Costs, Uncertainty and Durable Adjustments – Finnish Cross-Sectional Evidence from Automobile Purchases

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Jari Viitanen

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Jari Viitanen[1]

Government Institute for Economic Research

University of Joensuu, Department of Business and Economics

Abstract

Based on the Cobb-Douglas preferences and standard (S,s) model this study extends the previous theoretical frameworks by deriving explicitly the optimal consumption rule for durables. The model states that an agent has a desire to keep a fraction of his wealth to be invested in a durable. Because of the depreciation of the durable together with stochastic movements in prices and agent's wealth, the actual fraction deviates from that of the target. Including the possibility of uncertainty, the model shows that it is optimal for the agent to allow an inaction (S,s) band around the target level of durable to avoid irreversible investment costs, and not to adjust until the critical band trigger is reached. The implications of the model - the width of the inaction (S,s) band is positively related to an increase in uncertainty, while a higher depreciation rate leads to more frequent adjustment - were tested using four Household Budget Surveys from Finland. The results on automobile purchases did not reject the (S,s) model. Higher income uncertainty widens the inaction band and decreases the probability of adjustment, while an increase in repair costs increases the probability of adjustment.

Keywords: Uncertainty, irreversible investment, durable, (S,s) rule, idiosyncratic and systematic risk

[1]University of Joensuu, Department of Business and Economics, P.O. Box 111, FIN-80101 Joensuu, Finland. E-mail: *jari.viitanen@metla.fi*. I am indebted to Dr. Matti Estola and Prof. Mikael Linden for comments, Risto Sullström, Airi Pajunen, Veli-Matti Törmälehto and Pekka Tsupari for providing data, and Anssi Kähkönen for editorial assistance. Financial support from University of Joensuu, Faculty of Social Sciences and Yrjö Jahnsson Foundation is gratefully acknowledged.

1. INTRODUCTION

It is a well-known fact that uncertainty about future consumption possibilities determines also today's saving and consumption purchases. This has a direct link to business cycles. An increase in uncertainty contributes a fall in consumption and may lead the economy to a depression. In literature, large empirical and theoretical work has been carried out to understand the link between uncertainty and consumption. Especially two mechanisms have been found to be important in determining this link. The first one, originally introduced by Leland (1968), is known as a precautionary saving motive. An increase in uncertainty creates an incentive for an agent to increase the precautionary savings and, in Deaton's (1991) terminology, to exhibit buffer-stocks for a rainy day. This reduces today's consumption purchases.

The second mechanism which affects the timing of consumption purchases is the irreversibility mechanism which - besides on uncertainty - builds on the existence of transaction costs for durables. Typically, these costs should be paid every time when durables are purchased or sold, including searching and information cost (lemons problem), sales taxes and commissions to brokers, among others.[1] While the investments are often at least partially irreversible, it can sometimes be better to postpone the purchases until the consumer obtains more information about the future. As McDonald & Siegel (1986), Pindyck (1991) and Dixit & Pindyck (1994) have shown, uncertainty over income, asset and commodity prices, costs and other market conditions create an option value of waiting for new information to arise before adjusting consumption to the desired level. Therefore, there is often an incentive to delay the purchase/selling decisions to the future until new information arrives.[2]

At the individual level, uncertainty over future consumption possibilities may be divided into idiosyncratic and systematic risk. The first means a possibility that a consumer faces an unexpected shock in his nominal wealth due to, for example, accident or illness. The second means that he is conscious of the uncertainty which has an influence on the whole economy, and which may affect his wages and wealth position. An adverse supply shock which affects prices is a good example. As one or both of the uncertainty components increase, consumers postpone their decision-making more easily to the future and the stronger is the incentive to wait for new information. Clearly, if this increase in uncertainty affects many agents at the same time, the theory provides a link between individual purchases, aggregate consumption and business cycles.

[1] Bernanke (1985) included the consumer's distaste for shopping and learning how to use new durables. Lam (1989) argued that there exist no perfect resale market for durables which creates additional costs for consumers.

[2] High technology such as mobile phones and computers provide other examples. Typically, the newest versions of those goods provide features that the older ones do not. Thus, there is always a temptation to postpone purchases until the latest version arrives, which carries an option value to wait.

Recent consumption research has considered irreversible investment, transaction costs and uncertainty to be crucial reasons in explaining individual investments in durables. Pindyck (1991) and Dixit & Pindyck (1994) have argued that the level of risk may be even more important than taxes and interest rates for investment decisions. The irreversible investment decision under uncertainty can be featured by the (S,s) model. According to this model, the durable is adjusted to a target level when the state variable crosses the critical lower or upper band trigger. When the state variable is inside the inaction band, the optimal policy is not to adjust the durable. The attractiviness of the (S,s) model is in its implications such as, in most of the times, consumers do not adjust their stock of the durables, and when they do, the adjustments are substantial and lumpy. However, the former (S,s) models (see e.g. Grossman & Laroque (1990) and Lam (1991)) explaining the effect of uncertainty suffer from the fact that risk is typically defined as a constant parameter. In the series of papers, Hassler (1994, 1996a,b,c) has combined the preliminary work of option value of waiting by McDonald & Siegel (1986) and the standard (S,s) model of inventory. His contribution to the model is that the risk level is in itself a stationary stochastic process over time. An increase in risk increases the value of waiting and the purchases are more easily postponed to the future. The shorter the high-risk periods are expected to be, the stronger is the incentive to wait for new information.

This paper concentrates on the irreversibility mechanism.[3] The theoretical model extends the framework of Hassler (1994, 1996a) by deriving the intertemporal consumption rule explicitly. The foundation of the model comes from the famous optimisation results of the static Cobb-Douglas preferences which state that the optimum is a function of (relative) commodity prices, wealth and the (constant) parameter reflecting a fraction of total wealth spent on the good. This result is extended to the dynamic context so that a consumer has a desire to keep a fraction of his wealth close to a constant target value. Since the durable depreciates over time together with the stochastic movements in prices and consumer's wealth, the actual level of the fraction deviates from that of desired. While the transaction costs prevent an agent to adjust the stock of a durable continuously, the adjustment is not made until a critical lower (or upper) bound is reached. The model includes uncertainty over the future consumption possibilities by assuming that an agent faces the two types of risk as discussed above. First, he faces idiosyncratic risk. To model this, we follow Hassler (1994) and use standard Poisson process for unexpected jumps in prices and consumer's real wealth and show how a change in the expected time until the jump occurs will have an effect on the durable purchases. Second, he faces systematic risk which is defined as a switch between two states of the economy, namely, low and high risk state. The model shows how an increase in systematic risk will cause the inaction range to increase. Then, the purchases are postponed until the critical bounds are reached or there is a switch back to the low risk state.

[3] For the importance of precautionary saving motive and its empirical relevance, see Caballero (1990), Hubbard et al. (1995) and Carroll & Samwick (1996, 1998) and the referenced cited there.

The theoretical model implicates that an increase in uncertainty and adjustment costs should have a positive effect on the width of the (S,s) inaction bands while a greater depreciation should lead to more frequent adjustment. The validity of implications of the model is then tested using three distinct methods and data on automobile purchases from four cross-sectional Finnish Household Budget Surveys conducted by Statistics Finland. First, the width of the inaction (S,s) band is tested against the household-level uncertainty which is measured by the difference between the actual and predicted disposable income of the household, housing debt and expenditures on health. The repair costs are included to test the effect of depreciation rate on automobile purchases. The second method builds on the multiplicative heteroskedasticity approach to test a household-level variance of income. The third method is based on heteroskedastic probit analysis in order to identify the effect of the variables above on the probability of transaction.

This study is organised as follows. Section two illustrates the basic (S,s) rule and the augmented (S,s) model in which the uncertainty is included and divided into the components discussed above. Section three describes the data used in the study. In section four, we report the main results, evaluate them with respect to the international evidence, and discuss the problems concerning the model's goodness-of-fit and the measurement of household-level uncertainty. Finally, section five concludes the paper.

2. (S,s) MODEL

In this section we introduce the theoretical (S,s) model used in the study. First, in section 2.1 we introduce the standard (S,s) rule in order to get an intuition of discontinuous investment decisions. Then, section 2.2 reviews the irreversibility literature. In section 2.3 we include individual and systematic risk in the model and show how an increase in one of the risk components will affect the timing of the durable purchase.

2.1. Introduction to (S,s) Rule

Individual investment decisions together with transaction costs can be described by using a (S,s) model originally introduced by Arrow et al. (1951) for the study of inventories. According to the model, the state variable evolves stochastically over time and is allowed to deviate from the optimal target which can be interpreted as a frictionless target. The more it deviates, the more disutility the consumer obtains, and the temptation to readjust it back to the target level increases. However, every time when this readjustment is made, the consumer has to pay a lump sum adjustment cost which prevents him from updating continuously. Under uncertainty, consumer is unaware of his future wealth, prices and other market conditions. Thus, by postponing purchase decision to the future and waiting for new information to arrive, he can possibly make a better deal. This creates an option value of waiting. The state variable continues to deviate

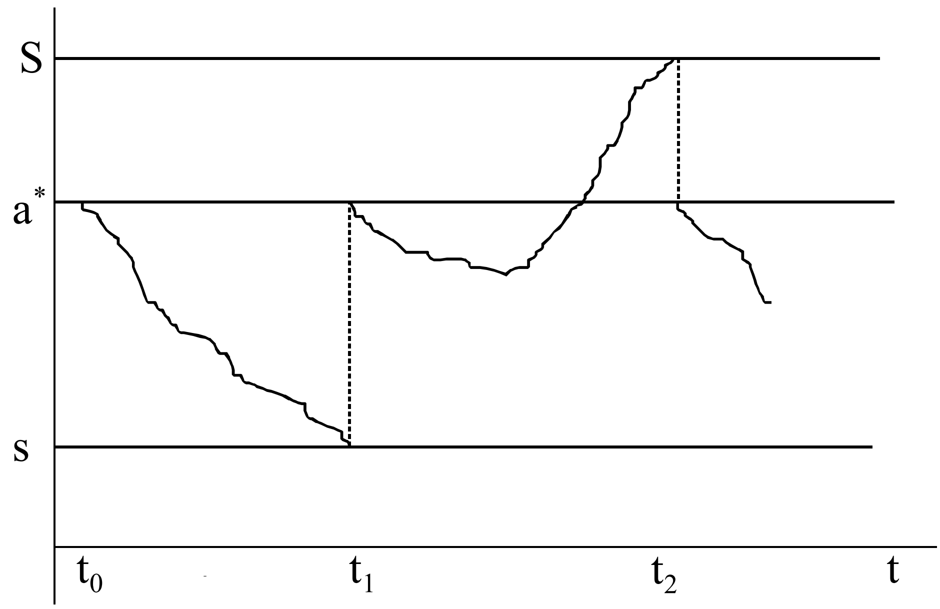

Figure 2.1: Standard (S,s) Model.

from the target until the lower/upper threshold/trigger is reached. Then it is readjusted back to the optimal level. The triggers can be seen as limits in which the temptation to adjust (utility gain) equals the adjustment costs plus the value of waiting. Inside the band the costs are higher than the utility gain from the updating, and no adjustment is made. Clearly, there is always a trade-off between adjusting now or waiting for new information before making the purchase decision.

Figure 2.1 illustrates the optimal (S,s) rule. For convenience, let us consider that the state variable is a stock of durable which depreciates over time.[4] Also, assume that the consumption flow is proportional to the stock of the good. The vertical axis represents the deviation of the current stock from some target level a^*. This frictionless target would be chosen if no transaction costs existed, that is, the stock of a durable would be continuously updated. However, continuous adjustment would entail infinite transaction costs and, therefore, cannot be optimal. After the purchase at the time t_0, the state variable is most of the time inside the inaction band (S,s) and the readjustment is not made until it reaches one of the triggers S or s. When a falls below some lower bound s, consumer pays a lumpy adjustment cost and the stock is readjusted back to a target size a^*. In figure 2.1, this readjustment is not made until at time t_1. Similarly, when the state variable increases above the upper trigger S, it is readjusted back to the target level a^*. This happens at time t_2. As long as the stock of the durable good remains inside the band, no action takes place.

[4]In this illustrative example, the stock of a durable can also be interpreted as a control variable.

Often, it is unrealistic to define the state variable only as a stock of durable which depreciates over time. Instead, consumer's wealth can be seen as one of the important variables determining the level of the stock. Typically, consumer's wealth grows over time and reflects the changes in consumer's spending. Even though the amount of the stock itself can remain unchanged, the quality of this stock may depend on the wealth level. Also, the relative prices of durables and nondurables are important determinants for consumption.[5] On the other hand, the optimal target level may depend on the quality and relative prices of goods, consumer's wealth, seasonal dummies and various demographic factors[6], while the size of the (S,s) band can be related, among others, to the size of the transaction cost, changes in household size, depreciation rate and to factors affecting the opportunity cost of deviating from the optimal level of durables. The use and decay characteristics of capital goods also differ because of the intensity of use, or the different extraction rates once the good has been purchased and installed.

