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ISBN 952-458-609-6
ISSN 1458-686X
no 22

Effort Extraction, Contract Length, and the Golden Hand Shake

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DECEMBER 2004

Abstract

A model of temporary job contract with effort extraction is introduced. The actual contract length is determined randomly. The worker's effort performance has a positive effect on the expected contract length and wage level. Without the layoff compensation the worker chooses optimal level of his effort that is higher than the level without random contract length. However, he has an low incentive to engage in a long contract with low wage level. The firm must offer the worker a compensation package, "a golden hand shake", in the case of job termination in order to overcome the worker's incentive problem.

Key Words: *Effort Extraction, Contract Length, Temporary Jobs.*

1 Introduction

A work contract shares the common feature of all contracts that they state explicit, deterministic termination dates at which contractual obligations and rights ceases. However in many firms we found job contracts where the contract length is left open, i.e. a contract without explicit termination date with both parties maintaining the right to quit at any time. The case gives interesting options for both parties since if the worker is valuable for the firm it must offer him or her on a regular basis re-contracting offer with better pay-off than last contract offered. This is the game where the worker has a first advantage. In more ordinary cases when the worker's value for the firm is not much higher than the firing costs and requirement costs of new worker the problem of contract form and its length is different. The firm can use the worker's low firm value as a cause to terminate the job contract. If worker's outside options (i.e. unemployment benefits or alternative job) are poor the firm has a dominance over the determination of contract and it's length.

The literature of contract theory surprisingly seldom deals the issue of contract length (see e.g. Macho-Stadler & Perez-Castrillo 1996, Hart & Holmström 1987, Wiggins 1991). The papers that deal explicitly the contract length are few. Cantor (1988) shows that contract length affects the worker's decision to put forth effort. Finite-length contracts are necessary in order to make effort incentive-compatible. The optimal contract length minimizes the sum of explicit contracting costs and the costs of shirking. Dye (1985) shows that if contract relevant new information is observable to at least one of the contracting parties, the explicit optimal contract termination dates are non-determined. End of contract or recontracting can occur when ever the one of contract parties chooses such action.

In following we build a model that tries to combine some of these results in analytically tractable fashion. Although our approach is quite different from Cantor's and Dye's we are able to show that (nominal) contract length offered by the firm is not optimal for the worker if he faces uncertainty concerning the actual job length. The firm must offer the worker a compensation for participation in finite length contract despite the fact that worker can affect the expected contract length with his effort performance.

2 Model

The worker has the following expected value or profit function for his work effort performance

$$V(e, T) = [w(e) - c(e)]E[\min\{t(e), T\}]$$

where $w(e)$ is his wage level determined by his work effort with $w_e > 0$ and $w_{ee} < 0$. Thus it is assumed that the worker is rewarded for good work performance and high productivity. High work effort is not without costs and function $c(e)$ $(c_e > 0, c_{ee} > 0)$ captures the dis-utility and opportunity costs of effort supply. $T \geq t(e)$ is the nominal offered contract length of the temporary job to which the worker is engaged. However, the job period can terminate randomly (as seen from the worker's point of view) for many different reasons (lay-off due haltered production, bad behaviour, personal problems etc.). Thus the contract is not binding in all circumstances and the firm has right to laid off the worker before T. In response to this the worker can show higher work effort and morale that lowers the probability of actual job termination. This means that $t \sim \exp[-\lambda(e)t]$ is a random variable representing the actual length of job contract. The exponential distribution is a natural choice in this context as the probability mass is concentrated to low values of t but the hazard rate (the probability that the job is terminated at time $t+\Delta t$ when it has last t periods) is constant. That is the worker faces the highest risk of losing the job at the beginning of his job term. The risk intensity of loosing job, $\lambda(e)$, is decreasing function of effort, $\lambda_e < 0, \lambda_{ee} > 0$, i.e. the worker can control in some extension his expected job length and career $E[t] = 1/\lambda(e)$ with high work morale.

