averse, -lower and -neutral investor. A risk-averse investor thus makes his decision on the basis of a lower rate of return of the risky asset he actually expects; the greater the value of b_i, the more risk averse the investor[6].

Now, $Z_i'(t)$ measures in units $\$/\Delta_i t$ (time is thus measured in units $\Delta_i t$) the instantaneous flow of money by which investor i is willing to buy (or sell) the studied securities. We assume that the net demand of a certain asset of any investor is a public offer to buy or sell the corresponding securities at the prevailing price[7] with a fixed amount of money. Assuming that $b_i, S_i, \Delta_i t, \tilde{p}_i(t+\Delta_i t)$ are fixed quantities known by investor i at time moment t, by taking the time derivative of (1) we get

$$\begin{aligned} R_i'(t) &= \frac{\partial R_i}{\partial p}p'(t) + \frac{\partial R_i}{\partial r}r'(t) + \frac{\partial R_i}{\partial Z_i}Z_i'(t) = -\frac{Z_i(t)\tilde{p}_i(t+\Delta_i t)}{p^2(t)\Delta_i t}p'(t) \\ &+ [S_i - Z_i]r'(t) + \left(\frac{\tilde{p}_i(t+\Delta_i t) - p(t)}{p(t)\Delta_i t} - b_i - r(t)\right)Z_i'(t). \end{aligned} \tag{2}$$

The aim of the investor is to increase his returns with time, i.e. to make $R_i'(t) > 0$. Now, the only variable by which the investor can affect his returns from period $\Delta_i t$ is Z_i, because p and r are outside his control. A revenue-seeking investor adjusts his portfolio with time so that the two quantities

[6] If the studied asset is the share of a company, expectations of possible dividends are assumed to be capitalized in the expected share price so that income from dividends is not treated separately. The role of dividends becomes the more important the longer the investment period, and long-term investments do not create short-term asset price movements we model here

[7] At stock market the buying and selling offers are made at fixed prices, and trade occurs when two opposite offers match. We simplify the real world by assuming that the offers are made at the prevailing price because only those offers get realized.

multiplied at the third term of Eq. (2) continue to be of equal sign, i. e.

$$
\begin{aligned}
Z_i'(t) > 0 \quad &\text{when} \quad \frac{\tilde{p}_i(t+\Delta_i t) - p(t)}{p(t)\Delta_i t} - b_i - r(t) > 0, \\
Z_i'(t) < 0 \quad &\text{when} \quad \frac{\tilde{p}_i(t+\Delta_i t) - p(t)}{p(t)\Delta_i t} - b_i - r(t) < 0 \quad \text{and} \\
Z_i'(t) = 0 \quad &\text{when} \quad \frac{\tilde{p}_i(t+\Delta_i t) - p(t)}{p(t)\Delta_i t} - b_i - r(t) = 0.
\end{aligned}
$$

The risk-corrected expected rates of return of the two investment possibilities are the benefits and costs the investor considers in his decision-making. Multiplying the above inequalities by $p(t)\Delta_i t$ they become as

$$Z_i'(t) > 0 \quad \text{when} \quad \tilde{p}_i(t+\Delta_i t) - [1 + (b_i + r(t))\Delta_i t]p(t) > 0 \quad \text{etc.}$$

The adjustment behavior of the investor can be expressed mathematically as

$$Z_i'(t) = f\Big(\tilde{p}_i(t+\Delta_i t) - [1 + (b_i + r(t))\Delta_i t]p(t)\Big), \quad f' > 0, \quad f(0) = 0, \quad (3)$$

where f is any function with the above requirements. The first order Taylor series approximation of function f in the neighborhood of the equilibrium point $\tilde{p}_i(t+\Delta_i t) - [1 + (b_i + r(t))\Delta_i t]p(t) = 0$ is

$$f(x) \approx f(0) + f'(0)(x - 0) = f'(0)x.$$

With this approximation we can write Eq. (3) as

$$Z_i'(t) = f'(0)\Big[\tilde{p}_i(t+\Delta_i t) - [1 + (b_i + r(t))\Delta_i t]p(t)\Big], \quad 0 \leq Z_i \leq S_i, \quad (4)$$

where $f'(0)$ is a positive constant. If we now denote $f'(0) = 1/m_i$, where m_i is a positive constant with unit $\Delta_i t/unit$, we can interpret m_i as the inertial factor ('mass') of investment of investor i. The numerical value of m_i measures the sensitivity of the monetary flow $Z_i'(t)$ with respect to the 'force' $\tilde{p}_i(t+\Delta_i t) - [1 + (b_i + r(t))\Delta_i t]p(t)$ acting upon the net demand of investor i of the studied asset[8]. All potential investors are assumed to behave