Unfortunately, the theoretical model which covers all those notions is technically complex. Therefore, the construction of the model must include compromises which are often unrealistic but necessary for analytical or numerical solutions. First, the stochastic process of the state variable must be characterised. Second, the target level and the trigger points have to be characterised. Third, one have to characterise how an increase in uncertainty will effect on the timing of the purchases. Finally, one has to characterise the source of uncertainty. This motivates the theoretical model in the section 2.3.

2.2. Review of Irreversibility Literature

Recently, irreversibility and uncertainty over future have been considered important factors in determining the timing of the durable purchases. Even though the link between uncertainty and consumption has been understood for a long time, the theoretical models concentrating on uncertainty, irreversibility mechanism and consumption did not appear until the mid-1980s.[7] In our opinion, there are three main reasons for this appearance. First, the dynamic nature of durable consumption seemed much more complex than that of the other components of consumption, and the former consumption models, including the standard permanent income hypothesis, were uncapable on explaining the large variations in consumption over the business cycles, especially for durables purchases.[8] Second,

[5] In literature, the state variable has been modelled as the ratio of the consumption good to wealth (Grossman & Laroque (1990) and Eberly (1994)), the ratio of the durable stock to nondurables (Attanasio (1995, 2000)) and the ratio of the durable to permanent income (Dunn (1998)), among others.

[6] In addition, the target level does not necessarily coincide with the optimal frictionless level without transaction costs. For such models, see Attanasio (1995, 2000) and Hassler (1996a).

[7] This review concentrates only on literature concerning uncertainty, irreversibility and consumption. See Hassler (1996c) for a general discussion of the connection between risk and consumption. Carruth et al. (2000) and the references cited there give a thorough survey on the literature of industrial investment decisions under uncertainty.

[8] Typically, new models appeared because the old ones were not able to describe the U.S. economy.

the irreversibility models were developed to test if the results from the successors of Hall (1978), such as Mankiw (1982), hold also under liquidity constraints and other market incompletenesses.[9] Finally, the framework of McDonald & Siegel (1986), Pindyck (1991) and Dixit & Pindyck (1994) enabled us to understand better the significant effect which uncertainty and expectations have on irreversible investment decisions. Instead of using these categories in the following, we, however, used the classification between microeconomic and aggregate level studies. The following survey by no means covers the whole literature. Rather, our purpose is to show the importance of the irreversibility mechanism and the role of uncertainty on the timing of consumption decision and to sketch the evolution of irreversibility theory.

Typically, the literature involving aggregation first studied convex (quadratic) costs of adjustment. For example, Bernanke (1985) were among the first ones to include adjustment costs in a partial-adjustment model to study the separability in utility between durables and nondurables and the persistence of aggregate durables expenditures. He concluded that the quadratic adjustment costs are not sufficient in explaining the excess sensitivity to transitory income in the aggregate time-series. Subsequently, many other studies based on the convex adjustment costs turned out to be unsuccesfull. The main reason for this poor performance is that the convex costs approach predict a smooth adjustment towards an equilibrium. Also, to avoid increasing costs, agents will adjust their stocks infrequently and by small amounts. This, of course, contradicts common observations that durables are typically purchased in lumpy increments and updated only infrequently.[10]

A direction of improvement was then to consider nonconvex costs of adjustment (or "kinked" adjustment costs as described by Bertola & Caballero (1990)), such as fixed or proportional costs. The model by Bar-Ilan & Blinder (1987, 1992) can be seen as a preliminary work towards inertial models of consumer expenditures and an extension of the standard LC/PIH framework to include consumer durables. In their model it was sometimes optimal for the agents "to do nothing" if the transactions involve lumpy costs and to choose a finite range rather than a single level for their durables. The study of Grossman & Laroque (1990) was also based on the idea of nonconvex costs. Their theoretical model extended an inventory model of Arrow et al. (1951) to study the portfolio choice and a single illiquid durable purchase. It was proved that the optimal strategy can be modeled as an (S,s) rule as in Figure 2.1 above. The attractiviness of this approach was that it supports the common observations, that is, in most periods consumers do not adjust their stock of durables and when they finally do, the adjustments

[9]Mankiw (1982) showed that under the rational expectations augmented permanent income hypothesis, consumer durables expenditures should follow an ARMA(1,1) process. However, this implication was strongly rejected by the aggregate data (see e.g. Attanasio (1998)) and new models were needed to understand why it did not work.

[10]See also Bar-Ilan & Blinder (1992).

are usually substantial and lumpy. A closely related approach to this was employed by Lam (1989) who used irreversibility mechanism and imperfect resale market approach, and showed that these cause substantial serial correlation on aggregate durable expenditures.[11] His interpretation was that when resales are costly, consumers are reluctant to adjust their stocks downward, and, because of the possibility of having to resell, hesitant in adjusting their stocks upward. Thus, consumers tolerate their actual stocks to deviate from their desired target levels over the business cycles.

As noted above, one purpose of the adaption of the micro-level models together with the assumption of incomplete markets was that they helped to understand why the time-series of aggregate expenditures behave unlike the prevailing theories (such as Mankiw (1982)) predicted. Based on the individual behaviour several studies have concentrated on the aggregate effects and the business-cycle dynamics. In his slow adjustment models Caballero (1990, 1993) showed how the lumpy purchases in microeconomic level can explain different features of the aggregate time-series behaviour of durable goods, and how shocks can have persistent effects when individuals follow (S,s) policies. Caballero & Engel (1993) used a model in which the probability that an agent adjusts his durable stock is increasing to the deviation of the state variable from its moving target. Leahy & Zeira (2000) used shocks in individual wealth and decline in productivity to show that the timing decision can serve as a mechanism for the amplification and propagation of aggregate shocks. A decline in wealth causes individuals to rebuild their wealth position and during this time they delay durable purchases, which reduces the total demand dramatically for some time.

Even though a common feature of all these studies is that they showed clearly the important connection between uncertainty, individual purchases and business cycles, the weakness of the theoretical presentation is that risk was typically defined as a constant parameter. To overcome this issue Hassler (1994, 1996a,b,c) presented a model where the risk level is in itself a stationary stochastic process over time. Based on the preliminary work of option value of waiting by McDonald & Siegel (1986) and the standard (S,s) model of inventory, he showed how an increase in risk increases the value of waiting, and the purchases are more easily postponed to the future. The shorter the high-risk periods are expected to be, the stronger is the incentive to wait for new information. Hassler also showed how an increase in uncertainty affects the dynamics of aggregate consumption.

One shortcoming of the theoretical (S,s) models based on the individual behaviour is that they are restrictive in assumptions and the characterisation of an individual behaviour is possible only under very special circumstances.[12] Also,

[11] See also House & Leahy (2000) who used adverse selection and lemons problem to study the effect of resale market imperfections.

[12] Bar-Ilan & Blinder (1992) pointed out that even simple generalisations of the (S,s) models are extremely difficult to analyse because one looses the possibility of having a single state variable.

these models typically do not have a closed-form solutions and they must be employed numerically. Nevertheless, several studies (see Lam (1989) and Carroll & Dunn (1997), among others) use calibration and simulation tests to show that the (S,s) models can explain the empirical data better than the previous theoretical models. To summarise, both theoretical and empirical studies based on (S,s) model have shown the importance of uncertainty and irreversibility mechanism on the timing of individual consumption purchases and, thus, aggregate dynamics of durables consumption.[13]

The following augmented (S,s) model introduces the effect of an increase in uncertainty on the timing of durable purchases. While the foundation of the model builds on the framework of Hassler (1994, 1996a), it derives the consumption rule more explicitly by using Dixit & Pindyck's (1994) methodology and by dynamicing the famous optimisation rule of the static Cobb-Douglas preferences.

2.3. Model

In a static context, the maximisation of the standard Cobb-Douglas preferences subject to the linear budget constraint produces an optimal amount of the commodity. This optimum is a function of prices, consumer's wealth and the constant fraction of wealth spent on that good. Thus, the fraction of the wealth can be expressed as

$$a_t(P_t, C_t, W_t) = \frac{P_t C_t}{W_t}, \tag{2.1}$$

where C_t is the stock of durable, P_t is the (relative) price of the durable and W_t is consumer's total wealth including income, real and financial assets. Consider an agent who continuously faces the problem when to update his stock of a durable in response to the stochastic movements of the variables on the right-hand side of the equation (2.1). Assume that he wants to follow an optimal rule where the stock of a durable is kept in a level where it costs a certain constant fraction of his total wealth. Typically, durables are expensive and their purchases include lumpy costs that are at least partially irreversible. This feature prevents him from continuous updating. In the absence of adjustment costs, the consumer is willing to update continuously his stock of a durable and keep a equal to the frictionless target a^*.[14] Together with the depreciation of the durable each of the variables in the right-hand side in (2.1) evolves stochastically according to the following geometric Brownian motions

$$\begin{aligned} dW &= \alpha_W W dt + \delta_W W dz_W, \\ dP &= \alpha_P P dt + \delta_P P dz_P, \end{aligned} \tag{2.2}$$

[13] In section 4.3 we discuss more on the empirical microeconomic evidence of the (S,s) model.

[14] It could be more appropriate to model consumer's behaviour such that a certain fraction of wealth is spent on consumption categories. This includes transportation, electronics, clothes etc. rather than a single commodity such as a car or a computer. However, the technical treatment, then, becomes more difficult. See Estola & Hokkanen (1999) for a more detailed discussion on this subject.

$$dC = -\delta_C C dt.$$

Typically, both consumer's wealth and the price of the durable are increasing over time. Thus, the parameters α_W and α_P can be interpreted as drift components. δ_W and δ_P are the standard deviations of the processes reflecting the uncertainty over future. The terms dz_k are the increments of the Brownian motion capturing the idea that the variables satisfy the Markov property so that their next period probability distributions are functions of their current stage only. Also, the variance of the prediction error grows linearly with the time horizon.[15] δ_C is the rate of the depreciation. In Appendix A we show that the fraction of wealth a evolves as well according to the geometric Brownian motion

$$da = \alpha_a a dt + \delta_a a dz_a. \tag{2.3}$$

Infrequently, the consumer faces unexpected idiosyncratic shocks in his total wealth such as unemployment or illness.[16] Also, an accident can cause an immediate depreciation of the commodity. We will use a Poisson process to capture the idea of unexpected jumps in the ratio $\frac{PC}{W}$. Letting λ denote the mean arrival rate of an event, during a time interval of (infinitesimal) length dt, the probability that a negative or positive shock will appear is given by $\frac{\lambda}{2}dt$. Thus, $\frac{PC}{W}$ shifts an amount $\pm\xi$ with equal probability. The probability that an event will not occur is given by $1 - \lambda dt$. The combined Poisson and Ito processes are given as

$$da = \alpha_a a dt + \delta_a a dz_a + a dq \tag{2.4}$$

in which

$$dq = \begin{cases} \xi & \text{with probability} \quad \frac{\lambda}{2}dt \\ 0 & \text{with probability} \quad 1 - \lambda dt \\ -\xi & \text{with probability} \quad \frac{\lambda}{2}dt \end{cases}. \tag{2.5}$$

The increments dz and dq are assumed to be independent such that $E[dzdq] = 0$ and $E[dzdz] = dt$. The expected length of time until the shock appears is λ^{-1}.

Correspondingly, the consumer faces the systematic risk concerning the economy as a whole. The risk comes, for instance, from the threat of war, stock markets or an adverse supply shock.[17] Hereafter, risk is defined to be synonymous with uncertainty regarding future events that are relevant for the agent's decision-making. Following Hassler (1994, 1996a, 2001) the systematic risk is defined such that the consumer expects the state of the economy to switch stochastically

[15] Typically, in the long run the trend component is the dominant determinant of the Brownian motion, whereas in the short run the volatility component of the process dominates. See Dixit (1993), Dixit & Pindyck (1994) and Merton (1999) for an introduction and for mathematical properties of Brownian motion.

[16] Bernanke (1985) noted that at the family level, the most important influences on income are non-systematic factors such as ability, education and inheritance, among others.