The worker faces the following maximization problem

$$\begin{aligned} \underset{\{e,T\}}{Max}\ V(e,T) &= [w(e)-c(e)]E[\min\{t(e),T\}] \\ s.t.\ V(e,T) &\geq 0 \ \text{ and } w(e)-c(e) \geq 0. \end{aligned}$$

We assume that T, the nominal duration of the contact, is held constant. This assumption is relaxed later. The first-order condition for the problem is given by

$$\frac{\partial V}{\partial e} = [w_e - c_e]E[\min\{t(e),T\}] + [w(e)-c(e)]\frac{\partial}{\partial e}E[\min\{t(e),T\}] = 0.$$

The first-order condition is reasonable only when $E[\min\{t(e),T\}] > 0$ and $w(e)-c(e) > 0$. This means that signs of $[w_e - c_e]$ and $\frac{\partial}{\partial e}E[\min\{t(e),T\}]$ must be opposite. We analyze first the latter term.

$$\begin{aligned} E[\min\{t(e),T\}] &= \int_0^T t\lambda(e)\exp\{-\lambda(e)t\}dt + T\int_T^\infty \lambda(e)\exp\{-\lambda(e)t\}dt \\ &= \frac{1-\exp\{-\lambda(e)T\}}{\lambda(e)} > 0. \end{aligned}$$

Now

$$\frac{\partial}{\partial e}E[\min\{t(e),T\}] = \frac{\partial}{\partial e}[\frac{1-\exp\{-\lambda(e)T\}}{\lambda(e)}]$$

$$= \frac{\lambda_e}{[\lambda(e)]^2}[\exp\{-\lambda(e)T\}(T\lambda(e)+1)-1].$$

The first part of solution, $\frac{\lambda_e}{[\lambda(e)]^2}$, is negative as $\lambda_e < 0$ and the second part is function like $H(\phi) = [e^{-\phi}(\phi+1)-1]$, where $H(0) = 0, H(\infty) = -1$ and $H_\phi = -e^{-\phi}\phi < 0 \ \ (\phi > 0)$. This shows that $H(\phi) < 0$ for all $\phi > 0$ and $\frac{\partial}{\partial e}E[\min\{t(e),T\}] > 0$.

Thus the condition

$$\frac{\partial V}{\partial e} = [w_e - c_e]E[\min\{t(e),T\}] + [w(e)-c(e)]\frac{\partial}{\partial e}E[\min\{t(e),T\}]$$

$$= [w_e - c_e][\frac{1-\exp\{-\lambda(e)T\}}{\lambda(e)}] + [w(e)-c(e)]\frac{\lambda_e}{[\lambda(e)]^2}[\exp\{-\lambda(e)T\}(T\lambda(e)+1)-1] = 0$$

can only be fulfilled when $[w_e - c_e] < 0$. That is, the marginal return of effort performance w_e is less than the marginal cost of it c_e. The worker is forced to elicit more effort than in the case without risky contract $(w_e - c_e = 0)$.

The second-order condition for maximization is

$$\frac{\partial^2 V}{\partial e^2} = [w_{ee} - c_{ee}]E[\min\{t(e),T\}] + 2[w_e - c_e]\frac{\partial}{\partial e}E[\min\{t(e),T\}]$$

$$+[w(e)-c(e)]\frac{\partial^2}{\partial e^2}E[\min\{t(e),T\}] < 0.$$

The negative sign is valid because $w_{ee} - c_{ee} < 0$ (by the assumptions), $E[\min\{t(e),T\}] > 0$, $2[w_e - c_e]\frac{\partial}{\partial e}E[\min\{t(e),T\}] < 0$ (see above), $[w(e)-c(e)] > 0$ (the optimality conditions), and only if $\frac{\partial^2}{\partial e^2}E[\min\{t(e),T\} < 0$, i.e.