[8]The term $b_i p(t)\Delta_i t$ in the 'force' with unit $\$/unit$ has the same role as static friction has in physics. If $b_i > 0$, the expected monetary returns from one risky asset must exceed the risk-free alternative by factor $b_i p(t)\Delta_i t$ before the investor becomes active in buying. The risk-averseness of investors thus discourages them in buying and encourages them in selling risky assets. In economics, this friction term has been usually identified as transaction costs, but concept 'friction' may contain also other factors than costs, such as rigid manners, laziness, risk-averse behavior etc.

as described by Eq. (4), although their expectations of the future asset price and planning time horizons may vary.

The above defined investment behavior is unstable, because the amount of money investor i is willing to invest in the securities increases without limit with time when $\tilde{p}_i(t+\Delta_i t)-[1+(b_i+r(t))\Delta_i t]p(t) > 0$ and vice versa. This is one explanation for the observed volatility of stock market as compared with goods' market, where decreasing marginal utility limits consumers' willingness to buy goods. The investment flows caused by investor i are restricted by the following constrains: $Z_i'(t)$ can be positive only when $Z_i < S_i$ and negative when $Z_i > 0$. These conditions could be released by allowing short-selling and lending. Allowing lending would, however, complicate the situation by bringing the interest costs of loans in the decision-making, which is the reason we assume the above viability conditions.

4 The Behavior of the Stock Market

We assume only one kind of investors in the market, who at certain moments are asset buyers and at other moments sellers. We study the price dynamics of only one asset, but because this can be any of the existing securities, the model can be applied to them all. If some securities are expected to be equally profitable, splitting wealth into these assets does not affect the expected returns, but decreases the risk of the portfolio in the case of non-perfectly correlated returns. This is a clear reason for splitting wealth.

The connection between micro- and macro-level modelling is made here so that macro-level equations are formulated by adding the adjustment equations of all investors with investor specific weights. We assume n potential investors in the market, and the weight of investor i is denoted by a dimensionless number w_i, $\sum_{i=1}^n w_i = 1$. Multiplying the investors' portfolio adjustment equations by weights w_i and adding them, we get

$$\sum_{i=1}^{n} w_i m_i Z_i'(t) = \sum_{i=1}^{n} w_i \Big[\tilde{p}_i(t+\Delta_i t) - [1+(b_i+r(t))\Delta_i t]p(t)\Big]. \qquad (5)$$

This is the macro version of Eq. (4); the right hand side is 'the market force acting upon the net demand of the risky asset'. We can approximate the left hand side of (5) as

$$\sum_{i=1}^{n} w_i m_i Z_i'(t) \approx m_Z \sum_{i=1}^{n} Z_i'(t) = m_Z Z'(t), \qquad (6)$$

where constant $m_Z = \sum_{i=1}^n w_i m_i$ with unit $\overline{\Delta}t/unit$ is a weighted average of inertia of all investors, $Z(t) = \sum_{i=1}^n Z_i(t)$ and $Z'(t)$ is the aggregate of net investment flows of all investors on the studied asset. The right hand side of (5) can be approximated as

$$\sum_{i=1}^{n} w_i \Big[\tilde{p}_i(t+\Delta_i t)-[1+(b_i+r(t))\Delta_i t]p(t)\Big] \approx \tilde{p}(t+\overline{\Delta}t)-[1+(b+r(t))\overline{\Delta}t]p(t), \quad (7)$$

where $\sum_{i=1}^{n} w_i \Delta_i t = \overline{\Delta}t$ is a weighted average of investors' time horizons, dimensionless quantity $\sum_{i=1}^{n} w_i b_i \Delta_i t$ is approximated by $b\overline{\Delta}t$, where $b = \sum_{i=1}^{n} w_i b_i$ is an average measure for the risk-averseness of the investors and $\tilde{p}(t+\overline{\Delta}t) = \sum_{i=1}^{n} w_i \tilde{p}_i(t+\Delta_i t)$ is the market expectation of the asset price at moment $t+\overline{\Delta}t$.

