

JOENSUUN YLIOPISTON YHTEISKUNTATIEETEELLISIÄ JULKAISUJA
UNIVERSITY OF JOENSUU PUBLICATIONS IN SOCIAL SCIENCES

N:o 66

Jari Viitanen

**ESSAYS ON INTERTEMPORAL CONSUMPTION
BEHAVIOUR IN FINLAND**

Joensuun yliopisto
Joensuu 2004

Julkaisija Publisher	Joensuun yliopisto University of Joensuu
Sarjan toimittaja Editor	VTT Mikko A. Salo
Vaihto	Joensuun yliopiston kirjasto/Vaihdot PL 107, 80101 JOENSUU Puh. 013-251 2677, Faksi 013-251 2691 Email:vaihdot@joensuu.fi
Exchange	Joensuu University Library/ Exchanges P.O. Box 107, FIN-80101 JOENSUU Tel. +358-13-2512677 Fax +358 13 251 2691 Email:vaihdot@joensuu.fi
Myynti	Joensuun yliopiston kirjasto/ Julkaisujen myynti PL 107, 80101 JOENSUU Puh. 013-251 2652, 013-251 2677 Faksi 013-251 2691 Email:joepub@joensuu.fi
Sale	Joensuu University Library/ Sales of publications P.O. Box 107, FIN-80101 JOENSUU, FINLAND Tel. +358-13-251 2652, 251 2677 Fax +358 13 251 2691 Email: joepub@joensuu.fi

Copyright © 2004 Joensuun yliopisto
1. painos

ISSN 0781-0350
ISBN 952-458-431-X

Kannen suunnittelu: Esko Puranen
Kansikuva: Jaakko Hosia, *Tripolaris* (1998; akryyli, 217 mm x 178 mm)
Kannen tekninen toteutus: Joensuun yliopistopaino
Paino: Joensuun yliopistopaino
Joensuu 2004

Preface

The roots of this dissertation lie in the early 1990s. When contemplating the subject of my pro graduate thesis in economics, Mr. Heikki Taimio encouraged me to study intertemporal consumption and asset pricing, with emphasis on the empirical determinants on intertemporal consumption decisions in Finland. Little did I know on the vastness of the task I was to undertake. Still, I owe a special gratitude to Heikki.

Even though this thesis has been composed independently and without regular supervisors, I have received suggestions and critique from several persons on the subject. Especially Dr. Matti Estola's insight and constructive comments have substantially improved the contents of the study and improved my understanding of dynamic optimisation problems. The suggestions of Professors Markku Rahiala, Mika Linden and Kyösti Pulliainen on the econometric part of the thesis were invaluable. Also, the suggestions and comments by Dr. Heikki Kauppi and helpful discussions with Dr. Heikki Niemeläinen are gratefully acknowledged.

I am also indebted to Dr. Timo Tammi, the Head of the Department of Business and Economics, for his help in administrative issues, Mr. Risto Sullström, Mrs. Airi Pajunen, Mr. Veli-Matti Törmälehto, Dr. Kari Takala and Mr. Pekka Tsupari for providing data. I would also like to thank the preliminary examiners of this dissertation, Dr. Jouko Vilmunen from Bank of Finland and Professor Timo Koivumäki from University of Oulu, for their comments which helped me to improve the final version of the thesis.

The completion of this study would not have been possible without the facilities offered by the Department of Business and Economics which provided a stimulating environment. I would also like to express my gratitude to the Finnish Postgraduate Programme in Economics for granting a graduate school fellowship. In addition, financial support from University of Joensuu, Faculty of Social Sciences, Yrjö Jahnsson Foundation, Osuuspankkiryhmän tutkimussäätiö and Finnish Cultural Foundation is gratefully acknowledged. My thanks also go to Mrs. Jaana Hokkanen for improving the English language, and Mr. Anssi Kähkönen for editorial assistance.

Last but not least, I would like to thank my wife Kristiina for her patience and understanding of my occasional absence of mind and intertemporal optimisation to avoid household duties, and my daughters, Kia and Marilla, for drawing my attention away from research issues.

Joensuu, January 2004

Jari Viitanen

Abstract

This dissertation is a collection of four essays which analyse intertemporal consumption behaviour, with the emphasis on the empirical determinants of intertemporal consumption decisions in Finland. Based on the rational consumer's intertemporal optimisation, the essays investigate intertemporal consumption from different perspectives.

The first essay discusses the theoretical and empirical shortcomings of standard consumption theories. The second essay studies how consumer's own consumption history, habit formation and durability of durables, affects his present consumption behaviour. Also, it investigates the role of other reference consumption levels, which may induce envy for the consumer and has an effect on the consumption expenditures. The third essay studies the magnitude and temporal evolution of relative risk aversion and elasticity of intertemporal substitution in Finland. Finally, the last essay concentrates on the role of uncertainty and irreversible investment costs on the timing of durable purchases. The empirical evidence of the first three essays is based on the private aggregate consumption data, while the results of the final essay is based on the four Finnish Household Budget Surveys.

The main findings of the study are the following. First, the Finnish aggregate consumption data from 1975-2001 does not mainly support the standard consumption theories based on the expected utility approach. Second, there has been a structural change in consumption behaviour in Finland due to the financial liberalisation in the middle of 1980s. After the deregulation, habit formation has dominated over the durability of durables. The results, however, do not support the hypothesis that Finnish consumers in average are envious with respect to the lagged total private consumption in Sweden or in OECD countries. Third, the elasticity of intertemporal substitution and risk aversion have increased after the deregulation. The results also reveal that consumers in aggregate dislike risk more than intertemporal fluctuations in consumption. Finally, higher income uncertainty leads consumers to postpone durable acquisitions, such as automobiles, to the future and decreases the probability of adjustments.

Key words: Intertemporal consumption, risk aversion, intertemporal substitution, habit formation, envy, irreversibility, uncertainty, (S,s) rule

Tiivistelmä

Tämä väitöskirja koostuu neljästä esseestä, joissa tarkastellaan rationaalisesti toimivan kuluttajan intertemporaaliseen optimointiin perustuvaa kulutuskäyttäytymistä eri näkökulmista. Tutkimuksen tarkoituksena on tuottaa havaintoaineistoon perustuvaa uutta tietoa suomalaisten yksityiseen kulutukseen vaikuttavista tekijöistä

Ensimmäisessä esseessä tarkastellaan yleisesti perinteisten kulutusmallien teoreettisia puutteita ja empirian heikkoutta. Toisessa esseessä tutkitaan kulutushistorian ja -tottumusten sekä kulutushyödykkeiden kestokulutusominaisuuksien merkitystä tämänhetkiseen kulutuskäyttäytymiseen. Lisäksi tarkastellaan niin sanottujen kateustekijöiden vaikutusta kulutuspäätöksiin. Kolmannessa esseessä tutkitaan suhteellisen riskin karttamisen ja intertemporaalisen substituutiojouston suuruutta ja ajallista kehitystä Suomessa. Viimeinen essee analysoi epävarmuuden ja palautumattomien hankintakustannusten vaikutusta kestokulutustavaroiden hankinta-ajankohtaan. Kolmen ensimmäisen esseen tulokset perustuvat yksityiseen kokonaiskulutusaineistoon. Viimeisen esseen tulokset perustuvat aineistoon neljästä eri kotitaloustiedustelusta.

Tutkimuksen keskeiset tulokset ovat seuraavat. Suomalaiseen kokonaiskulutusaineistoon perustuvat tulokset vuosilta 1975 - 2001 eivät pääsääntöisesti tue odotettuun hyötyyn perustuvia perinteisiä kulutusmalleja. Pääomamarkkinoiden vapauttaminen 1980-luvun puolivälissä on muuttanut rakenteellisesti kulutuskäyttäytymistä Suomessa. Säännöstelyn vapautumisen jälkeen kulutustottumusten merkitys yksityiseen kokonaiskulutukseen on hallitsevampaa kuin kestokulutushyödykkeiden kestävyysominaisuudet. Saadut tulokset eivät kuitenkaan tue sitä hypoteesia, että suomalaiset kuluttajat olisivat keskimäärin kateellisia Ruotsin tai OECD-maiden viivästetyille yksityiselle kokonaiskulutukselle. Intertemporaalinen substituutiojousto ja riskin karttaminen kulutuksen suhteen ovat kasvaneet pääomamarkkinoiden säännöstelyn vapauduttua. Tuloksien perusteella kuluttajat karttavat enemmän tulevaan kulutukseen liittyvää epävarmuutta kuin kulutuksen ajallista vaihtelua. Lisääntyvä epävarmuus tuloista saa kotitaloudet siirtämään kestokulutustavaroiden kuten autojen hankinta-ajankohtaa tulevaisuuteen ja pienentää hankinnan todennäköisyyttä.

Avainsanat: Kulutus yli ajan, riskin karttaminen, kulutuksen korvattavuus yli ajan, kulutustottumukset, kateus, palautumattomuus, epävarmuus, (S,s)-sääntö

General Contents

Preface	3
Abstract	4
Tiivistelmä	5
General Contents	6
List of Figures and Tables	7
Chapter 1: Consumption-Based CAP Model with Finnish Evidence: Introductory Essay with Summaries of Other Chapters	9
Chapter 2: Envy, Habit Formation and Durability in Aggregate Finnish Consumption	43
Chapter 3: Risk Aversion and Intertemporal Substitution in Aggregate Consumption: Finland 1975-2001	105
Chapter 4: Costs, Uncertainty and Durable Adjustments - Finnish Cross-Sectional Evidence from Automobile Purchases	141

List of Figures and Tables

List of Figures:

Chapter 2:

Figure A1: Habit Formation and Durability 97

Chapter 3:

Figure 2.1: Timing of Resolution of Uncertainty 112

Figure 4.1: Weights of Financial Wealth 121

Chapter 4:

Figure 2.1: Standard (S,s) Model 146

Figure 2.2: Augmented (S,s) Model 153

Figure 4.1: Economic Outlook and Consumer Confidence Survey 168

Figure B1: Fundamental Quadratic 178

List of Tables:

Chapter 1:

Table 1: Results for random walk hypothesis $\Delta c_t = \varepsilon_t$ on nondurables, 1975.1-2000.3 22

Table 2: Schwartz-Bayesian and Akaike's information criterion tests for ARIMA($p,1,q$) specification 23

Table 3: ARIMA($p,1,q$) estimates for expenditures on durables, 1975.1-2000.3 24

Table 4: Results of prediction of lagged information on durable expenditures, 1975.1-2000.3 25

Table 5: Moments for equation $E_t(r_{t+1}^i) - r_{t+1} = \gamma cov[r_{t+1}^i, g_{t+1}]$ 26

Chapter 2:

Table 1: Descriptive statistics on consumption data 1975.2-2001.2 70

Table 2: Descriptive statistics on real return data 1975.1-2001.2 71

Table 3: Correlations of consumption growth and real asset returns 1975.1-2001.2 72

Table 4: Correlations between instruments and variables in model, first panel covers 1975.1-2001.2, second 1975.2-1986.4, and third 1987.1-2001.2 74

Table 5: GMM results for model $E_t \beta \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^i \right\} = 1$ and its structural stability, durables and semidurables (DSeD)	76
Table 6: GMM results for model $E_t \beta \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^i \right\} = 1$ and its structural stability, nondurables and services (NDS)	78
Table 7: GMM results for model $E_t \beta \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left(\frac{C_{t+1-\varphi}}{C_{t-\varphi}} \right)^{\theta(\gamma-1)} R_{t+1}^i \right\} = 1$ and its structural stability, $\varphi = 1$, durables and semidurables (DSeD)	79
Table 8: GMM results for model $E_t \beta \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left(\frac{C_{t+1-\varphi}}{C_{t-\varphi}} \right)^{\theta(\gamma-1)} R_{t+1}^i \right\} = 1$ and its structural stability, $\varphi = 1$, nondurables and services (NDS)	80
Table 9: GMM results for model $E_t \left\{ \beta \left(\frac{c_{t+1}}{c_t} + b_1 \right)^{-\gamma} \left(R_{t+1}^i - b_1 \right) + \beta^2 b_1 \left(\frac{C_{t+2}}{C_t} + b_1 \frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^i \right\} - \left[1 + b_1 \left(\frac{c_{t-1}}{c_t} \right) \right]^{-\gamma} = 0$ and its structural stability, durables and semidurables (DSeD)	81
Table 10: International evidence of parameter values	85

Chapter 3:

Table 1: GMM results for model $\beta^{\frac{\gamma}{\sigma}} E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma}{\sigma}(\sigma-1)} \left(R_{t+1}^P \right)^{\frac{\gamma}{\sigma}-1} R_{t+1}^i \right\} = 1$, nondurables, portfolio 1	123
Table 2: GMM results for model $\beta^{\frac{\gamma}{\sigma}} E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma}{\sigma}(\sigma-1)} \left(R_{t+1}^P \right)^{\frac{\gamma}{\sigma}-1} R_{t+1}^i \right\} = 1$, services, portfolio 1	125
Table 3: GMM results for model $\beta^{\frac{\gamma}{\sigma}} E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma}{\sigma}(\sigma-1)} \left(R_{t+1}^P \right)^{\frac{\gamma}{\sigma}-1} R_{t+1}^i \right\} = 1$, nondurables, portfolio 2	126
Table 4: GMM results for model $\beta^{\frac{\gamma}{\sigma}} E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma}{\sigma}(\sigma-1)} \left(R_{t+1}^P \right)^{\frac{\gamma}{\sigma}-1} R_{t+1}^i \right\} = 1$, services, portfolio 2	127

Chapter 4:

Table 1: (S,s) target and triggers for state variable calculated as means in each category	159
Table 2: Determinants of lower band width, residual approach	163
Table 3: Determinants of upper band width, residual approach	164
Table 4: Determinants of band width when uncertainty is based on multiplicative heteroskedasticity approach	165
Table 5: Maximum likelihood estimates from heteroskedastic probit model	166
Table D1: Dummy variables used in estimation	183

Chapter 1

Consumption-Based CAP Model with Finnish Evidence: Introductory Essay with Summaries of Other Chapters

1. INTRODUCTION	11
2. EVOLUTION OF CONSUMPTION-BASED CAP MODEL	13
2.1. Basic CCAP Model	13
2.2. Empirical Failure of CCAP Model	18
3. PRELIMINARY RESULTS FROM FINNISH DATA	21
4. DIRECTIONS OF IMPROVEMENT	27
5. CONCLUSION WITH SUMMARIES OF OTHER CHAPTERS	32
References	34
Appendix A	40
Appendix B	41

Consumption-Based CAP Model with Finnish Evidence: Introductory Essay with Summaries of Other Chapters

by

Jari Viitanen

University of Joensuu, Department of Business and Economics

Abstract

This study surveys the historical evolution of consumption-based capital asset pricing model, its empirical failure and theoretical shortcomings. It also discusses alternative theoretical suggestions affecting intertemporal decision-making. In particular, we will show why the standard expected utility hypothesis is inappropriate in explaining the co-movement of consumption and asset prices together with the time-additive, time-separable power utility function. The empirical results revealed that the Finnish aggregate consumption data does not support the two famous theoretical cornerstones of the modern consumption theory, namely Hall's (1978) random walk theory of nondurables and services and Mankiw's (1982) ARMA(1,1) process for durable expenditures. Furthermore, the coefficient for relative risk aversion should be implausibly high to be consistent with the observed consumption behaviour, confirming the equity premium puzzle of Mehra & Prescott (1985).

Keywords: CCAPM, power utility, AR(1), ARMA(1,1)

1. INTRODUCTION

According to the Finnish National Accounts, the total private consumption expenditures form over half of the Gross Domestic Product. Therefore, to explain the economic fluctuations, it is important to understand the fluctuations in (aggregate) private consumption expenditures. Consumption decisions determine also savings which affect capital markets, investments, economic growth and development. Thus, the information on consumer behaviour, preference parameters underlying this behaviour, and the changes of determinants of consumption are important for anticipating consumption expenditures, for understanding the recent development of financial markets and the importance and need of social institutions, and for policy making.

For over two decades the dominant paradigm in the intertemporal consumption research has been based on the Euler equation approach and consumption-based capital asset pricing model (CCAPM). Since the seminal papers of Lucas (1978), Hall (1978) and Breeden (1979), the traditional CCAP model and its close successors have, however, confronted large empirical failure (see Mankiw (1982), Hansen & Singleton (1983) and Mehra & Prescott (1985), among others). Despite of its theoretical attractiveness, the standard model is seen to have two main problems. First, because of time-separability, only nondurables and services are allowed to be included in the theoretical formulation to explain the co-movements of asset returns and consumption. Second, the parametrisation of the standard model with a time-additive, time-separable isoelastic power utility function is too rigid and inadequate to uncover reliably the consumer's consumption behaviour. This, for example, culminates in the fact that the constant coefficient of relative risk aversion (CRRA) and the constant elasticity of intertemporal substitution (EIS) are, by definition, inversely related.

Recently, there has been great academic interest to overcome these theoretical and empirical problems. While the literature in this area is extensive and it is not even possible to summarise all the recent contributions, the topics covering liquidity constraints, precautionary saving motive, habit formation and irreversibility mechanism, to mention some, have deepened the understanding of intertemporal consumption and saving behaviour under uncertainty. Also, the rich parametrisation underlying these improvements together with improved data sets give a deeper empirical insight of intertemporal decision-making.

The purpose of this thesis is to improve the understanding of the determinants of intertemporal consumption decisions in Finland. The following essays study consumption behaviour from different perspectives. The first essay describes the basic CCAP model, its evolution, and its empirical failure. The knowledge of this evolution of CCAPM is essential in understanding why it is important to relax the standard assumption of time-separable preferences in intertemporal decision-making, and it motivates the other essays. The second essay is based on

the time-nonseparable preferences, and it evaluates how consumers' consumption history affects current consumption decisions. Also, it gives insight whether the consumers in average are envious, and have some reference consumption level on which they relate their current consumption. The third essay concentrates on disentangling the link between the coefficient of relative risk aversion and the elasticity of intertemporal substitution. Finally, the fourth essay studies an irreversibility mechanism and, especially, the effect of an increase in uncertainty on the durable consumption. All these four essays contain empirical results on Finnish consumption behaviour.

The motivation for this research comes from the fact that such studies have not been conducted with Finnish data. The recent papers on consumption research in Finland have almost entirely been based on the consumption function approach (see, for instance, Takala (1995, 2001)). They have tested which exogenous variables should be included in the consumption function. Also, they have studied if the time-series of these variables are cointegrated with consumption expenditures. Although this consumption function approach is important *per se* and is based on the principles of intertemporal optimisation, and while it considers the wealth effect on consumption (which is typically small), its insight is inadequate and narrow, because it neglects the dynamic nature of intertemporal consumption optimisation and consumer's expectation mechanism over future in an uncertain world. Accumulating wealth (positive or negative) offers a means through which rational consumers can allocate their resources to smooth consumption and to optimise their life-cycle consumption pattern. Even though some intertemporal consumption research has been carried out in Finland¹, the results of those studies concern mainly the period before the deregulation of financial markets and the recession years of the early 1990s, and they are, possibly, out of date. Some recent studies have applied the consumer's intertemporal decision-making (see e.g. Brunila (1997), Koivumäki (1999) and Takala (2001)) to explain consumer behaviour, but their aims have been different from those of the following essays.

This introductory essay is organised as follows. Section two outlines briefly the main theoretical microeconomic foundations based on the standard consumption-based CAP model and surveys the empirical failure of this model. Section three examines the time-series behaviour of Finnish durables and nondurables consumption expenditures, following the seminal papers of Hall (1978), Mankiw (1982) and Mehra & Prescott (1985). After observing similarities of those studies, section four discusses the directions of improvement of the basic model. In this section, we review in short the recent intertemporal consumption literature and present some remedies suggested to solve the empirical consumption puzzles. Finally, section five concludes the study with summaries of the other essays.

¹For instance, see Starck (1987), Stenius (1989), Svento (1990), Kostiainen & Starck (1991) and many papers of Koskela & Viren (1985, among others).

2. EVOLUTION OF CONSUMPTION-BASED CAP MODEL

2.1. Basic CCAP Model

Merton's (1973) study was the first attempt to simultaneously explain the intertemporal consumption decisions of a consumer and the choice of risky assets under uncertainty.² His contribution was that the wealth and assets are not goods as such, but represent a possibility to allocate savings to maximise the expected lifetime utility from consumption in an uncertain world. His multiperiod framework of asset pricing, based on continuous-time dynamic programming and a stochastic process on asset prices, pointed out that, unlike in a static one-period CAP model, a riskless asset is not constant over time.³ Also, each asset includes not only the covariance of return of asset with market portfolio, but also the covariances of return with all state variables in the model. This is widely known as the intertemporal CAP model (ICAPM).

However, no one until Hall (1978) had combined consumer's intertemporal decision-making with the rational expectations hypothesis which brought a revolution in consumption theory. Even though his framework, documented in more detail below, was naive by making simple assumptions of consumer's environment, it provided a fascinating way to introduce the first-order conditions - the Euler equations - without the need to explicitly derive the consumer's dynamic optimisation problem.

Partly based on the methodology of Hall, the basic consumption-based capital asset pricing model or CCAPM of Lucas (1978) and Breeden (1979) described the so-called "representative agent's" world where Merton's multi-beta ICAPM reduces to a single-beta model, where the excess return on any asset is proportional to its covariance with the aggregate real consumption. In this world, the market price of risk is given by the beta with respect to consumption, namely the consumption-beta.

Mathematically, the Lucas-Breeden model states that at any time t ($0 \leq t < \infty$) a single, infinitely long-lived consumer chooses his optimal consumption stream $\{c_t\}_{t=1}^{\infty}$ and the weights of portfolio by maximising the expected discounted utility of future consumption, subject to the standard budget constraint. Formally:

$$\max_{c_t, \lambda_{i,t}} E_t \left[\sum_{t=0}^{\infty} \beta^t U(c_t) \mid I_t \right] \quad (2.1)$$

subject to

²Of course, the life-cycle hypothesis of Modigliani & Brunberg (1954) and the permanent income hypothesis of Friedman (1957) considered asset returns. In their models, however, the uncertainty over future was resolved in a very simple way. See also Samuelson (1969) for discrete-time modelling.

³The static CAP model assumes that an asset's covariance is related to the return on all invested wealth, namely, market return (market portfolio) only. See Sharpe (1964) and Lintner (1965) for details.

$$W_{t+1} = (W_t + Y_t \Delta t - c_t \Delta t) R_{t+1}^P, \quad (2.2)$$

$$R_{t+1}^P = \sum_{i=1}^K \lambda_{i,t} R_{t+1}^i, \quad i = 1, \dots, K, \quad (2.3)$$

where c_t, Y_t and W_t are consumption expenditures, labour income and wealth at time t , respectively. The measurement units of these variables are as follows: $W_t: FIM$, $Y_t: FIM/\Delta t$ and $c_t: FIM/\Delta t$. $R_{t+1}^P = (1 + r_{t+1}^P \Delta t)$ is a K -dimensional *real return factor* on assets and r_{t+1}^P is a *real rate of return* on the portfolio between periods t and $(t + 1)$.⁴ $\beta = (1 + \rho \Delta t)^{-1}$ is the discount factor where ρ denotes the subjective rate of time preference. Hereafter, we assume that the time periods are of the same size, and the term Δt can be scaled to one. The weights, $\lambda_{i,t}$, are optimally chosen in period t such that $\sum_{i=1}^K \lambda_{i,t} = 1$. Some of the K assets might be risk-free so that the rate of return is not conditional on the realisation of the period $(t + 1)$ state of nature. $U(\cdot)$ is a one-period von Neumann-Morgenstern utility of consumption, with positive and decreasing marginal utility ($U'(c_t) > 0, U''(c_t) < 0$) and Inada-conditions ($\lim_{c \rightarrow 0} U'(c) \rightarrow \infty, \lim_{c \rightarrow \infty} U'(c) \rightarrow 0$). E_t denotes a mathematical expectation operator conditional upon the information set available to consumer at time t . Note that this kind of von Neumann-Morgenstern utility is additive and time-separable. An appropriate Bellman equation for dynamic programming maximisation is

$$V_t(W_t) = \max_{c_t, \lambda_t} [U(c_t) + \beta E_t V_{t+1}(W_{t+1})] \quad (2.4)$$

subject to budget constraints (2.2) and (2.3).

Regardless of which single asset the accumulated saving is invested in, the first-order condition of equation (2.4) for the periods t and $(t + 1)$ yields the optimal consumption stream:⁵

$$U'(c_t) = \beta E_t [U'(c_{t+1}) R_{t+1}^i], \quad (2.5)$$

or

$$E_t \left[\beta \frac{U'(c_{t+1})}{U'(c_t)} R_{t+1}^i - 1 \right] = 0 \quad (2.6)$$

or

⁴In literature these concepts are defined in many different ways: Among others, Boldrin et al. (1995) and Campbell (1996) define the term $(1 + r_{t+1}^i)$ as a (gross) rate of return. Carroll (1996), on the other hand, defines it as a (gross) interest rate, while de Brouwer (1996) defines the term r_{t+1}^i as a real interest rate. Note also that our definition of rate of return differs from the standard textbook definition $r = \frac{d}{p} \times 100$ (%), where d and p are dividends and market price of the asset, respectively.

⁵In Appendix A we briefly show how to derive the first-order condition (2.5). The FOC could also be derived for the total portfolio, R_{t+1}^P , but the single asset approach is sufficient for our illustrative purposes.

$$E_t (IMRS_{t+1} R_{t+1}^i) = 1, \quad (2.7)$$

where $U'(\cdot)$ is the marginal utility of consumption and $IMRS_{t+1}$ the intertemporal marginal rate of substitution between present and future consumption.⁶ The first-order condition in (2.5) is known as Euler equation in consumption, and it has a very simple intuitive interpretation: at any point in time t , for the consumption stream to be optimal, an infinitesimal perturbation in consumption should have no effect on the optimum, and leave the consumer indifferent whether to consume dc_t less today or an increase $R_{t+1}^i dc_t$ tomorrow. A reduction in consumption means a reduction $U'(c_t)dc_t$ in utility terms today. While an increase in next period's consumption means an increase $U'(c_{t+1})R_{t+1}^i dc_{t+1}$ in expected utility discounted at rate β . In optimum these two should be equal. Also, for any two risky assets simultaneously, the following orthogonality condition should be satisfied in equilibrium:

$$E_t [IMRS_{t+1} (R_{t+1}^i - R_{t+1}^j)] = 0, \quad i, j \in \mathbb{R}^K, \quad i \neq j. \quad (2.8)$$

In his seminal paper Hall (1978) assumed that the return factor R_{t+1} is constant and risk-free. Then the equation (2.5) becomes

$$E_t [U'(c_{t+1})] = (\beta R_{t+1})^{-1} U'(c_t). \quad (2.9)$$

Hall argued that the marginal utility of consumption should follow a univariate first-order Markov process. Then consumption itself is only a random walk process with first-order approximations of marginal utility of consumption, if the riskless rate of return and the subjective time preference are equal.⁷ Once lagged values of consumption are known, no other variables in consumer's information set can help to predict forthcoming values. Symbolically:

$$c_{t+1} = c_t + \varepsilon_{t+1}, \quad \varepsilon_t \sim i.i.d. (0, \sigma^2), \quad \forall t \in [0, \infty). \quad (2.10)$$

To the extent that information is available and relevant to predict c_{t+1} , it is already embedded in c_t . The error term ε_{t+1} reflects only the new information regarding permanent income at time $(t+1)$. If consumers form their estimates of permanent income rationally, then this error term should be serially uncorrelated, and consumption must follow an AR(1) process.

A stride to the realism was introduced by Grossman & Shiller (1981) who relaxed the assumption of constant return factor by assuming a time-varying real rate

⁶Generally in asset pricing models, this term is called the *stochastic discount factor* or the *pricing kernel*. In this consumption-based model the term is equivalent to the discounted ratio of marginal utilities, so it is natural to call it the *intertemporal marginal rate of substitution*. Since marginal utilities are positive, the IMRS is positive. For more discussion on this subject, see Cochrane & Hansen (1992). See also Sargent (1987) who documents the usage of this model on asset pricing more generally.

⁷As a special case, if the formula is reduced to $(\beta R_{t+1})^{-1} = 1$, then the marginal utility of consumption is a martingale (a stochastic process $\{c_t\}$ so that $E_{t+\tau}(c_{t+\tau+1}) = c_{t+\tau} \quad \forall \tau \in \mathbb{R}_+$).

of return. They assumed that an agent maximises a time-separable relative risk aversion (CRRA) isoelastic power utility⁸

$$U(c_t) = (1 - \gamma)^{-1} c_t^{1-\gamma}, \quad \forall t \in [0, \infty), \quad 0 < \gamma \neq 1, \quad (2.11)$$

where γ is the constant coefficient of relative risk aversion reflecting concavity of the consumer's utility function.⁹ Taking the derivatives with respect to consumption for periods t and $(t + 1)$ and substituting the results into (2.6), we get a moment restriction

$$E_t \left[\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^i \right] = 1, \quad (2.12)$$

which is directly testable by using some nonlinear estimation method. This Euler equation has several important implications. Grossman & Shiller (1981) derived an analogous discrete-time representation to Breeden's (1979) continuous-time model and showed that, in general, the expected excess real rate of return between risky and riskless assets are linked to covariances of aggregate consumption and real rate of return on assets as follows. Applying (2.12) to the risk-free rate of return, we can derive the approximation¹⁰

$$r_{t+1} = \rho + \gamma E_t(g_{t+1}) - \frac{1}{2} \gamma (1 + \gamma) \text{var}(g_{t+1}), \quad (2.13)$$

where g_{t+1} is the growth rate of consumption and $\text{var}(g_{t+1})$ its variance. The risk-free rate of return r_{t+1} depends positively on the rate of time preference ρ and growth rate of consumption, and negatively on the variance of the growth rate; the effect of the risk aversion coefficient is ambiguous. These results are interpreted as follows: other terms equal, the more heavily individuals discount the future, the higher their preference for present consumption over future consumption, and, therefore, the higher the rate of return must prevent consumers from borrowing to finance their current consumption expenditures. Other things equal, the future consumption is higher relative to current consumption in a growing trend, and, therefore, the higher the rate of return must be to prevent consumers from borrowing and converting future consumption to current consumption. The last term in equation (2.13) reflects the effect of uncertainty of future consumption. When this uncertainty (the variance of the growth rate of

⁸If $\gamma = 1$, then $U(c_t) = \log(c_t)$ according to L'Hopital's rule. According to the definition $\left(\frac{dU(c_t)}{dc_t} \frac{c_t}{U(c_t)} \right)$, the elasticity of utility with respect to consumption is constant and equals $(1 - \gamma)$.