[17] Bonoma & Garcia (1997) included infrequent information arrivals such as periodic releases of macroeconomic statistics and divident announcements.

between two levels.[18] The state 0 is defined as a low risk state, while 1 is high risk state. The state u is assumed to follow a first-order Markov process with a transition matrix

$$\begin{bmatrix} 1-\gamma_0 & \gamma_0 \\ \gamma_1 & 1-\gamma_1 \end{bmatrix}. \tag{2.6}$$

At time t, if the economy is in state $u_t = 0, 1$, it is assumed to switch to the state $\overline{u}_t$ with probability γ_u during the time dt. The probability that there does not exist a switch is $1 - \gamma_u$. The expected length until the risk switches is γ_u^{-1}.

The augmented (S,s) model in figure 2.2 illustrates the evolution of the variable a and the adjustment process. The vertical axis is defined as in figure 2.1. Assume that $S_1 > S_0 > a^* > s_0 > s_1$. At time t_0 the fraction of the wealth a is at the frictionless optimum a^*. Typically, consumer's real wealth is increasing over time (the ratio $\frac{P}{W}$ is decreasing) while the depreciation decreases the stock of the durable. This means that a decreases over time until the lower trigger s_0 is reached at time t_1. Then the consumer pays the adjustment costs and the stock is adjusted back to the optimal level a^*. Sometimes the opposite happens, so that the real wealth is decreasing over time and the depreciation of the durable is not high enough to force the variable a to decrease.[19] At time t_2 the upper trigger S_0 is hit and the stock of durable is adjusted back to the frictionless optimum.[20]

Occasionally, a consumer faces unexpected changes in his real wealth (idiosyncratic risk). For example, at time t_3 he confronts a reduction in real wealth which causes a upward jump on a variable a. However, this jump is not high enough for an adjustment to be made. At time t_4 a positive shock to real wealth occurs (for instance, a win in a lottery or a bequest) and shifts the variable below the lower trigger. This causes an upward adjustment of a durable back to the target level a^*. For convenience, if the shock occurs, it is hereafter assumed large enough to be optimal to adjust the variable immediately.[21] This assumption together with depreciation and positive growth of the real wealth imply that the variable is nearly always in the region between the lower trigger and target level. Thus, we will concentrate on the range $[s_1, a^*]$ and we will not formulate the situation such that between the times t_1 and t_2. At time t_5 an uncertainty concerning the systematic risk increases widening the inaction band to (S_1, s_1). Clearly, a con-

[18] The assumption of bivariate risk in economy is unrealistic but necessary for the technical treatment. To allow more states results in the multivariate systems of differential equations which are difficult to solve.

[19] Technically, depending on the sign of the drift parameter α_a, a is increasing or decreasing over time. See equation (A.6) in Appendix A.

[20] Hassler (1994,1996a) assumed that there exist upper and lower targets $\overline{a}$ and $\underline{a}$ such that if the state variable is hit by the upper trigger S_u, then it is readjusted to the upper target $\overline{a}_u$. Similarly, if the lower bound s_u is reached then the variable is readjusted to the lower target $\underline{a}_u$. For the technical treatment to become easier we assume that $\overline{a}_u = \underline{a}_u = a^*$.

[21] Of course, after the shock occurs the adjustment of a durable does not take place immediately. Instead, there is an "adjustment period" when, for instance, the consumer is collecting information about the prices and properties of durables available in the market. Even though it is possible to build models with deliberating time and jumps inside the bands without adjustment, they are difficult to treat technically.

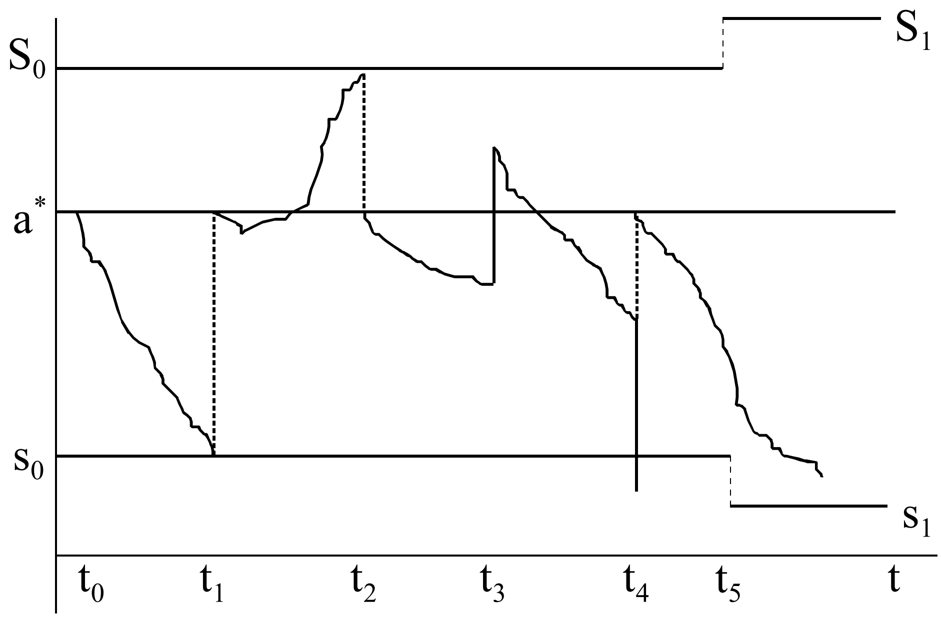

Figure 2.2: Augmented (S,s) Model.

sumer allows larger deviation from the target level before adjusting. The larger the uncertainty, the wider is the inaction range and the larger is the deviation between actual and target level.

The problem is to determine the critical points, triggers, at which it is optimal to pay the adjustment costs and adjust the commodity back to the optimum level. Since a evolves stochastically, we are not able to determine an explicit time when the investment is to be made. Instead, the investment rule which follows will take the form of a critical values (s_1, s_0) such that it is optimal to invest once the variable a goes below one of these band triggers.

As shown above, an agent obtains disutility if the state variable deviates from its frictionless optimum which is, for convenience, assumed to be a constant.[22] Let $x_t = a_t - a^*$ be the state variable denoting the gap between the actual and the target level, and let the loss of deviation be a quadratic distance from the target level.[23] If the total costs are the sum of expected discounted values of present- and future-period costs, then the optimisation problem for an infinitely

[22] This assumption is made to simplify the mathematical treatment of the model. However, the assumption is in harmony with the standard Cobb-Douglas preferences where the fraction of the wealth is a constant number. See Niemeläinen (1995) for details and other preferences where this fraction can be shown to depend on prices, wealth or both. See also Hassler (1996a) for a model which allows the target level to depend on wealth and the state of the economy.

[23] The quadratic loss function approach means that a consumer's disutility is symmetric around the target level. While this assumption can be questionable, the technical reasons prevent us to use asymmetric disutility approach.

long-lived consumer can be written as

$$\min \left[E_t \int_t^{\infty} e^{-rt} \left(\frac{x_t^2}{2} + I_t c \right) dt \right], \tag{2.6}$$

subject to the equations (2.4) and (2.5). E_t denotes the expectation operator conditional on the information set available to consumer at time t and r is the subjective discount rate. For convenience, the consumption flow is assumed to be proportional to the stock of the durable. I_t is a bivariate variable such that

$$I_t = \begin{cases} 1, & \text{if } a \text{ is adjusted} \\ 0 & \text{otherwise} \end{cases}. \tag{2.7}$$

It denotes that an adjustment cost c should be paid only when the adjustment is made. The optimal value function $V(x_t, u_t)$ is defined as the minimum of discounted expected total costs over the infinite future time horizon when the economy is in state u_t. In Appendix B we show that if the consumer is following the optimal policy, the Bellman equation

$$V(x_t, u_t) = \frac{x_t^2}{2} dt + e^{-rdt} E_t V(x_{t+dt}, u_{t+dt}) \tag{2.8}$$

can be rewritten as

$$\begin{aligned} rV(x_t, u_t) \;\; = \;\; & \frac{x_t^2}{2} + \left[\alpha_x x_t V'(x_t, u_t) + \frac{1}{2}\sigma^2 x_t^2 V''(x_t, u_t) \right] \\ & + \lambda \left[(V(a^*, u_t) + c) - V(x_t, u_t) \right] + \gamma_u \left[V(x_t, \overline{u}_t) - V(x_t, u_t) \right]. \end{aligned} \tag{2.9}$$

The left-hand side is the value of the cost function multiplied by the discount rate. The first term on the right-hand side is the utility loss during the time dt. The second term is the expected change in total costs if no shock nor state shift occur. The third term captures the idea that there exists a wealth shock during the time period dt. Then, by assumption, the durable is purchased and the state variable is adjusted to the target level a^*. The last term comes from the possibility of a state shift.[24] This causes the expected total costs to shift from $V(x_t, u_t)$ to $V(x_t, \overline{u}_t)$ where $u_t \neq \overline{u}_t$.

While $u_t = 0, 1$, the equation (2.9) results in the following system of second-order differential equations

$$\begin{aligned} & \frac{1}{2}\sigma^2 x_t^2 V''(x_t, 0) + \alpha_x x_t V'(x_t, 0) - (\lambda + r) V(x_t, 0) + \gamma_0 \left[V(x_t, 1) - V(x_t, 0) \right] \\ = \;\; & -\frac{x_t^2}{2} - \lambda (V(a^*, 0) + c), \end{aligned} \tag{2.10}$$

[24] The optimisation problem could be stated and solved by two different techniques: contingent claims analysis or stochastic dynamic programming. Even though in most applications they both give identical decision rules, their assumptions are different concerning discount rates and financial markets. Also, the interpretation of the equation (2.9) slightly differs. See Dixit & Pindyck (1994) or Pietola (1997) for more details.

$$
\begin{aligned}
&\frac{1}{2}\sigma^2 x_t^2 V''(x_t,1) + \alpha_x x_t V'(x_t,1) - (\lambda + r)V(x_t,1) + \gamma_1\left[V(x_t,0) - V(x_t,1)\right] \\
= \quad & -\frac{x_t^2}{2} - \lambda(V(a^*,1) + c).
\end{aligned}
$$

In Appendix B we show that the algebraic solution for (2.10) is

$$
\begin{aligned}
V(x_t,0) &= \frac{\gamma_0\gamma_1}{\gamma_0+\gamma_1}A_1 x^{\beta_1} - \frac{\gamma_0}{\gamma_0+\gamma_1}C_1 x^{\theta_1} \\
&\quad -\frac{1}{2(\sigma^2 + 2\alpha - (\lambda + r))}x^2 + \frac{\lambda c}{r}, \\
V(x_t,1) &= \frac{\gamma_0\gamma_1}{\gamma_0+\gamma_1}A_1 x^{\beta_1} + \frac{\gamma_1}{\gamma_0+\gamma_1}C_1 x^{\theta_1} \\
&\quad -\frac{1}{2(\sigma^2 + 2\alpha - (\lambda + r))}x^2 + \frac{\lambda c}{r},
\end{aligned}
\tag{2.11}
$$

in which the roots β_1 and θ_1 are defined as

$$
\begin{aligned}
\beta_1 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda + r)}{\sigma^2}} > 1, \\
\theta_1 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda + r + \gamma_0 + \gamma_1)}{\sigma^2}} > \beta_1.
\end{aligned}
\tag{2.12}
$$

The total cost functions in (2.11) are valid only for $x \in [s_0, a^*]$. For some values of $V(x_t, u_t)$, a switch of the state will lead to an immediate adjustment. For example, if x_t is in the region $[s_0, s_1]$, a switch from the high risk state to the low risk state causes an adjustment. Thus, in the region between s_1 and s_0 the cost function $V(x_t, 0)$ is a constant and equals $V(s_0, 0)$ because $x_t < s_0$. In the range $[s_1, s_0]$ the system of differential equations degenerates to

$$
\begin{aligned}
&V(x_t,0) = V(s_0,0), \\
&V(x_t,1) = D_1 x^{\mu_1} - \frac{1}{2(\sigma^2 + 2\alpha - (\lambda + r + \gamma_1))}x^2 + \frac{\gamma_1 V(a^*,0) + (\gamma_1 + \lambda)c}{r + \gamma_1},
\end{aligned}
\tag{2.13}
$$

in which the root μ_1 is defined as

$$
\beta_1 < \mu_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda + r + \gamma_1)}{\sigma^2}} < \theta_1. \tag{2.14}
$$

Economically, the system of cost functions in (2.13) shows the effect of the change in uncertainty on durable purchases. It also reveals how the consumer will benefit of the new information and do what is optimal then. If the state of the world is risky and the state variable is close to the lower band trigger s_1, a shift to low risk will cause an immediate adjustment. If a high number of consumers are close to this lower band trigger and the state of the economy is high risk, a switch to the low risk will cause an aggregate investment boom.