We assume that $\tilde{p}_i(t+\Delta_i t)$ consists of a finite expected value $\overline{p}_i\big(t+\Delta_i t; B_i(t)\big)$ and variance ${\sigma_i}^2(t)$, $i = 1, \ldots, n$, where $B_i(t)$ is the set of information investor i uses at moment t in formulating his forecast. The expected value and variance of $\tilde{p}(t+\overline{\Delta}t)$ are then

$$E[\tilde{p}(t+\overline{\Delta}t)] = \sum_{i=1}^{n} w_i \overline{p}_i\big(t+\Delta_i t; B_i(t)\big), \quad var[\tilde{p}(t+\overline{\Delta}t)] = \sum_{i<j} w_i w_j c_{ij}(t),$$

where $c_{ij}(t)$ is the covariance between expectations of investors i and j. With these assumptions, there exists at every time moment a unique measure corresponding to the market expectation of the future asset price and a well-defined distribution viewing its variability. The market expectation is a weighted average of a finite sum of random variables. Assuming that the weight of every investor is the same $w_i = 1/n$, $i = 1, \ldots, n$ — which holds approximately when there are enough investors — we can write

$$var\left(\frac{1}{n}\sum_{i=1}^{n} \tilde{p}_i(t+\Delta_i t)\right) = \frac{1}{n^2}\sum_{i=1}^{n} \sigma_i^2(t) + \frac{2}{n^2}\sum_{i<j} c_{ij} = \frac{n\overline{\sigma}^2}{n^2} + \frac{n^2-n}{n^2}\overline{c}_{ij},$$

where $\overline{\sigma}^2$ and $\overline{c}_{ij}$ are the average values of $\sigma_i^2(t)$ and $c_{ij}(t)$, respectively, $\forall\ i,j,\ i \neq j$. If the investors' expectations are independent, that is $c_{ij}(t) = 0\ \forall\ i,j,\ i \neq j$, the market expectation gets more concentrated around its mean with the number of investors. The market expectation of the future asset price is thus more uncertain in a thin than in a wide market. Investors can utilize this information in two ways: 1) In terms of the efficient market hypothesis, investors can believe that current prices are more accurately priced in wide than in thin markets; later we show that an 'equilibrium asset price' reflects the market expectation of its future price. 2) The knowledge of 1) motivates investors to search for information of fundamentals of assets in thin markets, because it is more likely that there exists a potential for abnormally high returns. If, however, the investors' expectations are correlated, this may distort the asymptotic accuracy of the market expectation when $\overline{c}_{ij}$ is great enough; this is in line with the Keynes's remark quoted above.

The asset price is assumed to react to the macro-level net demand as

$$p'(t) = g(Z'(t)), \quad g' > 0, \quad g(0) = 0, \tag{9}$$

where $p'(t)$ measures in units $(\$/unit)/\overline{\Delta}t$ the velocity of the security price and g is any function with the above requirements. Taking the Taylor series approximation of function g in the neighborhood of the equilibrium point $Z'(t) = 0$, we get

$$p'(t) = g'(0)Z'(t) \quad \text{where} \quad g'(0) > 0 \quad \text{is constant.} \tag{10}$$

Next we denote $g'(0) = 1/m_p$ and interpret the positive constant m_p $(unit)$ as the 'inertia of the asset price'. It measures the sensitivity of the asset price with respect to the 'force acting upon the asset price' $Z'(t)$. The number of the corresponding assets in circulation is one suitable measure for m_p because excess demand raises security prices inversely related to their scarcity. Eq. (10) is a linear representation of the law of demand and supply formulated by Samuelson (1941) written in a net form.

5 Asset Price Dynamics

Substituting (5) into (10) by taking account (6) and (7) gives the following equation for the dynamics of the asset price

$$m_Z m_p p'(t) = \tilde{p}(t + \overline{\Delta}t) - [1 + (b + r(t))\overline{\Delta}t]p(t), \quad t \geq 0. \tag{11}$$

Now (11) is the general form for our model, and it can explain various kinds of processes depending on investors' expectations. We suggest below a few conjectures for $\tilde{p}(t+\overline{\Delta}t)$ to study their implications. These should be understood as tests for these assumptions and not as rigorous attempts to describe real world behavior.

Conjecture 1. Let $\tilde{p}(t + \overline{\Delta}t)$ be a deterministic function of time. Then (11) is an ordinary differential equation with solution

$$p(t) = e^{-\int_0^t \left(\frac{1+(b+r(s))\overline{\Delta}t}{m_p m_Z}\right)ds} \left(p(0) + \int_0^t e^{\int_0^s \left(\frac{1+(b+r(h))\overline{\Delta}t}{m_p m_Z}\right)dh} \frac{\tilde{p}(s+\overline{\Delta}t)}{m_p m_Z} ds\right), 0 \leq s \leq t.$$