$$
\begin{aligned}
\frac{\partial^2}{\partial e^2}E[\min\{t(e),T\} &= \frac{\partial}{\partial e}[\frac{\lambda_e}{[\lambda(e)]^2}(\exp\{-\lambda(e)T\}(T\lambda(e)+1)-1)] \\
&= (\frac{\partial}{\partial e}\frac{\lambda_e}{[\lambda(e)]^2})(\exp\{-\lambda(e)T\}(T\lambda(e)+1)-1) \\
&\quad +(\frac{\lambda_e}{[\lambda(e)]^2})\frac{\partial}{\partial e}(\exp\{-\lambda(e)T\}(T\lambda(e)+1)-1) \\
&= \frac{\lambda_{ee}\lambda(e)^2-2\lambda_e^2}{[\lambda(e)]^4}(\exp\{-\lambda(e)T\}(T\lambda(e)+1)-1) \\
&\quad +\frac{\lambda_e}{[\lambda(e)]^2}(-\lambda_e T^2\lambda(e)\exp\{-\lambda(e)T\}).
\end{aligned}
$$

The latter term on the right hand side is negative because of $\lambda_e < 0$ but the former term is negative only when $\lambda_{ee} < 2\lambda_e^2/[\lambda(e)]^2 = 2\frac{1}{e^2}[\eta_{\lambda e}]^2$, where $\eta_{\lambda e}$ is the risk intensity elasticity with respect to effort. For typical cases with a steep risk function with low e $|\eta_{\lambda e}| > e\sqrt{\lambda_{ee}/2}$.

The effect of nominal contract length T on optimal effort rate is obvious from above analysis. When $T = \infty$ we have

$$
\frac{\partial V}{\partial e} = [w_e - c_e]/\lambda(e) - [w(e)-c(e)]\frac{\lambda_e}{[\lambda(e)]^2} = 0
$$

$$
\Longrightarrow \quad [c_e - w_e] = [w(e)-c(e)]\frac{|\lambda_e|}{\lambda(e)} > 0.
$$

Effort extraction is still valid with long contracts. Note also that at $T = 0$ first-order condition for e ($\frac{\partial V}{\partial e} = 0$) is not defined in terms of e.

Properties of $\frac{\partial^2 V}{\partial e \partial T}$ at $e = e^*$ reveal some interesting behaviour.

When $T \to \infty$

$$\frac{\partial^2 V}{\partial e \partial T} = \exp\{-\lambda(e)T\}[(w_e - c_e) - (w(e) - c(e))T\lambda_e]_{|e=e^*} \to 0 \downarrow,$$

and when $T = 0$

$$\frac{\partial^2 V}{\partial e \partial T} = (w_e - c_e) < 0,$$

implying that a $T = T'$ exists for which $\frac{\partial^2 V}{\partial e \partial T} = 0$ and the sign of $\frac{\partial^2 V}{\partial e \partial T}$ changes from negative to positive at T'. This means that the nominal contract length has effect on the effort the worker must elicit. When $T < T'$ increasing contract length has a offsetting effect on the effort. Note that $\partial E[\min\{t(e), T\}]/\partial T > 0$. Thus less effort is needed to have same expected contract length when T is increasing. However, when $T > T'$ the optimal level of effort must rise to sustain longer contract length.

Taking contract length as variable the worker also optimize respect to it. The first- and second-order conditions for maximization are given by

$$\begin{aligned}
\frac{\partial V}{\partial T} &= [w(e) - c(e)] \exp\{-\lambda(e)T\} = 0 \\
\frac{\partial^2 V}{\partial T^2} &= [w(e) - c(e)][-\lambda(e) \exp\{-\lambda(e)T\} < 0.
\end{aligned}$$

First order condition is valid only when $[w(e) - c(e)] = 0$ for all $T < \infty$ or when $[w(e) - c(e)] > 0$ and $T = \infty$. In the first case the worker will not take the job as any other alternative for it with positive net welfare will give him better pay-off. The case with $[w(e) - c(e)] > 0$ and $T = \infty$ gives $\frac{\partial V}{\partial T} = 0$ but now $\frac{\partial^2 V}{\partial T^2}$ is zero. Note that $\frac{\partial^2 V}{\partial T^2} < 0$ when $0 \leq T < \infty$.