Even though we are not able to find an algebraic solution for the integration constants in (2.11), (2.13) and band triggers (S_u, s_u) , it is possible to find numerical solution using the following conditions

$$\begin{aligned} Smooth-pasting &: \begin{cases} V'(s_0,0)=0 \\ V'(s_1,1)=0 \\ V'(a^*,0)=0 \\ V'(a^*,1)=0 \end{cases}, \\ Value-matching &: \begin{cases} V(s_0,0)=V(a^*,0)+c \\ V(s_1,1)=V(a^*,1)+c \end{cases}. \end{aligned} \tag{2.15}$$

The smooth-pasting conditions require that the derivatives or slopes of the functions meet tangentially at the boundaries and target level under the states $u_t = 0, 1$. The value-matching conditions match the values of the unknown function $V(x_t, u)$ to those of the known values if the adjustment is made.[25]

2.4. Increase in Risk and Timing of Purchases

Technically, the smooth-pasting and value-matching conditions enable us to find numerical solutions for the total cost functions in (2.11) and (2.13). Economically, the value-matching conditions imply that at the band triggers the temptation to adjust equals the temptation to wait an instant. This interpretation becomes more clear if we evaluate the equation (2.9) at the target point a^* and at the band trigger s_u. Following Hassler (1996a) it can be shown (see Appendix C) that the indifference condition becomes

$$\frac{(a_u - a^*)^2}{2} = \overbrace{rc}^{1} + \overbrace{\lambda c}^{2} + \overbrace{\gamma_u \left[c + V(a^*, \overline{u}) - V(s_u, \overline{u})\right]}^{3} \tag{2.16}$$

where $a_u - a^*$ means the deviation of the state variable from the target level evaluated at the band trigger point s_u. The left-hand side is the immediate temptation to adjust. An adjustment decreases the utility loss from deviating from the target point. The right-hand side is the temptation to postpone the purchase an instant dt.[26] It has three different parts. If a consumer does not adjust but invests the amount c in a safe asset, he receives an interest rate yield during the time dt. This is the first part of the right-hand side. The second one reflects the possibility of a wealth shock. Delaying the purchase will save one adjustment if the shock occurs after the time dt. The third term denotes the possibility of a state shift in economy. If it switches during the instant dt and a consumer waits until after that instant he can do what is optimal then. This creates an option value of waiting which is non-negative (see Appendix C).

It is straightforward to analyse the effects of the determinants on the size of the inaction band $[s_u, a^*]$. From (2.16) it is evident that if the adjustment costs

[25] See Dixit (1993) for a thorough discussion.

[26] The left-hand side is interpreted as an instantaneous cost of waiting in Hassler (1996a). The right-hand side is a value of waiting in Dixit & Pindyck's (1994) terminology.

increase, the consumer tolerates a larger deviation from the target before adjustment. The same happens if the probability λ of idiosyncratic risk increases or the expected time decreases until the personal shock will appear. Even though the state shift does not occur during the time dt, the decrease in the expected time until it may appear (or an increase in probability γ_u) will lead to the wider inaction band.

3. DATA DESCRIPTION

Because of the lack of micro-level data, the formal tests of the (S,s) model are rare. Ideally, the panel of household data should be long enough to track the stock of the durables over time so that one can identify times of adjustment, targets and adjustment triggers together with the information on income, wealth and heterogeneous characteristics of the households. Also, as pointed out by Attanasio (1998), it is desirable to follow households over some time to bound the inaction range by the households that are observed not to adjust. Such data is difficult to find.[27] Also, another reason for the lack of econometric analysis is the difficulty in defining irreversibility and uncertainty. Concerning the irreversible mechanism, the only studies that have utilised panel data are Lam (1991), Eberly (1994), Attanasio (1995) and Foote et al. (2000). In all these studies, however, there are problems in accounting properly the variables involved in the model.

In this study we used Household Budget Surveys conducted by Statistics Finland. The data is drawn from four cross-section Surveys made in Finland in the years 1985, 1990, 1994-96 and 1998. The data from 1994-96 is combined from three annual Surveys and processed such that it can be used as a one cross-section. The number of households in the Surveys are 8200 in 1985, 8258 in 1990, 6743 in 1994-96 and 4087 in 1998. The respondents in the Surveys were asked several questions concerning the characteristics of the household, income, liabilities, education, and detailed expenditures in different consumption categories.[28] Also, the respondents were asked if they owned a certain durable. Unfortunately, the Surveys did not follow the same households over time, and it is not possible to construct a panel tracking the stock of the durable and evaluate the depreciation rate together with the wealth position of the households over time.

The information on car ownership is best documented in the data because the respondents were asked the gross and net markka-value of the acquisition of the cars as well as the exchange value of the used cars. Also, the information of expenditures on repairs as well as other charges and costs are available. Therefore, we used the data from the car acquisitions to identify the (S,s) triggers and the

[27] In fact, the only appropriate large microeconomic data sets are the Consumer Expenditure Survey (CEX) from U.S. and Family Expenditure Survey (FES) from U.K.

[28] Suoniemi & Sullström (1995) provide a thorough description of the Surveys and of the change of the consumption structure in Finland.

target, and to evaluate the effects of the determinants on them. According to the data, the households can be categorised as follows.

1. Those who upgrade by buying a used car and give a car in exchange.
2. Those who upgrade by buying a used car without a trade-in car.
3. Those who upgrade by buying a new car and give a car in exchange.
4. Those who upgrade by buying a new car without a trade-in car.
5. Those who upgrade by buying both a used and a new car and give a car in exchange.
6. Those who downgrade to zero by selling a car.
7. Those who downgrade by selling a car and buy a cheaper one.
8. Those who do not engage in transaction.

For households engaging in a transaction so that they either buy a new or a used car and give a car in exchange, the width of the lower inaction band is observable. Correspondingly, the width of the upper band is observable only for those who downgrade by selling a car and buy a cheaper one. For households that upgrade by buying either a used or new car without a trade-in car, only the target level can be identified. For those who downgrade by selling a car, only the upper trigger can be identified. Due to the limited time-series information for those not engaging in a transaction, neither the triggers nor the target is observable. In the following section, these categories are referred to when analysing the data.

4. RESULTS

4.1. Estimation Methods

As a measure of the state variable, we used a ratio of the value of the car to the disposable income of the household. Even though this measure of wealth does not properly account for the total lifetime wealth, it still implicitly includes the unobservable human wealth, the existence of liquidity constraints, and gives the wealth position of the household. The triggers were calculated by dividing either the value of the trade-in cars with respect to the households disposable income depending whether the household updates its durables stock up (lower trigger, categories 1 and 3) or down (upper trigger, category 7), or with respect to the selling value of the car (upper trigger, category 6)[29]. The gross value of the purchased car with respect to the disposable income gives the target value for the state variable. Table 1 depicts the means and standard deviations for the triggers and target, and the number of observations in each categories. The last panel in Table 1 gives the results for the total number of entries of the triggers and target. The state variable values higher than one were omitted from the analysis for practical reasons.[30]

[29]In category 6, the low values of the state variable indicate perhaps a sort of a scrap value of the cars rather than (S,s) behaviour. Then, it is a matter of taste if this trigger should be treated as S or s.

[30]Typically, each category contained few values which were more than one. In the categories less than 100 observations, these outliers had a substantial effect on mean and standard error (the highest value for

Table 1
(S,s) target and triggers for state variable calculated
as means in each category

		1985	1990	1994-96	1998
Category 1	a*	0.31 (0.21)	0.34 (0.21)	0.31 (0.21)	0.33 (0.21)
	s	0.12 (0.12)	0.13 (0.13)	0.13 (0.13)	0.13 (0.12)
	n	886	610	438	285
Category 2	a*	0.14 (0.15)	0.17 (0.18)	0.15 (0.15)	0.16 (0.16)
	n	583	606	396	357
Category 3	a*	0.53 (0.17)	0.52 (0.21)	0.59 (0.19)	0.60 (0.19)
	s	0.25 (0.15)	0.24 (0.16)	0.26 (0.15)	0.27 (0.14)
	n	297	227	114	124
Category 4	a*	0.43 (0.17)	0.43 (0.23)	0.43 (0.23)	0.49 0.22
	n	48	67	30	33
Category 6	S	0.09 (0.10)	0.11 (0.16)	0.10 (0.14)	0.14 (0.16)
	n	78	82	123	36
Category 7	S	0.21 (0.19)	0.23 (0.18)	0.19 (0.20)	0.18 (0.15)
	a*	0.10 (0.11)	0.14 (0.13)	0.10 (0.12)	0.09 (0.08)
	n	74	68	78	34
Total	S	0.15 (0.16)	0.17 (0.18)	0.14 (0.17)	0.16 (0.16)
	n	152	150	201	70
	a*	0.29 (0.23)	0.30 (0.23)	0.27 (0.23)	0.30 (0.24)
	n	1888	1578	1056	833
	s	0.15 (0.14)	0.16 (0.15)	0.16 (0.14)	0.17 (0.14)
	n	1183	837	552	409

The state variable is the value of the car/disposable income. a, S, s and n are the target, upper and lower triggers and number of observations, correspond– ingly. Categories are as in Chapter 2. The standard deviations are given in parentheses.*

The results from the categories that reveal the width of the inaction band are

the state variable was 60), and some of them may be a consequence of data processing. Therefore, to make results comparable over time, the state variable is restricted between zero and one.

the most interesting. For example, the results from the category 1 imply that on the average the value of the trade-in cars is allowed to drift down to 13 percent of the disposable income before adjustment. Then, it is adjusted slightly above 30 percent. This behaviour clearly differs from that of buying a new car and giving a car in exchange (category 3). These households adjust when the value of the trade-in car is about one fourth of the disposable income, while the target is slightly above half. In both cases, the width of the inaction band slightly changes over time. Households that downgrade by selling a car and buy a cheaper one (category 7) tolerate the value of the car to increase to one fifth of the disposable income before adjusting it back to about 10 percent. However, it is noteworthy that the high standard errors indicate a high cross-sectional heterogeneity of behaviour.

The common feature across the categories is that, in aggregate, both the triggers and target seem to be quite stable over time implying only a small variation in the width of the inaction band. Somehow this is surprising since the Surveys are from the years of different economic circumstances. In Finland, in 1985 financial markets were regulated, in 1990 there was a boom and overheating of the economy, while the years 1994-96 and 1998 were times of economic recovery along with a high rate of unemployment. Casually, however, only the year 1990 seems to be an exception. The lower band width for the category 1 is slightly higher, and the upper trigger and target for the category 7 is somewhat higher compared to other years.

After identifying the upper and lower band width, we focused on the implication of the (S,s) model. That is, an increase in the adjustment costs and uncertainty leads households to purchase a durable less frequently. While the heterogeneous adjustment costs including searching and information costs, commissions to brokers etc. cannot be observed from the cross-sectional data, we use instead pure cost measures which are available in the Surveys, and which are assumed to have an effect on the timing of the purchases. These repair costs include expenditures on repair pairs, accessories, maintenance and other repairing. A higher rate of depreciation will indicate more need for repairing, which should lead to more frequent adjustment.

A problem for the applied econometrician is the identification and integration of uncertainty into the theory. Even though the theoretical model assumed idiosynchratic risk to follow Poisson process with immediate adjustment after the shock occurs, such behaviour is difficult to capture empirically. Instead, we study how idiosynchratic risk effects on the width of the inaction band. To measure the household-level uncertainty we used two distinct methods.[31] Following Eberly (1994) the first method builds on the difference between actual and predicted disposable income. First, we regressed disposable income of each household on

[31] See Carruth et al. (2000) who give a survey of uncertainty proxies in irreversible investment research. In time-series and panel data the conditional variances of the underlying variables (such as the growth rates of income, stock prices and inflation) are typical proxies.

a number of household characteristics including socio-economic status, province, local authority, total months of unemployed, type of the household, educational attainment and gender of the household head.[32] Then, we used these coefficients to impute a predicted income for each household in the Surveys. Finally, we calculated the difference between the actual and predicted disposable income, and used it as a measure of uncertainty. The advantage of this method is that it takes into account the households' heterogeneous characteristics. For example, if some of the household members are unemployed, it is reasonable to assume that the households' income is less than that of reference income, thus, affecting on the willingness to adjust the durable back to the target level.

The shortcoming of this method, however, is that the measured uncertainty has an asymmetric effect on the purchases. If the disposable income of the household is higher than that of predicted (that is, the residual is positive), the household can be assumed to be better-off than the average and it may be more willingness to update the durable back to the target sooner than those of the worse-off households. Thus, it is reasonably to assume that the coefficients for the worse-off households should be positive and statistically significant indicating a wider inaction band, while the coefficients for the better-off households are assumed to be statistically negative or insignificant.

While this measure of household-level uncertainty based on the residual method above may be questioned for many reasons[33], we added other cross-sectional factors that may be related to the households uncertainty concerning the future. These are housing debt and expenditures on health.[34] The magnitude of both of these measures implicitly include a sort of uncertainty.