This solution is stable if $\tilde{p}(t + \overline{\Delta}t) \to \tilde{p}(\infty + \overline{\Delta}t)$ and $r(t) \to r(\infty)$ when $t \to \infty$, the convergence is monotonous and $\tilde{p}(\infty+\overline{\Delta}t)$ and $r(\infty)$ are bounded positive quantities. Assuming $\tilde{p}(t+\overline{\Delta}t)=\tilde{p}$ and $r(s)=r$ to be constants, the above solution becomes in a more easily interpreted form

$$p(t)=\frac{\tilde{p}}{1+(b+r)\overline{\Delta}t}+\left(p(0)-\frac{\tilde{p}}{1+(b+r)\overline{\Delta}t}\right)e^{-\left(\frac{1+(b+r)\overline{\Delta}t}{m_p m_Z}\right)t},\quad t\geq 0.$$

The dynamic process is stable and the equilibrium price $\tilde{p}/[1+(b+r)\overline{\Delta}t]$ positively depends on the future price expectation, and negatively on the interest rate and the average risk-averseness of investors. The explanation for stability is that an increase in the asset price decreases the right hand side of Eq. (11) and vice versa; a price increase thus negatively affects the force acting upon the asset price. The equilibrium asset price balances the two growth rates so that the equilibrium state corresponds to zero force,

$$\lim_{t\to\infty}p(t)\equiv p_\infty=\frac{\tilde{p}}{1+(b+r)\overline{\Delta}t}\quad\Rightarrow\quad\frac{\tilde{p}-p_\infty}{p_\infty\overline{\Delta}t}-b=r.$$

In order to get another content for Eq. (11), we write it as follows

$$p'(t)=-\frac{[1+(b+r(t))\Delta t]}{m_p m_Z}\left(p(t)-\frac{\tilde{p}(t+\overline{\Delta}t)}{1+(b+r(t))\overline{\Delta}t}\right).\qquad(12)$$

Eq. (12) is known as Error Correction form in econometrics. In (12) the force is similar to that obeying Hooke's law in physics $F=-ky$, where y is the deviation of the moving body from its equilibrium position and constant k measures the strength of the spring causing the motion. In our case $p(t)-\tilde{p}(t+\overline{\Delta}t)/[1+(b+r(t))\overline{\Delta}t]$ is the deviation of the asset price from its equilibrium value, and the magnitude of constant $[1+(b+r)\Delta t]/m_Z$ measures the strength of the 'spring' causing the motion in the asset price.

Conjecture 2. Considering the market expectation $\tilde{p}(t+\overline{\Delta}t)$ as the position of a Brownian particle, various stochastic differential equations can be presented for $p(t)$. First we suppose that the expectation of every investor i, $i=1,\ldots,n$, of the growth rate of the asset price is

$$\frac{\tilde{p}_i(t+\Delta_i t)-p(t)}{p(t)\Delta_i t}=a(t)+\frac{\sigma(t)\xi_i}{p(t)\Delta_i t\sqrt{dt}}.$$

The expected growth rate consists of a deterministic component $a(t)$ measured in units $1/\overline{\Delta}t$ — which can be interpreted as the growth rate corresponding to the known fundamentals of the asset — and an investor specific component $\sigma(t)\xi_i/(p(t)\Delta_i t\sqrt{dt})$, where normally distributed stationary random variable $\xi_i\sim N\big(0,1/(nw_i^2)\big)$ represents the subjective information of investor i of the growth rate of the asset price; the greater the weight of investor i in the market expectation, the lower the variance of his subjective information. Parameter $\sigma(t)$ with unit $\$ \times \sqrt{\overline{\Delta} t}/unit$ characterizes the state of the market[9]. These assumptions give

$$\tilde{p}(t+\overline{\Delta}t) = \sum_{i=1}^{n} w_i \tilde{p}_i(t+\Delta_i t) = \sum_{i=1}^{n} w_i p(t) + \sum_{i=1}^{n} w_i a(t) p(t) \Delta_i t$$

$$+\frac{\sigma(t)}{\sqrt{dt}} \sum_{i=1}^{n} w_i \xi_i = p(t) + a(t)p(t)\overline{\Delta}t + \frac{\sigma(t)\xi}{\sqrt{dt}}, \qquad (13)$$

where $\xi = \sum_{i=1}^{n} w_i \xi_i$ is normal with expected value and variance as

$$E[\xi] = E\big(\sum_{i=1}^{n} w_i \xi_i\big) = 0, \quad var[\xi] = var\big(\sum_{i=1}^{n} w_i \xi_i\big) = \sum_{i=1}^{n} 1/n = 1,$$