⁹According to the definition $-\frac{U''(c_t)c_t}{U'(c_t)} = \gamma$. The larger the parameter γ , the more risk averse a consumer is and he prefers consumption in different states of the world to be similar.

¹⁰Depending on the specification of the utility function and the order of approximation, the results slightly differ from the following results (see Aiyagari (1993), Romer (1996), Campbell et al. (1997) and Carroll (1997), among others). Hansen & Singleton (1983) derived a log-linear counterpart on equation (2.12) by assuming that all the variables are jointly conditionally log-normally distributed and conditionally heteroskedastic. While their formulations are more cited in the literature, the results do not differ significantly from results (2.13) - (2.15) derived in appendix B.

consumption) increases, consumers typically have a strong incentive for precautionary savings and buffer stocks to smoothen their consumption and to prevent dramatic consumption declines. This increase in savings decreases the rate of return. Finally, for large values of the risk aversion coefficient, the negative effect of the variance in consumption growth (uncertainty) dominates the positive effect of the consumption growth. The more risk averse the consumer, the more he is willing to save in order to avoid unplanned consumption fluctuations. Hence, high values of γ decrease the risk-free rate of return. On the other hand, for small values of γ , the effect on risk-free rate of return is positive.

For any risky asset $i \in \mathbb{R}^J \subset \mathbb{R}^K$, the equation (2.13) becomes

$$E_t [r_{t+1}^i] = \rho + \gamma E_t(g_{t+1}) + \gamma cov(r_{t+1}^i, g_{t+1}) - \frac{1}{2}\gamma(1 + \gamma)var(g_{t+1}). \quad (2.14)$$

In addition to the previous interpretation, there is also a covariance term in the equation. The higher the covariance between the growth rate of consumption and asset's risky rate of return, the higher the expected rate of return. Finally, after subtracting (2.13) from (2.14), the expression for the equity premium for any risky and riskless asset is

$$E_t(r_{t+1}^i) - r_{t+1} = \gamma cov [r_{t+1}^i, g_{t+1}]. \quad (2.15)$$

This equation states that the expected return premium for any risky relative to the risk-free rate of return is proportional to the covariance of its rate of return with that of the growth rate of consumption. Indeed, this model has been called the consumption-based capital asset pricing model or the consumption CAPM, and the covariance term in equation (2.15) is known as the model's consumption beta. Intuitively, this equity premium, as well as the equation (2.14), can be interpreted as follows: Since consumers are typically risk averse, they try to avoid random (unpredictable) variations in consumption. If a risky asset has such a pattern of returns that it yields high returns when consumption is high and low returns when consumption is low, then holding such an asset tends to make consumption more volatile relative to holding an asset with no risk, which, regardless of the state of the world (low or high level of consumption) in the next period, always yields the same return. To hedge, a risk averse consumer has a clear preference for an asset with a negative correlation with the growth of consumption. Therefore, to compensate the agent for holding an asset with a return positively correlated with that of the growth of consumption, the expected rate of return must be higher than the risk-free rate of return. Moreover, the higher the aversion to risk, the higher the premium.

An undesirable feature of the basic model above is that its theoretical foundations are based on the assumption of time-separability in utility. Thus, the model can be tested with nondurables and services, but its theoretical foundations should be developed to study the expenditures on durables, the services of which are

typically divided over several subsequent periods together with the stocks depreciating over time. This was the approach of Mankiw (1982) who used a closely related approach to that of Hall (1978) to study the expenditures on durables. If the utility function is quadratic, then the stock of the durables k obeys the exact regression

$$k_{t+1} = \alpha_0 + \alpha_1 k_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim i.i.d. (0, \sigma^2), \quad \forall t \in [0, \infty) \quad (2.16)$$

in which the coefficients α_0 and α_1 are combinations of the real rate of return and subjective discount rate. Again, no other variable observed in period t or earlier will have a predictive power in explaining the evolution of the stock of the durables. Further, assuming that the stock of the durables depreciates geometrically at the rate δ , the stock and purchases are related through the fundamental identity

$$k_t = (1 - \delta)k_{t-1} + d_t, \quad (2.17)$$

where d_t is the expenditure on durable goods. Combining the equations (2.16) and (2.17) yields

$$d_{t+1} = \delta\alpha_0 + \alpha_1 d_t + \varepsilon_{t+1} - (1 - \delta)\varepsilon_t. \quad (2.18)$$

In other words, the expenditures on durables should follow ARMA(1,1) instead of AR(1) or random walk processes.¹¹ The moving average coefficient should be negative and equal to one minus the rate of depreciation.

2.2. Empirical Failure of CCAP Model

Since its appearance, Hall's (1978) paper has created a whole area of research where several specific utility functions, income variables, and real rates of return have been included and tested in Euler equations. For instance, in the spirit of Grossman & Shiller (1981), Hansen & Singleton (1982, 1983), Muellbauer (1983) and Hall (1988) assumed that the real rate of return is not constant over time. Mankiw et al. (1985) and Eichenbaum et al. (1988) concentrated on consumption-leisure choice and Finn et al. (1990) extended the basic Euler equations by cash-in-advance and money-in-the-utility terms.

The majority of these basic models have, however, come across a large empirical failure. Most of the studies have concentrated solely on the markets of the United States while some international evidence also tends to reject the theoretical formulations. For instance, Hall (1978) rejected his random walk hypothesis, and Mankiw (1982) did not find support for ARMA(1,1) process for durable expenditures. Hansen & Singleton (1982, 1984) rejected the nonlinear Euler equation restriction (2.12) on asset returns and the intertemporal marginal rate of substitution implied by the time- and state-separable preferences with several real

¹¹In Hall's insight, if the service flow is assumed to be proportional to the stock of the durables and the rate of subjective time preference equals the real rate of interest, the equation becomes $\Delta d_{t+1} = \varepsilon_{t+1} - (1 - \delta)\varepsilon_t$. Thus, the change in expenditures on durables should follow IMA(1) instead of random walk. The correlation coefficient between Δd_{t+1} and Δd_t should equal $(\delta - 1)/(2 - 2\delta + \delta^2)$ which is negative since $0 \leq \delta \leq 1$.

rate of return estimates. Mankiw & Shapiro (1986) rejected the CCAPM equity premium relation against the static CAPM equity premium relation with comovement of asset returns with the market return. Of course, there are studies, such as Hamori (1992a,b) with the Japanese data which report the empirical evidence consistent with the theoretical model.¹²

Although the use of CRRA power utility is attractive with its many properties, one theoretical problem is that the coefficients of relative risk aversion and the intertemporal elasticity of substitution are, by definition, inversely related. This is easy to see from the equation (2.13). Solving it with respect to the consumption growth yields

$$E_t(g_{t+1}) = -\frac{\rho}{\gamma} + \frac{1}{\gamma}r_{t+1} + \frac{1}{2}(1 + \gamma)var(g_{t+1}). \quad (2.19)$$

The effect of an increase in the risk-free rate of return on expected consumption growth is γ^{-1} .¹³ Following the arguments of Hall (1988), this inverse relation is an artificial one because the elasticity of intertemporal substitution measures the consumer's willingness to substitute consumption between different periods in time. It is also well-defined in the absence of uncertainty. On the other hand, the coefficient of relative risk aversion reflects the consumer's willingness to allocate consumption between different states of the world. It is also well-defined in a one-period model and in the absence of time dimension. According to these arguments, it is a contradiction to link these parameters inversely.¹⁴

One of the most relevant empirical rejections concerning the equity premium (2.15) was found by Mehra & Prescott (1985) with the U.S. data. They observed that the coefficient of relative risk aversion γ is too large to be sensitive in practice to explain this equity premium. This is easily perceived from the historical point of view, as the average annual real rate of return of stocks in U.S. markets has been nearly seven percent per year, and the average annual real rate of return to Treasury bills has been only about one percent per year. The equity premium is,

¹²It is surprising that these standard models are rejected and refined if they do not fit U.S. data, even though the same models fit data from other countries.

¹³Usually the definition of the intertemporal elasticity of substitution is $-\frac{\partial(\frac{c_{t+1}}{c_t})}{\partial(\frac{U'(c_{t+1})}{U'(c_t)})} \times \frac{\frac{U'(c_{t+1})}{U'(c_t)}}{\frac{c_{t+1}}{c_t}}$ and its logarithmic version is $-\frac{\partial \ln(\frac{c_{t+1}}{c_t})}{\partial \ln(\frac{U'(c_{t+1})}{U'(c_t)})}$. The link between the growth of consumption and the real rate of

return can be derived directly applying equation (2.6) in which case the corresponding logarithm definition becomes $\frac{\partial \Delta \ln c_{t+1}}{\partial r_{t+1}}$. Taking logarithms from (2.6) and assuming that r_{t+1} and ρ are small enough to justify the approximation, the formula reduces to $\Delta \ln c_{t+1} = \gamma^{-1}(r_{t+1} - \rho)$. This equation also shows how the planned consumption responds to the change in anticipated real rate of return, with the elasticity of intertemporal substitution equal to the reciprocal of coefficient of relative risk aversion γ .

¹⁴Moreover, if the uncertainty of future consumption possibilities is high, a risk averse consumer is willing, through the precautionary saving motive, to postpone consumption from present to future: both risk aversion and elasticity of intertemporal substitution are high, which is also a contradiction to the inverse relation between the parameters.

therefore, six percent. Over the sample period (1890-1979), the covariance of the growth of consumption with the market rate of return is 0.0024, which implies that the coefficient of relative risk aversion should be as large as $\gamma = 25$. The message is clear: the difference in the covariances of these real returns with real consumption growth is only large enough to explain the difference in the average returns or the equity premium, if the representative investor is implausibly risk averse. For different data sets some studies have reported even higher values for this coefficient (see Mankiw & Zeldes (1991), among others). To give an intuition for such large degree of risk aversion involved, let us consider a consumer with a relative risk aversion coefficient of 100. Then he would prefer to take one unit of consumption for certain rather than face a lottery involving a 40-percent chance of getting 0.99 units of consumption and a 60-percent chance of getting 1 million units of consumption.¹⁵ In other words, the growth rate of consumption appears to be too smooth with a too low covariance with the asset rate of return to justify the mean equity premium. If the consumer's risk aversion would be plausible, for example $\gamma < 5$, he could gain by arbitrary at the margin by borrowing at the riskless rate and investing in risky assets. This empirical anomaly is widely known as the “*equity premium puzzle*”.

Weil (1989) presented another puzzle concerning equation (2.13). According to the standard model of individuals' preferences with inverse relationship between risk aversion and the elasticity of intertemporal substitution, if agents want their consumption to be smooth over the states of the world, then they also desire smoothness of consumption over time. The former means that agents dislike risk (are highly risk averse), the latter that they also dislike consumption growth (low elasticity of intertemporal substitution). Weil (1989) used moments of historical U.S. data to show that if consumers indeed are highly risk averse, and even though Treasury bills offer only a low rate of return, instead of smoothing consumption over time by borrowing, individuals defer consumption expenditures for the future by saving now and *per capita* consumption grows rapidly. According to the model, however, the discount factor β should be greater than unity to fit the data. This implies that the rate of subjective time preference should be negative, which contradicts the standard theory. This anomaly is known as the “*risk-free rate puzzle*”.¹⁶

¹⁵These figures are from Aiyagari (1993). See also the illustrative calculations in Mankiw & Zeldes (1991). Most economists restrict the coefficient of the relative risk aversion to be less than ten, because higher values imply consumers' willingness to pay unrealistically large amounts of money to avoid risk in consumption.

¹⁶It is noteworthy that both of these puzzles contradict *quantitative* rather than *qualitative* implications of this particular asset pricing model. There are also other anomalies and puzzles, namely, the so-called *default-premium puzzle* and the *term structure puzzle*. Because these puzzles are not widely known and are of minor interest among economists, we will not analyze them in this study. See, for instance, Cochrane and Hansen (1992) for a detailed discussion of these issues. See also Aiyagari (1993) for other theoretical problems concerning the standard model.

3. PRELIMINARY RESULTS FROM FINNISH DATA

Even though the analytical results based on the standard CCAP model above may be questioned, this section presents some preliminary results from the Finnish aggregate quarterly data covering the time period of 1975.1-2000.3. Our purpose is not to give strict statements and interpretations of the reality. Rather, the purpose is to evaluate the results to those of Hall (1978), Mankiw (1982) and Mehra & Prescott (1985) to find out the possible similarities, problems and puzzles, and to motivate the theoretical remedies of the intertemporal consumption model in the following essays.

As was shown in equation (2.10), Hall (1978) argued that under some simplifying assumptions the consumption expenditures should follow a random walk process. The simplest implication of the hypothesis is that consumption should be unrelated to lagged consumption and any other economic variable that is observed in earlier periods. To find out if this holds true also with Finnish data, we ran a variety of regressions. Because of the time-separable structure of the intertemporal model, only the expenditure on nondurables and services can be used in regressions. While it is possible to select a lot of possible candidates as explanatory variables from the consumer's information set, we followed Hall (1978) and used lagged values of consumption measure, disposable income (YD) and Helsinki stock exchange (HEX) index. Different from Hall (1978), the variables are in difference form.¹⁷ If the random walk process holds true, these variables should not be statistically significant. However, if the lagged income has some predictive power, then the model is refuted and consumption is excessively sensitive to income.¹⁸ Also, if the HEX index turns out to be statistically significant, the stock prices reflect expectations of the future which affects also on consumption behaviour.

Table 1 gives results for the quarterly nondurables covering the time period 1975.1-2000.3. The standard errors of the coefficients are given in parentheses. In all regressions the coefficient for the first lag of consumption was statistically significant revealing an ARI(1) process and, thus, refuted the random walk hypothesis. The coefficients for second and third lags of consumption did not significantly differ from zero. The predictive power of changes in disposable income and HEX index turned out to be statistically insignificant.

The time period 1975-2000 included two significant events that could have caused consumers to change their consumption behaviour. The first was the dereg-

¹⁷Originally, Hall (1978) tested AR(1) process and used lagged levels of the variables. According to the theory, only the coefficient for the lagged consumption should be statistically significant. However, the levels of these variables are not stationary and provides a possibility for spurious results. The differences are stationary according to the *ADF* test statistics (not reported). The more precise description of the data follows in the second essay.

¹⁸Consumers can be unable to smooth consumption over transitory fluctuations in income because of the liquidity constraints and other rigidities. See Flavin (1981) for more on the theory of excess sensitivity of consumption.

Table 1
Results for random walk hypothesis $\Delta c_t = \varepsilon_t$ on nondurables,
1975.1-2000.3

	Δc_{t-1}	Δc_{t-2}	Δc_{t-3}	ΔYD_{t-1}	ΔYD_{t-2}	ΔHex_{t-1}	ΔHex_{t-2}
(1)	-0.252* (0.098)						
(2)	-0.230* (0.010)	-0.188 (0.010)					
(3)	-0.287* (0.102)	-0.167 (0.105)	0.071 (0.102)				
(4)				0.037 (0.028)			
(5)	-0.280* (0.098)			0.049 (0.028)			
(6)				0.029 (0.028)	0.047 (0.029)		
(7)						0.010 (0.011)	
(8)						0.004 (0.012)	0.011 (0.012)
(9)	-0.264* (0.098)					0.012 (0.010)	
(10)	-0.281* (0.098)			0.043 (0.033)		0.004 (0.012)	

Consumption is measured as quarterly real expenditures on nondurables deflated by 1995 prices. The data for nondurables (c) and disposable income (YD) is per capita and seasonally adjusted. YD is also deflated by 1995 prices. All variables are differenced once to get them stationary. The asterisks signify that the coefficients differ statistically from zero at 5% level of significance. The regressions do not include a constant term. The number of observations is 101.

ulation of financial markets which culminated towards the end of 1986. The second was the deep depression in the early 1990s. We selected the last quarter of 1986 and the second quarter of 1991 to be the breakpoints after which the time-series may be expected to differ from each other. To avoid excessive reporting we do not give these results in any Table. However, before the liberalization of the financial markets, the random walk hypothesis seemed to hold. After that the coefficients for the lagged HEX index became statistically significant imposing the strong effect of the deregulation on the stock markets, future expectations and consumption behaviour. After the depression in 1990s this effect had no predictive power. Instead, the coefficients for lagged consumption of nondurables became significant. It is also noteworthy, that the lagged disposable income seemed not to have any explanatory power on consumption of nondurables.¹⁹

¹⁹The diagnostic tests showed that the disturbances are homoskedastic and serially uncorrelated. We also experimented other variables such as GDP, investments, wages and salaries and the correspondent stationary differences of them. These variables turned out to be statistically insignificant. Also, the explanatory

We also ran the correspondent regressions for the category services. Again, these results are not reported in any Table to save the space. Even though the main results followed closely to those of nondurables, there are some exceptions that are noteworthy. First, either the first or second lag of disposable income turned out to be statistically significant in nearly all cases covering the different time periods. Before the financial liberalisation the first lag of consumption was significant, while after the deregulation the second lag was significant. Second, the coefficients for the lagged consumption were positive instead of those reported in Table 1. While this observation could be a result of the sampling variation alone, it can also reflect a sort of habit formation which is studied in the second chapter.

Next we tested Mankiw's (1982) hypothesis that the expenditure on durable goods should follow an ARMA(1,1) process with the MA coefficient equal to the negative of one minus the rate of depreciation of the durable goods stock. Intuitively, past purchases should affect current expenditure on those goods. Table 2 shows the results for the ARIMA(p, d, q) specification tests both for durables and semidurables covering the different time periods.²⁰ The tests are based on the Schwarz-Bayesian and Akaike's information criterion of the model specification.

Table 2
Schwarz-Bayesian and Akaike's information criterion
tests for ARIMA($p, 1, q$) specification

Time period	Durables	Semidurables	No. of obs.
1975.1-2000.3	IMA(1)	IMA(2)	101
1975.1-1986.4	ARI(1)	ARI(1)	46
1987.1-1991.2	RW	IMA(3)	18
1991.3-2000.3	RW	ARIMA(1,1,1)	37

Consumption measures are deflated by 1995 prices. The data is per capita and seasonally adjusted. The regressions include a constant term.

Contrary to the theory, both tests suggested that ARIMA(1,1,1) did not fit the data of expenditure on durables. After the financial deregulation the tests gave support for the random walk (RW) process. Only the time period after the depression for semidurables was according to the theory.²¹

To find support for the results in Table 2 we ran several regressions for the quarterly consumption of durables. The results are given in Table 3. If the subjective discount rate equals the real rate of return, the coefficient $\alpha_1 = 1$, and the

variables in Table 1 without any lags were statistically insignificant. The inclusion of the trend component did not change the results substantially.

²⁰Again, we used differenced values of the durables to avoid spurious results. Thus, we tested ARIMA(1,1,1) model instead of ARMA(1,1).

²¹Also, the tests for the levels of variables support the insight: Instead of ARMA(1,1), either AR(1) or MA(1) process fits better to the data. However, because of the small number of observations, any strict statements of the model specification should not be given.

change in expenditure follows pure IMA(1) process. The correlation coefficient between Δd_{t+1} and Δd_t was -0.05 implying the quarterly rate of depreciation to be 0.94 (see footnote 11). This is too high to be plausible. The first regression gives results for the ARI(1) process, second for the IMA(1), and third for the ARIMA(1,1,1).

The results were mixed. The coefficient for ARI(1) process was not statistically significant in the first regression implying that we cannot reject the random walk process of durable expenditures. While the rate of the depreciation in the second regression did not differ statistically from unity, the same conclusion remains. The third regression gave an estimate of the quarterly depreciation rate to be 0.239. Statistically, this estimate differs from unity and, together with the statistically significant ARI(1) coefficient, rejects the random walk hypothesis. These results were in favour of ARIMA(1,1,1) process and clearly contradicted the findings in Table 2. The fourth and fifth regressions tested if Δd_t can be predicted from its own recent lags. None of the estimates differed statistically significantly from zero and the F statistic for the regressions are 0.569 and 0.576, respectively. Thus, consumer expenditure on durable goods did not follow higher-order autoregressive process.²²

Table 3
ARIMA($p, 1, q$) estimates for expenditure on durables,
1975.1-2000.3

	Constant	Δ d_{t-1}	Δ d_{t-2}	Δ d_{t-3}	δ
(1)	10.645 (8.708)	-0.060 (0.101)			
(2)	10.135 (8.071)				0.942* (0.091)
(3)	2.139 (2.481)	0.828* (0.138)			0.239 (0.1357)
(4)	9.769 (8.774)	-0.055 (0.101)	0.089 (0.101)		
(5)	9.201 (8.823)	-0.063 (0.102)	0.092 (0.101)	0.078 (0.102)	

The data is measured as in Table 2. The asterisks signify that the coefficients differ statistically from zero at 5% level of significance. (1-δ) is equal to negative of the MA(1) coefficient and δ is the depreciation rate. The number of observations is 101.

The results from the correspondent regressions from the separate time periods did not differ substantially from those reported in Table 3. Instead, the regression

²²Caballero (1990a) and Hong (1996) have pointed out that, for some reason, it is possible that agents respond to a shock with some lags in their consumption behaviour. Then, the change in durable expenditures will follow a higher-order than a first-order MA process. Contrary to this, the results in Table 2 do not support this insight.

results based on semidurables were different. The ARIMA(1,1,1) model did not provide statistically significant coefficients together with the rates of quarterly depreciation being in the range of 0.27-1.33, which is implausible high. Also, some of the lags of the consumption measure were statistically significant.

To parallel further Hall's (1978) and Mankiw's (1982) works, we also tested if other economic variables can explain the expenditure on durables. Table 4 shows the results.

Table 4
Results of prediction of lagged information on durable expenditures,
1975.1-2000.3

	Constant	Δd_{t-1}	ΔYD_{t-1}	ΔYD_{t-2}	ΔHex_{t-1}	ΔHex_{t-2}	F
(1)	1.568 (9.161)		0.079* (0.037)				4.542 [3.94]
(2)	1.459 (9.146)	-0.118 (0.102)	0.091* (0.384)				2.945 [3.09]
(3)	2.599 (10.063)	-0.112 (0.105)	0.090* (0.039)	-0.011 (0.040)			1.970 [2.70]
(4)	6.945 (8.852)				0.014 (0.014)		1.064 [3.94]
(5)	7.402 (8.921)	-0.056 (0.101)			0.014 (0.014)		0.682 [3.09]
(6)	7.184 (9.040)	-0.058 (0.102)			0.013 (0.015)	0.003 (0.016)	0.462 [2.70]

The data is measured as in Tables 1 and 2. The asterisks signify that the coefficients differ statistically from zero at 5% level of significance. The critical values for F statistic (5%) are given in square brackets. The number of observations is 101.

According to the results, it seems that the change in durable expenditures can be explained by the change in lagged disposable income. The coefficients for the first-lagged values differed statistically from zero, though the magnitude was small. However, except for the first regression, the F statistic did not reject the joint hypothesis that these regressors other than the intercept term have zero coefficients. The change in the stock markets did not have statistically significant effect on the change in durable expenditures.

The regressions for the separate time periods revealed that the importance of the lagged disposable income was apparent after rather than before the depression years in early 1990s. Nevertheless, the F statistic did not reject the joint hypothesis of zero coefficients. The correspondent experiments for semidurables revealed different pattern. Before the financial liberalisation, the first lag of consumption and the first and second lags of disposable income were statistically significant together with the F statistic rejecting the joint hypothesis of zero coefficients.

After the deregulation, the first lags of consumption and HEX index proved to have a predictive power on semidurable expenditures.²³

The equity premium puzzle by Mehra & Prescott (1985) was that the coefficient for risk aversion should be implausibly high to match the equality between excess return and consumption beta. We tested if the equity premium puzzle holds also with Finnish data. The results are reported in Table 5. The covariance term includes a quarterly percentage change in real consumption of nondurables (g_{t+1}) and the risky real rate of return (r_{t+1}^i), which is defined as quarterly percentage change of HEX index added by an average quarterly effective dividend yield. As an riskless return we used the rates of return on government bonds, all bonds in markets and the average borrowing rate of commercial banks in Finland. All rates of return are tax-free.²⁴

Table 5
Moments for equation $E_t(r_{t+1}^i) - r_{t+1} = \gamma cov[r_{t+1}^i, g_{t+1}]$

Excess return	Period	Mean	$cov(r_{t+1}^i, g_{t+1})$	γ
Shares - GBond	1975.2-2000.3	-0.034	7.49E-05	-457.5
	1975.2-1986.4	-0.061	1.29E-04	-472.4
	1987.1-2000.3	-0.008	6.96E-05	-119.5
	1987.1-1991.2	-0.074	3.11E-05	-2379
	1991.3-2000.3	0.016	1.96E-04	81.43
Shares - Bond	1975.2-2000.3	-0.034	7.49E-05	-458.2
	1975.2-1986.4	-0.062	1.29E-04	-476.3
	1987.1-2000.3	-0.008	6.96E-05	-115.7
	1987.1-1991.2	-0.077	3.11E-05	-2483
	1991.3-2000.3	0.017	1.96E-04	86.43
Shares - Rrate	1975.2-2000.3	-0.002	7.49E-05	-29.91
	1975.2-1986.4	-0.001	1.29E-04	-77.03
	1987.1-2000.3	0.008	6.96E-05	115.90
	1987.1-1991.2	-0.082	3.11E-05	-2630
	1991.3-2000.3	0.037	1.96E-04	190.6

Consumption growth is the quarterly change in real consumption of nondurables. Shares include the quarterly percentage change of HEX index and dividends. GBond = Government Bonds, Bond = All bonds in market, Rrate = Average borrowing rate of commercial banks. All the rates of return are tax-free and deflated by private consumption prices on nondurables.

The moments show that before the breakpoint in 1991, the risk aversion coefficient is negative implying that the representative agent has been extreme risk

²³Without giving any specific results, we also found that the first or second lags (or both) of investments were statistically significant in explaining both the durable and semidurable expenditures for different time periods. For semidurables, we also found other lagged exogenous variables, such as GDP and wages and salaries, to have some explanatory power. In spite of the significance, the magnitude of the coefficients were small.

²⁴The more precise description of the rates of return and the modification of the data follows in the second chapter.

loving instead of risk averse in his investment and consumption decisions. Without doubt, this result is a consequence of the negative mean of excess real rate of return. Before the 1990s the stock markets in Finland were undeveloped, few consumers owned stocks, and the trading in markets was small. Also, the theory is based on the *ex ante* variables while the regressions are based on the *ex post* variables. After the depression years the excess return is positive. Still, to match the theory, the representative consumer should have been extreme risk averse in his investment and consumption behaviour.

To sum up the results, even these casual experiments show that the Finnish aggregate consumption data did not completely support the two cornerstones of the modern consumption theory, namely, Hall's (1978) random walk hypothesis of nondurables and services and Mankiw's (1982) ARMA(1,1) process for durable expenditures. Rather, it seems that the ARI(1) process can better explain the change in consumption on nondurables and services. This also confirms the results by Svento (1990) for the time period of 1960-1988.²⁵ While the regression results for durable expenditures exhibit support also for the ARIMA(1,1,1) process, the statistical significance of the lagged disposable income revealed that either Flavin's (1981) hypothesis of excess sensitivity of consumption on disposable income cannot be rejected. Also, the present results conform with the benchmark puzzle of Mehra & Prescott (1985): the coefficient for relative risk aversion should be implausibly high to be consistent with the observed consumption behaviour if the excess return is positive.

4. DIRECTIONS OF IMPROVEMENT²⁶

Because of the empirical failure of the Euler equations, and, especially, because of the equity premium puzzle, several theoretical and empirical attempts have been made to improve the models to fit the data. Some of these attempts are more or less for academic concern, but some attempts have raised important aspects of the operation of markets, new estimation techniques, and about the consumer behaviour under uncertainty. Even though most of the new theories cannot explain the puzzles and problems themselves, they offer a better understanding of how a rational consumer forms his consumption and saving decisions when allocating his scarce resources in order to maximise the total life-time utility. Even though one is not interested in solving these puzzles *per se*, the rich preference parametrisation underlying these improvements gives a deeper empirical insight of consumer behaviour. These suggestions can roughly be put in three main categories: relaxing the assumption of complete markets, problems on data

²⁵Most of the previous studies, including Koskela & Viren (1984, 1985) for international and Svento (1990) for Finnish evidence, used nonstationary variables. Consequently, the results can be suspected to be spurious, and the direct comparison to those of ours is not justified.

²⁶Excellent review articles are Cochrane & Hansen (1992), Kocherlakota (1996), Siegel & Thaler (1997), Campbell (1998) and Attanasio (1998). For textbook treatments, see also Deaton (1992), Romer (1996) and Campbell et al. (1997).

measurement and estimation, and alternative specification of preferences. Next we discuss briefly these remedies and their contribution to resolve the puzzles and also their ability to better understand intertemporal consumer behaviour. We also take a sophisticated look at the literature.²⁷

The first category considers the assumptions that the asset markets are complete and frictionless, which are the basics for the standard model described above. Because of its simplifying power, market completeness is a widely used (and accepted) cornerstone which creates a basis for many fundamental theories in the area of financial economics. However, if this was true, consumers could use financial markets to diversify away from any idiosyncratic differences in their consumption streams and, as a result, their consumption streams would look similar to each other and to *per capita* consumption. In this case, the representative consumer (defined and discussed later in chapter 2) approach is justified, and aggregation problems are solved. In fact, however, large empirical evidence rejects this assumption (see Fama & French (1988) and Granger (1992), among others) and points out that asset markets are not complete in practice. Even if the theoretical formulations in (2.5) and (2.15) are correct, they cannot describe the actual co-movement of asset prices and consumption in incomplete markets.