These results and the above ones for optimal effort supply mean that the worker hardly will take the job contract when some alternative income (for example, unemployment benefit) is available and if the contract length is long and finite ($T > T'$). The result speaks for job career alternative where the worker prefers many short jobs for one long one having flat wage function. Seen from the firm's side this means that the best choice is a nominally almost permanent job offer ($T' < T \to \infty$). However the long nominal contract must be supported with generous wage schedule. The worker must have some incentive to prefer long nominal contract over the short ones.

The analysis so far have shown that contract parties have conflicting views concerning the optimal contract length. In order to solve these contract problems the firm offers the worker a compensation package that depends on the contract length and performed effort level but is independent of laid off probability. The worker gets $aTe \leq M$ in all cases. a is some positive constant and M is some maximum compensation that the worker gets when his effort performance his high or/and if the contract length is very long. This "golden handshake" rule gives the worker the opportunity to solve his personal career in the job with different combinations of contract length and effort performance.

The worker faces now the following maximization problem

$$\begin{aligned} \underset{\{e,T\}}{Max}\ V(e,T) &= [w(e) - c(e)]E[\min\{t(e),T\}] \\ s.t\ \ aTe &\leq M, \quad V(e,T) \geq 0 \text{ and } w(e) - c(e) \geq 0. \end{aligned}$$

The first-order conditions for this Lagrangian problem $L(e,T) = V(e,t) - \mu(aTe - M)$ are with $\mu \geq 0$

$$\begin{aligned} \frac{\partial L}{\partial e} &= [w_e - c_e]E[\min\{t(e),T\}] + [w(e) - c(e)]\frac{\partial}{\partial e}E[\min\{t(e),T\}] - \mu aT = 0 \\ \frac{\partial L}{\partial T} &= [w(e) - c(e)]\frac{\partial}{\partial T}E[\min\{t(e),T\}] - \mu ae = 0 \\ \frac{\partial L}{\partial \mu} &= M - aTe \geq 0 \end{aligned}$$

With $\mu = 0$ we replicate the results obtained so far. With $\mu > 0$ when the compensation constraint $M - aTe$ is binding there exist values $0 < T < \infty$ and $e^* > 0$ for which $[w_e - c_e] = 0$ and $[w(e) - c(e)]\frac{\partial}{\partial e}E[\min\{t(e),T\}] - \mu aT = 0$ with $[w(e) - c(e)] > 0$.

Similarly, a value $0 < T^* < \infty$ with e^* exists for which $\frac{\partial L}{\partial T} = 0$ and $[w(e) - c(e)]\frac{\partial}{\partial T}E[\min\{t(e), T\}] > 0$. Actually this value is analytically solvable. That is

$$T^* = -\frac{1}{\lambda(e)}\ln[\frac{w(e) - c(e)}{\mu e}] > 0 \quad if$$

$$0 < \frac{w(e) - c(e)}{\mu e} < 1.$$

These results mean the worker's personal effort extraction is at optimal level and his has an incentive to sign the job contract although it involves the risk of ending before the contracted length T. He is optimally compensated for this risk with a "golden hand shake".

3 Discussion and extensions

A simple model of personal work contract was introduced to shed some light on contract length problems. Surprisingly the worker prefers short nominal contracts to longer ones. The firm policy is opposite. The result is due the facts that actual expected job contract may end before the nominal one because of the low effort performance and negative marginal payoff of effort. A risk bearing optimal nominal contract length exists for a worker that allows for reduced effort. Beyond that a higher effort performance is needed. The firm offers a compensation package to reduce the worker's risk-bearing in order to secure longer contract length. The layoff compensation depends on the worker's effort performance and the nominal contract length. This "golden hand shake" rule solves the worker's incentive problem.

The proposed model is naturally highly stylized. The wage determination depends solely on the effort performance. A more complete model needs a contract on the wage level independently of effort. The exponential distribution assumption concerning the actual expected length job duration is too restrictive. More general distribution would change the results. E.g. if the the layoff probability rises on worker's trial time before starting to decrease multiple optimal effort and contract lengths may exist. The most serious weakness of the model are the missing firm's profit maximization conditions. Thus we need firms's optimizing conditions for efficiency wage levels and contract lengths. The future work will elaborate these issues.

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