where ξ_i are assumed independent, $i = 1, \ldots, n$. Substituting (13) to (11) gives the following stochastic differential equation

$$dp(t) = K\left(p(t)[a(t) - b - r(t)]\overline{\Delta}t + \frac{\sigma\xi}{\sqrt{dt}}\right) dt, \quad p(0) = \overline{p}, \quad t \geq 0,$$

where $K = 1/(m_p m_Z)$ is a positive constant with unit $1/\overline{\Delta}t$; for simplicity $\sigma(t) = \sigma$ is assumed constant. Next we define $\xi dt/\sqrt{dt} = \xi\sqrt{dt} = dN(t)$, where $N(t)$ is a stochastic process following the Brownian motion; the increments $dN(t)$ are independent of the past behavior of $p(t)$ with $N(0) = 0$, $E[N(t)] = 0$, $E\{[dN(t)]^2\} = dt$, $E[dN(t)dt] = E[(dt)^2] = 0$. The weighted average of investors' subjective information of the future asset price scaled by factor $\sqrt{dt}$ thus follows the Brownian process. These assumptions give

$$dp(t) = K\overline{\Delta}t p(t)[a(t) - b - r(t)]dt + K\sigma dN(t), \quad p(0) = \overline{p}, \quad t \geq 0.$$

The obtained stochastic process is of Ornstein-Uhlenbeck form with solution

$$p(t) = \overline{p} e^{K\overline{\Delta}t \int_0^t [a(s)-b-r(s)]ds} + K\sigma \int_0^t e^{K\overline{\Delta}t \int_s^t [a(u)-b-r(u)]du} dN(s), \ 0 \leq s \leq t.$$

The first term is the deterministic and the second the stochastic component of the solution. If $a(s) - b - r(s) = 0$, $0 \leq s \leq t$, the deterministic component

[9] When modelling the dynamics of molecules as Brownian particles, Einstein (1905) treated the parameter σ^2 as a dimensional constant which depends on various physical macro-level quantities characterizing the state of the system; temperature (the phase of the business cycle), friction coefficient and viscosity of the resisting medium (inertia of investors), Avogadro's number (number of investors and assets circulating) etc.

equals $\overline{p}$ and price changes take place only due to new information. The anticipated asset price at time moment t is

$$E[p(t)] = \overline{p}e^{K\overline{\Delta}t\int_0^t [a(s)-b-r(s)]ds} \quad \text{since} \quad E\Big[\int_0^t e^{K\overline{\Delta}t\int_s^t [a(u)-b-r(u)]du} dN(s)\Big] = 0,$$

and the anticipated growth rate of the asset price is

$$\frac{dE[p(t)]/dt}{E[p(t)]} = K\overline{\Delta}t[a(t)-b-r(t)] = \frac{[a(t)-b-r(t)]\overline{\Delta}t}{m_p m_Z}.$$

Interest rate and the average measure of risk-averseness thus negatively affect the anticipated growth rate. Quantity $a(s)-b-r(s)$ measures the expected risk-corrected excess rate of return for investors from the risky investment. The greater this quantity the higher the anticipated growth rate of the asset price. This feature of the model — expectations fulfill themselves — is common in models with rational expectations. Assuming that $a(s)-b-r(s) > 0$, $s \leq t$, which holds for all 'attractive' risky assets, the smaller the inertial factor $m_p m_Z$ the greater the anticipated growth rate.

Using the basic isometry of Ito Integrals (Øksendal 1989 p. 18), we can write the variance of the asset price as

$$\begin{aligned} var[p(t)] &= K^2\sigma^2 E\Big[\int_0^t e^{K\overline{\Delta}t\int_s^t [a(u)-b-r(u)]du} dN(s)\Big]^2 \\ &= \frac{\sigma^2}{(m_p m_Z)^2}\int_0^t e^{\left(\frac{2\overline{\Delta}t}{m_p m_Z}\right)\int_s^t [a(u)-b-r(u)]du} ds. \end{aligned}$$

The variance positively depends on σ and quantity $a(u)-b-r(u)$, $s \leq u \leq t$ and if $a(u)-b > r(u)$, $s \leq u \leq t$, then the variance negatively depends on the inertial factor $m_p m_Z$.