One way to relax the assumption of complete markets is the introspection that a fraction of all consumers are liquidity constrained. In reality, because of the presence of uncertainty and default-risk of a borrower, most of consumers are not allowed to borrow today as much as they would like to optimise their life-time consumption pattern. Furthermore, the lending interest rate does not equal the borrowing interest rate in markets unlike often assumed in economic theory. For example, an impatient consumer who has a high rate of time preference may confront a binding budget constraint and is not allowed to borrow freely for consumption against future income. Therefore, his current consumption will be highly sensitive to current income. Depending on whether the consumption good is a durable (housing, car) or a nondurable one, the restrictions for borrowing can be different, because, in case of a durable good, the lender may accept a part of it as a security or collateral. Moreover, as Zeldes (1989) emphasised, even if the constraints are not currently binding, the possibility that they may bind in the future may cause a sharp reduction in current consumption. One fascinating theoretical contribution in this area is the paper of Campbell & Mankiw (1989). They divided population into two separate parts: a fraction λ of consumers are simply “hand-to-mouth” who spend their current income and the rest $(1 - \lambda)$ behave without any binding constraints as Hall’s (1978) random walk theory assumes. The total income is $Y_t = \lambda Y_t^1 + (1 - \lambda)Y_t^2$, where the constrained consumers consume a fraction λ of the total income and the other part the rest, respectively. Agents in the first group consume their current income, $c_t^1 = Y_t^1$, implying $\Delta c_t^1 = \Delta Y_t^1 = \lambda \Delta Y_t$, while according to the equation (2.10), consump-

²⁷The following survey will deal with only a fraction of recent literature, and we will concentrate only on those papers which, in our opinion, are the most relevant contributions.

tion of agents in other group follows the random walk, so $\Delta c_t^2 = (1 - \lambda)\varepsilon_t$. The change in total consumption is, therefore,

$$\Delta c_t = \Delta c_t^1 + \Delta c_t^2 = \lambda \Delta Y_t + (1 - \lambda)\varepsilon_t. \quad (4.1)$$

Remembering the restrictions on the information set, this equation can be easily tested empirically. According to the null hypothesis, $H_0: \lambda = 0$, liquidity constraints are not binding and all the consumers behave like the rational expectations permanent income hypothesis (REPIH) suggests. Currently, the role of liquidity constraints has become one of the main areas in consumption research and the literature concerning liquidity constraints is large, investigating the causes, extent and possible effects on binding constraints.²⁸

The risk-free rate puzzle discussed in the previous section was that the consumers save even though the risk-free rate is low. One solution to this can be found from Kimball (1990) who introduced a fascinating way to interpret the third-order derivative from consumer's utility function to represent a sophisticated risk or a "prudence". With a quadratic utility function and assumptions of both zero rate of return and time preference (or $(\beta R_{t+1}^i)^{-1} = 1$), the Euler equation in (2.5) becomes

$$U'(c_t) = E_t [U'(c_{t+1})] = U' [E_t(c_{t+1})], \quad (4.2)$$

because the marginal utility is linear. But if we assume that $U'''(\cdot)$ is positive, then the marginal utility, $U'(c_t)$, is a convex function of c_t and $E_t [U'(c_{t+1})]$ exceeds $U' [E_t(c_{t+1})]$. This means, however, that in the case of smooth consumption where c_t equals $E_t(c_{t+1})$, the term $E_t [U'(c_{t+1})]$ exceeds $U'(c_t)$, and a marginal reduction in consumption in period t increases the expected utility from period $t + 1$. Clearly, a mean preserving spread in uncertainty in future income makes a consumer to defer consumption to the future and save now, so that it is not just expectations of future income but also the uncertainty about that income which affect the consumption profile. This type of saving is widely known as a "precautionary saving".²⁹

Another way to examine the market incompleteness is to consider an uninsurable uncertain future income and other market frictions, such as asymmetric information and trading costs. Although these lines of study were not developed directly to solve the puzzles described above, but to extend or to present alternatives to the permanent-income hypothesis, they are valuable in helping to solve the

²⁸See also Hayashi (1985), Hubbard & Judd (1986), Deaton (1991, 1992), Jappelli & Pagano (1994), Guiso et al. (1996) and Constantinides et al. (1998), among others. Takala (2001) found that in Finland approximately one fourth of the consumers were liquidity constrained by the end of 1990s, and that the excess sensitivity of consumption was higher in Finland than in Sweden.

²⁹Kimball himself refers to $-c_t U'''(c_t)/U''(c_t)$ as a coefficient of relative prudence, in analogy to the coefficient of relative risk aversion, $-c_t U''(c_t)/U'(c_t)$. Conversely, if marginal utility is a concave function, an increase in risk will increase current consumption expenditures. For more on precautionary saving motive, see also Hassler (1996a) who gives a nontechnical illustration on this area, and Caballero (1990b), Hubbard et al. (1995) and Haliassos & Hassapis (1998).

puzzles as well. For instance, Weil (1992) presented a two-period economy where individuals are not capable of insuring themselves against the fluctuations in future labour income and, therefore, in the consumption stream. He argued that due to this, consumers generate a greater demand to transfer resources to the future, that is, they save now. This increase in savings in incomplete markets increases the demand of assets, and decreases the rates of return below those in the complete markets. However, Huggett (1993) pointed out that in an infinite horizon economy, the possibility of self-insurance leads to a difference in rates of return between complete and incomplete markets, which is not large enough to have a considerable effect on the consumption stream.

One crucial deficiency of basic models is that they assume transaction costs to be insignificant. In reality, however, an individual is nearly always forced to pay taxes, brokerage fees, bid-ask spreads, information costs, and all other kinds of trading costs which are not symmetric across the markets. Depending on the magnitude of these market costs, the standard model without any assumption as such could be far away from reality and lead an empirical economist to wrong conclusions. Aiyagari & Gertler (1991, 1998), Aiyagari (1993), He & Modest (1995) and Heaton & Lucas (1996) are the most promising papers in this area.

To summarise, incompleteness in markets may lead consumers to change their consumption behaviour (perhaps far) away from the optimal profile of the standard model. Because of uncertainty about future, liquidity constraints, and other market incompletenesses, consumers prepare themselves for the future by increasing savings now, or with Deaton's (1991) terminology, exhibiting "buffer-stocks", to avoid a possible reduction in future income. This behaviour leads to extra savings even though the risk-free rate of return is low, and the risk-free rate puzzle is resolved. On the other hand, if consumers face binding constraints for borrowing, they cannot smooth consumption, and the variability of consumption increases. This provides one explanation for the equity premium puzzle.

The second category considers the estimation methods and the measurement of data as explanations on why the models do not fit the data. The early rejections of the standard model were based on the linearised Euler equations with suitable approximations (see Hansen & Singleton (1983) among others). However, Carroll (1997) has recently abandoned these methods because they cannot successfully cover the true structural parameters of the model. Another way to estimate the standard model was introduced by Hansen (1982) who described the generalized method of moments (GMM) estimation to estimate directly the orthogonality conditions (2.12) without the need of linearisation. Thereafter, a large number of empirical research has solely been based on this method. GMM has a lot of attractive properties but it presents some disadvantages as well which limit its usefulness in estimating preference parameters. In chapters 2 and 3 we will discuss these subjects in more detail. Also, the problems in data measurements could be a reason for the empirical failure. For example, Mankiw & Zeldes (1991)

suggested that the aggregation of consumption of stockholders and nonstockholders may result in the equity premium puzzle. They found that the consumption of stockholders is three times more sensitive to fluctuations in stock markets than that found in aggregate data. This extra sensitivity reflects a higher covariance term in equation (2.15) and may help to resolve the puzzle.

Finally, the last category concentrates solely on modifying the stochastic properties of the intertemporal marginal rate of substitution, namely, the consumer's preferences. The standard model with time-additive, time-separable CRRA power utility (2.11) seems to be too severe to describe the consumer's behaviour in reality.³⁰ While it takes into consideration only nondurables and services, which create only a part of total consumption expenditures, the role of durables and non-separable preferences are omitted. In Finland, for example, durables form nearly one fifth of total consumption, and omitting this can cause important implications on empirical research.³¹

As noted above, one crucial implication of the standard model with CRRA utility is that the coefficient of relative risk aversion is by definition the reciprocal of elasticity of intertemporal substitution. Highly risk averse consumers view the consumption in different time periods as highly complementary. Clearly, there is no possibility to disentangle these two parameters from each other in the standard model. To overcome this issue, several preference modifications have been made. Epstein & Zin (1989, 1991) and Weil (1989, 1990) allowed more general types of utilities, that is, non-separable, non-expected recursive utilities, based on the framework of Kreps & Porteus (1978, 1979a, 1979b), to enter in representative agent's models.³² In their model, consumers care about the timing of resolution of uncertainty, and the preference parameters can be separated from each other. The model generalises an ordinary model, and with a suitable selection of parameters it collapses back to the standard von Neumann-Morgenstern power utility with a constant RRA. The empirical advance of these models is that they allow the possibility to explain both a high equity premium and low real interest rate.

Sundaresan (1989) and Constantinides (1990) represented a habit formation model, which generalised models already appeared in the literature on consumption. These works relaxed the time-separability of von Neumann-Morgenstern

³⁰The problem is that the aggregate consumption is too smooth to explain the large equity premium. So, any preference modification that makes intertemporal marginal rate of substitution more variable without increasing risk aversion can help to explain the puzzles.

³¹In order to understand fluctuations in consumption and business cycles, it is more important to understand changes in expenditures on durables and semidurables, because the time-series of these variables are more volatile than the time-series of nondurables and services. Especially, in the depression years of the early 1990s in Finland, it was the expenditure of durables which collapsed most and, in turn, deepened depression.

³²See also Attanasio & Weber (1989), Giovannini & Weil (1989), Farmer (1990), Kocherlakota (1990), Kandel & Stambaugh (1991), Jorion & Giovannini (1993) and Weil (1990).

preferences to allow consumer's utility to depend also on past consumption expenditures. Therefore, the contribution of this habit persistence is that it takes also consumption durables into consideration. Abel (1990), on the other hand, introduced a "catching up with the Joneses" model where consumers care about their relative position with respect to some reference consumption level and not necessarily the history of their own consumption.

Mankiw's (1982) hypothesis of durable consumption is typically strongly rejected by the aggregate data. One reason to explain this poor performance could be the aggregation problems. Typically, in most periods consumers do not adjust their stock of durables, and when they finally do, adjustments are lumpy and substantial. The aggregation of such data may cause systematic biases in the estimation of structural parameters and perhaps reveal nothing on what are the determinants that affect this lumpy behaviour. Therefore, a great deal of recent research has concentrated on studying household-level consumption behaviour for durables. One way to do this is to use (S,s) models considered in the optimal inventory literature at the 1950s. Eberly (1994) and Hassler (1996b), among others, have shown how the irreversible investment costs and uncertainty affect on the durable purchases. For example, when the uncertainty affecting the household's future wealth position increases, it is optimal for a household to do nothing and wait until new information on future arrives.

5. CONCLUSION WITH SUMMARIES OF OTHER CHAPTERS

The aim of this essay was to introduce the theoretical basics of the standard consumption-based capital asset pricing model and its close successors, their theoretical weaknesses and empirical failure, and to supply some Finnish evidence based on these basic frameworks. Also, we summarised briefly the recent theoretical contributions in intertemporal consumption literature. Especially, the emphasis was to understand why it is important to relax the assumption of time-separable preferences and to allow a richer decision mechanism to enter consumer's utility function.

The empirical tests with Finnish aggregate data on consumption did not support the random walk model for nondurables by Hall (1978) nor the MA(1) model for durables of Mankiw (1982). In both cases, there existed lagged economic variables which had some statistically significant power to explain both consumption expenditures. The tests, however, supported the equity premium puzzle by Mehra & Prescott (1985) which states that the relative risk aversion coefficient should be implausibly high to explain the excess returns between risky and riskless assets. Thus, even these simple tests show that there is clearly a need for more accurate estimation results based on the alternative preferences and model specification.

The following three essays study intertemporal consumption behaviour in Finland from different perspectives. The models involved in these essays can be

seen as improvements or alternatives for the standard Euler equation approach with power utility. The second essay is based on habit formation, durability and envy. We show that the representative consumer's decision-making is not entirely based on economic but also on psychological behaviour. The theoretical model extends the standard Euler equation framework by introducing the presence of consumption externalities where a consumer's interdependent preferences consist of both internal and external effects on consumption. Following Ferson & Constantinides (1991) and many others, the internal effect means that the consumer's own consumption history, namely, habit persistence and durability of durables, affects as a subsistence level to his present consumption expenditures. The external effect says that the consumer is envious and has some reference consumption level to which he relates his own consumption. At an individual level, this reference consumption could be the average consumption of his neighbourhood, socio-economic class, or the *per capita* consumption.

The parametrisation of the model leads to the first-order conditions which are then tested empirically by using Hansen's (1982) generalized method of moments (GMM) estimation and real aggregated Finnish quarterly consumption (durables and semidurables, nondurables and services) and asset data from 1975 to 2001. The results revealed that there has been a structural change in preference parameters because of the financial deregulation in the mid-1980s. Under the regulation, either the durability of durables has dominated the habit persistence, or both the durability and habit persistence have been insignificant. After the deregulation, habit formation has been the dominant force. The estimations for the external effect showed that the representative agent in Finland is slightly envious or he is not at all, with respect to the total private consumption growth in Sweden. Also, in spite of its theoretical weaknesses, the standard Euler equation with power utility fits reasonably well on data.

The third essay builds on the non-expected utility methodology as proposed by Epstein & Zin (1989, 1991) and Weil (1990). This utility presentation helps to disentangle the above described link between the parameters of relative risk aversion and elasticity of intertemporal substitution in consumption. The GMM estimation results on real aggregated Finnish quarterly consumption on nondurables and services, several real rates of return and optimally constructed portfolio confirm the observation of the second essay: In Finland, there has been a structural change in consumption and investment decisions due to the financial liberalisation and change in economic environment in the mid-1980s. The elasticity of intertemporal substitution in consumption has increased over time, and is in the range of 2-10. While the estimates for the relative risk aversion have been statistically insignificant under the regulation, they are statistically between zero and one after the liberalisation. The relative magnitude of these parameters reveal that the consumers on average dislike risk more than intertemporal fluctuations in consumption. The results also show that an investment in shares has been the most risky investment decision across time and policy regimes.

The last essay studies the irreversible mechanism. The theoretical model builds partly on the preliminary work of Hassler (1996b) who presented a model where an increase in uncertainty affects agents to postpone their durable purchases. While the investments are often at least partially irreversible and the continuous updating is costly, it can be sometimes better to wait until new information from future arrives before adjusting. We extend this model by deriving explicitly the optimal rule for durables. The model, which is based on the (S,s) rule and standard Cobb–Douglas preferences in dynamic context, states that an agent has a desire to keep a fraction of his wealth to be invested in a certain durable. While the durable depreciates over time together with stochastic movements in prices and agent’s wealth, the actual fraction deviates from that of desired. This creates an inaction band around the target level and the adjustment is not made until the critical bound is reached. The model shows that an increase in uncertainty and adjustment costs should widen the width of the (S,s) inaction bands, thus, inducing an agent to tolerate larger deviations from the target level. A greater depreciation should lead to more frequent adjustment.

To test the implications of the model we used four Finnish Household Budget Surveys conducted by Statistics Finland. The results on automobile purchases do not reject the (S,s) behaviour. A higher income uncertainty widens the inaction band and decreases the probability of adjustment while an increase in repair costs increases the probability of adjustment.

REFERENCES

- Abel, A.B. (1990): “Asset Prices under Habit Formation and Catching up with the Joneses.” *American Economic Review* 80, 38-42.
- Aiyagari, R.S. (1993): “Explaining Financial Market Facts: The Importance of Incomplete Markets and Transaction Costs.” *Federal Reserve Bank of Minneapolis Quarterly Review* 17(1), 17-31.
- Aiyagari, R.S. & Gertler, M. (1991): “Asset Returns with Transaction Costs and Uninsured Individual Risk.” *Journal of Monetary Economics* 27, 311-331.
- Attanasio O.P. (1998): “Consumption Demand.” NBER Working Paper No. 6466.
- Attanasio, O.P. & Weber, G. (1989): “Intertemporal Substitution, Risk Aversion and the Euler Equation for Consumption.” *Economic Journal* 99, supplement, 59-73.
- Benveniste, L. & Scheinkman, J. (1979): “On the Differentiability of the Value Function in Dynamic Models in Economics.” *Econometrica* 47, 727-732.
- Boldrin, M., Christiano, L.J. & Fisher, J.D.M. (1995): “Asset Pricing Lessons for Modeling Business Cycles.” Working Paper No. 560. Federal Reserve Bank

of Minneapolis.

Breeden, D. (1979): "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment." *Journal of Financial Economics* 7, 265-296.

Brunila, A. (1997): "Fiscal Policy and Private Consumption - Saving Decisions: European Evidence." Bank of Finland Studies E:8.

Caballero, R.J. (1990a): "Expenditure on Durable Goods: A Case for Slow Adjustment." *Quarterly Journal of Economics* 105, 727-743.

Caballero, R.J. (1990b): "Consumption Puzzles and Precautionary Savings." *Journal of Monetary Economics* 25, 113-126.

Campbell, J.Y. (1996): "Consumption and the Stock Market: Interpreting International Evidence." *Swedish Economic Policy Review* 3, 251-299.

Campbell, J.Y. (1998): "Asset Prices, Consumption and the Business Cycle." NBER Working Paper No. 6485.

Campbell, J.Y., Lo, A.W. & MacKinlay, A.G. (1997): *The Econometrics of Financial Markets*. Princeton University Press. Princeton, New Jersey.

Campbell, J.Y. & Mankiw, G.N. (1989): "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence." *NBER Macroeconomics Annual* 4, 185-216.

Carroll, C.D. (1996): "Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis." NBER Working Paper No. 5788.

Carroll, C.D. (1997): "Death to the Log-linearized Consumption Euler-equation! (And Very Poor Health to the Second-order Approximation)" NBER Working Paper No. 6298.

Cochrane, J. & Hansen, L.P. (1992): "Asset Pricing Explorations for Macroeconomics." in *NBER Macroeconomics Annual 1992*. Eds.: Olivier Blanchard and Stanley Fischer. Cambridge, MA: MIT Press, 1992, 115-182.

Constantinides, G. M. (1990): "Habit Formation: A Resolution of the Equity Premium Puzzle." *Journal of Political Economy* 98, 519-543.

Constantinides, G. M., Donaldson, J.B. & Mehra, R. (1998): "Junior Can't Borrow: A New Perspective on the Equity Premium Puzzle." NBER Working Paper 6617.

de Brouwer, G. (1996): "Consumption and Liquidity Constraints in Australia and East Asia: Does Financial Integration Matter?" Research Discussion Paper 9602. Economic Research Department, Reserve Bank of Australia.

Deaton, A. (1991): "Saving and Liquidity Constraints." *Econometrica* 59, 1221-1248.

Deaton, A. (1992): *Understanding Consumption*. Clarendon Press. Oxford, UK.

Eberly, J.C. (1994): "Adjustment of Consumers' Durable Stocks: Evidence from

- Automobile Purchases.” *Journal of Political Economy* 102, 403-436.
- Eichenbaum, M.S., Hansen, L.P. & Singleton, K.S. (1988): “A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty.” *The Quarterly Journal of Economics* 103, 51-78.
- Epstein, L & Zin, S. (1989): “Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns I: A Theoretical Framework.” *Econometrica* 57, 937-969.
- Epstein, L & Zin, S. (1991): “Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns II: An Empirical Analysis.” *Journal of Political Economy* 99, No. 2, 263-286.
- Fama, E.F. & French, K.R. (1988): “Permanent and Temporary Components of Stock Prices.” *Journal of Political Economy* 96, 246-273.
- Farmer, R.E.A. (1990): “RINCE Preferences.” *The Quarterly Journal of Economics*, 43-60.
- Ferson, W.E. & Constantinides, G.M. (1991): “Habit Persistence and Durability in Aggregate Consumption.” *Journal of Financial Economics* 29, 199-240.
- Finn, M.G., Hoffman, D.L. & Schlagenhauf, D.E. (1990): “Intertemporal Asset-Pricing Relationship in Barter and Monetary Economies: An Empirical Analysis.” *Journal of Monetary Economics* 25, 431-451.
- Flavin, M. (1981): “The Adjustment of Consumption to Changing Expectations about Future Income.” *Journal of Political Economy* 89, 974-1009.
- Friedman, M. (1957): *A Theory of Consumption Function*. Princeton University Press. Princeton.
- Giovannini, A. & Weil, P. (1989): “Risk Aversion and Intertemporal Substitution in the Capital Asset Pricing Model.” NBER Working Paper No. 2824.
- Granger, C.W.J. (1992): “Forecasting Stock Market Prices: Lessons for Forecasters.” *International Journal of Forecasting* 8, 3-13.
- Grossman, S.J. & Shiller, R.J. (1981): “The Determinants of the Variability of Stock Market Prices.” *American Economic Review*, 222-227.
- Guiso, L., Jappelli, T. & Terlizzese, D. (1996): “Income Risk, Borrowing Constraints, and Portfolio Choice.” *American Economic Review* 86, 158-172.
- Haliassos, M. & Hassapis, C. (1998): “Borrowing Constraints, Portfolio Choice, and Precautionary Motives: Theoretical Predictions and Empirical Complications.” Centre for Studies in Economics and Finance Working Paper No. 111.
- Hall, R.E. (1978): “Stochastic Implications of the Cycle-Permanent Income Hypothesis: Theory and Evidence.” *Journal of Political Economy* 86, No. 6, 971-987.
- Hall, R.E. (1988): “Intertemporal Substitution in Consumption.” *Journal of Political Economy* 96, 221-273.

- Hamori, S. (1992a): "Test of C-CAPM for Japan: 1980-1988." *Economics Letters* 38, 67-72.
- Hamori, S. (1992b): "On the Structural Stability of Preference Parameters Obtained from Japanese Financial Market Data." *Economics Letters* 40, 459-464.
- Hansen, L.P. (1982): "Large Sample Properties of Generalized Method of Moments Estimators." *Econometrica* 50, 1029-1054.
- Hansen, L.P. & Singleton, K.J. (1982): "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." *Econometrica* 50, 1269-1286.
- Hansen, L.P. & Singleton, K.J. (1983): "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns." *Journal of Political Economy* 91, 249-265.
- Hansen, L.P. & Singleton, K.J. (1984): "Errata." *Econometrica* 52, 267-268.
- Hassler, J. (1996a): "Risk and Consumption." *Swedish Economic Policy Review* 2, 303-337.
- Hassler, J. (1996b): "Variations in Risk and Fluctuations in Demand: A Theoretical Model." *Journal of Economic Dynamics and Control* 20, 1115-1143.
- Hayashi, F. (1985): "The Effect of Liquidity Constraints on Consumption: A Cross-Sectional Analysis." *The Quarterly Journal of Economics* 100, 183-206.
- He, H. & Modest, D.M. (1995): "Market Frictions and Consumption-Based Asset Pricing." *Journal of Political Economy* 103, 94-117.
- Heaton, J. & Lucas, D. (1996): "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing." *Journal of Political Economy* 104, 668-717.
- Hong, K. (1996): "A Comment on Durable Goods Consumption." *Journal of Monetary Economics* 37, 381-391.
- Hubbard, R.G. & Judd, K.L. (1986): "Liquidity Constraints, Fiscal Policy, and Consumption." *Brookings Paper on Economic Activity*, Vol 1.
- Hubbard, R.G., Skinner, J. & Zeldes, S.P. (1995): "Precautionary Saving and Social Insurance." *Journal of Political Economy* 103, 360-399.
- Huggett, J. (1993): "The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies." *Journal of Economic Dynamics and Control* 17, 953-969.
- Jappelli, T. & Pagano, M. (1994): "Saving, Growth, and Liquidity Constraints." *Quarterly Journal of Economics* 110, 83-109.
- Jorion, P. & Giovannini, A. (1993): "Time-Series Tests of a Non-Expected Utility Model of Asset Pricing." *European Economic Review* 37, 1083-1100.
- Kandel, S. & Stambaugh, R.F. (1991): "Asset Returns and Intertemporal Preferences." *Journal of Monetary Economics* 27, 39-71.

- Kimball, M.S. (1990): "Precautionary Saving in the Small and the Large." *Econometrica* 58, 53-73.
- Kocherlakota, N.R. (1990): "Disentangling the Coefficient of Relative Risk Aversion from the Elasticity of Intertemporal Substitution: An Irrelevance Result." *The Journal of Finance* 45, 175-190.
- Kocherlakota, N.R. (1996): "The Equity Premium: It's Still a Puzzle." *Journal of Economic Literature*, Vol. 34, 42-71.
- Koivumäki, T. (1999): "Uncertainty and the Dynamics of Demand." Ph.D. Thesis, University of Oulu, Serie G5.
- Koskela, E. & Viren, M. (1984): "Testing the Stability of the Hall Consumption Function Specification." *Economics Letters* 14, 289-293.
- Koskela, E. & Viren, M. (1985): "Time-Varying Hall Consumption Function: Some Empirical Evidence." *Economics Letters* 17, 321-325.
- Kostiainen, S. & Starck, C.C. (1991): "Does Government and Corporate Saving Influence Household Saving in Finland?" *Liiketaloudellinen Aikakauskirja* 2/91, 128-144.
- Kreps, D.M. & Porteus, E.I. (1978): "Temporal Resolution of Uncertainty and Dynamic Choice Theory." *Econometrica* 46, 185-200.
- Kreps, D.M. & Porteus, E.I. (1979a): "Temporal von Neumann-Morgenstern and Induced Preferences." *Journal of Economic Theory* 20, 81-109.
- Kreps, D.M. & Porteus, E.I. (1979b): "Dynamic Choice Theory and Dynamic Programming." *Econometrica* 46, 1426-1445.
- Lintner, J (1965): "The Valuation of Risky Assets and the Selection of Risky Investment in Stock Portfolios and Capital Budgets." *Review of Economics and Statistics* 47, 13-37.
- Lucas, R.E. (1978): "Asset Prices in Exchange Economy." *Econometrica* 46, 1426-1445.
- Mankiw, G.N. (1982): "Hall's Consumption Hypothesis and Durable Goods." *Journal of Monetary Economics* 10, 417-425.
- Mankiw, G.N., Rotemberg, J.J. & Summers, L.H. (1985): "Intertemporal Substitution in Macroeconomics." *The Quarterly Journal of Economics*, 225-251.
- Mankiw, G.N. & Shapiro, M.D. (1986): "Risk and Return: Consumption Beta versus Market Beta." *Review of Economics and Statistics* 68, 452-459.
- Mankiw, G.W. & Zeldes, S.P. (1991): "The Consumption of Stockholders and Non-Stockholders." *Journal of Financial Economics* 29, 97-112.
- Mehra, R. & Prescott, E.C. (1985): "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15, 145-161.
- Merton, R.E. (1973): "An Intertemporal Asset Pricing Model." *Econometrica*

41, 867-887.

Modigliani, F. & Brunberg R. (1954): "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data." in Kenneth Kurihara (eds.), *Post-Keynesian Economics*, New Brunswick N.J. Rutgers University Press, 388-436.

Muellbauer, J. (1983): "Surprises in the Consumption Function." *Economic Journal, a supplement*, 34-50.

Romer, D. (1996): *Advanced Macroeconomics*. McGraw-Hill Companies, Inc.

Samuelson, P.A. (1969): "Lifetime Portfolio Selection by Dynamic Stochastic Programming." *Review of Economics and Statistics* 51, 239-246.

Sargent, T.J. (1987): *Dynamic Macroeconomic Theory*. Harvard University Press, Cambridge, MA.

Sharpe, W (1964): "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." *Journal of Finance* 19, 425-424.

Siegel, J.J. & Thaler, R.M. (1997): "Anomalies: The Equity Premium Puzzle." *Journal of Economic Perspectives* 11, 191-200.

Starck, C.C. (1987): "Intertemporal Substitution in Consumption: Some Empirical Evidence from Finnish Data." *Liiketaloudellinen Aikakauskirja* 2/87, 128-143.

Stenius, M. (1989): "The Consumption Based Capital Asset Pricing Model: Some Cross-Country Evidence." Discussion Paper No. 274. University of Helsinki, Department of Economics.

Sundaresan, S.M. (1989): "Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth." *Review of Financial Studies* 2, 73-88.

Svento, R. (1990): "Tutkimus kulutusfunktion tulokäsitteestä." Research Reports No. 34. University of Oulu, Department of Economics. (In Finnish)

Takala, K. (1995): "Permanent Income Hypothesis and Saving in Finland." Bank of Finland Discussion Papers, Economics Department, No. 15/95.

Takala, K. (2001): "Studies in Time Series Analysis of Consumption, Asset Prices and Forecasting." Bank of Finland Studies E:22.

Weil, P. (1989): "The Equity Premium Puzzle and the Risk-Free Rate Puzzle." *Journal of Monetary Economics* 24, 401-421.

Weil, P. (1990): "Unexpected Utility in Macroeconomics." *The Quarterly Journal of Economics*, 29-42.

Weil, P. (1992): "Equilibrium Asset Prices with Undiversifiable Labor Income Risk." *Journal of Economic Dynamics and Control* 16, 769-790.

Zeldes, S.P. (1989): "Consumption and Liquidity Constraints: An Empirical Investigation." *Journal of Political Economy* 97, 305-346.

APPENDIX A

The first-order conditions could be derived directly from the standard intertemporal maximisation by taking appropriate derivatives for adjacent periods. The other way is to use dynamic programming in which case the recursive Bellman equation in its simplest form becomes

$$V_t(W_t) = \max_{c_t, \lambda_t} [U(c_t) + \beta E_t V_{t+1}(W_{t+1})] \quad (\text{A.1})$$

subject to the equation of motion

$$W_{t+1} = (W_t + Y_t - c_t) R_{t+1}^P, \quad (\text{A.2})$$

where

$$R_{t+1}^P = \sum_{i=1}^K \lambda_{i,t} R_{i,t+1}. \quad (\text{A.3})$$

The symbols and measurement units are as in the main text, c_t and W_t are the control and the state variable, respectively, and $V_t(W_t)$ is the value function at time t . By differentiating the value function with respect to wealth and concentrating only on one asset i , the appropriate Benveniste-Scheinkman equation becomes³³

$$V'_t(W_t) = \beta E_t V'_{t+1}(W_{t+1}) \left(\frac{\partial W_{t+1}}{\partial W_t} \right) = \beta E_t V'_{t+1}(W_{t+1}) R_{t+1}^i. \quad (\text{A.4})$$

By substituting the equation of motion (A.2) to the Bellman equation (A.1) we get

$$V_t(W_t) = \max_{W_{t+1}} \left\{ U \left[W_t + Y_t - \left(\frac{W_{t+1}}{R_{t+1}^i} \right) \right] + \beta E_t V_{t+1}(W_{t+1}) \right\} \quad (\text{A.5})$$

and by taking derivatives with respect to wealth, W_t , at time t yields

$$V'_t(W_t) = U'(c_t). \quad (\text{A.6})$$

This should hold for every period $t \in [0, \infty)$. Thus, for period $(t + 1)$:

$$V'_{t+1}(W_{t+1}) = U'(c_{t+1}). \quad (\text{A.7})$$

Combining the results (A.6), (A.7) and (A.4) we get the FOCs (2.5) as in the main text

$$U'(c_t) = \beta E_t [U'(c_{t+1}) R_{t+1}^i], \quad (\text{A.8})$$

where the investor's portfolio consists only of one risky asset, $\lambda_{i,t} = 1, \lambda_{j,t} = 0 \forall j \in [1, K] \setminus i$.