The second method is based on the Harvey's (1976) multiplicative heteroskedasticity, which can be seen as an stochastic volatility type method without the time-dimension structure in error terms. Analysis of the OLS residuals of the first method reveals that depending on the Survey the error variance is mostly related to the education of the household head and/or socio-economic status of the household. Therefore, the skedastic function is

$$\sigma^2_{M,i} = \exp\left(\beta_0 + \beta_1 Dummy(Education)_i + \beta_2 Dummy(Status)_i\right), \qquad (4.1)$$

where i denotes each household in the Survey. The maximum likelihood estimation procedure involves deriving first derivatives of the log-likelihood function

[32]The regression results from these dummy variables are available from the author by request. See Appendix D for the description of the variables. We also tried other candidates which may affect the disposable income such as education of the spouse. For all of these, however, the coefficients turned out to be statistically insignificant.

[33]For example, Pagan (1986) has shown that in time-series analysis these two-stage/step regressions with expectations provide consistent parameter estimates but the covariance matrix of the parameter estimates is usually inconsistent. To correct the estimation, one should use instrument variables. Even when such expectations of the future variables do not exist explicitly in our cross-sectional regressions, it is likely to assume that there may exist a sort of inconsistency in the variances of the parameter values as well.

[34]Instead of housing debt in 1985 we use total debt because of the lack of data.

with respect to mean equation parameters and skedastic function parameters. The resulting conditional variance estimates ($\hat{\sigma}_i^2$) were used as a proxy of the household-level uncertainty and entered as regressors in the band estimation.

The analysis above utilises only a subset of the Surveys disregarding the categories 2, 4, 6 and 8, that is, the households who only sell or buy a car, or do not engage in transaction at all. Thus, it is fertile to include these observations into the analysis to identify the effect of uncertainty and repair costs on the probability of transaction. To evaluate this behaviour, the third method is based on a probit analysis to get the adjustment probits. However, while the Surveys include households which are heterogeneous in their characteristics, the standard probit estimation generates parameter coefficients which are both biased and inconsistent. To improve the statistical performance of the estimates, we used heteroskedastic probit model, where the skedastic function is

$$\sigma_{P,i}^2 = \exp(\beta_1 DI_i)^2. \tag{4.2}$$

DI_i is the disposable income of the household explaining the variation in the error terms.[35]

4.2. Estimation Results

Table 2 presents the results of the determinants on the lower band width based on the residual method. While it is difficult to interpret quantitatively the standard linear regression coefficients, we regressed log-linearized versions which give the direct percent changes in the band width. All the independent variables in the model were divided by the disposable income to scale the variables and to get consistent measurement units in regression. To control the asymmetric behaviour between the worse-off and better-off households, we added dummies for the constant and uncertainty (Dconstant and Dincome) for the better-off households.

According to the results, all the intercepts were statistically significant for the worse-off households. The intercepts for the better-off households differed from those of the worse-off only for the years 1985 and 1990 indicating a narrower inaction band.[36] The measured income uncertainty effect was statistically significant for both household types only for the years 1985 and 1990. In 1985 an increase of one percent in the income uncertainty increased the lower band width for 6.3 percent for the worse-off households but decreased it for 6.9 percent for the better-off households. These results did not reject the (S,s) model. The other

[35] See Harvey (1976) or Greene (2003) for a theoretical justification of the methods.

[36] The exclusion of some of the regressors did not change the statistical interpretations for the remaining coefficients. The sum of the constant and Dconstant, and the sum of Income and Dincome are the intercept and the coefficient of the measured income uncertainty for the better-off households, respectively. The antilogs of the intercepts give the standard constant terms.

uncertainty variables (housing debt and health) seemed not to perform well statistically, except for the years 1994-96, which seemed to generate reverse results in general. An increase in housing debt with respect to the disposable income seemed to decrease the band width, but statistically this was significant only in 1994-1996. As expected, throughout the Surveys an increase in repair costs typically seemed to decrease the inaction band for few percents, but these coefficients did not either differ statistically from zero. The coefficient of determination (R^2) was typically less than 0.10.

Table 2

Determinants of lower band width, residual approach

	Dependent variable is $\log(a^* - s)$			
	1985	1990	1994-96	1998
Constant	-1.741**	-1.568**	-1.835**	-1.833**
	(0.129)	(0.156)	(0.188)	(0.243)
Dconstant	-0.333**	-0.406**	0.209	-0.096
	(0.103)	(0.127)	0.167	(0.220)
$\widehat{Income}$	0.063*	0.096**	-0.079*	0.022
	(0.032)	(0.040)	(0.048)	(0.065)
$\widehat{Dincome}$	-0.132**	-0.159**	0.087	-0.042
	(0.043)	(0.057)	(0.076)	(0.105)
Debt	-0.001	-0.011*	-0.024**	-0.011
	(0.007)	(0.007)	(0.009)	(0.010)
Health	-0.010	0.028	0.044*	0.029
	(0.015)	(0.024)	(0.023)	(0.022)
Repair	-0.002	-0.009	0.026**	-0.025
	(0.009)	(0.011)	(0.013)	(0.019)
n_1	542	399	266	184
n_2	641	438	286	225

*Income is the absolute value of the difference between predicted and observed disposable income. All the regressors are divided by the disposable income and are in logarithms. n_1 and n_2 denote the number of the worse−off and better−off households, respectively. The asterisks * and ** denote that the coefficients differ statistically from zero at 10% and 5% levels of significance, respectively. The standard errors in parantheses are heteroskedasticitycorrected.*

Table 3 presents the corresponding results for the upper band width. The common feature of these results is that they performed poorly statistically. In most cases we even cannot reject the hypothesis that all the coefficients are zero (Survey 1990). Except of the intercepts, only the income uncertainty for the worse-off households in years 1985 and 1994-96 were statistically significant. The magnitude of these coefficients, however, was unconvincing. Even though the coefficients of the other uncertainty measures typically had the right sign, statistically they were irrelevant for the width of the upper band. Also, the coefficients for the repair costs had mostly the expected sign, but they did not differ statistically from zero. The coefficients of determination for the regressions were low.

Nevertheless, the number of observations is small and one should avoid making too strict interpretations of the results.

Table 3
Determinants of upper band width, residual approach

	Dependent variable is $\log(S - a^*)$			
	1985	1990	1994-96	1998
Constant	-5.673**	-2.540	-2.667**	-5.572*
	(1.651)	(1.878)	(1.187)	(3.360)
Dconstant	-0.047	1.098	-0.579	1.342
	(1.700)	(2.116)	(1.393)	(2.583)
$\widehat{Income}$	-0.542**	-0.069	0.920**	0.014
	(0.261)	(0.667)	(0.325)	(1.121)
$\widehat{Dincome}$	-0.280	0.846	-0.264	-0.267
	(0.651)	(1.036)	(0.703)	(1.201)
Debt	0.170	-0.015	0.048	0.175
	(0.135)	(0.087)	(0.068)	(0.130)
Health	0.072	0.281	0.033	-0.218
	(0.240)	(0.324)	(0.166)	(0.192)
Repair	-0.074	0.054	-0.065	-0.028
	(0.152)	(0.128)	(0.102)	(0.191)
n_1	43	41	43	15
n_2	31	27	35	19

See Table 2.

Based on the multiplicative heteroskedasticity approach, Table 4 gives the results both on the upper and lower band widths. Again, the independent variables were scaled by the disposable income, and the log-linearised version of the models were estimated.[37] The results seemed highly consistent with those in Tables 2 and 3 with one exception: the model for the lower band width in 1994-96 seemed to fit the data well. The other coefficients and their statistical interpretations were closely related to those of Tables 2 and 3. Again, the magnitude and the statistical relevance of the coefficients for the upper band width were dubious because of the small sample properties in estimation. An interesting feature was revealed by the coefficients for the housing debt ratio: an increase in this ratio seemed to increase the upper inaction band while decreasing the lower one. Typically, these coefficients, however, did not statistically differ from zero. Also, the coefficients for repairing costs were generally of the expected sign, but were statistically insignificant. Even though not reported, the coefficients of the

[37]Instead of $\log(\hat{\sigma}_i^2)$ the uncertainty measure is also scaled like the other explatory variables, and is $\log(\hat{\sigma}_i^2 / DI_i)$, where DI_i is the disposable income of the household. When regressing the model without scaling the variables involved in the model, the results were in accordance with those reported in Table 4. Thus, the scaling does not distort the statistical significance and interpretations of the parameters.

skedastic equation (4.1) turned out to be statistically significant at 5% level of significance.

Table 4
Determinants of band width when uncertainty is based on multiplicative heteroskedasticity approach

		Regressors (in logs)				
	Band	**Constant**	**log** $(\hat{\sigma}^2_{M,i}/DI_i)$	**Debt**	**Health**	**Repair**
1985	Upper	5.424	-1.027	0.186	0.126	-0.046
		(7.796)	(0.836)	(0.129)	(0.239)	(0.142)
	Lower	-4.319**	0.267**	0.004	-0.012	-0.002
		(0.503)	(0.054)	(0.007)	(0.015)	(0.009)
1990	Upper	-2.996	-0.022	-0.022	0.236	0.015
		(4.685)	(0.411)	(0.084)	(0.319)	(0.122)
	Lower	-2.941**	0.123**	-0.011*	0.036	-0.008
		(0.431)	(0.043)	(0.006)	(0.024)	(0.011)
1994-96	Upper	1.976	-0.678	0.008	0.079	-0.096
		(5.482)	(0.584)	(0.072)	(0.184)	(0.107)
	Lower	-3.149**	0.156**	-0.018**	0.041*	-0.027**
		(0.648)	(0.066)	(0.009)	(0.023)	(0.013)
1998	Upper	0.593	-0.484	0.215*	-0.190	0.020
		(6.799)	(0.642)	(0.120)	(0.204)	(0.214)
	Lower	-2.722**	0.080*	-0.012	0.027	-0.026
		(0.547)	(0.048)	(0.010)	(0.022)	(0.019)

See Table 2.

Table 5 presents the results from the heteroskedastic probit analysis. The uncertainty measure was calculated as in equation (4.2). The first row corresponding to each Surveys gives the standard probit estimates. The second row reports the marginal effects around the means of the independent variables.

With a few exceptions, the coefficients were statistically significant either in 10 percent or 5 percent level of significance. Except for the year 1990, the effect of an increase in uncertainty on the probability of adjusting was negative, as predicted by the (S,s) model. An increase in housing debt, on the other hand, had a statistically significant positive effect on the probability of purchase, which seems to contradict the theory. The health effect had negative effect while an increase in repair costs affected positively on the probability to adjust. Both of these were in accordance with the theory. The likelihood-ratio (LR) test of heteroskedasticity which tests the model with heteroskedasticity against the model without it was highly significant in all cases. Even though the coefficients for the uncertainty term differed statistically from zero, their magnitude on the probability of adjustment was only few percents.

Table 5
Maximum likelihood estimates from heteroskedastic probit model

	Regressors					
	Constant	$\hat{\boldsymbol{\sigma}}^2_{P,i}$	**Debt**	**Health**	**Repair**	**LR**
1985	-2.056*	-0.013	0.304**	-2.509**	0.191	121.62
	(1.223)	(0.115)	(0.045)	(0.921)	(0.379)	
		-0.001	0.030**	-0.245**	0.019	
		(0.011)	(0.004)	(0.090)	(0.037)	
1990	-2.887**	0.092	0.133**	-1.034	4.184**	165.45
	(0.639)	(0.057)	(0.030)	(0.813)	(0.655)	
		0.011*	0.015**	-0.123	0.498**	
		(0.006)	(0.004)	(0.098)	(0.075)	
1994-96	1.073**	-0.234**	0.053*	-0.849	0.886**	39.05
	(0.356)	(0.034)	(0.028)	(0.728)	(0.356)	
		-0.044**	0.010*	-0.159	0.166**	
		(0.007)	(0.005)	(0.136)	(0.059)	
1998	-0.140	-0.139**	0.177**	-4.614**	1.043**	91.67
	(0.541)	(0.051)	(0.055)	(1.161)	(0.477)	
		-0.019**	0.024**	-0.630**	0.142**	
		(0.007)	(0.008)	(0.155)	(0.065)	

See Table 2. The 5% critical value for the LR test is $\chi^2(1) = 3{,}84$.

This held true also for the housing debt. Instead, the measures of health and repair costs generated probabilities, which seem somewhat unreasonably large: the estimates indicate even as large as 60 percent effect on the probability of adjustment.