Conjecture 3. Suppose the market expectation of the asset price is

$$\tilde{p}(t+\overline{\Delta}t) = p(t) + \left(a(t)\overline{\Delta}t + \frac{\sigma(t)\xi}{\sqrt{dt}}\right)p(t),$$

where $a(t)$ and ξ are as before but now parameter $\sigma(t)$ characterizing the market situation is measured in units $\sqrt{\overline{\Delta}t}$. This corresponds to the following market expectation of the growth rate of the asset price

$$\frac{\tilde{p}(t+\overline{\Delta}t)-p(t)}{p(t)\overline{\Delta}t} = a(t) + \frac{\sigma(t)\xi}{\overline{\Delta}t\sqrt{dt}}.$$

Assuming this, Eq. (11) takes the form

$$dp(t) = Kp(t)\left([a(t)-b-r(t)]\overline{\Delta}t + \frac{\sigma(t)\xi}{\sqrt{dt}}\right)dt, \quad p(0) = \overline{p}, \quad t \geq 0, \quad (17)$$

where $K = 1/(m_p m_Z)$ is as before. Assuming $\xi dt/\sqrt{dt} = \xi\sqrt{dt} = dN(t)$ as before and $\sigma(t) = \sigma$ to be a constant, Eq. (17) takes the form

$$dp(t) = K\overline{\Delta}tp(t)[a(t) - b - r(t)]dt + Kp(t)\sigma dN(t), \quad p_0 = \overline{p}, \quad t \geq 0.$$

This Geometric Brownian process has solution

$$p(t) = \overline{p}e^{K\int_0^t\left([a(s)-b-r(s)]\overline{\Delta}t - \frac{\sigma^2 K}{2}\right)ds}e^{\sigma KN(t)}, \quad 0 \leq s \leq t.$$

The stochastic component of the solution is now multiplicative while previously it was additive. The anticipated asset price and its growth rate are

$$E[p(t)] = \overline{p}e^{K\overline{\Delta}t\int_0^t[a(s)-b-r(s)]ds}, \quad \frac{dE[p(t)]/dt}{E[p(t)]} = \frac{[a(t) - b - r(t)]\overline{\Delta}t}{m_p m_Z}.$$

These results are identical as in the previous case, but variance is now

$$var[p(t)] = \overline{p}^2 e^{2K\overline{\Delta}t\int_0^t[a(s)-b-r(s)]ds}\left(e^{K^2\sigma^2 t} - 1\right).$$

The variance positively depends on $\overline{p}$ and σ, and if $a(s) - b \geq r(s)$, $0 \leq s \leq t$, then the variance positively depends also on $K = 1/(m_p m_Z)$.

Conjecture 4. Our final conjecture is that the market expectation is made by extrapolating the current price movement till time moment $t + \overline{\Delta}t$ corrected by term $\sigma(t)p(t)\xi/\sqrt{dt}$, which represents the subjective information of investors. This corresponds to the following market expectation of the future asset price and its growth rate

$$\begin{aligned} \tilde{p}(t + \overline{\Delta}t) &= p(t) + \frac{dp(t)}{dt}\overline{\Delta}t + \sigma(t)p(t)\xi\sqrt{dt} \quad \Leftrightarrow \\ \frac{\tilde{p}(t + \overline{\Delta}t) - p(t)}{p(t)\overline{\Delta}t} &= \frac{dp(t)/dt}{p(t)} + \frac{\sigma(t)\xi}{\overline{\Delta}t\sqrt{dt}}. \end{aligned}$$

Assuming $\xi dt/\sqrt{dt} = \xi\sqrt{dt} = dN(t)$ as before and $\sigma(t) = \sigma$ to be constant, Eq. (11) takes the form

$$dp(t) = \sigma Hp(t)dN(t) - [r(t) + b]\overline{\Delta}tHp(t)dt, \quad p(0) = \overline{p}, \quad t \geq 0,$$

where $H = 1/(m_p m_Z - \overline{\Delta}t)$ is a constant with unit $1/\overline{\Delta}t$. The solution of this Geometric Brownian process is

$$p(t) = \overline{p}e^{-H\int_0^t\left([r(s)+b]\overline{\Delta}t + \frac{\sigma^2 H}{2}\right)ds}e^{\sigma HN(t)}, \quad 0 \leq s \leq t.$$

The anticipated asset price and its growth rate are now

$$E[p(t)] = \overline{p}e^{-H\overline{\Delta}t\int_0^t [r(s)+b]ds}, \quad \frac{dE[p(t)]/dt}{E[p(t)]} = -\frac{[r(t)+b]\overline{\Delta}t}{m_p m_Z - \overline{\Delta}t}.$$

Now the anticipated behavior of the asset price critically depends on the sign of H. If $m_p m_Z - \overline{\Delta}t > 0$, H is positive, and the asset price is anticipated to decrease with time. If H is negative, the asset price has the above given anticipated positive growth rate. The explanation for these results is the same as for hyper-inflation. It occurs in an inflationary economy when people rush to buy goods with raising prices. Similarly, if investors buy (and sell) assets on the basis of their observed price velocities, assets with the greatest (positive) price velocities are most wanted. This will speed up their price velocity. On the other hand, assets with low or negative price velocity will have a negative net demand which still decreases their price velocity.