³³Benveniste-Scheinkman equation is based on the envelope theorem. For a proof, see Benveniste & Scheinkman (1979) or Sargent (1987).

APPENDIX B

The CRRA utility function is

$$U(c_t) = (1 - \gamma)^{-1} c_t^{1-\gamma}, \quad \forall t \in [0, \infty), \quad 0 \leq \gamma \neq 1, \quad (\text{B.1})$$

where the concavity parameter γ measures consumer's relative attitude towards risk. By taking the marginal utility for periods t and $(t + 1)$ and substituting these to Euler equation (2.5), the expression becomes

$$c_t^{-\gamma} = \beta E_t [c_{t+1}^{-\gamma} R_{t+1}^i], \quad (\text{B.2})$$

or, by re-expressing the terms,

$$(1 + \rho) = E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^i \right] = E_t [(1 + g_{t+1})^{-\gamma} R_{t+1}^i], \quad (\text{B.3})$$

where g_{t+1} denotes the growth rate of consumption from period t to $(t + 1)$. Moreover, if the (expected) return on a risky asset is stated as $(1 + r_{t+1}^i)$ (one plus the real rate of return), $i \in [1, J]$, the formula reduces to

$$E_t [(1 + g_{t+1})^{-\gamma} (1 + r_{t+1}^i)] = 1 + \rho. \quad (\text{B.4})$$

To proceed, let us take a second-order approximation of the left hand side of this formula around $r_{t+1}^i = g_{t+1} = 0$. After computing the relevant derivatives³⁴, the expression becomes

$$(1 + g_{t+1})^{-\gamma} (1 + r_{t+1}^i) = (1 + r_{t+1}^i) - \gamma g_{t+1} - \gamma g_{t+1} r_{t+1}^i + \frac{1}{2} \gamma (1 + \gamma) g_{t+1}^2. \quad (\text{B.5})$$

Inserting this result back to equation (B.4) and utilising the statistical relations $E(xy) = E(x)E(y) + cov(x, y)$ and $E(x^2) = E(x)^2 + var(x)$, the formula (B.4) can be re-expressed as

$$1 + E_t [r_{t+1}^i] - \gamma E_t [g_{t+1}] - \gamma [E_t (r_{t+1}^i) E_t (g_{t+1}) + cov(r_{t+1}^i, g_{t+1})] + \frac{1}{2} \gamma (\gamma + 1) \{ [E_t (g_{t+1})]^2 + var (g_{t+1}) \} \cong 1 + \rho \quad (\text{B.6})$$

As the time interval decreases to infinitesimally small, the terms $E_t [r_{t+1}^i] E_t [g_{t+1}]$ and $\{E_t [g_{t+1}]\}^2$ become small relative to others. After omitting these and solving the equation above for a risky asset $E_t [r_{t+1}^i]$ yields

$$E_t [r_{t+1}^i] = \rho + \gamma E_t [g_{t+1}] + \gamma cov (r_{t+1}^i, g_{t+1}) - \frac{1}{2} \gamma (1 + \gamma) var (g_{t+1}). \quad (\text{B.7})$$

³⁴The familiar formula for a second-order approximation is $f(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) + \frac{1}{2} [f_{xx}(x_0, y_0)(x - x_0)^2 + 2f_{xy}(x_0, y_0)(x - x_0)(y - y_0) + f_{yy}(x_0, y_0)(y - y_0)^2]$ around the point $(x_0, y_0) = (0, 0)$.

For a riskless asset $R_{t+1} = (1 + r_{t+1})$ the derivation is similar except that we omit the covariance term. Thus,

$$E_t [r_{t+1}] = r_{t+1} = \rho + \gamma E_t [g_{t+1}] - \frac{1}{2} \gamma (1 + \gamma) \text{var} (g_{t+1}). \quad (\text{B.8})$$

Finally, after subtracting (B.8) from (B.7), the expression for the equity premium between any risky ($i = 1, \dots, J$) and riskless asset is

$$E_t [r_{t+1}^i] - r_{t+1} = \gamma \text{cov} (r_{t+1}^i, g_{t+1}), \quad i \in [1, J], \quad (\text{B.9})$$

which corresponds to the equation (2.15) in the main text.³⁵

³⁵While the interpretation remains the same, the results are slightly sensitive to the degree of approximation and other assumptions concerning the random variables. For such results, see Hansen & Singleton (1983), Aiyagari (1993) or Campbell (1996), among others.

Chapter 2

Envy, Habit Formation and Durability in Aggregate Finnish Consumption

1. INTRODUCTION	45
2. INTERNAL AND EXTERNAL EFFECTS	47
2.1. Background of Model	47
2.2. Model	51
3. ESTIMATION METHODOLOGY	57
3.1. Generalized Method of Moments Estimation	57
3.2. Discussion	62
4. DATA DESCRIPTION	65
4.1. Discussion	65
4.2. Consumption Data	68
4.3. Real Return Data	68
5. EMPIRICAL RESULTS	69
5.1. Descriptive Statistics	69
5.2. Diagnostic Tests	72
5.3. Results from Nested Models	75
5.4. Discussion and Evaluation of Results	82
6. CONCLUDING REMARKS	88
References	90
Appendix A	96
Appendix B	98
Appendix C	100

Envy, Habit Formation and Durability in Aggregate Finnish Consumption

by

Jari Viitanen

Finnish Postgraduate Programme in Economics

University of Joensuu, Department of Business and Economics

Abstract

This study explains private aggregate consumption in Finland by using time-nonseparable preferences of a representative consumer which relax the standard time-separability assumption in consumer's intertemporal environment, and allow durabilities to enter in his utility function. With suitable parametrisation, the model divides consumption decisions into internal and external effects. According to the first, the consumer's own consumption history, habit formation and durability of durables, affects as a subsistence level to his present consumption. The second implies that the consumer is envious and has a reference consumption level on which he relates his current consumption. Together with the standard budget constraint, the Euler equations for the optimal life-time consumption pattern with internal and external effects were derived. The empirical results based on GMM estimation and *per capita* consumption showed that there has been a structural change in preference parameters due to financial liberalisation. Under financial regulation, either the durability of durables has dominated habit persistence, or both the durability and habit persistence have been insignificant. After the deregulation, habit formation has been the dominant force. Also, the attitude towards consumption risk has slightly increased over time. Results for the external effect showed that the representative consumer is either slightly envious or he is not at all, with respect to the total private consumption growth in Sweden. In spite of its theoretical weaknesses, the standard model fits reasonably well on the data.

Keywords: CCAPM, internal and external effect, habit formation, durability, envy, reference consumption, representative agent, GMM

1. INTRODUCTION

Unlike the standard time-separable preferences which assume that current utility depends only on current consumption, it is more likely to assume that consumption decisions are based on a richer decision mechanism. One such extension is to assume time-nonseparable preferences with habit formation and durability which imply that past consumption patterns and levels form a threshold level to which current consumption should be compared. Another extension of this is to allow consumer's preferences to depend on the consumption of other consumers.

The objective of this study is to explain private aggregate consumption expenditures in Finland by using a representative agent approach. We test a model which takes durables and semidurables into consideration by relaxing the standard assumption of time-separability in consumers' intertemporal decision-making under uncertainty. In this study, we show that the representative consumer's decision-making is not entirely based on economic, but also on psychological behaviour. The model introduces the presence of consumption externalities in which the consumer's interdependent preferences consist of both internal and external effects on consumption. The internal effect means that the consumer's own consumption history, namely, habit persistence and durability of durables, affects as a subsistence level to his present consumption expenditures. The external effect means that the consumer is envious and has a reference level to which he relates his own consumption. At the individual level, this reference level could be the average consumption of his neighbourhood, socio-economic class, or the *per capita* consumption.

Based on the earlier works of Ferson & Constantinides (1991) and many others, the habit persistence implies that the coefficients of lagged consumption expenditures are negative in the model, whereas durability implies positive coefficients. Moreover, if both effects are present, the dominant one determines the sign. Intuitively, the inverse results between the habit formation and durability can be understood as follows: Under durability, the past consumption expenditures accumulate positively in the argument of utility function. Thus, the higher the durability of durables (the accumulation of wealth in durable terms), the smaller the need for current expenditures (consider a senior consumer who already owns a house, a car, and household appliances). The opposite happens under habit formation: the higher the previous consumption expenditures, the higher the habit and subsistence level, and the more consumption the consumer will require to have the same utility as in the adjacent time periods. In aggregate, if durable effects outperform the habit formation ones, consumption will fluctuate widely. In contrast, if habit formation is the dominant force, consumption will change relatively smoothly. Thus, the fluctuations in aggregate consumption expenditures may be explained and understood by the arguments in a consumer's utility function.

The model for external effect on consumption utilises the theoretical foundations of Abel (1990), Gali (1994) and Carroll et al. (1997). If a consumer realises that the (lagged) reference consumption level has risen, he will gain a higher marginal utility from an additional unit of consumption today to “catch up with the Joneses”. We develop this model further so that it can be empirically tested if there exists some reference consumption that affects Finnish aggregate private consumption. As a whole, the model with both effects is an extension of the standard von Neumann-Morgenstern power utility, and with a suitable parametrisation, it reduces to the standard constant relative risk aversion utility.

Theoretically, in addition to explaining consumption fluctuations, these consumption externalities present several new insights with respect to the standard model. Firstly, depending upon the magnitude of externalities, to maintain his relative consumption level with respect to subsistence and/or reference level, an agent may choose more risky assets in his portfolio and be less risk averse than otherwise. Secondly, if a consumer has fixed habits, he can involve more debt than otherwise to maintain his standard of living even though he has confronted a reduction in his income level.¹ Thirdly, when there are externalities, the equity premium between a risky and a riskless asset can differ in equilibrium from the case of non-externalities. Finally, one can disentangle the coefficient of relative risk aversion and the elasticity of intertemporal substitution from each other, and deal with these parameters as time-varying.

We test the internal and external effects as well as the standard model by using Hansen’s (1982) generalized method of moments (GMM) estimation and Finnish quarterly consumption (durables and semidurables, nondurables and services) and asset data. The estimated parameters, which reveal the representative consumer’s behaviour, are relative risk aversion, subjective discount factor, durability of durables/habit formation, and envy. As an external reference consumption level, we use a lagged average of the total private consumption data from Sweden and OECD. In Finland, after the financial deregulation of mid-1980s, it has been possible for consumers to allocate resources without binding restrictions, and follow their optimal consumption pattern.² We select this period as a possible breakpoint and test the structural stability of the parameter estimates before and after the deregulation.

This study is organised as follows. In section two, we present the theoretical discrete-time version of CCAP-model based on nonseparable preferences with both internal and external effects. Section three gives the background for the

¹Among other things, it can provide one explanation for the overheating of the Finnish economy during late 1980s and early 1990s together with the bubble in stock markets. To maintain their standard of living with respect to habits and others, “the yuppies” could have been risk takers rather than risk averse in their investment decisions.

²We omit the possibility of binding liquidity constraints. Takala (2001) observed that the proportion of the liquidity-constrained consumers in Finland is around 25%.

estimation, and presents the principles of the generalized method of moments estimation. In section four the data is described and the aggregation problems are discussed. In section five the main empirical results are presented and evaluated. Finally, section six concludes the study. Because the mathematics behind the main results is sometimes somewhat technical, the detailed derivations are given in the Appendices.

2. INTERNAL AND EXTERNAL EFFECTS

2.1. Background of Model

Habit formation, consumption durability, and utility comparison have a long history in the consumer behaviour research. Smith (1776) and Veblen (1899) were among the first ones to argue that an individual's utility is determined by comparing his current consumption to some standard societal consumption level observable in the economy. The first formalized version, known as the relative income hypothesis, goes as far as to Duesenberry (1949).³ He criticised the Keynesian consumption function approach because one of its assumptions is that every individual's consumption is independent of other individuals. Duesenberry's arguments were based on two hypotheses. First, consumer's preferences are not only defined on his absolute level of consumption expenditures, but also on his consumption level relative to the rest of population or the *per capita* consumption. Mathematically, the consumer's utility function can be formulated as

$$U_t = U_t \left(\frac{c_t}{C_t} \right), \quad \forall t \in [0, \infty), \quad (2.1)$$

where c_t and C_t are individual consumption and the weighted average of the rest of the population's consumption, respectively. The utility increases only if the individual's consumption increases relative to the *per capita* consumption. Duesenberry also argued that, in terms of utility, the sense of deprivation on a percentage decline in relative consumption is much stronger than the sense of elation from a corresponding percentage increase in relative consumption. Second, it is not the present absolute or relative level of consumption alone that affects consumption behaviour, but also the history of own consumption attained in previous periods. In this case, the appropriate utility function can be expressed as

$$U_t = U_t \left(c_t, \sum_{\tau=1}^{\infty} \delta_{\tau} c_{t-\tau} \right), \quad 0 < \delta < 1. \quad (2.2)$$

The first case arises more from psychological than economical behaviour, because consumers are aware of their consumption level with respect to some reference

³For a historical perspective on habit formation hypothesis, see Messinis (1999a). See also Ackerman (1997) for a non-technical review of history of dissenting economic perspectives on consumption (including the relative income theory) against the conventional neoclassical theory. In general, Bowles (1998) discusses on the role of institutions and endogeneous preferences in consumer theory.

consumption, namely, that of socio-economic status, neighbourhood, or *per capita* consumption. Thus, from the behavioural point of view, the utility function (2.1) reflects implicitly some sort of envy or jealousy. For convenience, we refer to this type of consumption hereafter as external. The second case contains implicitly habit formation and durability and is referred to as internal.⁴ Once a certain level of consumption is attained, the consumer becomes fixed in his habits, and then it becomes more difficult to reduce the consumption expenditures in the future.⁵ For a given level of current expenditures, past purchases contribute to a habit stock, and it is only an increase of current consumption over this habit stock which increases current utility.

In Duesenberry's macroeconomic model these type of preferences imply a "ratchet effect". When income or asset returns falls, consumption drops less than it rises as income or asset returns grow along a trend. Thus, consumers are reluctant to reduce consumption but not hesitant in increasing consumption. This, in turn, creates an inertia in consumer's response to the changes in current income, and has an important implication for aggregate consumption. An increase in current consumption in response to an increase in permanent income increases current utility, but, *ceteris paribus*, decreases utility in future periods. If consumers behave rationally, they are aware of these effects and will response to an increase in wealth with a more moderate increase in consumption. In the opposite case, following a negative income shock, a consumption decrease will be delayed because of unwillingness to reduce the standard of living. Both of these effects will explain the observed excess smoothness of aggregate consumption. Rational consumers with habit persistence prefer small changes rather than big jumps in consumption.⁶

To be more precise and to understand the idea behind our formalised consumption model in the following section, consider the following power utility function for two periods:

$$U = (1 - \gamma)^{-1} (c_t - \lambda c_{t-1})^{1-\gamma} + \beta(1 - \gamma)^{-1} (c_{t+1} - \lambda c_t)^{1-\gamma}, \quad (2.3)$$

$$0 < \gamma < 1.$$

⁴In literature, the terminology for these effects is rich and confusing. Internal effect is also known as "*inward-looking utility*" and "*internal habit formation*" while external effect is sometimes called "*outward-looking utility*", "*external habit formation*", "*relative consumption model*" and "*interdependent preferences*". The general class of utility, comparison or reference utility, is often called "*endogenous utility*" or "*positional utility*". From its ideological foundation, the external model is very close to the concepts of "*bandwagon effect*" and "*snob effect*".

⁵For convenience, we use separate names for the consumption which consumer uses when comparing his relative position. In external model this is called the reference consumption and in internal model as subsistence consumption.

⁶In literature, it is under discussion whether consumers are behaving rationally or myopically under the time-nonseparable preferences. See Messinis (1999b), among others.

The parameters γ , β and λ reflect the concavity of the utility function, discount factor, and relative importance of previous consumption on momentary utility, respectively. Depending on the sign of λ , the consumer's periodical utility is decreasing or increasing with respect to the last period's consumption. If $\lambda > 0$, he needs more consumption today to make him as satisfied as yesterday. The marginal utility with respect to c_t is

$$U'(c_t) = (c_t - \lambda c_{t-1})^{-\gamma} - \beta \lambda (c_{t+1} - \lambda c_t)^{-\gamma}. \quad (2.4)$$

This marginal utility of period t consumption is an increasing function of the previous period's consumption. The first term on the right side captures the effect that if the consumer purchases something valuable today, he is certainly better off today than yesterday. The second term reflects that if the consumer has purchased something valuable yesterday, he got used to that level of consumption, and will desire more for tomorrow. Habit persistence tends to reduce the volatility of marginal utility. With habit persistence, a consumer smooths consumption more than in the standard time-separable model. In other words, consumption is complementary over time. In reality, the extreme examples are those of smokers and alcoholics.⁷ On the other hand, if $\lambda < 0$, current utility increases with past consumption expenditures, i.e., consumption is durable or substitutable over time, the opposite of that implied by habit formation.⁸

An important question concerning equations (2.1) and (2.2) is how to specify a functional form between the consumer's own and relative consumption level. In the 1990s, the consumption literature has seen an expansion of specifications of habit formation and durability. Even though these effects on consumption can be modelled in several ways, two competing specifications usually exist in literature. In ratio models, utility is based on a power function of the ratio c_t/S_t (see Abel (1990), Harbaugh (1996), Carroll et al. (1997) and Fuhrer (1998) among others), where S_t is the reference or subsistence consumption. In difference models, as in (2.3), utility is based on the power of $(c_t - S_t)$ (see Boldrin et al. (1995) and Alesie & Lusardi (1997) among others). Selection between ratio and difference models is significant, because in ratio models risk aversion is constant over time whereas in difference models risk aversion is time-varying. Recently, Campbell et al. (1997), among others, have shown that the models with time-varying risk aversion generate a better predictability in excess returns, and match better with the observed evidence in volatility of stock prices even with a smooth consumption and a constant riskless interest rate. On the other hand, for utility to be well-defined in difference models, a consumer's current consumption must always be above the reference or subsistence consumption, while in ratio models this is not required. Evidently, there is always a trade-off in assumptions between these models.

⁷While these examples are psychologically closer to the concept addiction than habit formation, they provide a good understanding what habit formation means in practice. See Becker & Murphy (1988) and Gruber & Köszegi (2000) for a more detailed discussion on the theory of addiction.

⁸Empirical evidence of these two effects is conflicting. In section 5.4 we discuss briefly these results.

The next issue is the specification of the functional form of reference and subsistence consumption S_t . In external models two alternatives have emerged. Abel (1990) presented his “catching up with the Joneses” model in which the reference consumption is modelled with lagged aggregate consumption. Gali (1994), on the other hand, presented a sophisticated “keeping up with the Joneses” model where preferences depend on current instead of lagged *per capita* consumption. In habit formation models, the problems are the speed with which habits/durability enter in consumers momentary utility and how to specify the consumer’s own consumption history. A usual solution to this is to assume a process for habit stock S_t evolving as follows:

$$S_t = (1 - \xi)S_{t-1} + c_t, \quad (2.5)$$

where ξ may be considered as a depreciation parameter.

Since Duesenberry’s framework, both internal and external effects on consumption have had an important effect on other fields of economics. Especially in the 1990s, the applications of habit formation have increased substantially and the trend seems to continue. For instance, Ryder & Heal (1973) introduced the notion of adjacent and distant complementarity, and discussed the stability of a two-factor growth model in the presence of habit persistence. Carroll et al. (1997) studied the dynamics of endogenous growth models. Stiglitz & Becker (1977) argued that, instead of taking consumer’s preferences as exogenous, these should be taken as endogenous, and investigated for reasons that explain the observed differences and changes in behaviour. Dunn & Singleton (1984), Salyer (1995) and Abel (1999) modelled the term-structure of interest rates using nonseparable preferences together with durables, nondurables, and services. Eichenbaum et al. (1988) and Seckin (2001) concentrated on consumption and leisure choice with nonseparable preferences, while Wathieu (1997) found out that habit persistence can account for the most striking anomalies in the classical discounted utility theory. On the importance of habit formation in explaining business cycle facts with consumption-labour choice and the effect of taxation, the papers of Lettau & Uhlig (1995), Beaudry & Guay (1996) and Ljungqvist & Uhlig (1996) are noteworthy. Fuhrer (1998) showed the essence of habit formation when modeling for monetary policy analysis. Naik & Moore (1996) observed that habitual food consumption equals about one-half of total food consumption, with much of the habitual consumption was explained by an individual-specific permanent component. Fuhrer & Klein (1998) explained international portfolio diversification and international consumption correlations with habit formation in consumption, and Mansoorian (1998) studied current account dynamics. Recent papers by Campbell & Cochrane (1999a,b), Boldrin et al. (1995, 1997), Jermann (1998) and the series of papers by Seckin (2000a,b,c) suggested that external and internal effects in consumer’s utility are useful tools to understand and explain consumption and asset pricing puzzles.

Our basic model in the next section is based on the foundations of Abel (1990),

Ferson & Constantinides (1991), Braun et al. (1993), Gali (1994), Ermini (1994), Carroll et al. (1997), Campbell & Cochrane (1999a,b) and Campbell (1998). The habit formation models in these papers, on the other hand, are originally based on Sundaresan (1989) and Constantinides (1990), who presented a continuous-time habit formation model which generalised models already published in consumption literature. The cornerstone of these works is that they relaxed the typical assumption of time-separability in von Neumann-Morgenstern preferences and allowed a consumer's utility to depend also on past consumption expenditures. A contribution of habit persistence is that it takes also consumption durables, as well as the rate of depreciation, into consideration. However, unlike the papers above, which examined either of these effects, we extend those models to allow a consumer's utility to depend both on internal and external effect, and we derive several nested equilibrium models which may be tested empirically.⁹

2.2. Model

We study a stylised discrete-time economy where expenditures on goods at time t by a consumer are denoted by $c_t = \sum_{i=1}^N d_{i,t}$.¹⁰ The good i is a durable, and the durability entails that the consumer consumes a flow of services out of the current stock of goods purchased. As goods depreciate, the current flow of services provided by past and present expenditures can be expressed as

$$S_t = \sum_{\tau=0}^{\infty} \delta_{\tau} c_{t-\tau}, \quad (2.6)$$

where S_t is the amount of services provided by all consumption expenditures $c_{t-\tau}$, $\tau \geq 0$. The parameter δ_{τ} , $0 \leq \delta_{\tau} < 1$, $\sum_{\tau=0}^{\infty} \delta_{\tau} = 1$, is the rate of durability and it measures the fraction - or the rate of depreciation - of goods purchased at time $t - \tau$ that still survive at time t .¹¹ Consumer's temporal utility is assumed to exhibit time-nonseparable preferences:

$$U(c_t, C_{t-\varphi}) = (1 - \gamma)^{-1} \left[\alpha Z_t + (1 - \alpha) \left(\frac{c_t}{C_{t-\varphi}^{\theta}} \right) \right]^{1-\gamma}, \quad (2.7)$$

$$\forall t \in [0, \infty), \quad 0 < \gamma \neq 1, \quad 0 \leq \theta \leq 1, \quad 0 \leq \alpha \leq 1, \quad \varphi > 0.$$

⁹Messinis (1999b,c) also derives several nested models, but his aims and empirical methods are different from those of this research.

¹⁰Based on dynamic programming, continuous-time models with stochastic asset prices apply Brownian motion and Ito's lemma. Most frequently, the solutions based on these methods cannot be solved analytically but only numerically. Furthermore, under temporal aggregation, the continuous-time modelling of a consumer's decisions is as arbitrary as assuming no temporal aggregation at all, while the data for empirical estimation is in aggregate form. Therefore, we prefer a discrete-time model to derive an empirically testable model. For models based on continuous time, see Grossman & Laroque (1990), Detemple & Zapatero (1991), Duffie (1992) and Dybvig (1995), among others.

¹¹ $\delta_{\tau} = (1 - \xi_{\tau})$, where $0 \leq \xi_{\tau} \leq 1$ is the rate of depreciation. For notational convenience, the rate of depreciation, ξ_{τ} , is a mean of depreciation rates of all N durable goods in the consumption bundle from time $(t - \tau)$ to t . In Appendix A we give more intuition for a perplexed reader and discuss the background of equations (2.6) and (2.8).

The first part in square brackets implies the internal effect, and the second part the external effect on consumption. The term $C_{t-\varphi}$ is the observed level of external lagged reference consumption at time t . For a consumer, this consumption is exogenous and θ reflects the importance of relative consumption.¹² The closer the parameter is to unity, the more envious the consumer is. α is the fraction between internal and external effect, and γ is the utility curvature parameter. To induce both habit persistence and durability on internal preferences over consumption goods to the utility function, the term Z_t is modelled as

$$Z_t = S_t - h \sum_{s=1}^{\infty} a_s S_{t-s}. \quad (2.8)$$

This internal effect denotes the gap between past and present flow of services. Habit formation entails that temporal utility depends on the deviation of the current flow of services S_t from the accumulation of past consumption patterns which are formed into a habit of consumption. Thus, the function $h \sum_{s=1}^{\infty} a_s S_{t-s}$ reflects a subsistence level - or a "bliss" level - where the habit parameter $h \geq 0$ represents the fraction of the weighted sum of lagged consumption flows that establishes a subsistence level of consumption. The fractions a_s , $0 \leq a_s < 1$, on the other hand, measure the persistence of lagged consumption flows in the subsistence level such that $\sum_{s=1}^{\infty} a_s = 1$. As a whole, this utility reconstruction is not time-separable, because the consumption choice at any period reflects the future habit subsistence level in the utility of all future periods.

The model has several important implications and, with a suitable selection of parameters, consists of several nested subutilities.

- i) If $\alpha = 0$ and $\theta = 0$, the standard RRA utility applies.
- ii) If $\alpha = 0$, utility is based on external effect only. The closer the parameter θ to unity, the higher the relative importance of reference consumption.
- iii) If $\alpha = 1$, only the internal effect on consumption is present in utility.
- iv) If $\alpha = 1$ and $h = 0$, the utility function (2.7) reduces to the standard von Neumann-Morgenstern time-separable utility function in consumption flows with the concavity parameter γ reflecting consumer's relative attitude toward risk. Note, however, that the utility function is not time-separable in consumption expenditures.
- v) If $\alpha = 1$, $h = 0$, $\delta_0 = 1$ and $\delta_\tau = 0$, $\tau \geq 1$, the model is also time-separable in consumption expenditures.

¹²Unlike in Gali (1994), the consumer's information set may possibly not contain the contemporaneous reference consumption level. The lagged consumption values are, therefore, more appropriate. Our model differs also from Abel (1990) where the preferences are formed as $U(c_t, v_t) = (c_t/v_t)^{1-\gamma}/(1-\gamma)$, $v_t = (c_{t-1}^D C_{t-1}^{1-D})^\gamma$ where c_t , v_t and C_t are own, reference, and *per capita* consumption, respectively. Note that we cannot have $\theta > 1$ since this would imply that the steady-state utility is decreasing with own consumption.

Consider first the case $\alpha = 0$ where only the external effect is present in (2.7).¹³ $U_{c_t}(c_t, C_{t-\varphi}) > 0$ and $U_{c_t c_t}(c_t, C_{t-\varphi}) < 0$ imply the standard properties that utility is an increasing and concave function of c_t with any given level of lagged reference consumption. Furthermore, according to the following derivatives

$$\frac{-U_{c_t c_t}(c_t, C_{t-\varphi})c_t}{U_{c_t}(c_t, C_{t-\varphi})} = \gamma, \quad (2.9)$$

$$\frac{\partial U(c_t, C_{t-\varphi})}{\partial C_{t-\varphi}} = -\theta \left(c_t^{1-\gamma} C_{t-\varphi}^{\theta\gamma-\theta-1} \right) < 0, \quad (2.10)$$

and

$$\frac{\partial U_{c_t}(c_t, C_{t-\varphi})}{\partial C_{t-\varphi}} = \theta(\gamma - 1) \left(c_t C_{t-\varphi}^{-\theta} \right)^{-\gamma} C_{t-\varphi}^{-\theta-1}. \quad (2.11)$$

These results can be interpreted as follows: The parameter γ is the measure of the constant relative risk aversion.¹⁴ An increase in lagged reference consumption decreases consumer's utility at period t thus making him worse-off. If the consumer is risk averse so that $\gamma > 1$ and if $\theta > 0$, an increase in lagged reference consumption level increases the marginal utility of his own consumption. Heuristically, an addition to his current level of consumption becomes more valuable, because it is required to avoid envy and to keep the consumer relative to his reference consumption. Once again, if $\theta = 0$ and $\alpha = 0$, the utility does not contain external effect, and, furthermore, the utility is similar to that of the standard von Neumann-Morgenstern time-additive utility function.

Consider next only the internal effect ($\alpha = 1$). In the presence of the subsistence level, the close connection between the parameter γ and relative risk aversion can be shown according to the surplus service flow ratio

$$SFR = \frac{S_t - h \sum_{s=1}^{\infty} a_s S_{t-s}}{S_t}. \quad (2.12)$$

This surplus ratio increases with current consumption expenditures c_t and, therefore, with the service flow S_t at time t . If the habit term $h \sum_{s=1}^{\infty} a_s S_{t-s}$ is held fixed as the present service flow varies, the local coefficient of relative risk aversion

¹³One interesting presentation could be the quadratic loss function approach $L = \frac{1}{2}f(c_t - C_t^*)^2$, where C_t^* is the level of reference consumption. Consider two examples: a member of a group will behave like all the other members and a deviation from this could lead to serious consequences. An economist in the Bank of Finland should wear a suit like all other economists; he cannot wear jeans, but, on the other hand, a black suit might be too solemn. A second attractive way to model a consumer's behaviour could be using game theory. At time t (or in dynamic context as well), if agents are aware of each others consumption level, a finite Nash-equilibrium may result.