4.3. Discussion and Evaluation of Results

The interpretation of the above results is not straightforward and requires discussion. A major failure of the results concerning the estimates on the upper band width are most likely related to the small number of observations and, therefore, the estimates cannot give a reliable picture of the adjustment behaviour and they should be interpreted as preliminary rather than strictly concluding.[38] On the other hand, the estimates for the lower band width as well as the estimates for the probability of adjustment give more plausible explanation between the uncertainty and on the frequency of adjustment. Even though the residual method dividing the households into two groups - worse-off and better-off households - was able to found statistically significant evidence only for the years 1985 and 1990, the other methods found more systematic significance. According to these results, a percent increase in uncertainty increases the inaction band more than

[38] As mentioned earlier, the low values of the state variable may indicate a scrap value of the vehicle rather than voluntary adjustment.

10 percent while an unit increase in uncertainty decreases the probability of adjustment for few percents. Even though the magnitude of the latter result seems to be more realistic than the former, both are in favour of the (S,s) model.

The other uncertainty measures generated ambiguous results. The coefficients of the housing debt on the band width and on the adjustment probabilities were typically either statistically insignificant or of the wrong sign. This is hard to interpret. We also tried total debt as a regressor, but the results were parallel. One explanation for the latter is that in Finland the expensive net purchases (such as cars) are typically financed by taking out a loan, thus, generating a positive correlation between the borrowings and net purchases. The regression results based on the amount of total consumer credit support this insight. Also, while the expenditures on health certainly describe a kind of individual uncertainty and even though the estimates are of the right sign and in accordance with the (S,s) model, statistically they cannot explain the behaviour in automobile market.

The negative sign of the coefficients for the repair cost indicating a sort of depreciation of the cars turned out to be in favour of the infrequent adjustment theory. However, only in few cases the effect on the inaction band width was statistically significant. Instead, an increase in repair costs had a statistically significant positive effect on the probability of adjustment. When adding other user costs (automobile tax, inspection fee, traffic insurance charge and other costs including expenditures on gasoline) to explain the effect on the inaction band width, the coefficients turned out to be positive but statistically insignificant. On the other hand, the effect on the adjustment probabilities was even more statistically consequential than the pure repair cost effect.

One explanation for the poor performance of the regressors on the width of the inaction bands is the possibility that the regressors have a parallel effect both on the triggers and target which remains the band width unchanged, but changes the location of the whole (S,s) band. To control this possibility we run separate regressions for the triggers and target (results are not reported). According to these results, however, the different uncertainty measures as well as the repair costs seem not to have a statistically significant effect on the location of the (S,s) band. Only on few cases, the coefficients became statistically significant, but not systematically.

So far we have not discussed anything concerning the effect of general economic situation in Finland on the estimation results. The Household Budget Surveys are collected under different economic circumstances and it is reasonable to assume that the economic environment matters on the intertemporal consumption decisions. To evaluate the effect of the general economic confidence, Figure 4.1 presents two different indicators concerning the expectations of future in Finland. The first is Finnish industrial confidence indicator (FICI) collected by the Confederation of Finnish Industry and Employers, and the second is consumer confidence indicator (CCI) supplied by Statistics Finland[39]. Both indicators reveal that years 1985 and 1990 contained more systematic risk concerning the future than the later years. Especially, the end of year 1990 generated negative expectations reflecting the forthcoming deep depression in Finland. The expectations were most positive in 1994 according to the economic outlook. CCI considers year 1998 to contain least uncertainty with respect to the other years.

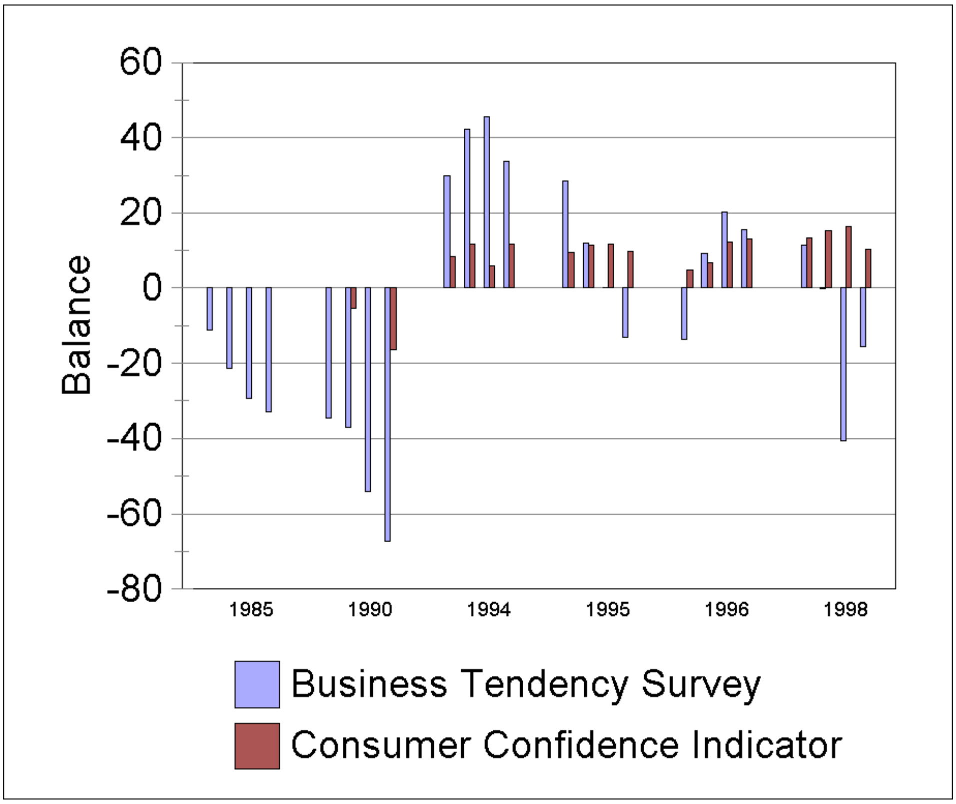

Figure 4.1: Economic Outlook and Consumer Confidence Survey

Although it is suspected that the investment decisions will be more sensitive to variations in household-level uncertainty than to increases in risk which affects all households in general, the occasional dominance of the latter may result to behaviour which cannot be revealed from the estimation. This insight can explain some of the statistical performance of the income uncertainty on the inaction band width. The income uncertainty was statistically significant in years 1985 and 1990 (high risk according to the indicators) while in 1998 (low risk according to the indicators) it was hard to find connection between income uncertainty and frequence of adjustment. This observation justifies also the assumption that the household-level uncertainty and the general economic situation may be highly correlated. Thus, while no household-level uncertainty exists as such, the general negative expectations of the future may induce precautionary saving behaviour which the cross-sectional data cannot reveal.

[39]Consumer confidence indicator was collected semiannually since 1987. In 1991-1995 it was collected quarterly and monthly thereafter.

Most earlier empirical studies of durable purchases were based on the aggregate consumption data (see Bertola & Caballero (1990), Caballero (1990, 1993) and Hassler (2001), among others). Evidently, this was because of the lack of appropriate micro-level data. However, there are few studies which have utilized individual data. Lam (1991) used threshold adjustment model and panel data to study the consumption behaviour in an automobile market. He found that the resale market imperfections and liquidity constraints have important effects on automobile expenditures. Also, the upward adjustments are substantially quicker than downward adjustments. His interpretation of this asymmetry in differencies between the upper and lower bands from the desired level implies that the efficiency of policy depends on its direction. A policy change that increases the desired stock can be expected to be more effective than a policy that reduces the desired stock by the same magnitude. Using U.S. panel data on automobile purchases, Eberly (1994) conducted similar results regarding an increase in uncertainty. One of her findings was that the width of the inaction band is positively related to the income variability. Carroll & Dunn (1997) studied the effect of an unemployment risk on durable and nondurable spending and household balance sheets. They found that the durable expenditures are very robustly correlated with lagged unemployed expectations. Dunn (1998) used household level data from 1983 and 1992 and found similar results to that of Eberly (1994): households with a higher probability of becoming unemployed are less likely to have recently purchased home or an automobile. Thus, the inaction range will be wider for those who face greater unemployment risk.

Using Finnish quarterly data from 1979 to 1992 and the conditional variance of the innovations in the aggregate income and the change in unemployment rate as a source of systematic risk, Koivumäki (1999) found statistical evidence that increased income uncertainty has suppressed consumption growth in Finland. Also, he found a negative relationship between consumption and unemployment rate. Correspondingly, Foote et al. (2000), using adjustment probits and panel of U.S. automobile holdings, found that more variable income leads to less frequent adjustment while more miles driving indicating a greater rate of depreciation leads households to adjust more often. All of these findings are in accordance to our findings and support the (S,s) behaviour. The only exception to these mainstream conclusions is Attanasio (2000) who showed that it is difficult to characterise the time-series properties of aggregate expenditure from the estimated (S,s) rules.

Even though we found some evidence of the importance of uncertainty to postpone automobile purchases, it is likely to assume that all the identified uncertainty measures and repair costs are not adequate proxies to emulate the real uncertainty and depreciation rate, respectively. To obtain more reliable results, one should improve the data by bringing time structure into the empirical analysis. While such microeconomic data does not exist, one fertile approach may be to construct an artificial panel by dividing each Survey into the groups, say, according to the income deciles, and then follow each group over time to study if the income variance of the groups have any effect on adjustment. Evidently, this approach needs restrictive assumptions of preferences and may lead to the further problems, for example, because of the households movement between income deciles. Also, the other long-lived durables should be used to test the validity of the (S,s) model. These are, however, left for the further research.

5. CONCLUSION

This study investigated implications of uncertainty, depreciation and adjustment costs on the timing of adjustment and purchases of durable goods. The model based on the (S,s) rule extended the theoretical framework by Hassler (1994, 1996a) by deriving an (S,s) rule explicitly from the Cobb-Douglas preferences. The model states that an agent has a desire to keep a certain fraction of his wealth to be invested in one (expensive) durable. Because of the depreciation of the good and stochastic movements in prices and agent's wealth over time, the actual level of fraction deviates from that of target. While the continuous updating is costly and the purchases include costs that are at least partially irreversible, it is optimal for an agent to allow an inaction band around the frictionless target and not to adjust until the actual fraction goes outside the band. Including the possibility of idiosynchratic and systematic risk, the model states that the width of the inaction band is positively related to the systematic risk. An increase in risk increases the option value of waiting, and the purchases are postponed to the future. On the other hand, a greater depreciation should lead to more frequent adjustment.

Using four different cross-sectional Household Budget Surveys from the years 1985, 1990, 1994-96 and 1998, the empirical consumption behaviour based on the Finnish automobile purchases was in most cases in favour of the (S,s) rule. A percent increase in household-level income uncertainty increases the inaction band more than 10 percent, while an unit increase in uncertainty decreases the probability of adjustment with few percents. The other uncertainty measures - housing debt and expenditures on health - did not perform well statistically, and typically did not affect on the width of the inaction band. An increase in depreciation of automobiles measured by repair costs increases the probability of adjustment, which is consistent with the infrequent adjustment theory.

The finding that income uncertainty has a large role in household's decision-making and affects intertemporal consumption behaviour is not surprising, but the results help to understand better the effect of uncertainty on the magnitude of saving and business cycles. While the lack of data prevented us to study other uncertainty measures, durable goods and the dynamic nature of the purchases, it is likely to assume that including these elements into the study would even strengthen the importance of uncertainty on the timing of durable purchases.

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APPENDIX A

Proposition:

If the variables P, W and C are evolving over time according to the following geometric Brownian motions

$$\begin{aligned} \frac{dW}{W} &= \alpha_W dt + \delta_w dz_W, \\ \frac{dP}{P} &= \alpha_P dt + \delta_P dz_P, \\ \frac{dC}{C} &= -\alpha_C dt, \end{aligned} \tag{A.1}$$

then the function $a_t(P_t, C_t, W_t) = \frac{P_t C_t}{W_t}$ follows geometric Brownian motion

$$\frac{da}{a} = \alpha_a dt + \delta_a dz_a. \tag{A.2}$$

The terms α and δ may be interpreted as drift and variance parameters of the processes, respectively. Especially, the term α_C is the rate of depreciation. The terms dz are the increments of a Wiener process such that $dz_k = \varepsilon_k \sqrt{dt}$. While $\varepsilon_k \sim N(0,1)$, $E\left(dz_k\right) = 0$ and $Var\left(dz_k\right) = E\left[\left(dz_k\right)^2\right] - \left[E\left(dz_k\right)\right]^2 = dt$.