In which direction the above described unstable process starts depends on the difference between the length of the average investment period and the magnitude of the inertial factor $m_Z m_p$. We can understand this as a battle between the magnitude of price inertia and the length of the investors' average investment horizon. The shorter the investors' average investment horizon and the higher the price inertia, the more certain it is that this price will decrease with time and vice versa.

The variance of the asset price

$$var[p(t)] = \overline{p}^2 e^{-2H\overline{\Delta}t\int_0^t [r(s)+b]ds}\left(e^{H^2\sigma^2 t} - 1\right)$$

positively depends on $\overline{p}$ and σ, but the effect of H on it is ambiguous.

From this section we can conclude that the described evolution of the asset price may be stable or unstable. Unstable behavior results if the observed velocities of asset prices have a role great enough in investors' expectations of future asset prices. Any rational investor investing in a risky asset must expect a higher rate of return from this investment compared with a risk-free alternative (a premium from taking the risk). The expected growth rate of the price of a risky asset should thus be great enough to attract investors. The greater the expected risk premium of a risky asset, the higher anticipated growth rate its price will have. The variability of our results implies that more information of the expectation-making process of investors is needed for more accurate results. Moreover, the assumption of Gaussian distribution for the subjective information of individual investors can be replaced by any other distribution, if we have evidence that the stochastic processes asset prices follow are not Brownian.

6 Conclusions

We analyzed the dynamics of one asset price and showed the existence of 'economic forces' which either push the price toward its equilibrium value, or keep the price in motion with time. The equilibrium asset price equates the investors' average expectation of the rate of return of holding these securities as compared with other investment possibilities. The evolution of the asset price was analyzed as a stochastic process where the randomness originates from investors' information and beliefs concerning the future asset price. Asset prices may or may not have a deterministic trend, and the randomness of the process causes deviations from this trend.

According to our model, the evolution of an asset price eliminates investors' expected profit-making potential concerning this asset, and trading takes place as long as there are investors believing this potential remains. At the equilibrium state there may exist investors willing to invest in this asset and those willing to sell these assets, and the sum of these monetary flows equals zero. The steady state is thus a flow equilibrium where the opposite flows (forces) cancel each other, and no force is acting upon the price. In the modelling we did not specify the variables investors apply in making their expectations of asset prices. We can believe that these sets contain measurable and non-measurable firm, industry and economy level quantities with investor-specific weights, assumptions of other investors' decisions etc. It is a subject for empirical studies to specify the information sets.

Our model explains the observed mean reverting behavior of asset prices and the negative relation between interest rate and asset prices. Our model has also three explanations for the observed positive correlation between rates of return of holding shares of various companies (the basic assumption of the CAP -model): 1) Investors adjust their portfolios with time so that securities with highest expected rates of return are most wanted. Because security prices adjust according to their net demands, price changes equalize the rates of return of holding any securities. 2) An increase in interest rate decreases the net demand of all securities thus reducing their prices. 3) If various investors keep the same macro-variables — the growth rate of GDP, the effective exchange rate of domestic currency etc. — in their information sets, then, for example, a general expectation of the growth of GDP positively affects the net demand of various shares thus increasing their prices.

References

Becker, G.S., 1976. The Economic Approach to Human Behaviour. The University of Chicago Press, Chicago.

Bentham, J., 1963. An Introduction to the Principles of Morals and Legislation. New York: Hafner.

Cox, J.C., Ingersoll J.E.Jr., Ross, S.A., 1985. An Intertemporal General Equilibrium Model of Asset Prices. Econometrica Vol. 53, No. 2.

de Jong, F., 1967. Dimensional Analysis for Economists. Amsterdam, North-Holland.

Duffie, D., Huang, C-F., 1985. Implementing Arrow-Debreu Equilibria by Continuous Trading of Few Long-Lived Securities. Econometrica Vol. 53, No. 6.

Einstein, A., 1956. (Original work 1905) Investigations on the theory of the Brownian movement. Dover Publications, New York.

Estola, M., 2001. A Dynamic Theory of a Firm: An Application of 'Economic Forces'. Advances in Complex Systems, Vol. 4 No. 1.

Fisher, F.M., 1983. Disequilibrium Foundations of Equilibrium Economics. Cambridge University Press.