¹⁴In the external case we prefer the ratio model to make a clear separation between the internal and external model, even though it results to a time-invariant coefficient of relative risk aversion.

with respect to the service flow becomes

$$\frac{-U''(S_t)S_t}{U'(S_t)} = \frac{\gamma}{\frac{S_t - h \sum_{s=1}^{\infty} a_s S_{t-s}}{S_t}} = \frac{\gamma}{SFR}. \quad (2.13)$$

Unlike in standard power utility, the relative risk aversion is time-varying and reflects the periodical attitude towards consumption risk. In extremely bad times the surplus ratio is near zero, while in good times the ratio is close to unity. Following the standard assumption, we assume hereafter that this ratio is always non-negative.¹⁵ If $SFR = 1$, the utility curvature parameter γ equals the time invariant coefficient of RRA. Intuitively, this expression has a clear message: in bad times, as the present flow of services from durables declines toward the subsistence level, or as the surplus flow ratio declines, the consumer becomes more risk averse.¹⁶¹⁷ For notational convenience, the expression between present flow of services and the subsistence level is rewritten as

$$Z_t = S_t - h \sum_{s=1}^{\infty} a_s S_{t-s} = \sum_{\tau=0}^{\infty} b_{\tau} c_{t-\tau}, \quad (2.14)$$

where $b_{\tau} = \delta_{\tau} - h \sum_{i=1}^{\tau} a_i \delta_{\tau-i}$ ($b_0 = 1$) is time-varying coefficient consisting of preference parameters δ_{τ} , h and a_s . So far we have not discussed how the rates of durability and habit formation are modelled. Following Dunn & Singleton (1984), Hayashi (1985), Eichenbaum & Hansen (1990) and Ferson & Constantinides (1991), the durability of goods and habit persistence are assumed to exhibit an exponential decay. That is, $\delta_{\tau} = (1 - \delta)\delta^{\tau}$ and $a_s = (1 - \eta)\eta^{s-1}$ where $\delta = (1 - \xi)$ and $0 \leq \delta, \eta \leq 1$. After some calculus¹⁸, the coefficient b_{τ} can be re-expressed as

$$b_{\tau} = \xi(1 - \xi)^{\tau} \left[1 - h \left(\frac{1 - \eta}{1 - \xi - \eta} \right) \right] + h\eta^{\tau}\xi \left(\frac{1 - \eta}{1 - \xi - \eta} \right). \quad (2.15)$$

¹⁵This means that the current flow of services cannot fall below the habit-forming past consumption flows. Technically, however, if we maintain the assumption of positive risk aversion, it is possible to allow SFR to be negative. Then we should require that the utility is a convex function of the flow of services, which is against the conventional consumer theory. See also Shrikhande (1996) for a non-additive continuous-time model that allows SFR to be negative.

¹⁶Note that the coefficient of relative risk aversion is measured here with respect to the service flow, and not with respect to consumption expenditures, which is the usual experimentation. In standard power utility, the intertemporal elasticity of substitution in consumption equals the inverse of the RRA coefficient. Generally, with habit persistence and difference models, $h > 0$. Ferson & Constantinides (1991) have shown that γ approximately equals the RRA coefficient but may differ substantially from the intertemporal elasticity of substitution in consumption. Constantinides (1990) proved that γ is less than the inverse of elasticity of substitution.

¹⁷This construction of time-varying risk aversion has enabled to explain the evidence why the equity risk premium seems to be higher at business cycle troughs than in peaks. This also gives a solution to the equity premium puzzle and stock market volatility puzzle discussed in Campbell (1998). As consumption declines toward the subsistence level in a business cycle trough, risk aversion rises and the expected returns on risky assets rise and risky asset prices fall. For more on these subjects, see Campbell & Cochrane (1999a).

¹⁸See Appendix B for discussion and proof. See also Ermini (1994) whose assumption of geometric process for durability and habit formation, $\delta_{\tau} = \delta^{\tau}$ and $a_s = \eta^s$, implies results slightly different from ours.

This coefficient contains the following cases:

i) With absence of habit persistence ($h = 0$) and presence of durability, this equation is reduced to $b_\tau = \xi(1 - \xi)^\tau = (1 - \delta)\delta^\tau > 0$, $\tau \geq 1$.

ii) With habit persistence ($h > 0$) and no durability ($\delta = 0$), the equation implies that $b_\tau = -h(1 - \eta)\eta^{\tau-1} < 0$, $\tau \geq 1$.

iii) When both habit persistence and durability are present, the time-varying coefficient b_τ is positive or negative depending on the relative magnitudes of the preference parameters h , η and δ . If $(1 - \xi) \geq \eta + h(1 - \eta)$, the coefficient is positive for all $\tau \geq 0$. If $(1 - \xi) \leq h(1 - \eta)$, then b_τ is negative for all $\tau \geq 1$. Finally, if $h(1 - \eta) < \delta < \eta + h(1 - \eta)$, b_τ is positive for recent lags and negative for more distant ones.¹⁹ This formulation illustrates the opposing forces of habit formation and durability on the coefficients of lagged consumption expenditures. Because of the exponential decay for both habit formation and durability, it shows that if durability is dominated by a habit persistence for a given lag τ , $b_\tau < 0$, then it is dominated also by all greater lags as well, $b_{\tau-\nu} < 0$, $\nu > 0$.

Based on the fertile parametrization above, the consumer's maximisation problem is

$$\max_{d_{i,t}, \lambda_{i,t}} \left[E_t \sum_{t=0}^{\infty} \beta^t U(c_t, C_{t-\varphi}) \mid I_t \right] \quad (2.16)$$

together with budget constraint

$$W_{t+1} = (W_t + Y_t \Delta t - c_t \Delta t) R_{t+1}^P,$$

in which

$$\begin{aligned} R_{t+1}^P &= \sum_{i=1}^K \lambda_{i,t} R_{t+1}^i, \\ c_t &= \sum_{i=1}^N d_{i,t}, \end{aligned} \quad (2.17)$$

and utility specification (2.7). In addition to the symbols above, Y_t and W_t are labour income and wealth at time t , respectively. The measurement units of the variables are as follows: $W_t: FIM$, $Y_t: FIM/\Delta t$ and $c_t: FIM/\Delta t$. $R_{t+1}^P = (1 + r_{t+1}^P \Delta t)$ is a K -dimensional real return factor on assets and r_{t+1}^P is a real rate of return on the portfolio between periods t and $(t + 1)$. $\beta = (1 + \rho \Delta t)^{-1}$ is the discount factor where ρ denotes the subjective rate of time preference. Hereafter, we assume that the time periods are of equal length, and the term Δt can be scaled to one. The weights, $\lambda_{i,t}$, are optimally chosen in period t such that $\sum_{i=1}^K \lambda_{i,t} = 1$. Some of the K assets might be risk-free such that the rate of return is not conditional on the realisation of the period $(t + 1)$ state of nature. E_t

¹⁹See Ferson & Constantinides (1991, pp. 202-203).

denotes a mathematical expectation operator conditioned on the information set, I_t , available to consumer at time t . In Appendix C we show that the first-order conditions, the Euler equations, for the nested models i)-v) presented above can be expressed as²⁰

$$\begin{aligned}
E_t \beta \left\{ \left(\frac{c_{t+\tau}}{c_t} \right)^{-\gamma} R_{t+1}^i \right\} &= 1, & (2.18) \\
E_t \left\{ \beta \left(\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left(\frac{C_{t+1-\varphi}}{C_{t-\varphi}} \right)^{\theta(\gamma-1)} \right) R_{t+1}^i \right\} &= 1, \\
E_t \beta^\tau \left\{ \sum_{\tau=1}^{\infty} \left(\frac{Z_{t+\tau}}{Z_t} \right)^{-\gamma} (b_{\tau-1} R_{t+1}^i - b_\tau) \right\} &= 1, \\
E_t \beta^\tau \left\{ \sum_{\tau=1}^{\infty} \left(\frac{S_{t+\tau}}{S_t} \right)^{-\gamma} (\delta^{\tau-1} R_{t+1}^i - \delta^\tau) \right\} &= 1, \\
E_t \beta \left\{ \left(\frac{c_{t+\tau}}{c_t} \right)^{-\gamma} R_{t+1}^i \right\} &= 1.
\end{aligned}$$

In the absence of the internal and external effects ($\alpha = 0$, $\theta = 0$), the first orthogonality condition is reduced to the standard time-separable model, in which consumption is measured with expenditures on durables rather than nondurables and services. The second Euler equation is based only on the external part of the utility ($\alpha = 0$) and envy. The third equilibrium condition reflects only the internal effect ($\alpha = 1$) with both habit persistence and durability present. The next equation is based on the internal effect with only durability ($\alpha = 1$, $h = 0$) present. The last Euler equation has been derived with the absence of both habit formation and durability ($\alpha = 1$, $h = 0$, $\delta_0 = 1$, $\delta_\tau = 0$).

These Euler equations have similar explanations as in the basic model, except that in the presence of internal effect, the recursive structure is still left. According to the perturbation argument, the infinitesimal reduction in consumption expenditures at any date t reduces current utility by the amount $U'(Z_t)dZ_t$. Furthermore, the reduction in utility in every future period is $U'(Z_{t+\tau})b_\tau dZ_t$ because of the decumulation $b_\tau dZ_t$ of service flow and habit formation, and the change in the subsistence level in the future. The amount not consumed at period t can be invested in a portfolio with either risky or riskless assets and expected return factor R_{t+1}^i . Correspondingly, this increases the future utility by the amount of $U'(Z_{t+\tau})b_{\tau-1}R_{t+1}^i dZ_t$. The external effect, on the other hand, is derived from the standard maximisation problem. For a consumption path to be optimal, the present value of the expected utility loss must equal the expected utility gain. In the absence of internal and external effects, the first-order conditions reduce back to the standard form of the basic CCAPM with no recursive structure. However,

²⁰A usual experiment in empirical work is to test the first-order condition of the (whole) theoretical model. Then, one can test the significance of the parameters and whether the model can be reduced to the nested models. Unfortunately, the Euler equation including both internal and external effects is too nonlinear for such empirical purposes.

it is noteworthy that the standard time-separable model is based on expenditures on nondurables and services while the first and last equations here are based on durable goods.

3. ESTIMATION METHODOLOGY

3.1. Generalized Method of Moments Estimation

The stochastic Euler equations derived in the previous section are highly non-linear functions in their parameters. To estimate these structural parameters, a common and widely accepted method in the consumption literature is to derive log-linearised consumption Euler equations and estimate the parameters by using Maximum Likelihood (ML) method.²¹ This method has, however, two apparent problems. First it is necessary to specify the stochastic processes of the underlying variables, namely, the conditional distribution (usually specified as lognormal distribution) of all variables in the model. If this conditional distribution is misspecified, the parameter estimates may be inconsistent and biased. Second, most often it is necessary to use a constrained optimisation technique in which case the likelihood estimation requires numerical integration where the computations may become a difficult task. Moreover, in his critical paper, Carroll (1997) has recently shown that these linearised approximations cannot successfully uncover the true structural parameters. With simulated data he showed that these methods cannot produce consistent estimates of the true parameter values.

To avoid such problems, we adopt Hansen's (1982) generalized method of moments (GMM) estimation technique where these parameters can be estimated without any assumptions of the distributions of variables. Also, while the Euler equations are based on the expectations, it is likely that the variables involved in the model and the disturbance term are correlated. Because GMM takes advantage of instrument variables, this problem can be avoided. GMM has also several other advantages which are summarised below.

- i) A complete, explicit presentation of economic environment is not required.
- ii) A general form of conditional heteroskedasticity in disturbance terms is allowed.
- iii) A serial correlation in disturbance terms is allowed.

For our purposes, the first remark means that we do not have to explicitly solve the stochastic Euler equations with linear approximations. The third remark is perhaps most important. While economic time-series (e.g. interest rates) often contain strong autocorrelation and while consumption is based on consumer durables (which provide services not only for the current period but also for the future periods), the disturbance terms are almost certainly serially correlated.

²¹See e.g. Hansen & Singleton (1983).

Although GMM has become a well-known estimation method among economists, we will briefly outline the basic ideas behind GMM and its statistical properties in order to understand the estimation results presented in section 5.²² The purpose of GMM is to generate a family of moment conditions that mimic the true population moment conditions in order to construct a quadratic criterion function. The GMM estimator is the one which minimises this criterion function. The criterion function is constructed so that the GMM estimator is consistent, asymptotically normal, and has an asymptotic covariance matrix which can be estimated consistently. For an econometrician, more moment conditions are typically available for use in estimation than there are unknown parameters to be estimated. These overidentified restrictions can be tested using Hansen's J -test to find out if these extra moment conditions are correctly specified.

Suppose that a nonlinear rational expectation model can be described by the function

$$E_t f(x_{t+\tau}; \Theta), \quad \tau \geq 1, \quad (3.1)$$

where $x_{t+\tau}$ is a strictly stationary²³ k -dimensional vector of all random variables in the model observed at time $(t+\tau)$, Θ is the true value of unknown l -dimensional parameter vector and $f(x_{t+\tau}; \Theta)$ is a differentiable vector of functions from $R^k \times R^l$ to R^m . $E_t[\cdot | I_t]$ is the conditional expectation operator conditioned with the information set I_t known at time t . For our purposes, the equation (3.1) can be thought as emerging from the first-order conditions of a consumer's optimisation in an uncertain world. We refer to

$$f(x_{t+\tau}; \Theta) = u_{t+\tau} \quad (3.2)$$

as the disturbance term in GMM estimation. Because the parameter vector Θ is true,

$$E_t [u_{t+\tau}] = E_t [f(x_{t+\tau}; \Theta)] = 0. \quad (3.3)$$

Since all information at time t is available when rational expectations are formed, applying the law of iterated expectations the model yields

$$E [f(x_{t+\tau}; \Theta)] = E [E_t f(x_{t+\tau}; \Theta)] = 0, \quad (3.4)$$

where E is an unconditional expectation operator. Let z_t denote a q -dimensional vector of variables in the consumer's information set. If agents behave rationally, they use all available information at time t to form their expectations. It follows that if variables $z_t \in I_t$ and $x_{t+\tau} \notin I_t, \forall \tau \geq 1$, then $E_t(z_t x_{t+\tau}) = z_t E_t(x_{t+\tau})$. If $E_t(z_t x_{t+\tau}) = 0$, then also $z_t E_t(x_{t+\tau}) = 0$. Next define the function

²²Hansen & Singleton (1982) was the first attempt to apply this method to nonlinear rational expectations Euler equations. See also Hamilton (1994), Mátyás (1999) and Hayashi (2000) for textbook treatment. In our opinion, however, Hall (1993) and Ogaki (1993a) are the best references of GMM for a perplexed reader.

²³A time series is strictly stationary, if the joint distribution of series (x_1, \dots, x_t) is the same as the distribution of the series $(x_{1+\tau}, \dots, x_{t+\tau})$ for all $\tau > 0$.

$$h(x_{t+\tau}; \Theta, z_t) = f(x_{t+\tau}; \Theta) \otimes z_t, \quad (3.5)$$

where h maps $R^k \times R^l \times R^q$ to R^r and \otimes is the Kronecker product. The equation (3.4) implies that the unconditional moment condition (population moment condition)²⁴ can be expressed as

$$E[h(x_{t+\tau}; \Theta, z_t)] = E[u_{t+\tau}, z_t] = 0. \quad (3.6)$$

The ultimate purpose of the GMM is to mimic these population moment restrictions together with sample data. Suppose that the sample size n is large. Then the law of large numbers implies that the corresponding sample moment

$$g_n(\Theta_0) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{t=1}^n f(x_{t+\tau}, z_t; \Theta_0), \quad \Theta_0 \in R^l \quad (3.7)$$

converges with probability one to the population moment condition:

$$g_n(\Theta_0) \rightarrow E[h(x_{t+\tau}; \Theta, z_t)], \quad (3.8)$$

where Θ is the true parameter vector of the model.²⁵ The equation (3.6) implies that $g_n(\Theta_0) = 0$ at parameter values $\Theta = \Theta_0$ and $g_n(\Theta_0)$ should be close to zero for large values of n . Hansen (1982) showed that the GMM estimator of Θ_0 , Θ_{GMM} , which makes $g_n(\Theta_0)$ close to zero, can be obtained by minimising the following quadratic loss function

$$J_n(\Theta_0) = \min_{\theta} [g_n(\Theta_0)' W_{GMM} g_n(\Theta_0)] \quad (3.9)$$

with respect to Θ_0 . The apostrophe denotes transposition and W_{GMM} is a $(r \times r)$ positive definite symmetric matrix. Since the problem is nonlinear, typically, this minimisation must be performed numerically. The first-order condition is

$$D_n(\Theta_0)' W_{GMM} g_n(\Theta_0) = 0, \quad (3.10)$$

in which $D_n(\Theta_0)$ is a matrix of partial derivatives defined by

$$D_n(\Theta_0) = \frac{\partial g_n(\Theta_0)}{\partial \Theta_0}. \quad (3.11)$$

In general, the GMM estimator is given as the solution of

$$\Theta_{GMM} = \arg \min_{\Theta_0} [g_n(\Theta_0)' W_{GMM} g_n(\Theta_0)]. \quad (3.12)$$

²⁴Hansen (1982) refers to these conditions as orthogonality conditions. This term arises from the close relationship to the instrumental variable framework, in which the moment conditions are based on the orthogonality of variables u_t and z_t .

²⁵This is based on the uniformity of the convergence. See McCabe & Tremayne (1993) or Mátyás (1999) for detail.

The asymptotic covariance matrix for this estimator Θ_{GMM} clearly depends on this distance matrix W_{GMM} . To construct an estimator with the smallest asymptotic covariance matrix, one needs a convergence criterion

$$\lim_{n \rightarrow \infty} W_{GMM} = W, \quad (3.13)$$

where W is the true positive definite symmetric matrix defined by

$$\begin{aligned} W &= S^{-1}, \\ S &= \left[\sum_{j=-\tau+1}^{\tau-1} E [h(x_{t+\tau}; \Theta, z_t) h(x_{t+\tau-j}; \Theta, z_t)'] \right]; \end{aligned} \quad (3.14)$$

τ denotes the number of population autocovariances in the disturbance term u_t . Both W_{GMM} and W are also referred to as the weighting or the distance matrix. The resulting asymptotic covariance matrix for the estimator Θ is given by $\Lambda = (D'WD)^{-1}$, where

$$D = E \left[\frac{\partial f}{\partial \Theta}(x_{t+\tau}, \Theta) \otimes z_t \right], \quad (3.15)$$

which is assumed to have a full rank. However, even though D and W are not directly observable, Hansen (1982, lemmas 3.2 and 3.3) showed that D and S can be estimated consistently from any sample data by using

$$D_{GMM} = \frac{1}{n} \sum_{t=1}^n \frac{\partial f}{\partial \Theta}(x_{t+\tau}, \Theta_{GMM}) \otimes z_t \quad (3.16)$$

and

$$S_{GMM} = \left[\sum_{j=-\tau+1}^{\tau-1} \frac{1}{n} \sum_{t=1+j}^n h(x_{t+\tau}; \Theta_{GMM}, z_t) h(x_{t+\tau-j}; \Theta_{GMM}, z_t)' \right]. \quad (3.17)$$

Under regular conditions specified in Hansen (1982), it can be shown that the matrix W_{GMM} converges almost surely to a constant nonsingular matrix W , the GMM estimator is consistent and, according to the central limit theorem, it converges in distribution to a normally distributed random vector with mean zero and a covariance matrix Λ_{GMM} .

$$\sqrt{n}(\Theta_{GMM} - \Theta) \xrightarrow{d} N(0, \Lambda_{GMM}). \quad (3.18)$$

Unfortunately, there are still some difficulties in estimation. Firstly, in a finite sample, only a finite number of autocovariances can be estimated. Secondly, sometimes the structure of the autocorrelation cannot be identified. Thirdly, there is no guarantee that the matrix S_{GMM} is necessarily positive definite. To overcome these problems, we adopt the following estimator, suggested by Newey

& West (1987), to get a consistent covariance matrix of estimators in the presence of generally non-specified autocorrelation:

$$S_{NW} = n^{-1} \sum_{t=1}^n h_{t+\tau} h'_{t+\tau} + \sum_{j=1}^{\tau-1} \left(1 - \frac{j}{\tau}\right) \left[n^{-1} \sum_{t=1+j}^n h_{t+\tau} h'_{t+\tau-j} + n^{-1} \sum_{t=1+j}^n h_{t+\tau-j} h'_{t+\tau} \right] \quad (3.19)$$

where $h_{t+\tau-j} = h(x_{t+\tau-j}; \Theta_{GMM}, z_{t-j})$. The purpose of the weights $(1 - j/\tau)$ is to guarantee that S_{NW} is positive definite and nonsingular to obtain the positive definite weighting matrix W_{GMM} . Moreover, the matrix is consistent because the downweighting higher-order autocovariances disappear asymptotically.²⁶

Since the equilibrium conditions in (2.18) are nonlinear functions of the parameters, this minimisation is easiest to perform numerically with nonlinear algorithms. We adopt the multi-stage optimisation procedure documented in Hall (1993). The first-stage GMM estimator is obtained by setting $W_1 = I$ (identity matrix). Although this estimator is inefficient, it is still consistent and we may employ it in constructing a new weighting matrix, W_2 . The second stage estimator is obtained using W_2 . This estimator can be used to form a new weighting matrix W_3 etc. This iterative procedure can be continued until the selected convergence criteria have been fulfilled.

Typically, it is desirable to test whether the data is consistent with the economic model before focusing on inference of the results of that model. It is evident from the above description of the GMM estimator that the desirable statistical properties of Θ_{GMM} rely crucially on the validity of the sample moment conditions. When the GMM estimation procedure sets the l linear combinations of the r moment conditions to minimise the objective function (3.9), and when $r > l$, there remains $(r - l)$ linearly independent moment conditions which are not used in estimation. If the model is correctly specified, these $(r - l)$ remaining conditions should be close to zero, that is, one would expect $\frac{1}{n} \sum_{t=1}^n z_t f(x_{t+\tau}; \Theta_{GMM}) \approx 0$. This provides a basis for a goodness-of-fit test of the model specification. Hansen (1982) proposed a J -test for these overidentified restrictions to measure how close to zero the sample moment conditions are:

$$nJ_n(\Theta_{GMM}) \rightarrow \chi^2(r - l), \quad (3.20)$$

where Θ_{GMM} is the value which minimises the loss function. Under the null hypothesis $E[h(x_{t+\tau}; \Theta_{GMM}, z_t)] = 0$, the test statistic is asymptotically distributed as a chi-square with $(r - l)$ degrees of freedom. Subsequently, Hansen & Jagannathan (1989) expressed that this test rejects the true model too frequently if

²⁶See Hamilton (1994) and Campbell et al. (1997) for a more detailed discussion. An appropriate lag, suggested in literature, is $\tau = n^{1/4}$.

the number of observations is small. They proposed that the criteria should be based on r degrees of freedom.

Sometimes a shift of regime or a change in economic environment may cause economic agents to change their behaviour. To examine these changes, we adopt Andrews & Fair's (1988) test statistic to test the structural stability of the estimated parameters. This test is based on the moments of two subsamples, that is, before and after the structural breakpoint. Analogously, one can compute the corresponding estimates from these subsamples. Let n_0 denote the possible breakpoint consisting of the sample size of the first subsample and of $n_1 = n - n_0$, the size of the second subsample. Let $\pi = n_0/n$. Again,

$$\begin{aligned}\sqrt{n}(\Theta_{GMM}^0 - \Theta^0) &\stackrel{d}{\sim} N(0, \Lambda_{GMM}^0/\pi), \\ \sqrt{n}(\Theta_{GMM}^1 - \Theta^1) &\stackrel{d}{\sim} N(0, \Lambda_{GMM}^1/(1 - \pi)),\end{aligned}\tag{3.21}$$

where the superscripts refer to subsamples. The test statistic is

$$\begin{aligned}AF &= n(\theta_{GMM}^0 - \theta_{GMM}^1)' [\pi^{-1}\Lambda_{GMM}^0 + (1 - \pi)\Lambda_{GMM}^1]^{-1} (\theta_{GMM}^0 - \theta_{GMM}^1) \\ &\rightarrow \chi^2(l)\end{aligned}\tag{3.22}$$

for testing the null hypothesis $\Theta_{GMM}^0 = \Theta_{GMM}^1$. This null hypothesis, as well as the restrictions for the overidentification, are rejected when the test statistics (3.20) and (3.22) are higher than their critical values obtained from the appropriate χ^2 distribution.²⁷

Often there is a need to test hypotheses about the value of a parameter vector θ_{GMM} that has been estimated by GMM. Consider a null hypothesis $H_0 : R(\theta_{GMM}) = r$ involving s linear or nonlinear restrictions on θ_{GMM} . This null hypothesis is tested against an alternative hypothesis $H_1 : R(\theta_{GMM}) \neq r$. The Wald test of the null hypothesis is

$$\begin{aligned}n(R(\theta_{GMM}) - r)' &\left[\left(\frac{\partial R(\theta)}{\partial \theta'} \Big|_{\theta=\theta_{GMM}} \right) \Lambda_{GMM} \left(\frac{\partial R(\theta)}{\partial \theta'} \Big|_{\theta=\theta_{GMM}} \right)' \right]^{-1} \\ &\times (R(\theta_{GMM}) - r) \sim \chi^2(s)\end{aligned}\tag{3.23}$$

where θ_{GMM} is the unrestricted GMM estimator. Under H_0 , Wald test statistic follows the χ^2 distribution with degrees of freedom equal to the number of restrictions s . This hypothesis is rejected when the test statistics is higher than its critical value.

²⁷There are several other candidates to test structural stability, such as Ghysels & Hall's (1990a,b) test. Hall (1993) and Hamilton (1994) surveyed these tests in GMM environment. See also Hall & Sen (1999) for a wider discussion on the methodology of structural stability tests of GMM.

3.2. Discussion

Although GMM has attractive properties, it also has disadvantages which limit its usefulness. We point out some of these problems in the finite sample properties of GMM that should be considered in our estimation and evaluation of the results (see Hall (1993) for further discussion). In GMM, only a subset of an economic environment is needed to be specified for the estimation. Thus, any variable in I_t is an appropriate candidate for forming the sample moment conditions. This abundance may create problems because the parameter estimates are usually highly sensitive to the choice of z_t . In his simulation test for a time-separable model, Tauchen (1986), for instance, found that the GMM estimator is sensitive to the choice of the number of lags in an instrument set. As the number of lags is increased, the variance of estimators decreased with an increase in bias. Kocherlakota (1990) conducted similar small sample results with iterated multi-stage GMM estimation. His conclusion was that the null hypothesis is rejected too frequently, and GMM performs worse when larger instrument sets and number of lags are used. Recent results by Nelson & Startz (1990) revealed that the quality of the instruments (the correlation between a regressor and an instrument) can cause the sensitivity in J_n . If the instrument set is of very “poor” quality, then the null hypothesis (3.20) tends to be rejected too frequently.²⁸ Even though no simulation tests for the nonseparable models have been made, it is reasonable to assume that the parallel results hold true.

Ermini (1994) pointed out another problem closely related to the choice of instruments in testing the moment restrictions. In his opinion, the GMM test is only “a test against nature”. The moment conditions are either rejected or not, but the model cannot be tested against any other alternative. Clearly, it provides no useful insight as to which direction a researcher should pursue to improve the model, or which instrumental variables from the consumer’s information set should be used instead.

To summarise, all these observations suggest that while the model to be estimated is correctly specified and while the asymptotic properties of the estimator are correct, the finite sample properties of GMM are highly sensitive to the quality of the instrumental variables, sample size, the number of moment restrictions and the number of lags used in the instrument set. The conclusion is that a small number of instruments with recent lags rather than a large number of instruments is to be recommended when *ad hoc* instruments are used to form the sample moment conditions. The quality of instruments should be confirmed by selecting variables which are correlated with regressors as much as possible. The estimation is more difficult when durables are involved as a measure of consumption expenditures. Several authors (see e.g. Mankiw (1982) and Campbell and Mankiw (1990)) have shown that because of durables and time aggregation, the disturbances in a regression of current expenditures on lagged expenditures ex-

²⁸A thorough discussion on this is found in section 5.2.

hibit an ARMA(1,1) structure. Therefore, the error terms may also correlate with the once-lagged instruments in a nonlinear estimation. Moreover, Ferson & Constantinides (1991) argued that the measurement error and other data problems may result in spurious correlations between the consumption and the real rates of return, if their own recent lags are used as instruments. This may bias the parameter estimates and lead to a spurious rejection of the moment conditions. They suggested other (financial) variables that are lagged at least twice rather than once to avoid such problems.

Our model contains three technical problems of estimation. Firstly, there are computational restrictions. For empirical purposes, the first-order conditions based on high-tailed internal effect are highly nonlinear functions. Thus, finding the parameter estimates to give a global saturation point (optimum) for the consumer is difficult with the standard nonlinear algorithms and GMM. Following Eichenbaum & Hansen (1990), Ferson & Constantinides (1991) and Braun et al. (1993), we limit to a one nonseparable lag model, $b_\tau = 0$, $\tau \geq 2$, to make the computation accessible. Secondly, to avoid economically implausible negative utility, the term $c_t + b_1 c_{t-1}$ should be non-negative for $\forall t$ with probability one. In the internal model with habit persistence dominance, this problem may arise if parameter b_1 is close to minus one. Therefore, we must restrict $c_t/c_{t-1} \geq b_1$.²⁹ The same notion should hold under durability reflecting that it is not plausible for b_1 to be greater than one even though utility is well-defined. The restriction $b_1 \leq 1$ means that the consumption in the previous period cannot be more important than the consumption at current period for that period's utility. While we cannot determine the distribution of consumption to fulfill these restrictions, we ensure that these conditions are satisfied for all realisations in the samples evaluated with our estimate for the parameter b_1 . Thirdly, perhaps the most important problem is that, in order to avoid biased estimators, the time-series should be stationary - one of the basic assumptions of GMM. Usually the (aggregate) consumption is growing over time, so we cannot assume the time-series of c_t to be stationary.³⁰ Another undesired feature of the moment conditions is that they can be satisfied with trivial solutions. To illustrate, consider the internal model with one lag ($b_\tau = 0$, $\tau \geq 2$). The Euler equation for the agent's portfolio allocation is then given by (see equation (C.16) in Appendix C)

$$E_t \left[\frac{\beta \left(Z_{t+1}^{-\gamma} + \beta b_1 Z_{t+2}^{-\gamma} \right) R_{t+1}^i}{Z_t^{-\gamma} + \beta b_1 Z_{t+1}^{-\gamma}} \right] = 1, \quad (3.24)$$

in which $Z_t = c_t + b_1 c_{t-1}$. This can be rewritten with respect to the error term u_{t+2} :

$$u_{t+2} = E_t \left[\beta \left(Z_{t+1}^{-\gamma} + \beta b_1 Z_{t+2}^{-\gamma} \right) R_{t+1}^i \right] - E_t \left(Z_t^{-\gamma} + \beta b_1 Z_{t+1}^{-\gamma} \right). \quad (3.25)$$

²⁹Especially in the depression years of early 1990s this problem could arise because $c_t < c_{t-1}$.