Proof:

In this proof we apply the Fundamental Theorem of stochastic calculus which is expanded to functions of several Ito processes.[40] In general, in the presence of several Wiener processes the differential dF for a function $F(t, x_1, ..., x_m)$ is given as

$$dF = \frac{\partial F}{\partial t} dt + \sum_i \frac{\partial F}{\partial x_i} dx_i + \frac{1}{2} \sum_i \sum_j \frac{\partial^2 F}{\partial x_i \partial x_j} dx_i dx_j \tag{A.3}$$

where dx_i and dx_j, $i, j = 1, ..., m$; $i \neq j$, are independent Ito processes. Inserting the derivatives and noting that there does not exist time explicitly in the function $a(P, C, W)$, the expression (A.3) becomes

$$\begin{aligned} da &= \left(\frac{C}{W} dP + \frac{P}{W} dC - \frac{PC}{W^2} dW\right) + \frac{1}{2}\left\{\frac{1}{W}(dP)(dC)\right. \\ &\quad - \frac{C}{W^2}(dP)(dW) + \frac{1}{W}(dC)(dP) - \frac{P}{W^2}(dC)(dW) + \frac{2PC}{W^3}(dW)^2 \\ &\quad \left. - \frac{C}{W^2}(dW)(dP) - \frac{P}{W^2}(dW)(dC)\right\} \\ &= \frac{C}{W} dP + \frac{P}{W} dC - \frac{PC}{W^2} dW + \frac{1}{W}(dP)(dC) - \frac{C}{W^2}(dP)(dW) \\ &\quad - \frac{P}{W^2}(dC)(dW) + \frac{2PC}{W^3}(dW)^2. \end{aligned} \tag{A.4}$$

[40] See Malliaris & Brock (1982) and Dixit & Pindyck (1994) who give more background for the stochastic calculus and describe the properties of the Ito processes in more detail.

After substituting the Ito processes from (A.1) the expanded form of the equation becomes

$$\begin{aligned} da &= \frac{C}{W}\left(\alpha_P P dt+\delta_P P dz_P\right)-\frac{P}{W}\left(\alpha_C C dt\right) \\ &-\frac{PC}{W^2}\left(\alpha_W W dt+\delta_W W dz_W\right)-\frac{1}{W}\left(\alpha_P P dt+\delta_P P dz_P\right)\left(\alpha_C C dt\right) \\ &-\frac{C}{W^2}\left(\alpha_P P dt+\delta_P P dz_P\right)\left(\alpha_W W dt+\delta_W W dz_W\right) \\ &+\frac{P}{W^2}\left(\alpha_C C dt\right)\left(\alpha_W W dt+\delta_W W dz_W\right)+\frac{2PC}{W^3}\left(\alpha_W W dt+\delta_W W dz_W\right)^2 . \end{aligned} \tag{A.5}$$

All the terms $(dt)^{\frac{3}{2}}$ and $(dt)^2$ go to zero faster than dt as time increments become infinitesimal small, so these terms are ignored. Noting that the term $E\left[dz_i dz_j\right]=\rho_{ij}dt$ is the coefficient of correlation[41] between the two processes the expression can be rewritten as

$$\begin{aligned} da &= \frac{PC}{W}\left(\alpha_P-\alpha_C-\alpha_W\right)dt+\frac{PC}{W}\left(\delta_P dz_P-\delta_W dz_W\right) \\ &-\frac{PC}{W}\delta_P\delta_W\rho_{PW}dt+\frac{2PC}{W}\delta_W^2 dt \\ &= \left[\alpha_P-\alpha_C-\alpha_W-\delta_P\delta_W\rho_{PW}+2\delta_W^2\right]adt+\left[\delta_P dz_P-\delta_W dz_W\right]a \\ &= \alpha_a adt+\delta_a adz_a . \end{aligned} \tag{A.6}$$

This is the equation (2.3) in the main text. It is easy to show that the mean and the variance of this process are

$$\begin{aligned} E\left(\frac{da}{a}\right) &= \alpha_a dt, \\ Var\left(\frac{da}{a}\right) &= E\left[(da)^2\right]-E\left[(da)\right]^2=\delta_a^2 dt. \end{aligned} \tag{A.7}$$

APPENDIX B

Suppose that each time increment is of length Δt and denote $x_t=a_t-a^*$, then $\Delta x=\Delta a$. The Bellman equation for the problem is

$$V(x_t,u_t)=\frac{x_t^2}{2}\Delta t+e^{-r\Delta t}E_t\left[V(x_{t+\Delta t},u_{t+\Delta t})\right], \tag{B.1}$$

in which $V(x_t,u_t)$ denotes the total cost function and $V(x_{t+\Delta t},u_{t+\Delta t})=V(x_t+\Delta x,t+\Delta t,u+\Delta u)$. Using the approximation $e^{-r\Delta t}\approx(1+r\Delta t)^{-1}$ and multiplying

[41] Note that because Wiener processes have variances and standard deviations per unit of time equal to one, ρ_{ij} is also the covariance per unit of time between the processes.

(B.1) with the term $(1 + r\Delta t)$ gives

$$\begin{aligned} rV(x_t, u_t)\Delta t &= \frac{x_t^2}{2}\Delta t(1 + r\Delta t) + E_t \left[V(x_{t+\Delta t}, u_{t+\Delta t}) - V(x_t, u_t)\right] \quad \text{(B.2)} \\ &= \frac{x_t^2}{2}\Delta t(1 + r\Delta t) + E_t \left[dV\right]. \end{aligned}$$

Dividing by Δt and letting it approach zero we get

$$rV(x_t, u_t) = \frac{x_t^2}{2} + \frac{1}{dt} E_t \left[dV\right]. \qquad \text{(B.3)}$$

The right-hand side of the equation can be interpreted as a current flow of disutility plus the expected rate of change of the total cost function. Using the version of Ito's lemma for combined Brownian and Poisson processes[42], the expectation of the differential V is given by

$$\begin{aligned} E\left[dV\right] &= \left[\alpha x_t V'(x_t, u_t) + \frac{1}{2}\sigma^2 x_t^2 V''(x_t, u_t) + h.o.t\right] dt \qquad \text{(B.4)} \\ &\quad + \lambda \left[(V(a^*, u_t) + c) - V(x_t, u_t)\right] dt + \gamma_u \left[V(x_t, \overline{u}_t) - V(x_t, u_t)\right] dt, \end{aligned}$$

where $h.o.t$ means higher order terms which approach zero faster than dt as $dt \to 0$. These terms are omitted. The second term on the right-hand side captures the idea that a Poisson shock occurs with probability λdt. Then, by assumption, the variable a is adjusted back to the target level a^* after the lumpy sum cost is paid. Note that while the immediate utility loss is zero at the target level, the term $V(a^*, u_t) \neq 0$ because of the expectation of the future deviations from the target. The term γ_u is the probability of the switch of the economy. If the switch occurs during the time increment dt, then the expected total costs shift from $V(x_t, u_t)$ to $V(x_t, \overline{u}_t)$.[43] Inserting the previous equation to (B.3) we get

$$\begin{aligned} &\frac{1}{2}\sigma^2 x^2 V''(x_t, u_t) + \alpha x V'(x_t, u_t) - (\lambda + r)V(x_t, u_t) + \gamma_u \left[V(x_t, \overline{u}_t) - V(x_t, u_t)\right] \\ = &-\frac{x_t^2}{2} - \lambda \left(V(a^*, u_t) + c\right). \qquad \text{(B.5)} \end{aligned}$$

While $u_t = 0, 1$, (B.5) constitutes a system of two second-order differential equations with two unknown functions. Even though this system is quite complex its set of solutions can be found using the following procedure. The system of the second-order differential equations can be rewritten as

$$\frac{1}{2}\sigma^2 x_t^2 V''(x_t, 0) + \alpha x_t V'(x_t, 0) - (\lambda + r)V(x_t, 0) + \gamma_0 \left[V(x_t, 1) - V(x_t, 0)\right]$$

[42] See Dixit & Pindyck (1994), Merton (1999) or Cochrane (2000) for a thorough mathematical treatment.

[43] To be precise, the terms $V(a^*, u_t)$ and $V(x_t, \overline{u}_t)$ should be written as $V(0, u_{t+1})$ and $V(x_t, \overline{u}_{t+1})$ to capture the idea of a jump or a switch between the times t and $t + 1$. However, in an infinite context these two are equal. Also, at the target level the state variable is $x_t = a^* - a^* = 0$. To avoid confusion later on, however, we use the notation $V(a^*, u_t)$ to describe the (expected) total costs at the target level when the state of the economy is u_t.

$$
\begin{aligned}
&= -\frac{x_t^2}{2} - \lambda\left(V(a^*,0)+c\right), && \text{(B.6)}\\
&\frac{1}{2}\sigma^2 x_t^2 V''(x_t,1) + \alpha x_t V'(x_t,1) - (\lambda+r)V(x_t,1) + \gamma_1\left[V(x_t,0)-V(x_t,1)\right] &&\\
&= -\frac{x_t^2}{2} - \lambda\left(V(a^*,1)+c\right). &&
\end{aligned}
$$

This is the equation (2.10) in the main text. Define two new functions (without the subscripts) such that

$$
\begin{aligned}
K(x) &= V(x_t,0) - V(x_t,1), && \text{(B.7)}\\
J(x) &= \frac{V(x_t,1)}{\gamma_1} + \frac{V(x_t,0)}{\gamma_0}. &&
\end{aligned}
$$

Then

$$
\begin{aligned}
&\frac{1}{2}\sigma^2 x^2 K''(x) + \alpha x K'(x) - (\lambda+r+\gamma_0+\gamma_1)K(x) && \text{(B.8)}\\
&= \lambda\left(V(a^*,0)-V(a^*,1)\right), &&\\
&\frac{1}{2}\sigma^2 x^2 J''(x) + \alpha x J'(x) - (\lambda+r)J(x) &&\\
&= -\frac{\gamma_0+\gamma_1}{2\gamma_0\gamma_1}x^2 - \lambda\left(\frac{1}{\gamma_0}(V(a^*,0)+c) + \frac{1}{\gamma_1}(V(a^*,1)+c)\right). &&
\end{aligned}
$$

The first equation comes from subtracting the first equation from the second in (B.6) and using (B.7). Adding up the equations in (B.6) and using (B.7) results the second equation in (B.8). Each of the equations in (B.8) yields an 'independent' solution. Consider the second equation in (B.8) and guess that the solution for the homogeneous part is of the general form

$$
J(x) = Ax^{\beta}, \tag{B.9}
$$

where A is a constant to be determined. Then, the homogeneous part can be rewritten as

$$
Ax^{\beta}\underbrace{\left(\frac{1}{2}\sigma^2\beta^2 + (\alpha - \frac{1}{2}\sigma^2)\beta - (\lambda+r)\right)}_{Q_\beta(\beta)} = 0\,. \tag{B.10}
$$

The roots of the fundamental quadratic $Q_\beta(\beta)$ are

$$
\begin{aligned}
\beta_1 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2 + \frac{2(\lambda+r)}{\sigma^2}} > 1, && \text{(B.11)}\\
\beta_2 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2 + \frac{2(\lambda+r)}{\sigma^2}} < 0. &&
\end{aligned}
$$

Thus, the general solution of the homogeneous part is

$$
J_H(x) = A_1 x^{\beta_1} + A_2 x^{\beta_2}. \tag{B.12}
$$

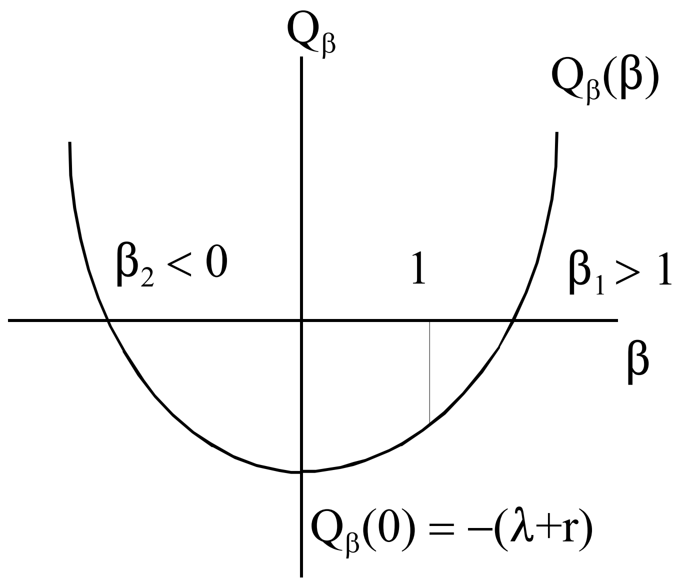

Figure B1: Fundamental Quadratic

The parameter A_2 can be determined as follows. Evaluating the fundamental quadratic at points (0,1) results $Q_\beta(0) = -(\lambda+r) < 0$ and $Q_\beta(1) = \alpha-(\lambda+r) < 0$.[44] Thus, $\beta_1 > 1$ and $\beta_2 < 0$. This result can be understood from the figure B1. The limiting behaviour near zero gives one condition. When a is expected to remain at its target value, there is no utility and adjustment costs. This gives the condition $J(0) = 0$. However, when $a \to a^*$, that is, when $x \to 0$ and $\beta_2 < 0$, the term $A_2 x^{\beta_2} \to \infty$. To ensure that $J(x)$ goes to zero as $x \to 0$, we set the coefficient of the negative power of x equal to zero, that is, $A_2 = 0$.