Grossman, S.J., 1976. On the Efficiency of Competitive Stock Markets where Trades have Diverse Information. The Journal of Finance, Vol. XXXI, No. 2.

Grossman, S.J., Stiglitz, J.E., 1976. Information and Competitive Price Systems. American Economic Association, Vol. 66 No. 2.

Grossman, S.J., Stiglitz, J.E., 1980. On the Impossibility of Informationally Efficient Markets. The American Economic Review, Vol. 70 No. 3.

Huang, C-F., 1987. An Intertemporal General Equilibrium Asset Pricing Model: The Case of Diffusion Information. Econometrica Vol. 55, No. 1.

Hull, J.C., 1989. Options, futures, and other derivative securities. Prentice Hall International editions.

Keynes, J.M., 1970. The General Theory of Employment, Interest and Money. First edition 1936, University Press, Cambridge.

Lo, A.W., MacKinlay, A.C., 1988. Stock Prices do not follow Random Walks: Evidence from a Simple Specification Test. Review of Financial Studies 1, 41-66.

Merton, R.C., 1973. An Intertemporal Capital Asset Pricing Model. Econometrica, Vol. 41. No. 5.

Mirowski, P., 1989. The Rise and Fall of the Concept of Equilibrium in Economic Analysis, Recherches Economiques de Louvain, Vol. 55:4.

Poterba, J.M., Summers, L.H., 1988. Mean Reversion in Stock Prices — Evidence and Implications. Journal of Financial Economics 22, 27-59.

Risager, O., 1998. Random Walk or Mean Reversion: The Danish Stock Market Since Wold War I, Institute of Economics & EPRU, Copenhagen Business School. Paper presented at the 'International Symposium of Economic Modelling', Vienna July 1998.
Samuelson, P.A., 1941. The stability of equilibrium, Econometrica 9, in: The Collected Scientific Papers of Paul A. Samuelson I, 1966. Ed. by Joseph E. Stiglitz, Cambridge Mass, M.I.T. Press, pp. 539-562.
Øksendal, B., 1989. Stochastic Differential Equations, An Introduction with Applications, Second Edition, Springer-Verlag.

Appendix

The force acting upon the net demand of a particular asset can also be formulated according to the neoclassical tradition. Investor i is assumed to buy securities when he expects this to be profitable. This occurs when he expects that the rate of return during time period $\Delta_i t$ from buying $k > 0$ assets at price $p(t)$ exceeds the interest returns for this money. The rule to make this decision is

$$k[\tilde{p}_i(t + \Delta_i t) - p(t)] > rkp(t)\Delta_i t.$$

On the left hand side are the expected monetary returns from the investment, and on the right hand side are the lost interest returns for the invested money $kp(t)$ at period $\Delta_i t$. We can transform the above inequality as

$$\frac{\tilde{p}_i(t + \Delta_i t) - p(t)}{p(t)\Delta_i t} - r > 0,$$

which quantity we used as the 'force acting upon the net demand of a risky asset' of a risk-neutral investor.

We get the above decision rule also in the case of an infinite investment horizon. Let time be divided into intervals of length $\Delta_i t$. Suppose that investor i believes a constant change $E_i[\Delta p] = \tilde{p}_i(t + \Delta_i t) - p(t)$ in the asset price and a fixed interest rate r during period $\Delta_i t$ throughout the infinite future. The expected present value S_l (\$) from investing into $k > 0$ securities during l time periods is then

$$S_l = \sum_{j=1}^{l} \left(\frac{1}{1 + r\Delta_i t} \right)^j kE_i[\Delta p] = kE_i[\Delta p] \sum_{j=1}^{l} \left(\frac{1}{1 + r\Delta_i t} \right)^j.$$

Letting $l \to \infty$, the infinite sum of this geometric series becomes

$$\lim_{l \to \infty} S_l = kE_i[\Delta p] \left(\frac{\frac{1}{1+r\Delta_i t}}{1 - \frac{1}{1+r\Delta_i t}} \right) = \frac{kE_i[\Delta p]}{r\Delta_i t}.$$

This investment is profitable, if the present value of expected income from the investment exceeds its costs $kp(t)$. This corresponds to

$$\frac{kE_i[\Delta p]}{r\Delta_i t} > kp(t) \quad \Leftrightarrow \quad \frac{E_i[\Delta p]}{p(t)\Delta_i t} > r,$$

which inequality we derived above. The decision-making of an investor can thus be formulated by assuming him to hold the assets he buys for an infinite time, if dividend returns are assumed to be capitalized in the asset price.