³⁰In recent years, while the Euler equation methodology has become a dominant paradigm in the consumption and asset pricing research, more attention has been paid on developing the properties of data.

If the model is true, $E_t[u_{t+2} | I_t] = 0$. This error term, however, cannot be used in the GMM estimation because both the stationary condition and identification assumptions are violated. If one selects $\gamma = 0$ and parameters β and b_1 so that $\beta b_1 = -1$, the moment conditions are trivially satisfied. To avoid nonstationary and trivial solutions, Ogaki (1993b) suggested that the error term u_{t+2} should be normalised by a scaling factor $Z_t^{-\gamma} (1 + \beta b_1)$ to induce a new disturbance term u_{t+2}^* .³¹ Since $Z_t^{-\gamma}$ is in I_t , u_{t+2}^* still satisfies moment conditions $E_t[u_{t+2}^* | I_t] = 0$. This scaled disturbance term is now a function of $Z_{t+\tau}/Z_t$ and R_{t+1}^i . If the ratio of consumption c_{t+1}/c_t and R_{t+1}^i are assumed to be stationary, the scaled disturbance is a function of stationary values.

Due to the computational restrictions, we use the scaling factor $c_t^{-\gamma}$ instead of the methodology above.³² After some straightforward calculus, the disturbance term can be expressed as

$$u_{t+2}^* = E_t \left\{ \beta \left(\frac{c_{t+1}}{c_t} + b_1 \right)^{-\gamma} \left(R_{t+1}^i - b_1 \right) + \beta^2 b_1 \left(\frac{c_{t+2}}{c_t} + b_1 \frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^i \right\} - \left[1 + b_1 \left(\frac{c_{t-1}}{c_t} \right) \right]^{-\gamma}. \quad (3.26)$$

Even though the trivial solution still holds, the advantage of this scaled disturbance term is that all the variables in the model are now stationary. To avoid a trivial solution, we use Wald test and ensure that the estimated parameter values do not fulfill the joint restriction $\gamma = 0$ and $\beta b_1 + 1 = 0$.

4. DATA DESCRIPTION

4.1. Discussion

The stochastic Euler equations in section two are based on the theory of a single consumer behaviour. Because the purpose of this study is to explain aggregate consumption behaviour in Finland, the problems and links between the individual consumer theory and its use with the aggregate time-series data are to be noted. For most empirical economists, the only consumption data available is the aggregate one, in which the aggregation means across consumers, time and commodities. It is a general practice to resolve the first aggregation problem by using a theoretical model based on a single consumer behaviour, then assuming that individuals can be aggregated into a single representative agent, thus, using aggregate data in estimation. If all individuals have identical preferences and production possibilities, their consumption profiles over time are identical, and the use of the aggregate consumption is justified. Unfortunately, this assumption of homogeneity is seldom realistic and many economists distrust the empirical

³¹This suggestion is also applied by Eichenbaum & Hansen (1990) and Hansen et al. (1996).

³²On the other hand, Ni (1993) showed that the parameter estimates of the model can be highly sensitive to the choice of this scaling factor.

results of these models.³³ Empirical results (see e.g. Deaton (1991)) indicate that the consumption of individuals is much more volatile, and does not very highly correlate with the aggregate consumption. These observations are against the use of representative agent models. Nevertheless, the arguments for these representative agent models came from Constantinides (1982) who showed that even if the consumers are heterogeneous in preferences and the levels of initial wealth, it may be possible to find some utility function for the representative agent which satisfies the nonlinear Euler equations. The only requirement is that asset markets must be complete so that the agents can diversify any idiosyncratic risk in consumption. According to Constantinides (1982), if this assumption is satisfied, it is possible to construct a representative agent who becomes marginally homogeneous even though consumers are initially heterogeneous.³⁴

The second aggregation, the temporal aggregation, raises an important methodological question, whether a consumer's decision-making can be separated to discrete-time dimensions when time itself is measured continuously. It is unrealistic to assume that a consumer makes his consumption decision together with asset trading, say, once a month (according to a monthly data), four times a year (quarterly data) or once a year (annual data), not to mention daily decisions. Generally, whatever the selected time period, it measures an average of consumption expenditures during the time period rather than is a point-in-time observation. Consumption is, however, a continuous process and the volatility of consumption, perhaps including sharp peaks and bottoms between the decision dates, is not observed. Furthermore, as noted by Brunila (1996), in the case of durables it is important to make a distinction between the concepts of consumption and consumption expenditures. At any point in time the consumption of purchased durable goods yields utility for a consumer without any acquisition of durable goods. On the other hand, the utility from current purchases of durable goods is distributed over several periods and is not restricted to the time of purchase. Due to these reasons, the theoretical presentations are typically based on the idea that durable goods yield a flow of services to a consumer. Unfortunately, such measures are difficult to be constructed in practice, and one has to rely on data of consumption expenditures.

Finally, the commodity aggregation is a problem still in addition to aggregations studied above. With heterogeneous consumers, the consumption bundles of individuals may significantly differ from those of *per capita* measures. Even though widely used in consumption and financial literature, it is evident that the

³³Deaton (1992) is more critical against these representative agent models with his words: "Representative agents have two great failings: they know too much, and they live too long (pp. ix)." and "The main puzzle is not why these representative agent models do not account for the evidence, but why anyone ever thought that they might, given the absurdity of the aggregation assumptions that they require (pp. 70)."

³⁴Even though the assumption of complete markets seems a little unrealistic, it is widely used in finance and macroeconomic models because of its considerable ease of obtaining qualitative and quantitative predictions. See Aiyagari (1993) for a model allowing the presence of incomplete markets and transaction costs.

aggregate data can never correctly characterise the true behaviour of individual consumption.³⁵

In spite of these problems, we used aggregate data and a representative agent approach in this study. According to Attanasio (1998, pp. 1-2), consumption decisions should be modelled with a well-specified and coherent optimisation model. Also, even though the link between individual consumer theory and aggregation is somewhat inappropriate, it is important to analyse the aggregate time-series to make statements about the observed behaviour or to evaluate the effect of any proposed change in economic policy. Hereafter, we define the concept representative agent as an average consumer whose demand is an average of the total demand. The idea behind this *per capita* approach is that all consumers are assumed to have nearly similar preferences.³⁶ Mathematically, when there are H utility-maximizing consumers in an economy, the demand of the representative consumer for an aggregate good c (for example all durable goods) at time t can be formulated as follows:

$$c_t = \frac{1}{H} \sum_{i=1}^N \sum_{j=1}^H c_{ij,t}(p, m_j), \quad (4.1)$$

in which $c_{ij,t}(p, m_j)$ is a demand of a single consumer j for commodity i , p is the price vector for N commodities and m_j is the consumer's income.

In this study, we employed only the rate of return from financial assets. The returns from real assets (such as housing or real estates) as well as from human capital form most of the consumers' total wealth in Finland. Nevertheless, these cannot be used in this study because the yield is distributed over several time periods and because of the difficulty to measure the obtained yield in practice.³⁷ Also, the investments in foreign assets were omitted. The reason for this is that until mid-1980s (covering nearly half of our research period) there were capital controls in Finland, thus preventing individuals to hold foreign assets and liabilities.

³⁵A thorough discussion of these aggregation problems is too wide and beyond the scope of this study. See Deaton & Muellbauer (1980), Blundell (1988), Kirman (1992), Stoker (1993) and Nurminen (1999) for detailed insights.

³⁶In the static environment, Nurminen (1999) surveyed the different theoretical possibilities to create a representative agent's demand function for a single commodity. Especially, he showed that in spite of the unpredictable random income and arbitrary distribution of a demand of a good of a single consumer, it is possible to construct a representative agent whose probability distribution is normally distributed and which is a good proxy for a normally distributed market demand. Under certain assumptions, Nurminen showed that the average consumption can be used as an estimate of the representative consumer's demand if the size of the population in the economy is large enough.

³⁷Takala et al. (1991) calculated real rates of return for several assets covering the time period 1960-1988. Those figures are, however, based on annual observations, and transforming them for our quarterly purposes is difficult. Moreover, financial assets are typically more liquid than real assets and these can be better used to smooth out the unexpected changes in consumers' income and wealth.

The variables in the models above are based on the expected real ratio of consumption measured at two successive points of time and one period ahead of the expected real rate of return. While the statistics on time-series of expected real rate of return and consumption are not comprehensively compiled, the measurement must be based on *ex post* rather than *ex ante* variables. The expected real rate of return is the quarterly nominal market rate, adjusted for taxes, less the expected rate of change of price level. Evidently, the longer the time horizon, the greater the measurement error between expected and observed rates.

4.2. Consumption Data

The consumption data consisted of quarterly real consumption expenditures on durables, semidurables, nondurables, and services.³⁸ To mimic a representative agent's behaviour and to reflect demographical changes, the real aggregate expenditures were divided by the number of total population to obtain real *per capita* consumption expenditures. The time period for our examination was 1975.1-2001.2. The consumption data was taken from the database of the Research Institute of the Finnish Economy, and the population data was from the Statistics Finland. The data was seasonally adjusted.

As an external reference consumption level we used the data from Sweden and the Organization for Economic Cooperation and Development (OECD) countries. The data from Sweden was measured and classified with consumption expenditures corresponding to that of Finland and was taken from the database of the Research Institute of Finnish Economy. The data from OECD countries comprises the average of total private consumption expenditures and was taken from the OECD database.

4.3. Real Return Data

We examined the quarterly real tax-free rates of return on government bonds and all bonds in the market (including corporations, commercial banks, local and state government), equities traded in the Helsinki Stock Exchange, and the average borrowing rate of commercial banks in Finland. From these, only the investment in stocks can be considered as risky, while the others are riskless.³⁹ Originally the returns of holding government bonds until the year 1988 were tax-free while after this year returns are taxable. The latter series were converted

³⁸The separation between these categories is not self-evident and evidently creates data measurement problems. A good example is a holiday trip. While it is physically perishable, and, therefore, should be treated as a nondurable, it has a long lasting psychological effect on preferences with, for instance, a better motivation and effort on working. Thus, it has some property of a durable good as well. Another example is dental services which usually are categorised as services (nondurables and services) while it is physically a long-lasting investment in dental care and should be treated as a durable.

³⁹To be precise, these assets are only nominally riskless if they are held in maturity. To the extent that there is uncertainty regarding inflation, they are not riskless in real terms. However, over short periods in time (like quarters of a year in our examination), this uncertainty is fairly small, and, these assets can be regarded as riskless in real terms as well.

to tax-free serieses by using the average of marginal tax rate from year 1989 to 1993 and capital tax rate after year 1993.⁴⁰ Until the year 1988 the returns on all bonds were closely related to the yields on government bonds. This is due to the fact that the government was almost the sole emitter until this year while after 1988 other institutions became considerable emitters as well. The time-series were converted to tax-free series correspondently.

The capital gains from the stocks were measured as percentage changes of Helsinki Stock Exchange Index (HEX). To obtain the total rate of return of stocks, we added the average effective dividend yields to these capital gains. For example, if the quarterly growth rate in HEX index was 3.2% and the average dividend yields were 2.1% for the corresponding time period, the total rate of return from stocks was 5.3%. HEX index was obtained by chaining the monthly observations of UNITAS index. The time-series of UNITAS index was collected from the UNITAS publication. The quarterly data was obtained from monthly observations by calculating a three-month moving average. The other rates of return were collected from several publications of the Statistics Finland.

The nominal rates of return on all these assets were converted to real by using an appropriate consumption price deflator. For instance, if the consumption expenditures were measured by the data of durables, asset returns were deflated by the deflator of consumer durables.

5. EMPIRICAL RESULTS

5.1. Descriptive Statistics⁴¹

Table 1 presents descriptive statistics from the consumption data. We selected the breakpoint to be the time of financial deregulation in Finland. This deregulation culminated towards the end of 1986 when a major part of the regulation was liberalised. We report separate statistics before and after this breakpoint to get an intuition whether there has been a structural change in consumer behaviour.⁴² The variables are the real *per capita* consumption growth rates with respect to the previous quarter. The results show that all the consumption serieses were fairly smooth with a small standard deviation. Especially, the smoothness of real growth of expenditures on nondurables and services imply that the acquisition of these goods is not easily substitutable over time, therefore, the behaviour is relatively smooth. The expenditures on durables is the most volatile component of total consumption.⁴³ This can be interpreted by the well-known fact that

⁴⁰Negative rates of return are not taxed.

⁴¹The purpose of this section is to give a general view of the real development of the time-series. More detailed results are available from the author by request.

⁴²Intuitively, one possible breakpoint could be the early years of 1990s when the first anticipations and forecasts of the forecoming depression appeared.

⁴³Koivumäki (1999) reported similar results from time period 1961-1994 with annual data.

when the economic situation is uncertain, consumers postpone the acquisition of durables until the future. Also, acquisition requires advance savings. The same arguments hold for semidurables which are more volatile than the other two series. Services have the highest and smoothest growth rate.

Table 1

Descriptive statistics on consumption data 1975.2-2001.2

Variable	1975.2-2001.2		1975.2-1986.4		1987.1-2001.2	
	Mean	Std.	Mean	Std.	Mean	Std.
D	1.0058	0.0525	1.0076	0.0431	1.0044	0.0594
SeD	1.0044	0.0234	1.0046	0.0255	1.0041	0.0218
ND	1.0030	0.0149	1.0031	0.0135	1.0029	0.0161
S	1.0068	0.0099	1.0072	0.0099	1.0064	0.0100
DSeD	1.0048	0.0328	1.0060	0.0273	1.0039	0.0370
NDS	1.0050	0.0088	1.0052	0.0079	1.0048	0.0095

D=Durables, SeD=Semidurables, ND=Nondurables, S=Services, DSeD=Durables+Semidurables, NDS=Nondurables+Services

The correspondent statistics for the subsamples 1975.2-1986.4 and 1987.1-2001.2 reveal that the mean for every consumption measure was higher in the first subsample than in the second. Conversely, the volatility of the consumption variables was somewhat higher in the second subsample. These findings show that there seems to have been a shift in the distribution of consumption pattern. Also, these findings are in contrast to the hypothesis that consumers can smooth consumption more freely when they do not face binding borrowing constraints. The reasons behind these observations can be found in the boom of late 1980s and the unanticipated deep depression in the early 1990s. For example, the maximum value (12%) of the growth of expenditures on durables can be found from the first quarter of 1989. The minimum value (-17%) is at the first quarter of 1992. The descriptive statistics for the real asset returns are depicted in Table 2, reported as one plus the rate of return. A few issues are worth mentioning. First, under the financial regulation, nominal returns were typically low and inflation was high implying a negative average real rate of return. Second, the average real rate of return has been higher for government bonds as well as for other bonds when compared with shares. This implies that the excess average return, the equity premium, is negative. Third, the 5 percent standard deviation of share returns implies that investment in shares has been the most risky investment decision.

The statistics for the second subsample reveal that the first and second remarks above are culminated to the breakpoint in the mid-1980s. After the deregulation, the real rates of return as well as the equity premium turned positive. The third remark is also true across time periods. It is also noteworthy that after 1986 the volatility of share returns has increased while that of other assets has decreased.⁴⁴

⁴⁴The highest quarterly real return on shares (1.36) was in the early 2000 while the lowest value (0.77) in the early 2001.

Thus, according to these notions, a risk averse investor should have invested only in bonds. This seems to be a contradiction in evidence, since daily observations as well as empirical results indicate that investments in shares have increased considerably in the 1990s. However, one must keep in mind that the realisation of the rates of return was not known when making investment decisions.

Table 2
Descriptive statistics on real return data 1975.1 - 2001.2

		1975.1-2001.2		1975.1-1986.4		1987.1-2001.2	
Variable	Deflator	Mean	Std.	Mean	Std.	Mean	Std.
GBOND	PCD	1.0038	0.0135	0.9973	0.0147	1.0091	0.0096
	PCND	1.0024	0.0131	0.9954	0.0146	1.0081	0.0081
	PCS	1.0013	0.0094	0.9956	0.0086	1.0060	0.0072
BOND	PCD	1.0038	0.0135	0.9973	0.0147	1.0091	0.0096
	PCND	1.0024	0.0131	0.9954	0.0146	1.0082	0.0081
	PCS	1.0013	0.0093	0.9956	0.0086	1.0060	0.0071
RRATE	PCD	1.0016	0.0137	0.9945	0.0147	1.0075	0.0094
	PCND	1.0002	0.0133	0.9926	0.0146	1.0065	0.0078
	PCS	0.9992	0.0094	0.9928	0.0086	1.0044	0.0062
SHAR	PCD	1.0087	0.0934	0.9908	0.0482	1.0235	0.1168
	PCND	1.0073	0.0954	0.9889	0.0514	1.0225	0.1185
	PCS	1.0062	0.0953	0.9891	0.0476	1.0204	0.1199

GBOND=Government bonds, BOND=All bonds in market, RRATE=Average borrowing rate of commercial banks, SHAR=Shares. Deflators are: PCD=Private consumption prices on durables, PCND=Private consumption prices on nondurables & semidurables, PCS=Private consumption prices on services.

To test if there is evidence of a co-movement between the growth rate of consumption and the real rates of return, we calculated the correlation coefficients between these variables. Table 3 shows the results. The real returns were deflated by the correspondent consumption price measure. Typically, the correlations between asset returns and consumption variables are low or even negative (semidurables).⁴⁵ Under financial regulation, the correlations are higher for durables and nondurables than for semidurables and services. They also differ statistically from zero at 10% level of significance. The correlations between real rates of return and semidurable goods are low implying a weak co-movement.⁴⁶ After 1986 the correlations remain nearly the same except those for durables which have decreased. Under financial regulation, the correlations between the real rates of return are high (not reported in the Table). The correlation coefficients between bonds, government bonds, and borrowing rates are above 0.9, while the correlations between shares and other returns are about 0.35. After

⁴⁵It is noteworthy that the correlations are calculated here for the stationary rather than level variables. In next section we will discuss how this will affect the magnitude of the correlation coefficients.

⁴⁶At a constant utility the negative correlations can be interpreted by the substitution effect dominance - consumers are postponing their acquirement of consumption goods into the future.

1986 all correlation coefficients are somewhat lower. Ignoring the returns from shares, the correlations between financial assets are in the range 0.80-0.96. The correlations between shares and other returns are in the range -0.17-0.30.

Table 3
Correlations of consumption growth and real asset returns,
1975.1-2001.2

		D	SeD	ND	S
1975.1-2001.2	GBOND	0.269*	0.011	0.203*	0.141
	BOND	0.265*	0.008	0.204*	0.140
	RRATE	0.228*	-0.023	0.189*	0.096
	SHAR	0.047	0.049	0.026	0.221*
1975.1-1986.4	GBOND	0.481*	-0.012	0.286*	0.180
	BOND	0.480*	-0.011	0.287*	0.184
	RRATE	0.473*	-0.014	0.283*	0.178
	SHAR	0.186	-0.030	0.233	0.333*
1987.1-2001.2	GBOND	0.206	0.073	0.219*	0.209
	BOND	0.197	0.061	0.221*	0.205
	RRATE	0.131	-0.034	0.197	0.125
	SHAR	0.023	0.093	-0.032	0.222*

The variables are as in Tables 1 and 2. The asterisks mean that the coefficients differ statistically from zero at 10% level of significance.

Even though the figures in Tables 1-3 are quite casual, it seems that the data after the deregulation is qualitatively different than before the liberalisation.

5.2. Diagnostic Tests

Theoretically, a simple autocorrelation structure can reveal important patterns of consumer behaviour. Durability of consumption expenditures induces a negative autocorrelation. This is due to the fact that if a consumer purchases a long-lasting durable in one period, he is unlikely to purchase another one for several periods. On the contrary, habit persistence induces positive autocorrelation. A consumer smooths his consumption more than it would be optimal with time-separable preferences. A similar structure can be found from aggregate consumption series. For the subsample 1975.2-1986.4 the first-order autocorrelations of consumption growth are negative for all consumption measures (not reported in Tables). Only the coefficient of the semidurables turns out to be statistically significant. For the second subsample 1987.1-2001.2, all autocorrelations are positive except for the consumption growth of nondurables. Only the autocorrelation coefficient of nondurables is statistically significant. The signs for the first-order autocorrelations suggest that the durability of consumption goods dominates before the breakpoint, and habit persistence is dominant after that.

These results, however, should be treated as preliminary rather than concluding because the measurement error of consumption data may cause a spurious positive or negative autocorrelation.

To implement the GMM estimation, one needs to identify the set of instrument variables from the consumer's information set. Typically, it is quite likely that the instruments should be highly correlated with the regressors but not significantly correlated with the error of measurement. A usual experiment in asset pricing models is to use lagged values of consumption and asset returns in order to create the orthogonality conditions.⁴⁷ This selection of instrument variables is, however, not unique. As concluded in section 3.2, other variables than those in the model should be used as instruments. We follow this suggestion and use a constant, the growth rates of gross domestic product (GDP) and disposable income (YD) as instruments.⁴⁸ Referring to the arguments above, we use as small an instrument set as possible. Moreover, the instruments are lagged at least twice to avoid spurious correlations.⁴⁹ Economically, the link between gross domestic product, disposable income, and consumption is evident. Under financial regulation, the connection between instruments and real rates of return can be justified through political decision-making. For example, if an economy is in boom, political decision-makers can raise the administrative interest level to cool the overheated economy and *vice versa*. The lags in instruments are appropriate to capture this argument. Under liberalisation, financial markets react to the changes in economic fundamentals, such as GDP and YD.

To confirm that the instrument variables are indeed appropriate we performed several tests. Table 4 shows the correlations between the instruments and variables involved in the Euler equations. The asterisks denote that the coefficients differ from zero at 10% level of significance. The first panel shows the correlations for the whole time span, the second for the first subsample and the third for the second subsample.

⁴⁷Among others, see Hansen & Singleton (1982) and Epstein & Zin (1991). Typically, several studies employed *ad hoc* variables as instruments without paying any attention to the quality of the instruments used.

⁴⁸After considerable effort and a number of experiments discussed below in the main text, these variables were selected as the instrument set. Generally, it was difficult to find variables which meet the demands placed on instruments. That is, the stationarity of time-series, correlates as much as possible with the variables in Euler equations and are orthogonal with each other. In particular, it was hard to find any (lagged) variables which predict the future asset returns. This is partly due to the financial regulation, when the interest rates were based on administrative decisions rather than changes in economic fundamentals. However, our choice of instruments can be supported by the fact that when we calculated the correlations between the variables involved in the Euler equations and their recent lags, they were not even that high as are reported in the following Table 4. Only the asset returns (except shares) correlated highly with their own lags.

⁴⁹Takala (2001) noted that at the aggregate level the inertia in reporting data can prevent consumers to use information, lagged by one quarter, when current consumption decisions are made. This also confirms the use of twice-lagged instruments.

Durables, semidurables and services are somewhat correlated with the lags of GDP and YD while nondurables are not. As expected, the real returns are not as correlated with the instruments as consumption measures. Even though the correlations are small, one must keep in mind that they have been calculated from stationary variables (growth rates) rather than the levels of time-series that is the usual experiment in an instrumental variable estimation.⁵⁰ The exclusion of the trend component weakens correlations. When we are using the levels of time-series, the correlations are much higher. For instance, one cannot find correlations between instruments and consumption measures less than 0.7. Also, the correlations between the levels of instruments and variables in the model are slightly higher for the second subsample after the breakpoint in 1986 than for the first subsample.

Table 4

Correlations between instruments and variables in model, first panel covers 1975.1-2001.2, second 1975.2-1986.4, and third 1987.1-2001.2

	D	SeD	ND	S	GBOND	BOND	RRATES	SHARES
GDP(-1)	0.43*	0.41*	0.18*	0.46*	0.14	0.14	0.08	0.33*
GDP(-2)	0.35*	0.39*	0.18*	0.37*	0.08	0.08	0.02	0.22*
GDP(-3)	0.32*	0.38*	0.15	0.30*	0.05	0.05	0.01	0.17*
YD(-1)	0.26*	0.33*	0.06	0.28*	0.34*	0.34*	0.30*	0.20*
YD(-2)	0.18*	0.29*	0.19*	0.22*	0.28*	0.28*	0.25*	0.15
YD(-3)	0.19*	0.16	0.15	0.05	0.24*	0.24*	0.21*	0.11
GDP(-1)	0.23	0.13	0.12	0.41*	0.30*	0.30*	0.29*	0.37*
GDP(-2)	0.01	0.11	0.16	0.17	0.03	0.03	0.03	0.19
GDP(-3)	0.05	0.01	0.08	0.04	-0.14	-0.14	-0.14	0.13
YD(-1)	0.21	0.24*	0.09	0.37*	0.20	0.20	0.18	0.40*
YD(-2)	0.21	0.20	0.14	0.20	0.10	0.11	0.10	0.30*
YD(-3)	0.26*	0.02	0.13	-0.03	0.08	0.08	0.08	0.23
GDP(-1)	0.50*	0.62*	0.22*	0.51*	0.10	0.09	-0.04	0.33*
GDP(-2)	0.47*	0.59*	0.19	0.48*	0.16	0.15	0.03	0.24*
GDP(-3)	0.41*	0.60*	0.18	0.43*	0.21	0.20	0.10	0.18
YD(-1)	0.30*	0.43*	0.05	0.24*	0.41*	0.41*	0.32*	0.10
YD(-2)	0.18	0.38*	0.22*	0.26*	0.36*	0.35*	0.28*	0.08
YD(-3)	0.17	0.25*	0.17	0.11	0.28*	0.27*	0.21	0.04

GDP = Gross domestic product, YD = Disposable income. The figures in parantheses are the number of lags. The asterisks signify that the coefficients differ statistically from zero at 10% level of significance.

One basic assumption for proper instruments is that they should not correlate

⁵⁰One of the assumptions of the time-series to be used in an GMM estimation is that all variables in the model are stationary. While the levels of time-series of consumption measures and instruments are not stationary, the growth rates of these variables are. This is not surprising since the growth rates are concentrating around the attractor one. Still, we confirmed this assumption by running a simple Dickey-Fuller (DF) test. The test statistics reveal that all the consumption measures in the model are stationary. Consequently, all other variables in the estimation are scaled around unity, so, they are also stationary.

with each other. The correlations between stationary instruments were around 0.5 and they differ statistically from zero. When regressing a simple time-series regression between the instrument variables, the results were somewhat mixed. In some cases the autocorrelation and heteroskedastic consistent t -statistics for the coefficients were statistically significant, and in some cases not. So we cannot claim with certainty that the instruments are not multicollinear with each other. Using simple Dickey-Fuller and Engle-Granger tests, we also found that the levels of time-series of different consumption measures are cointegrated with the time-series of GDP and YD. This also supports the use of these instruments.

Even though the results are partly mixed and not unambiguously interpretable, we found some evidence that the variables in the model are predictable to some extent using the lagged instruments.

5.3. Results from Nested Models

A few remarks are worth mentioning before reporting the estimation results. First, while the objective functions are highly nonlinear in their parameters, we tried a variety of starting values and selected the ones that fulfilled the convergence criteria and produced the smallest value for the loss function. Nevertheless, this does not guarantee that we indeed found the global minimum. Second, in some cases, both in external and internal models, the minimum value of the loss function was found in the area where the parameters were not specified. When we tried to force the procedure to converge in the specified area, this typically led to implausible values for the other preference parameters and/or high values for the loss function. Therefore, we report the estimates as such, but evaluate their statistical properties. Third, the GMM estimation as a system of several Euler equations turned out to be too difficult to be computed directly. While the rates of return from government and all bonds in market are highly collinear, we excluded the government bonds in the system estimation. Also, in some cases in system estimation, the weighting matrix was estimated nearly singular, and the estimation procedure produced only the standard errors and the value of the loss function without any iteration taking place from the given starting values. To get more reliable estimates, we first estimated the system as a nonlinear seemingly unrelated regression without paying any attention to heteroskedasticity or serial correlation properties. These estimates were then used as starting values in the GMM estimation to produce Newey & West (1987) autocorrelation and heteroskedasticity consistent estimates.

Although not reported in the following Tables, we also tried, as an exercise, several other instrument sets together with a variety of starting values. Typically, larger instrument sets led to higher values for the objective functions and, therefore, to the rejection of the models. In some experiments, different instrument sets produced smaller values for the loss function and “better” estimates for the underlying preference parameters. To make the interpretation comparable in re-

porting we, throughout, used only those instruments and their appropriate lags as argued above. We will discuss some of the other experiments and their results in the forthcoming footnotes and in the next section.

Tables 5 and 6 show the results for the standard model. Theoretically, because of the time-separable structure, the model should be estimated using only the expenditures of nondurables and services (NDS). On the other hand, the model was nested and it was derived from the time-nonseparable structure. This argument supports the use of durables and semidurables (DSeD). We use both these consumption measures in the estimation.⁵¹

Table 5
GMM results for the model $E_t\beta \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^i \right\} = 1$ and its structural stability, durables and semidurables (DSeD)

Asset	Period	β	γ	J^*	AF
GBOND	1975.2-1986.4	1.004* (0.003)	0.205 (0.180)	0.161	
	1987.1-2001.2	0.995* (0.011)	0.362 (0.767)	12.43	60.92
BOND	1975.2-1986.4	1.004* (0.003)	0.208 (0.182)	0.264	
	1987.1-2001.2	0.994* (0.010)	0.331 (0.651)	15.34	76.57
RRATE	1975.2-1986.4	1.007* (0.003)	0.195 (0.182)	0.135	
	1987.1-2001.2	0.995* (0.010)	0.263 (0.686)	11.55	83.78
SHAR	1975.2-1986.4	1.012* (0.014)	1.287* (0.497)	9.220	
	1987.1-2001.2	0.981* (0.017)	1.388* (0.515)	5.853	145.5
ALL	1975.2-1986.4	1.121* (0.056)	14.74* (5.407)	2074.6	
	1987.1-2001.2	0.983* (0.003)	0.704* (0.188)	1927.5	714.8

The instrument set is (One, GDP₋₂, YD₋₂) for single models and (One, GDP₋₂, YD₋₂, GDP₋₃, YD₋₃) for the system. The asterisks signify that the coefficients differ statistically from zero at 5% level of significance. The goodness-of-fit statistics is calculated as in equation (4.20): $J^ = nJ_n(\Theta_{GMM})$. For single models, the critical values are $\chi^2(3) = 11.345$ for the HJ test and $\chi^2(1) = 6.635$ for the J test. For the system, the correspondent values are $\chi^2(20) = 37.566$ and $\chi^2(18) = 34.805$. The critical value for the AF test is $\chi^2(2) = 9.210$.*

Table 5 depicts the results using only nondurables and services. The parameters

⁵¹For brevity, we do not give separate results for the series of D, SeD, ND and S. Even though the results slightly differ from those of the forthcoming Tables, the main conclusions remain.

to be estimated are the subjective discount factor β and the degree of relative risk aversion γ . The first panel of Table 5 shows results for government bonds, the second for all bonds in the market, the third for the average borrowing rate of commercial banks, the fourth for the stock returns, and the last one for the system. The autocorrelation and heteroskedasticity consistent standard errors of parameters are given in parentheses.