The particular solution can be found by using the method of undetermined coefficients. Guess that the solution is of the form

$$J(x) = B_2 x^2 + B_1 x + B_0. \tag{B.13}$$

Inserting the correspondent derivates to (B.8) and comparing the coefficients result

$$\begin{aligned} B_2 &= -\frac{\gamma_0+\gamma_1}{2\gamma_0\gamma_1(\sigma^2+2\alpha-(\lambda+r)}, \\ B_1 &= 0, \\ B_0 &= \frac{\lambda}{\lambda+r}\left(\frac{1}{\gamma_0}(V(a^*,0)+c)+\frac{1}{\gamma_1}(V(a^*,1)+c)\right). \end{aligned} \tag{B.14}$$

[44]The assumption $\alpha < \lambda + r$ ensures that there exists finite time when it is optimal to adjust. Otherwise, waiting longer would always be a better policy, and the optimum would not exist. See Dixit & Pindyck (1994, pp.137-138, pp.171-173) for illustrative calculations.

The general solution for $J(x)$ is

$$\begin{aligned} J(x) &= A_1 x^{\beta_1} - \frac{\gamma_0+\gamma_1}{2\gamma_0\gamma_1(\sigma^2+2\alpha-(\lambda+r)}x^2 \\ &\quad + \frac{\lambda}{\lambda+r}\left(\frac{1}{\gamma_0}(V(a^*,0)+c)+\frac{1}{\gamma_1}(V(a^*,1)+c)\right). \end{aligned} \tag{B.15}$$

Consider next the first equation in (B.8). Following the same steps as above the homogeneous part can be rewritten as

$$\underbrace{Cx^\theta\left(\frac{1}{2}\sigma^2\theta^2+(\alpha-\frac{1}{2}\sigma^2)\theta-\eta\right)}_{Q_\theta(\theta)}=0, \tag{B.16}$$

where $\eta=\lambda+r+\gamma_0+\gamma_1$. Thus, the general solution of the homogeneous part is

$$K_H(x)=C_1x^{\theta_1}+C_2x^{\theta_2}, \tag{B.17}$$

where the roots are

$$\begin{aligned} \theta_1 &= \frac{1}{2}-\frac{\alpha}{\sigma^2}+\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2(\lambda+r+\gamma_0+\gamma_1)}{\sigma^2}}>\beta_1, \\ \theta_2 &= \frac{1}{2}-\frac{\alpha}{\sigma^2}-\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2(\lambda+r+\gamma_0+\gamma_1)}{\sigma^2}}<\beta_2. \end{aligned} \tag{B.18}$$

It is easy to show that $Q_\theta(0)<Q_\beta(0)<0$ and $Q_\theta(1)<Q_\beta(1)<0$. This results $\theta_1>\beta_1>1$ and $\theta_2<\beta_2<0$. The coefficients C_1 and C_2 can be determined as above, leaving only C_1 to be determined. The particular solution is easy to find. It is

$$K_P(x)=-\frac{\lambda\left[V(a^*,1)-V(a^*,0)\right]}{\eta}. \tag{B.19}$$

The general solution for $K(x)$ is

$$K(x)=C_1x^{\theta_1}-\frac{\lambda\left[V(a^*,1)-V(a^*,0)\right]}{\eta}. \tag{B.20}$$

The solutions for $V(x_t,0)$ and $V(x_t,1)$ can be found by using (B.7), (B.15) and (B.20). Thus,

$$\begin{aligned} V(x_t,0) &= \frac{\gamma_0\gamma_1}{\gamma_0+\gamma_1}\left\{A_1x^{\beta_1}-\frac{\gamma_0+\gamma_1}{2\gamma_0\gamma_1(\sigma^2+2\alpha-(\lambda+r)}x^2\right. \\ &\quad \left.+\frac{\lambda}{\lambda+r}\left(\frac{1}{\gamma_0}(V(a^*,0)+c)+\frac{1}{\gamma_1}(V(a^*,1)+c)\right)\right\} \\ &\quad -\frac{\gamma_0}{\gamma_0+\gamma_1}\left\{C_1x^{\theta_1}-\frac{\lambda\left[V(a^*,1)-V(a^*,0)\right]}{\eta}\right\}, \\ V(x_t,1) &= \frac{\gamma_0\gamma_1}{\gamma_0+\gamma_1}\left\{A_1x^{\beta_1}-\frac{\gamma_0+\gamma_1}{2\gamma_0\gamma_1(\sigma^2+2\alpha-(\lambda+r)}x^2\right. \end{aligned} \tag{B.21}$$

$$+\frac{\lambda}{\lambda+r}\left(\frac{1}{\gamma_0}(V(a^*,0)+c)+\frac{1}{\gamma_1}(V(a^*,1)+c)\right)\Bigg\}$$
$$+\frac{\gamma_1}{\gamma_0+\gamma_1}\left\{C_1x^{\theta_1}-\frac{\lambda\left[V(a^*,1)-V(a^*,0)\right]}{\eta}\right\}.$$

The functions $V(a^*,0)$ and $V(a^*,1)$ can be found by evaluating (B.21) at the point a^* (then $x=0$). After some rigorous calculus[45], the solution becomes

$$\begin{aligned} V(x_t,0) &= \frac{\gamma_0\gamma_1}{\gamma_0+\gamma_1}A_1x^{\beta_1}-\frac{1}{2(\sigma^2+2\alpha-(\lambda+r)}x^2 \\ &\quad -\frac{\gamma_0}{\gamma_0+\gamma_1}C_1x^{\theta_1}+\frac{\lambda c}{r}, \\ V(x_t,1) &= \frac{\gamma_0\gamma_1}{\gamma_0+\gamma_1}A_1x^{\beta_1}-\frac{1}{2(\sigma^2+2\alpha-(\lambda+r)}x^2 \\ &\quad +\frac{\gamma_1}{\gamma_0+\gamma_1}C_1x^{\theta_1}+\frac{\lambda c}{r}. \end{aligned} \tag{B.22}$$

This is the equation (2.11) in the main text. However, the equation (B.22) is valid only in the range $x_t\in[s_0,a^*]$. If the state of the economy is low ($u_t=0$) and $x_t\le s_0$, the durable is immediately adjusted. Also, it the state variable is in the range $[s_1,s_0]$, a switch from the high risk state to the low risk state will cause an immediate adjustment. Thus, $V(x_t,0)$ is a constant for $x_t\in[s_1,s_0]$. The system of differential equations degenerates to

$$\begin{aligned} V(x_t,0) &= V(s_0,0), \\ V(x_t,1) &= \frac{x_t^2}{2}\Delta t+e^{-r\Delta t}E_t\left[V(x_{t+\Delta t},u_{t+\Delta t})\right]. \end{aligned} \tag{B.23}$$

Following the same steps as above the solution for the function $V(x_t,1)$ is

$$\begin{aligned} V(x_t,1) &= D_1x^{\mu_1}-\frac{1}{2(\sigma^2+2\alpha-(\lambda+r+\gamma_1)}x^2 \\ &\quad -\frac{\gamma_1V(a^*,0)+(\gamma_1-\lambda)c}{(r+\gamma_1)}, \end{aligned} \tag{B.24}$$

in which the root μ_1 is defined as

$$\beta_1<\mu_1=\frac{1}{2}-\frac{\alpha}{\sigma^2}+\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2(\lambda+r+\gamma_1)}{\sigma^2}}<\theta_1. \tag{B.25}$$

To summarise:

1^0 If $x_t\in[s_0,a^*]$ and the state of the economy is $u_t=0$, $V(x_t,0)$ is given by the equation (B.22).

2^0 If $x_t<s_0$ and the state of the economy is $u_t=0$, $V(x_t,0)$ is constant and

[45] We have benefited from Scientific Workplace in calculus.

equals $V(s_0, 0)$.

3^0 If $x_t \in [s_0, a^*]$ and the state of the economy is $u_t = 1$, $V(x_t, 1)$ is given by the equation (B.22).

4^0 If $x_t \in [s_1, s_0]$ and the state of the economy is $u_t = 1$, $V(x_t, 1)$ is given by the equation (B.24).

5^0 If $x_t < s_1$ and the state of the economy is $u_t = 1$, $V(x_t, 1)$ is constant and equals $V(s_1, 1)$.

Analytically, we are not able to find the unknown integration constants and band limits in (B.22) and (B.24). However, using the following smooth-pasting and value-matching conditions we are able to find the solutions numerically.

$$\begin{aligned} Smooth-\ pasting &: \begin{cases} V'(s_0, 0) = 0 \\ V'(s_1, 1) = 0 \\ V'(a^*, 0) = 0 \\ V'(a^*, 1) = 0 \end{cases}, \\ Value-matching &: \begin{cases} V(s_0, 0) = V(a^*, 0) + c \\ V(s_1, 1) = V(a^*, 1) + c \end{cases}. \end{aligned} \tag{B.26}$$

APPENDIX C

From Appendix B the equations (B.3) and (B.4) result

$$\begin{aligned} rV(x_t, u_t) &= \frac{x_t^2}{2} + \left[\alpha x_t V'(x_t, u_t) + \frac{1}{2}\sigma^2 x_t^2 V''(x_t, u_t)\right] \\ &\quad + \lambda\left[(V(a^*, u_t) + c) - V(x_t, u_t)\right] + \gamma_u\left[(V(x_t, \overline{u}_t)) - V(x_t, u_t)\right]. \end{aligned} \tag{C.1}$$

Evaluating this equation at point a^* and trigger bands s_u, $u = 0, 1$, gives

$$\begin{aligned} rV(a^*, u_t) &= 0 + 0 + 0 + \lambda\left[(V(a^*, u_t) + c) - V(a^*, u_t)\right] \\ &\quad + \gamma_u\left[(V(a^*, \overline{u}_t)) - V(a^*, u_t)\right], \\ rV(s_u, u_t) &= \frac{s_u^2}{2} + \alpha s_u V'(s_u, u_t) + \frac{1}{2}\sigma^2 s_u^2 V''(s_u, u_t) \\ &\quad + \lambda\left[(V(a^*, u_t) + c) - V(s_u, u_t)\right] + \gamma_u\left[(V(s_u, \overline{u}_t)) - V(s_u, u_t)\right]. \end{aligned} \tag{C.2}$$

The first three terms in the first equation are zero because at the target level a^* the state variable is $x_t = a^* - a^* = 0$. Subtracting the first equation from the second, using second-order Taylor approximation for the term $V''(s_u, u_t)$ and the smooth-pasting and value-matching conditions (B.26), and after some rearrangement gives

$$\frac{(a_u - a^*)^2}{2} = \overbrace{rc}^{>0} + \overbrace{\lambda c}^{>0} + \gamma_u\left[c + V(a^*, \overline{u}_t) - V(s_u, \overline{u}_t)\right] \tag{C.3}$$

On the left-hand side we have utilised the information that at the trigger band point the state variable becomes $(x_t)\mid_{s_u}= a_u - a^*$ denoting the largest deviation from the target level. The left-hand side is the temptation to adjust, and the right-hand side is the value of waiting still an instant dt. The first two terms on the right-hand side are positive. The third term can be rewritten as

$$\overbrace{\gamma_u}^{>0}\overbrace{[c - (V(s_u, \overline{u}_t) - V(a^*, \overline{u}_t))]}^{\geq 0} \geq 0. \tag{C.4}$$

This inequality can be understood as follows: If $x_t < s_u$, then $c \geq V(x_t, \overline{u}_t) - V(a^*, \overline{u}_t)$. If $x_t = s_u$, then $c = V(x_t, \overline{u}_t) - V(a^*, \overline{u}_t)$ according to the value-matching condition.

APPENDIX D

Table D1
Dummy variables used in estimation

Constant	
Province	Southern Finland Western Finland Eastern Finland Oulu Lapland Åland
Local Authority	City Commune
Gender of household head	Male Female
Type of household	Family without children One-parent family Family with children Aged family Other families
Educational attainment	Basic Lower middle-level Higher middle-level Lowest high-level Lower candidate-level Higher candidate-level Researcher or similar Unknown Employee
Socio-economic status	Subordinate official Superior official Entrepreneur Agricultural entrepreneur Student Retired Long-term unemployed Others
Months of unemployed	1-3 months 4-6 months 7-9 months 10-12 months More than 12 months