For the first subsample, the discount factor was slightly above unity implying that the subjective discount rate of the representative consumer is negative.⁵² The second subsample produced robust estimates for the discount factor. All these point estimates differed statistically significantly from zero. Except for the system estimation for the first subsample, the estimates of relative risk aversion were qualitatively and quantitatively sensitive in all cases, and it seems that these coefficients were slightly increasing over time. However, only the estimates of simultaneous equations and of the shares were significantly different from zero. According to the goodness-of-fit tests (Hansen's J test and Hansen & Jagannathan's HJ test), the single models were accepted for the first subsample. The opposite happened for the second subsample. Only the model for shares was accepted while the others were slightly rejected according to the HJ test. The model as a system was strongly rejected for both subsamples. Andrews and Fair test (AF test hereafter) statistic indicated a structural change in parameter values in all cases. Table 6 shows the corresponding results for nondurables and services. The results are quite similar to those in Table 5 with some exceptions. Again, except the first period for the shares, the discount factor is theoretically sensitive in the second subsample while the first subsample implies a negative discount rate. All these estimates are statistically significant. The estimates for the coefficient of relative risk aversion are typically negative in the first research period, but do not differ statistically significantly from zero. In the second subsample, they are positive and statistically different from zero. It is also noteworthy that these estimates for risk aversion seem to be higher for nondurables and services when compared with durables and semidurables. Also, as in Table 5, the risk aversion estimates for the model with shares are higher than for the other models. In Table 6, these estimates are extremely high. With the exception of the model for shares, either the J test, HJ test or both tests accept the overidentifying restrictions for the single models. The model as a system is again strongly rejected for both time periods. The AF test indicates that the null hypothesis for the structural stability of the parameters is rejected.

As for the standard model, the external model can also be tested by using both DSeD and NDS as measures for the consumption expenditures. In Table 7, consumption is measured as real expenditures on durables and semidurables. Parameters to be estimated include the envy parameter θ in addition to the

⁵²Throughout the empirical work, we also tried to fix the discount factor equal to 0.99, a usual experiment in the literature. Typically, this exercise led to implausible high values of the objective functions and the rejection of the models.

subjective discount rate β and the degree of relative risk aversion γ . The reference consumption level is the real private total consumption in Sweden lagged by one period ($\varphi = 1$).

Table 6
GMM results for model $E_t\beta \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^i \right\} = 1$ and its structural stability, nondurables and services (NDS)

Asset	Period	β	γ	J^*	AF
GBOND	1975.2-1986.4	1.002* (0.006)	-0.485 (0.874)	0.875	
	1987.1-2001.2	0.994* (0.002)	0.408* (0.176)	1.255	550.1
BOND	1975.2-1986.4	1.002* (0.006)	-0.467 (0.871)	0.871	
	1987.1-2001.2	0.994* (0.003)	0.359* (0.125)	7.051	655.5
RRATE	1975.2-1986.4	1.004* (0.007)	-0.525 (0.887)	0.583	
	1987.1-2001.2	0.995* (0.003)	0.360* (0.127)	6.895	677.9
SHAR	1975.2-1986.4	1.039* (0.019)	5.536* (2.223)	13.59	
	1987.1-2001.2	1.018* (0.023)	9.927* (4.676)	48.74	130.5
ALL	1975.2-1986.4	1.031* (0.005)	-1.282 (0.919)	2168.2	
	1987.1-2001.2	0.995* (0.002)	0.516* (0.255)	2060.2	1143.7

The instrument set is (One, GDP_{-1}, YD_{-1}) for single models and $(One, GDP_{-1}, YD_{-1}, GDP_{-2}, YD_{-2})$ for the system. The asterisks signify that the coefficients differ statistically from zero at 5% level of significance. The goodness-of-fit statistics is calculated as in equation (4.20): $J^ = nJ_n(\Theta_{GMM})$. For single models, the critical values are $\chi^2(3) = 11.345$ for the HJ test and $\chi^2(1) = 6.635$ for the J test. For the system, the correspondent values are $\chi^2(20) = 37.566$ and $\chi^2(18) = 34.805$. The critical value for the AF test is $\chi^2(2) = 9.210$.*

The parameter estimates for the discount factor and relative risk aversion follow typically the same pattern as in the standard model. For single models, the discount factor was higher in the first than in the second subsample, the model for shares being an exception. For the system, the estimates for the discount factor implied a negative discount rate for both research periods. Again, all these estimates were statistically significant. The point estimates for the coefficient of relative risk aversion were typically higher in the first than in the second period, but only the latter turned out to have a statistical power. Thus, the risk aversion seems to be increasing over time, except for the shares. In most cases, the point estimates for the envy parameter seem to decrease over time. However, we cannot

reject the hypothesis that they differed from zero, the model for the borrowing rates after the deregulation and the system for the first subsample being the only exceptions.⁵³

Table 7
GMM results for model $E_t\beta \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left(\frac{C_{t+1-\varphi}}{C_{t-\varphi}} \right)^{\theta(\gamma-1)} R_{t+1}^i \right\} = 1$ and its structural stability, $\varphi = 1$, durables and semidurables (DSeD)

Asset	Period	β	γ	θ	J^*	AF
GBOND	1975.2-1986.4	1.010* (0.020)	0.350 (0.503)	0.560 (1.749)	115.8	
	1987.1-2001.2	0.996* (0.004)	0.181* (0.061)	0.468 (0.259)	240.9	1541.9
BOND	1975.2-1986.4	1.011* (0.020)	0.355 (0.505)	0.560 (1.783)	113.2	
	1987.1-2001.2	0.996* (0.004)	0.175* (0.059)	0.463 (0.250)	243.9	1624.4
RRATE	1975.2-1986.4	1.011* (0.019)	0.300 (0.466)	0.409 (1.457)	118.2	
	1987.1-2001.2	0.997* (0.003)	0.101 (0.053)	0.376* (0.186)	273.1	1797.8
SHAR	1975.2-1986.4	0.990* (0.045)	3.100* (1.392)	0.760 (1.328)	161.6	
	1987.1-2001.2	0.992* (0.023)	1.599* (0.526)	0.300 (1.944)	98.37	118.6
ALL	1975.2-1986.4	1.001* (0.002)	0.363* (0.076)	0.183* (0.076)	205.5	276.0
	1987.1-2001.2	1.009* (0.006)	0.476* (0.225)	2.776 (2.011)	2089.1	

The instrument set is (One, GDP₋₂, YD₋₂, GDP₋₃, YD₋₃) both for the single models and for the system. The asterisks signify that the coefficients differ statistically from zero at 5% level of significance. The goodness-of-fit statistics is calculated as in equation (4.20): $J^ = nJ_n(\Theta_{GMM})$. For the single models, the critical values are $\chi^2(5) = 15.086$ for the HJ test and $\chi^2(2) = 9.210$ for the J test. For the system, the correspondent values are $\chi^2(20) = 37.566$ and $\chi^2(17) = 33.409$. The critical value for the AF test is $\chi^2(3) = 11.345$.*

The goodness-of-fit tests strongly rejected the performance of the models for both research periods. According to the structural stability test, however, there has been a structural change in the parameter values between the two periods.

Table 8 shows the results for nondurables and services. With a few exceptions, the parameters seem to behave similarly as in the case of durables and semidurables. The discount factor was robust for the latter time period while the first period estimates implied negative discount rate. All these estimates were statistically significant.

⁵³It is noteworthy that after the financial deregulation some of these estimates were accepted at 10% level of significance. This, on the other hand, implies an increase in awareness of the reference consumption.

Again, with a few exceptions, the point estimates for the relative risk aversion seem to be increasing over time. These estimates, however, were not statistically significant for single models in both time periods, the estimate for the shares in the first subsample being an exception. The simultaneous estimation produced statistically significant estimates for the RRA. It is also noteworthy that the estimates for the RRA parameter are higher when share returns were used as a measure of real returns. This observation holds also in Table 7. Even though the envy parameter θ did not converge in the defined area in some cases, we cannot reject the hypothesis that it differs from zero. This nonsignificancy holds across different time periods even though the point estimates seem to be decreasing over time. According to the HJ test, the single models were accepted for the first research period and strongly rejected for the second. Again, the AF test rejected the null hypothesis of stable parameter values.

Table 8
GMM results for model $E_t\beta \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left(\frac{C_{t+1-\varphi}}{C_{t-\varphi}} \right)^{\theta(\gamma-1)} R_{t+1}^i \right\} = 1$ and its structural stability, nondurables and services (NDS)

Asset	Period	β	γ	θ	J^*	AF
GBOND	1975.2-1986.4	1.018* (0.012)	0.380 (0.406)	1.014 (0.612)	13.46	
	1987.1-2001.2	0.996* (0.005)	0.497 (0.475)	0.101 (0.502)	480.44	138.3
BOND	1975.2-1986.4	1.018* (0.012)	0.396 (0.403)	1.033 (0.629)	13.33	
	1987.1-2001.2	0.995* (0.005)	0.468 (0.459)	0.090 (0.446)	469.10	148.1
RRATE	1975.2-1986.4	1.021* (0.012)	0.338 (0.429)	0.959 (0.553)	12.83	
	1987.1-2001.2	0.994* (0.003)	0.080 (0.300)	-0.029 (0.120)	292.20	449.0
SHAR	1975.2-1986.4	1.015* (0.028)	5.514* (1.633)	0.249 (0.316)	15.53	
	1987.1-2001.2	0.981* (0.039)	5.327 (4.765)	0.219 (0.520)	177.30	84.99
ALL	1975.2-1986.4	1.017* (0.002)	0.391* (0.190)	0.536* (0.175)	205.4	
	1987.1-2001.2	0.993* (0.004)	1.024* (0.442)	0.091 (5.150)	2239.8	444.9

The instrument set is (One, GDP₋₂, YD₋₂, GDP₋₃, YD₋₃) both for the single models and for the system. The asterisks signify that the coefficients differ statistically from zero at 5% level of significance. The goodness-of-fit statistics is calculated as in equation (4.20):

$J^ = nJ_n(\Theta_{GMM})$. For the single models, the critical values are $\chi^2(5) = 15.086$ for the HJ test and $\chi^2(2) = 9.210$ for the J test. For the system, the correspondent values are $\chi^2(20) = 37.566$ and $\chi^2(17) = 33.409$. The critical value for the AF test is $\chi^2(3) = 11.345$.*

Because of the nonseparable utility structure of the internal model, only durables

and semidurables were used as measures of consumption expenditures. The results are shown in Table 9.

Table 9

GMM results for model

$$E_t \left\{ \beta \left(\frac{c_{t+1}}{c_t} + b_1 \right)^{-\gamma} \left(R_{t+1}^i - b_1 \right) + \beta^2 b_1 \left(\frac{c_{t+2}}{c_t} + b_1 \frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^i \right\} - \left[1 + b_1 \left(\frac{c_{t-1}}{c_t} \right) \right]^{-\gamma} = 0$$

and its structural stability, durables and semidurables (DSeD)

Asset	Period	β	γ	b_1	J^*	Wald	AF
GBOND	1975.2-1986.4	0.995* (0.007)	1.472 (0.976)	0.082 (0.095)	140.5	142.4	
	1987.1-2001.1	0.991* (0.027)	-0.009 (0.027)	-0.893* (0.030)	0.780	19.48	5090.2
BOND	1975.2-1986.4	0.995* (0.007)	1.430 (0.942)	0.008 (0.095)	140.0	140.2	
	1987.1-2001.1	0.991* (0.022)	-0.008 (0.023)	-0.893* (0.055)	1.067	18.36	3881.6
RRATE	1975.2-1986.4	0.996* (0.008)	1.028 (0.733)	0.062 (0.111)	129.1	103.6	
	1987.1-2001.1	0.994* (0.039)	-0.010 (0.015)	-0.829* (0.008)	0.186	9.634	3929.1
SHAR	1975.2-1986.4	1.011* (0.012)	1.281* (0.372)	0.287 (0.558)	21.19	17.14	
	1987.1-2001.1	0.981* (0.047)	0.003 (0.601)	-0.893* (0.003)	24.99	9.390	786.23
ALL	1975.2-1986.4	1.063* (0.006)	0.004 (0.022)	-0.945* (0.004)	1719.3	0.031	
	1987.1-2001.1	0.995* (0.006)	-0.008 (0.004)	-0.893* (0.000)	high	3.167	high

The instrument set is (One, GDP₋₂, YD₋₂, GDP₋₃, YD₋₃) both for the single models and for the system. The asterisks signify that the coefficients differ statistically from zero at 5% level of significance. The goodness-of-fit statistics is calculated as in equation (4.20): $J^ = nJ_n(\Theta_{GMM})$. For the single models, the critical values are $\chi^2(5) = 15.086$ for the HJ-test and $\chi^2(2) = 9.210$ for the Wald- and J-test. For the system, the correspondent values are $\chi^2(20) = 37.566$ and $\chi^2(17) = 33.409$. The critical value for the AF-test is $\chi^2(3) = 11.345$.*

The lag structure is assumed to be $b_\tau = 0, \tau \geq 2$. The parameters to be estimated are the subjective discount factor β , the degree of utility curvature γ and the time-nonseparable coefficient b_1 . The Wald statistic tests the joint hypothesis $H_0 : \gamma = 0, \beta b_1 + 1 = 0$. With the exception of the first period in the fourth and fifth panel, the subjective discount factor was robust in all cases. The estimates decline over time. Contrary to the previous results, the estimates for the relative risk aversion were decreasing over time. However, except for those for the shares in the first research period, the estimates did not differ statistically significantly from zero. The point estimates of the nonseparability parameter b_1 were typically slightly

positive for the first subsample. This provides evidence that the durability of consumption expenditures dominates habit persistence. The estimates, however, did not significantly differ from zero. For the second period, all estimates were statistically significantly negative. This provides evidence for the dominance of habit persistence. The estimates from simultaneous estimation were negative and statistically significant.

According to the goodness-of-fit tests, the models did not seem to fit the data under the financial regulation. After it, the first three single models were accepted.⁵⁴ According to the Wald-test, the trivial result $\gamma = 0$, $\beta b_1 + 1 = 0$ cannot be rejected in simultaneous estimation. The AF-test indicates that there has been a structural change in the parameter values.

5.4. Discussion and Evaluation of Results

Keeping in mind the problems in the small sample properties of GMM and the computational restrictions, we refrain from making strong conclusions from the results. In many cases, the standard errors of parameter estimates were remarkably high. This implies that we cannot reject the hypothesis that they are zero. For example, consumers may be risk neutral rather than risk averse in their consumption decisions. However, the reason for high standard errors can be due to the small sample properties of GMM estimation. As we briefly discussed in section 3.2, many studies (for example Tauchen (1986) and Nelson & Startz (1990)) have performed Monte Carlo simulations to investigate the properties of t ratios and J test for the overidentifying restrictions. These studies suggest that even if the model is correctly specified, the finite sample performance of GMM is sensitive to both the number of moment conditions and the sample size. One consequence is that the J test tends to reject the models too frequently compared with their asymptotic properties, and standard errors may be too large in small samples. We also found that the standard errors and the value of the objective function were sensitive to the starting values, the number of instruments, and the number of observations while the parameter estimates remained nearly unchanged. This holds true for the system of equations. Therefore, we believe more in the point estimates than the mistakes related to them, if the mistakes are not too large. This interpretation may, however, be misleading.

The results given in Tables 5 and 6 were based only on the standard model. In spite of the theoretical weaknesses, the models seem to fit the data reasonably well, especially for the period before the financial liberalisation. The goodness-of-fit statistics accept the single models in most cases. After the liberalisation the models are slightly rejected for the DSeD, but accepted for the NDS. Typically, the first subsample until the year 1987 produced a discount factor greater than one. This means that the subjective discount rate for a representative consumer

⁵⁴It is noteworthy that in spite of the rates of return, the procedure converged toward the same parameter values in the second research period together with low values for the loss functions.

is negative. Even though Kocherlakota (1990) has shown that it is theoretically possible that the discount factor can be greater than one in a growing economy, in our opinion, the reason for this is that under the financial regulation consumers were unable to follow their optimal consumption patterns. However, in most cases we cannot reject the hypothesis that these point estimates differ significantly from figures slightly below one, which is consistent with the theory.

It is also not difficult to interpret the magnitude of the risk aversion in consumption in Finland. The point estimates of the standard model were typically lower in the first than in the second subsample. These results, as well as the structural stability tests, reveal that the financial liberalisation has changed the representative consumer's attitude toward risk in consumption. He was less risk averse before than after the liberalisation. In most cases, we cannot even reject the hypothesis that the representative consumer is risk neutral under financial market regulation. Also, the results indicate that an investment in shares has been the most risky investment decision. If consumers had followed the rational life cycle-permanent income hypothesis and wanted to smooth their consumption by investing in shares, which are the most volatile and unpredictable component of all assets, it has been a risky decision. Especially, one has tried to avoid this unanticipated risk when consumption is based on the NDS.

In the external model, the interpretation of the subjective discount factor and relative risk aversion remained mostly the same. Explaining the magnitude and significance of the envy parameter is more difficult. The point estimates of this parameter for both subperiods imply that envy has decreased over time. However, while the high standard errors of the parameters imply that we cannot statistically claim that consumers on average have been envious at the aggregate level, the statistical significance of the envy parameter is strongly rejected when nondurables and services were used as a measure of consumption. After the deregulation, the significance is accepted at 10% level for durables and semidurables. This may reflect the view that consumers are aware of changes on durables and semidurables as reference consumption, but are not adjusting their consumption according to the changes on NDS.⁵⁵ As a whole, the models do not fit as well as in the case of the standard model.

Theoretically, it is hard to define the precise length of the lag for the reference consumption. We also permitted the representative consumer to have an additional period to adjust his consumption to changes in his reference consumption. That is, we set $\varphi = 2$. Even though this experiment slightly changed the parameter values of the model, the qualitative interpretation of the results did not essentially change from those of the once-lagged model. We also tested the external models using the correspondently classified and lagged data on DSeD and

⁵⁵Intuitively, consumers are aware of what kind of cars, real estates, computers, and other durable goods are available abroad, but they do not pay attention to what kind of food, clothes, or other nondurables and services are consumed abroad.

NDS of Swedish consumption and the total average private consumption from OECD countries. Also, while the magnitude of the parameter values slightly changed, the results did not significantly differ from those reported: the standard errors of the parameters as well as the values of the objective functions were high. This supports the view that consumers on average are either slightly envious or not envious at all.

The results from the internal model imply the same time-variant structure for the subjective discount factor. That is, the subjective discount rate has increased over time. The relative risk aversion⁵⁶ reflects a different pattern compared to the other models. The point estimates indicate that the RRA has decreased over time. However, according to the statistical significance, the representative consumer may also be classified as risk neutral. The point estimates of the non-separability parameter b_1 are typically slightly positive in the first subsample and negative in the second. This provides evidence on that the durability of durables and semidurables has dominated habit persistence before the breakpoint, while habit persistence has been the dominant factor after it. The simple autocorrelation structure confirms this interpretation, and the AF-test reveals that there has been a structural change in parameter values. However, the estimates for the first subperiod do not statistically differ from zero, which means that perhaps neither of these effects has been significant.

In estimation, we tried to find parameter values that avoid the trivial solution of the parameter restrictions $\gamma = 0$, $\beta b_1 + 1 = 0$. Especially for the system estimation, this was hard.⁵⁷ These restrictions mean that the representative consumer is slightly risk averse or risk neutral, and, while the discount factor is close to unity, consumption expenditures are based on the strong habit persistence and b_1 is close to minus one. It is possible that this, indeed, is true, and we have rejected the right parameter values because of the test statistics. However, the results in Table 9 and the interpretation of the parameter values above do not contradict this conclusion. If the real world behaves as the point estimates indicate, this can be interpreted as follows. Firstly, even if the habit persistence would have been a latent force during the whole examination period, the inability to adjust consumption before the deregulation has forced consumers to act differently. Secondly, after deregulation there has been a change in the structure of DSeD which may have changed the consumption pattern. In the 1970s new versions of durables were relatively seldom, whereas in the 1990s new versions appeared every year.⁵⁸ Thirdly, the structural change and centralising of the whole

⁵⁶As shown by Constantinides (1990) and Ferson & Constantinides (1991), the utility curvature parameter in an internal model is a good approximation for the relative risk aversion. Therefore, we use this terminology.

⁵⁷Another restriction $c_t/c_{t-1} \geq b_1$ ensures that the utility is positive and well-defined. According to the data, this restriction is satisfied.

⁵⁸In practice, this means that decades ago a durable was bought and used for a long time until the service flow gained from it was so small that the consumer had to buy a new one. Even though this is partly true also today, some of the durables are now bought because one wants more feature which only the latest versions of the product can offer. A mobile phone is a good example. The financial liberalisation has allowed

economy has forced consumers to change their consumption behaviour. For most consumers and households, it is nowadays nearly a must to own durables such as mobile phones, cars, laundry machines etc. and to ensure that they are not out of date.

Table 10
International evidence of parameter values

Research	Data	Period	Method	γ	b_1
Mankiw (1981)	U.S.	1948(I)-1980(IV)	INST	3.9-5.8	
Hansen & Singleton (1982,1984)	U.S.	1959(2)-1979(12)	GMM/ML	0-1	
Hansen & Singleton (1983)	U.S.	1959(2)-1979(12)	ML	-0.4-4.1	
Shapiro (1984)	U.S.	Panel data	OLS	0.5	
Hall (1988)	U.S.	1959-1983	INST	2.9-15.2	
Bufman & Leiderman (1990)	Israel	1978(II)-1986(IV)	GMM	0.3-0.6	
Epstein & Zin (1991)	U.S.	1959(4)-1986(12)	GMM	0-1	
Ferson & Constantinides (1991)	U.S.	1948(II)-1986(II)	GMM	-3.3-6.3	negative
Hamori (1992a,b)	Japan	1980(1)-1988(12)	GMM	0-1.5	
Braun et.al. (1993)	U.K.	1970(IV)-1988(IV)	GMM	0.9-6.3	negative
	U.S.			0.2-3.1	negative
	France			1-1.2	negative
	Canada			1-12.7	negat/posit
	Germany			1.2-2.1	negative
	Japan			0.1-3.1	negative
Hamori & Tokunaga (1999)	Japan	1971(1)-1993(12)	GMM	0-1.7	positive

The methods are: INST = Instrumental variable estimation, GMM = Generalized method of moments, ML = Maximum likelihood, OLS = Ordinary least squares. The parameters are: γ = Coefficient of relative risk aversion/Utility curvature parameter, b_1 = Time-nonseparable coefficient.

To compare our results, Table 10 gives a selective survey of the international evidence. As one can see from Tables 5-9, the parameter values depend on the underlying theoretical model, the instrument set and the adopted consumption measure. Therefore, the results in Table 10 are not directly comparable to those of ours, but they show if our results can be regarded as robust. While most of

consumers to finance this behaviour.

the studies have concentrated solely on U.S. data, we have tried to select studies which give as good a description as possible from the international evidence.

The international evidence concerning the consumption based asset pricing models is rather confusing and mixed. The studies before the 1990s have typically concentrated on testing the standard model and its extensions (see chapter 1), while the later studies have concentrated on the improved models. According to the goodness-of-fit tests, the standard models are typically rejected with the U.S. data, while in the other countries they are often accepted. In many studies, it is a common feature that the estimate of the discount factor is slightly greater than unity. So, our results do not contradict to these findings. It also seems that the magnitude of the degree of relative risk aversion in this study is in accordance with international findings. Some studies presented in the Table 10, as well as many others, reported negative values for this risk parameter, which contradicts the assumption of well-behaving utility function. Some studies avoided these problems by restricting parameters to the values which are according to the theory. Without any restrictions, we also found some evidence of risk loving and risk neutral consumption behaviour. The standard errors of the parameter estimates also turned out to be high in some studies.

The structural stability tests have been examined only in few studies. Epstein & Zin (1991) using U.S. data, and Hamori (1992b) using Japanese data, reported that the preference parameters in the standard model remained invariant across time and policy regimes, contradictory to our results.

The international results from the internal model exhibit that the estimated parameter values for the coefficient b_1 are typically negative. This suggests that the habit formation has been the dominant factor in many countries. Hamori & Tokunaga's (1999) study from Japan is the only exception. Our findings of habit persistence for the second subperiod are roughly in accordance with the international evidence. However, most of the studies performed that the extension of nonseparability significantly improves the fit of the models. In contrast, we found that the standard model turned out to be statistically better. According to our knowledge, no other empirical studies concerning the external model have been conducted.

Most studies mentioned in the Table 10, and many others, have used lagged consumption and real rates of return as instruments in the GMM estimation. For comparison, we used also these variables to generate the orthogonality conditions and performed several experiments with various instrument sets. Even though in some experiments the point estimates of the preference parameters changed or even turned opposite, their interpretations remained in most cases nearly the same. Also, the values of the loss functions were typically lower, and the models were more easily accepted. Therefore, we conclude that these experiments rather strengthened than weakened our conclusions above.

No similar study has been conducted earlier with Finnish data. Starck (1987) showed that the intertemporal elasticity of consumption with respect to the expected real interest rate has been small in Finland, and did not differ significantly from zero before the financial liberalisation. Kostianen & Starck (1991) found a similar result for the time period 1961-1988. Svento (1990) found that it is harder to explain the Finnish consumption behaviour by the returns from the shares than by the returns from government bonds. If the reciprocal of the coefficient of intertemporal substitution is interpreted as a coefficient of relative risk aversion, these results contradict our findings of low risk aversion for the first subsample, but confirm the difficulties we had to fit the real return data of shares into the model.

To sum up the results from Tables 5-9:

1⁰ *Attitude towards the risk of consumption has slightly increased over time. Under the financial regulation, we cannot make a distinction whether consumers are risk averse or risk neutral on the average.*

2⁰ *Investment in shares has been the most risky investment decision across time and policy regimes.*

3⁰ *On the average, consumers are either not envious or slightly envious with respect to the growth of the lagged reference consumption.*

4⁰ *Before the financial liberalisation, either the durability of durables and semidurables has been the dominant factor, or both the durability and habit persistence have been insignificant, while after the deregulation habit formation has been the dominant factor.*

5⁰ *There has been a structural change in the consumption-investment decisions due to the financial liberalisation.*

One can find various arguments to falsify these results. In our opinion, this critique can be divided into three categories. Firstly, under uncertainty, the consumption-based capital asset pricing models based on a single consumer behaviour together with the assumption of a representative consumer (or consumers on average), and aggregate data are not appropriate to explain the decision-making of real consumers. Also, as argued by Bufman & Leiderman (1990), the Euler equation approach typically requires a volatile economic environment before it can reveal anything from consumers preferences. Under the financial regulation, the Finnish economy was relatively tranquil, and it is likely that the data cannot reveal the true consumption behaviour. Secondly, if these equilibrium models are accepted as a way to explain consumption decisions, the functional forms and assumptions concerning these models are not appropriate to mimic the true decision-making of consumers. One such strict assumption is that the model excludes the possibility of kinks in budget constraints. For instance, Takala (2001) reported that even though the excess sensitivity of con-

sumption to current income has decreased in the late 1990s, the share of the liquidity-constrained households may have been 30-50% during 1980-1998. This clearly prevents consumers to follow their optimal consumption paths. Also, because of the unanticipated boom and deep depression in late 1980s and early 1990s, these models are not capable of mimicing the true consumption decisions based on forward-looking expectations. Thirdly, the data and the restrictions in estimation may cause errors in our results. This last category may be argued by the following statements:

1⁰ *The estimates are highly sensitive to the starting values, the instrument set, and the number of observations.*

2⁰ *The instrument set is not valid to mimic the true orthogonality conditions.*

3⁰ *The seasonally adjusted aggregate data creates measurement errors and is too smooth to describe the individual decision-making.*

4⁰ *Only financial assets are used while the returns from real assets and human capital form most of the consumers' total wealth.*

5⁰ *Only a few consumers own shares, and so the use of the real rate of return from shares is inconvenient.⁵⁹*

6⁰ *The equilibrium models are based on ex ante variables while the estimation results are based on ex post variables.*

7⁰ *The estimation methodology is incorrect.*

Even though we can give both theoretical and practical arguments for the use of the models, data, and estimation methodology, these problems are amenable to further refinement and research. However, while we are aware of the problems on theoretical and empirical work and the interpretation of the results, in our opinion, we have found some new theoretical arguments on how the consumption decisions of an individual consumer are based also on psychological rather than only on economical reasons, which is the standard cornerstone of neoclassical economics. Also, we found some new empirical results on Finnish aggregate consumption behaviour.

6. CONCLUDING REMARKS

While the influence of social and institutional factors has a long history in the theory of consumption, its use in the literature has increased immensely in the 1990s. This is mainly due to the inability of the standard consumption or utility

⁵⁹Although the U.S. financial markets are among the most developed markets in the world, Mankiw & Zeldes (1991) documented that only one fourth of U.S. households own stocks either directly or through pension funds. This market segmentation means that only a subset of investors should be considered when testing the Euler equations. Even though the possession of shares has increased in Finland during the last decade, it is still a fairly small fraction of total investments.

based asset pricing model to resolve several empirical puzzles. This study has analysed private aggregate consumption in Finland by using a consumption-based capital asset pricing model. We tested the existence of a structural change in a representative agent's preference parameters due to the financial deregulation which culminated towards the end of 1986. The model, based on a representative agent's behaviour, relaxes the standard time-separable assumption in consumer's intertemporal preferences and utilises time-nonseparable preferences. This, unlike the standard model, allows durables and semidurables to be included in consumer's utility. This preference modification enables us to understand why consumption decisions may not be entirely based on economic but also on psychological reasons. As a whole, the model divides consumption decisions based on internal and external effects.

The internal effect means that a consumer's own consumption history, his habit persistence in preferences and durability of consumption goods, affects as a subsistence level to his present consumption decisions. Under durability, the past consumption expenditures accumulate positively in the argument of consumer's utility function. Thus, the higher the accumulation, the smaller the need for current expenditures. Under habit formation, the current utility depends on the deviation of the current expenditures (or a flow of services attained from durables and semidurables) from the accumulation of past consumption expenditures, the subsistence level. In Euler equations, habit persistence implies that the coefficients of the lagged consumption expenditures are negative, whereas durability implies positive coefficients. If both effects are present, the signs of the coefficients signify the dominance. In aggregate, if habit persistence outperforms durability, consumption would change relatively smoothly. On the other hand, if durability outperforms habit persistence, aggregate consumption has a tendency to fluctuate widely.

The external effect affecting consumption is that the consumer is envious and has some reference consumption to which he relates his own. Theoretically, this reference consumption can be that of his neighbourhood, socio-economic class, or *per capita* consumption. With suitable parametrisation, we derived nested first-order conditions for the standard, internal and external models which we tested empirically.

In the GMM estimation, we used aggregate Finnish quarterly data from 1975 to 2001 and several real rates of return. Firstly, the results show that there has been a structural change in preference parameters due to financial liberalization. Secondly, attitude towards consumption risk has slightly increased over time. Before the liberalisation, consumers have either been slightly risk averse or even risk neutral in their consumption-investment decisions. After the deregulation, they have been slightly risk averse. This change in behaviour reflects the fact that under the financial regulation, the Finnish economy was rather tranquil and the theoretical model perhaps was not able to reveal the true consumption

behaviour. After the liberalisation, consumers have been able to allocate resources by lending/borrowing and to follow their optimal consumption pattern more freely. Thirdly, as a textbook example, the investment in shares has been the most risky investment decision over times. Fourthly, we did not find any strong evidence that consumers on the average are envious compared to changes in the Swedish and OECD consumption measures. Finally, before the deregulation, either the durability of durables was the dominant effect or both the durability and habit formation were insignificant in consumption decisions, while after the liberalisation, the habit formation has been the dominant effect. This result leads to the fact that nowadays the aggregate consumption has a tendency to smooth itself if there is no exogeneous adverse shock, such as the deep depression in the early 1990s.

REFERENCES

- Abel, A.B. (1990): "Asset Prices under Habit Formation and Catching up with the Joneses." *American Economic Review* 80, 38-42.
- Abel, A.B. (1999): "Risk Premia and Term Premia in General Equilibrium." *Journal of Monetary Economics* 43, 3-34.
- Ackerman, F. (1997): "Consumed in Theory: Alternative Perspectives on the Economics of Consumption." *Journal of Economic Issues*, Vol XXXI, No. 3, 651-664.
- Aiyagari, R.S. (1993): "Explaining Financial Market Facts: The Importance of Incomplete Markets and Transaction Costs." *Federal Reserve Bank of Minneapolis Quarterly Review* 17(1), 17-31.
- Alesie, R. & Lusardi, A. (1997): "Consumption, Saving and Habit Formation." *Economics Letters* 55 103-108.
- Andrews, D.W.K. & Fair, R.C. (1988): "Inference in Nonlinear Econometric Models with Structural Change." *Review of Economic Studies* 55, 615-640.
- Attanasio, O.P. (1998): "Consumption Demand." NBER Working Paper No. 6466.
- Beaudry, P. & Guay, A. (1996): "What Do Interest Rates Reveal about the Functioning of Real Business Cycles?" *Journal of Economic Dynamics and Control* 20, 1661-1682.
- Becker, G.S. & Murphy, K. (1988): "A Theory of Rational Addiction." *Journal of Political Economy* 96, 675-700.
- Blundell, R. (1988): "Consumer Behaviour: Theory and Empirical Evidence - A Survey." *The Economic Journal* 98, 16-65.
- Boldrin, M., Christiano, L.J. & Fisher, J.D.M. (1997): "Habit Persistence and Asset Returns in an Exchange Economy." Working Paper 4/97. Federal Reserve Bank of Chicago.

- Bowles, S. (1998): "Endogenous Preferences: The Cultural Consequences of Markets and other Economic Institutions." *Journal of Economic Literature*, Vol 36, 75-111.
- Braun, P.A., Constantinides, G.M. & Ferson, W.E. (1993): "Time Nonseparability in Aggregate Consumption: International Evidence." *European Economic Review* 37, 897-920.
- Brunila, A. (1996): "Fiscal Policy and Private Consumption - Saving Decisions: Evidence from Finland." Bank of Finland Discussion Papers, Economics Department, No. 28/96.
- Bufman, G. & Leiderman, L. (1990): "Consumption and Asset Return under Non-expected Utility." *Economics Letters* 34 231-235.
- Campbell, J.Y. (1998): "Asset Prices, Consumption, and the Business Cycle." NBER Working Paper No. 6485.
- Campbell, J.Y. & Cochrane, J.H. (1999a): "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy* 107, 205-251.
- Campbell, J.Y. & Cochrane, J.H. (1999b): "Explaining the Poor Performance of Consumption-Based Asset Pricing Models." NBER Working Paper 7237.
- Campbell, J.Y., Lo, A.W. & MacKinlay, A.G. (1997): *The Econometrics of Financial Markets*. Princeton University Press. Princeton, New Jersey.
- Campbell, J.Y. & Mankiw, G.N. (1990): "Permanent Income, Current Income, and Consumption." *Journal of Business and Economic Statistics* 8, 265-278.
- Carroll, C.D. (1997): "Death to the Log-linearized Consumption Euler-equation! (And Very Poor Health to the Second-order Approximation)" NBER Working Paper No. 6298.
- Carroll, C.D., Overland, J. & Weil, D.N. (1997): "Comparison Utility in a Growth Model." NBER Working Paper No. 6138.
- Constantinides, G. M. (1982): "Intertemporal Asset Pricing with Heterogeneous Consumers and Without Demand Aggregation." *Journal of Business* 55(2), 253-267.
- Constantinides, G. M. (1990): "Habit Formation: A Resolution of the Equity Premium Puzzle." *Journal of Political Economy* 98, 519-543.
- Deaton, A. (1991): "Saving and Liquidity Constraints." *Econometrica* 59, 1221-1248.
- Deaton, A. (1992): *Understanding Consumption*. Clarendon Press, Oxford, UK.
- Deaton, A. & Muellbauer, J. (1980): *Economics and Consumer Behavior*. Cambridge University Press, Cambridge.
- Detemple, J.B. & Zapatero, F. (1991): "Asset Prices in an Exchange Economy with Habit Formation." *Econometrica* 59, 1633-1657.

- Duesenberry, J.S. (1949): *Income, Saving and the Theory of Consumer Behavior*. Harvard University Press, Cambridge.
- Duffie, D. (1992): *Dynamic Asset Pricing Theory*. Princeton University Press. Princeton, New Jersey.
- Dunn, K.B. & Singleton, K.J. (1984): "Modeling the Term Structure of Interest Rates under Non-Separable Utility and Durability of Goods." *Journal of Financial Economics* 17, 27-55.
- Dybvig, P.H. (1995): "Duesenberry's Ratcheting of Consumption: Optimal Dynamic Consumption and Investment Given Intolerance for any Decline in Standard of Living." *Review of Economic Studies* 62, 287-313.
- Eichenbaum, M.S., Hansen, L.P. & Singleton, K.S. (1988): "A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty." *The Quarterly Journal of Economics* 103, 51-78.
- Eichenbaum, M.S. & Hansen, L.P. (1990): "Estimating Models with Intertemporal Substitution Using Aggregate Time Series Data." *Journal of Business and Economic Statistics* 8, 53-69.
- Epstein, L & Zin, S. (1991): "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns II: An Empirical Analysis." *Journal of Political Economy* 99, No. 2, 263-286.
- Ermini, L. (1994): "A Discrete-Time Consumption-CAP Model under Durability of Goods, Habit Formation and Temporal Aggregation." Working Paper ECO No. 94/29. European University Institute, Florence.
- Ferson, W.E. & Constantinides, G.M. (1991): "Habit Persistence and Durability in Aggregate Consumption." *Journal of Financial Economics* 29, 199-240.
- Fuhrer, J.C. (1998): "An Optimizing Model for Monetary Policy Analysis: Can Habit Formation Help?" Working Paper 1/98. Federal Reserve Bank of Boston.
- Fuhrer, J.C. & Klein, M.W. (1998): "Risky Habits: On Risk Sharing, Habit Formation, and the Interpretation of International Consumption Correlations." NBER Working Paper No. 6735.
- Gali, J. (1994): "Keeping up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices." *Journal of Money, Credit and Banking* 26, 1-8.
- Ghysels, E. & Hall, A. (1990a): "A Test for Structural Stability of Euler Conditions Parameters Estimated via the Generalized Method of Moments Estimator." *International Economic Review* 31, 355-364.
- Ghysels, E. & Hall, A. (1990b): "Are Consumption-Based Intertemporal Capital Asset Pricing Models Structural?" *Journal of Econometrics* 45, 121-139.
- Grossman, S.J. & Laroque, G. (1990): "Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Consumption Goods." *Econometrica* 58, 25-51.

- Gruber, J. & Köszegi, B. (2000): "Is Addiction "Rational"? Theory and Evidence." NBER Working Paper No. 7507.
- Hall, A.R. (1993): "Some Aspects of Generalized Method of Moments Estimation." in *Handbook of Statistics (Econometrics)* ed. by Maddala, G.S., Rao, C.R. & Vinod, H.D., 393-417. Elsevier Science Publishers B.V., The Netherlands.
- Hall, A.R. & Sen, A. (1999): "Structural Stability Testing in Models Estimated by Generalized Method of Moments." *Journal of Business and Economic Statistics* 17, 335-348.
- Hall, R.E. (1988): "Intertemporal Substitution in Consumption." *Journal of Political Economy* 96, 221-273.
- Hamilton, J.D. (1994): *Time Series Analysis*. Princeton University Press. Princeton, New Jersey.
- Hamori, S. (1992a): "Test of C-CAPM for Japan: 1980-1988." *Economics Letters* 38, 67-72.
- Hamori, S. (1992b): "On the Structural Stability of Preference Parameters Obtained from Japanese Financial Market Data." *Economics Letters* 40, 459-464.
- Hamori, S. & Tokunaga, T. (1999): "Habit Formation and Durability and Consumption: Some Evidence from Income Quintile Groups in Japan." *Applied Economics Letters* 6, 397-402.
- Hansen, L.P. (1982): "Large Sample Properties of Generalized Method of Moments Estimators." *Econometrica* 50, 1029-1054.
- Hansen, L.P., Heaton, J. & Yaron, A. (1996): "Finite Sample Properties of Some Alternative GMM Estimators." *Journal of Business and Economic Statistics* 14, 262-280.
- Hansen, L.P. & Jagannathan, R. (1989): "Restrictions on Intertemporal Marginal Rates of Substitution Implied by Asset Returns." Working Paper 12, Department of Finance, Northwestern University.
- Hansen, L.P. & Singleton, K.J. (1982): "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." *Econometrica* 50, 1269-1286.
- Hansen, L.P. & Singleton, K.J. (1983): "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns." *Journal of Political Economy* 91, 249-265.
- Hansen, L.P. & Singleton, K.J. (1984): "Errata." *Econometrica* 52, 267-268.
- Harbaugh, R. (1996): "Falling behind the Joneses: Relative Consumption and the Growth-Savings Paradox." *Economics Letters* 53, 297-304.
- Hayashi, F. (1985): "The Permanent Income Hypothesis and Consumption Durability: Analysis Based on Japanese Panel Data." *The Quarterly Journal of Economics* 100, 1083-1113.

- Hayashi, F. (2000): *Econometrics*. Princeton University Press. Princeton, New Jersey.
- Jermann, U.J. (1998): “Asset Pricing in Production Economies.” *Journal of Monetary Economics* 41, 257-275.
- Kirman, A.P. (1992): “Whom or What the Representative Individual Represent?” *Journal of Economic Perspectives* 6, 117-136.
- Kocherlakota, N.R. (1990): “On the ‘Discount’ Factor in Growing Economies.” *Journal of Monetary Economics* 25, 43-48.
- Koivumäki, T. (1999): “Uncertainty and the Dynamics of Demand.” Ph.D. Thesis, University of Oulu, Serie G5.
- Kostiainen, S. & Starck, C.C. (1991): “Does Government and Corporate Saving Influence Household Saving in Finland?” *Liiketaloudellinen Aikakauskirja* 2/91, 128-144.
- Lettau, M. & Uhlig, H. (1995): “Can Habit Formation Be Reconciled with Business Cycle Facts?” Tilburg University, CentER Discussion Paper No. 9554.
- Ljungqvist, L. & Uhlig, H. (1996): “Catching Up with the Keynesians.” Tilburg University, CentER Discussion Paper 9696.
- Mankiw, G.N. (1981): “The Permanent Income Hypothesis and the Real Interest Rate.” *Economics Letters* 7, 307-311.
- Mankiw, G.N. (1982): “Hall’s Consumption Hypothesis and Durable Goods.” *Journal of Monetary Economics* 10, 417-425.
- Mankiw, G.W. & Zeldes, S.P. (1991): “The Consumption of Stockholders and Non-Stockholders.” *Journal of Financial Economics* 29, 97-112.
- Mansoorian, A. (1998): “Habits and Durability in Consumption, and the Dynamics of the Current Account.” *Journal of International Economics* 44, 69-82.
- Mátyás, L. (1999): *Generalized Method of Moments Estimation*. Cambridge University Press, Cambridge, UK.
- McCabe, B. & Tremayne, A. (1993): *Elements of Modern Asymptotic Theory with Statistical Applications*. Manchester University Press, Manchester and New York.
- Messinis, G. (1999a): “On the History of Habit Formation in Consumption.” University of Melbourne, Department of Economics. Research Paper No. 673.
- Messinis, G. (1999b): “Rational Habit Formation in Consumption: The Role of Durables and Credit.” University of Melbourne, Department of Economics. Research Paper No. 681.
- Messinis, G. (1999c): “Credit and Habit Modification in the USA: 1959-1997.” University of Melbourne, Department of Economics. Research Paper No. 682.
- Naik, N.Y. & Moore, M.J. (1996): “Habit Formation and Intertemporal Substitu-

- tion in Individual Food Consumption.” *The Review of Economics and Statistics*, 321-328.
- Nelson, C. & Startz, R. (1990): “The Distribution of Instrumental Variables Estimator and its t-ratio when the Instrument is a Poor One.” *Journal of Business* 63, 125-140.
- Newey, W.K. & West, K.D. (1987): “A Simple, Positive Semi-Definite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix.” *Econometrica* 55, 703-708.
- Ni, S.X. (1993): “Habit Formation as a Resolution to the Equity Premium Puzzle: What Is in the Data, What Is Not.” *Southern Economic Journal* 59, 749-759.
- Nurminen, M.R. (1999): “Market Demand and Representative Consumer: A Stochastic Approach.” Manuscript. (The earlier version of this paper was accepted as a Master Thesis.). University of Jyväskylä, School of Business and Economics.
- Ogaki, M. (1993a): “Generalized Method of Moments: Econometric Applications.” in *Handbook of Statistics (Econometrics)* ed. by Maddala, G.S., Rao, C.R. & Vinod, H.D., 455-488. Elsevier Science Publishers B.V., The Netherlands.
- Ogaki, M. (1993b): “GMM, A User Guide.” Rochester Center for Economic Research. Working Paper No. 348.
- Ryder, H.E.Jr. & Heal, G.M. (1973): “Optimum Growth with Intertemporally Dependent Preferences.” *Review of Economic Studies* 40, 1-33.
- Salyer, K.D. (1995): “Habit Persistence and the Nominal Term Premium Puzzle: A Partial Resolution.” *Economic Inquiry* 33, 672-691.
- Seckin, A. (2000a): “Consumption with Habit Formation.” Working Paper 2000s39. Centre Interuniversitaire de Recherche en Analyse des Organisations, Montreal.
- Seckin, A. (2000b): “Consumption with Liquidity Constraints and Habit Formation.” Working Paper 2000s41. Centre Interuniversitaire de Recherche en Analyse des Organisations, Montreal.
- Seckin, A. (2000c): “Habit Formation: A Kind of Prudence.” Working Paper 2000s42. Centre Interuniversitaire de Recherche en Analyse des Organisations, Montreal.
- Seckin, A. (2001): “Consumption-Leisure Choice with Habit Formation.” *Economics Letters* 70, 115-120.
- Shapiro, M.D. (1984): “The Permanent Income Hypothesis and the Real Interest Rate. Some Evidence from Panel data.” *Economics Letters* 14, 93-100.
- Shrikhande, M.M. (1996): “Nonaddictive Habit Formation and the Equity Premium Puzzle.” Working Paper 96-1. Federal Reserve Bank of Atlanta.

- Smith, A. (1776): “An Inquiry into the Nature and Causes of the Wealth of Nations.” *In The Wealth of Nations: The Cannan Edition*, ed. by E. Cannan. Modern Library, New York 1937.
- Starck, C.C. (1987): “Intertemporal Substitution in Consumption: Some Empirical Evidence from Finnish Data.” *Liiketaloudellinen aikakauskirja* 2/87, 128-143.
- Stiglitz, G.J. & Becker, G.S. (1977): “De Gustibus Non Est Disputandum.” *American Economic Review* 67, 76-90.
- Stoker, T.M. (1993): “Empirical Approaches to the Problem of Aggregation Over Individuals.” *Journal of Economic Literature* Vol 31, 1827-1874.
- Sundaresan, S.M. (1989): “Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth.” *Review of Financial Studies* 2, 73-88.
- Svento, R. (1990): “Tutkimus kulutusfunktion tulokäsitteestä.” Research Reports No. 34. University of Oulu, Department of Economics. (In Finnish)
- Takala, K. (2001): “Studies in Time Series Analysis of Consumption, Asset Prices and Forecasting.” Bank of Finland Studies E:22.
- Takala, K., Kostianen, S. & Hämäläinen, T. (1991): “Kotitalouksien varallisuuden koostumus, tuotot ja verotus Suomessa vuosina 1960-89.” Bank of Finland Discussion Papers No. 13/91. (in Finnish)
- Tauchén, G. (1986): “Statistical Properties of Generalized Method of Moments Estimators of Structural Parameters Obtained from Financial Market Data.” *Journal of Business and Economics Statistics* 4, 397-425.
- Veblen, T. (1899): *The Theory of the Leisure Class*. The Modern Library, New York.
- Wathieu, L. (1997): “Habits and Anomalies in Intertemporal Choice.” *Management Science* 43, 1552-1563.

APPENDIX A

The purpose of this appendix is to give the reader an understanding and intuition of consumer behaviour under internal consumption externalities. The underlying consumption process is formulated in equations (2.6) and (2.8) in the main text.

Figure A1 presents the formation of habit persistence and durability of consumption expenditures. The time dynamics is described on the horizontal axis. Consider first the durability of consumption durables. At any time $t - \tau$, $\tau \geq 0$, the flow of services, $S_{t-\tau}$, is the accumulated sum of previous consumption purchases, $c_{t-\tau-k}$, $k \geq 0$ where $c_t = \sum_{i=1}^N d_{i,t}$. However, durables will depreciate after the date of acquisition, and the parameter $0 \leq \xi_\tau \leq 1$ is the mean depreciation rate for all N durables between the times $(t - \tau)$ and t . The process is

$$S_t = \delta_0 c_t + \delta_1 c_{t-1} + \delta_2 c_{t-2} + \delta_3 c_{t-3} + \dots \quad (\text{A.1})$$

or

$$S_t = (1 - \xi_0)c_t + (1 - \xi_1)c_{t-1} + (1 - \xi_2)c_{t-2} + (1 - \xi_3)c_{t-3} + \dots \quad (\text{A.2})$$

Clearly, the rate of durability and the rate of depreciation are inversely proportional. The higher the rate of depreciation ξ , the lower the rate of durability δ , and *vice versa*. Often, the first intuition can be that the more previous expenditures should have more durability weight than the past ones. Namely, $\delta_0 > \delta_1 > \delta_2 > \dots$ or, inversely, $\xi_0 < \xi_1 < \xi_2 < \dots$. However, while in most cases accurate, this intuition is misleading because the rate of depreciation surely depends on the quality of the underlying expenditures $d_{i,t}$. Moreover, another intuition could be that the depreciation rate ξ_0 is zero. This postulation is misleading as well because the goods could be “out-of-date” already at the date of acquisition (computer and telephone technology, for instance).

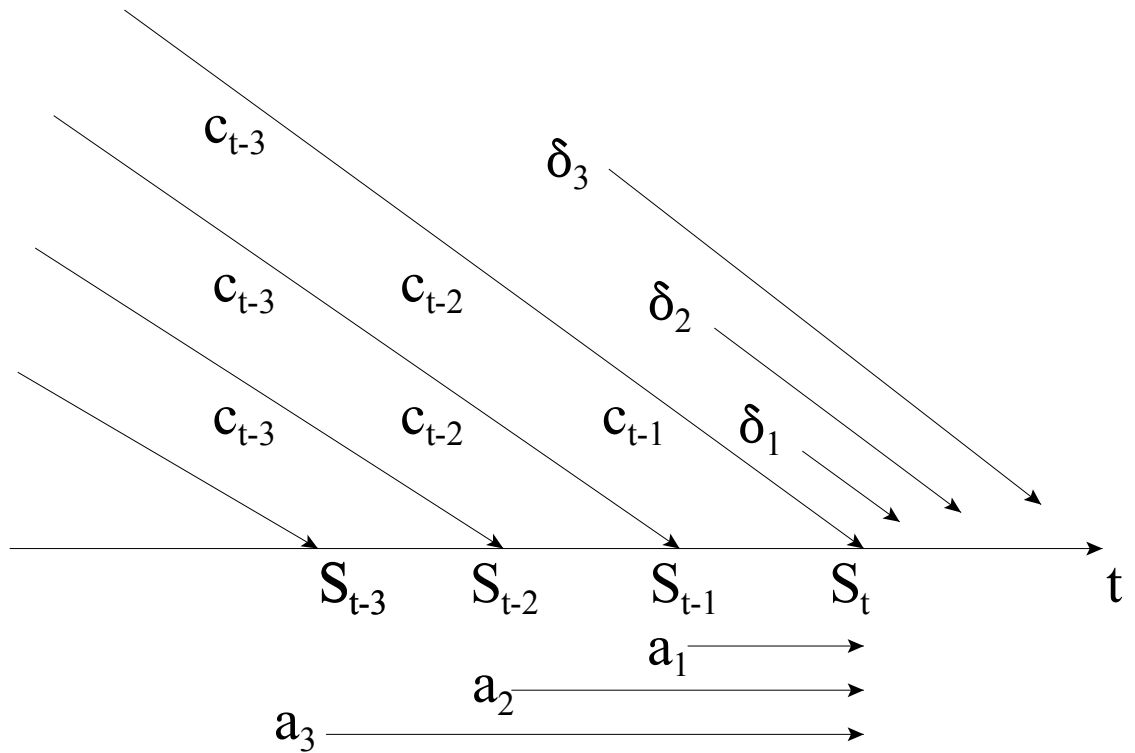


Figure A1: Habit Formation and Durability

An example illustrates this: At time t , a consumer will invest in a new computer and in a new automobile. Instantaneously after the purchase, he notices that the computer is old-fashioned and out-of-date ($\xi_{i,t} > 0$). Four years later, after producing some flow of services for the owner, however, because of rust, looseness in clutch and split in windscreen, the automobile is not new any more. The car has depreciated at some rate while the computer is “junk” (new programs do not run, most of the cells in hard disk are out of order etc.). The mean of the

depreciation for these two durables is ξ_{t-4} and the rate of durability is δ_{t-4} . For convenience, we assume that the infinite sum of δ_τ equals one, that is,

$$\sum_{\tau=0}^{\infty} \delta_\tau = 1. \quad (\text{A.3})$$

Consider next the habit formation in Figure 1. At any time $t - \tau$, $\tau \geq 0$, a consumer has a flow of services $S_{t-\tau}$ from durable goods. With respect to the flow of services at any time $t - \tau$, $\tau \geq 0$, a consumer has a subsistence level, which is a weighted sum of the past flow of services with weights, a_s , $s \in [1, \infty)$, summing up to unity. Again, the first intuition may be that more recent lags have more weight when considering the subsistence level, that is, $a_1 > a_2 > \dots > a_k > \dots$, but this assumption is again misleading. A good year five years ago might be more valuable for the consumer than the previous bad year, so, $a_5 > a_1$. Moreover, to obtain an equation for the estimation, the parameter $h \geq 0$ reflects the persistence of habit formation. As a special case, $h = 0$, there is no internal effect and the model collapses to the standard time-separable utility function.

A brief example (“Those were the days”) illustrates this habit formation: three years ago, when the agent travelled to work by his own car may have more weight and give more felicity than the previous year when he travelled in public transportation. Moreover, if he is used to smoke a package of cigarettes per day, only a few cigarettes per day may induce disutility to him.

APPENDIX B

First we discuss briefly why the selection of exponential decay is applicable in this context. Consider the case where the technology of producing a service flow l_t from durable goods is assumed to be linear and proportional to the sum of the stock of durable goods held by the consumer at the beginning of the period under examination, (k_{t-1}) , and purchases of durable good during this period, c_t . Thus, $l_t = \xi(k_{t-1} + c_t)$, $0 \leq \xi \leq 1$, $\forall t \in [1, \infty)$, k_0 given, where $c_t = \sum_{i=1}^N d_{i,t}$. Also, the stock of durables k_t is equal to $(k_{t-1} + c_t)$ less the amount needed to produce services, that is, $k_t = (1 - \xi)(k_{t-1} + c_t)$. Combining the equations of technology and stock results $l_t = \xi(1 - \xi)^{-1}k_t$. A straightforward recursive manipulation implies that $l_t = \xi \sum_{\tau=0}^{\infty} (1 - \xi)^\tau c_{t-\tau}$ where the coefficient $(1 - \xi)$ is exponentially decaying. If we now redefine the parameter ξ as the depreciation rate and assume that the rate of depreciation and the rate of durability are inversely related, $\xi(1 - \xi)^\tau = (1 - \delta)\delta^\tau$, the latter can be treated as in the following proof:

From the main text the equation (2.14) is

$$Z_t = \sum_{\tau=0}^{\infty} b_\tau c_{t-\tau}, \quad (\text{B.1})$$

in which

$$b_\tau = \delta_\tau - h \sum_{i=1}^{\tau} a_i \delta_{\tau-i}, \quad \tau \geq 1. \quad (\text{B.2})$$

The rate of durability and habit formation were assumed to exhibit exponential decay of the form $\delta_\tau = (1-\delta)\delta^\tau$ where $\delta = (1-\xi)$ and $a_i = (1-\eta)\eta^{i-1}$. Inserting these in the equation (B.2) gives

$$\begin{aligned} b_\tau &= (1 - (1 - \xi))(1 - \xi)^\tau - h \sum_{i=1}^{\tau} (1 - \eta)\eta^{i-1}(1 - (1 - \xi))(1 - \xi)^{\tau-i} & (\text{B.3}) \\ &= \xi(1 - \xi)^\tau - h(1 - \eta) \sum_{i=1}^{\tau} \eta^{i-1} \xi(1 - \xi)^{\tau-i} \\ &= \xi(1 - \xi)^\tau \left\{ 1 - h(1 - \eta) \sum_{i=1}^{\tau} \eta^{i-1} (1 - \xi)^{-i} \right\} \\ &= \xi(1 - \xi)^\tau \left\{ 1 - h(1 - \eta) \frac{1}{\eta} \sum_{i=1}^{\tau} \left(\frac{\eta}{1 - \xi} \right)^i \right\} \\ &= \xi(1 - \xi)^\tau \left\{ 1 - h(1 - \eta) \frac{1}{\eta} \left[\frac{\eta}{1 - \xi} + \left(\frac{\eta}{1 - \xi} \right)^2 + \left(\frac{\eta}{1 - \xi} \right)^3 + \dots \right. \right. \\ &\quad \left. \left. \dots + \left(\frac{\eta}{1 - \xi} \right)^{\tau-2} + \left(\frac{\eta}{1 - \xi} \right)^{\tau-1} + \left(\frac{\eta}{1 - \xi} \right)^\tau \right] \right\}. \quad (*) \end{aligned}$$

The contents in square brackets (B.3) can be separated in to two geometric series which can be treated separately.⁶⁰ Thus, the sum of the first series reduces to

$$\frac{\eta}{1 - \xi} + \left(\frac{\eta}{1 - \xi} \right)^2 + \left(\frac{\eta}{1 - \xi} \right)^3 + \dots = \frac{\eta/(1 - \xi)}{1 - \eta/(1 - \xi)} = \frac{\eta}{1 - \xi - \eta}. \quad (\text{B.4})$$

The second series can be expressed as

$$\begin{aligned} &\left(\frac{\eta}{1 - \xi} \right)^\tau + \left(\frac{\eta}{1 - \xi} \right)^{\tau-1} + \left(\frac{\eta}{1 - \xi} \right)^{\tau-2} + \dots & (\text{B.5}) \\ &= \eta^\tau (1 - \xi)^{-\tau} + \eta^{\tau-1} (1 - \xi)^{1-\tau} + \eta^{\tau-2} (1 - \xi)^{2-\tau} + \eta^{\tau-3} (1 - \xi)^{3-\tau} + \dots \\ &= \eta^\tau (1 - \xi)^{-\tau} \left(1 + \eta^{-1} (1 - \xi)^1 + \eta^{-2} (1 - \xi)^2 + \eta^{-3} (1 - \xi)^3 + \dots \right) \\ &= \eta^\tau (1 - \xi)^{-\tau} \left[1 + \left(\frac{1 - \xi}{\eta} \right) + \left(\frac{1 - \xi}{\eta} \right)^2 + \left(\frac{1 - \xi}{\eta} \right)^3 + \dots \right] \\ &= \left(\frac{\eta}{1 - \xi} \right)^\tau \left[\frac{1}{1 - (1 - \xi)/\eta} \right] \\ &= \left(\frac{\eta}{1 - \xi} \right)^\tau \left[\frac{\eta}{\eta - 1 + \xi} \right]. \end{aligned}$$

Substituting these results back to the equation (*) implies

$$b_\tau = \xi(1 - \xi)^\tau \left\{ 1 - h(1 - \eta) \frac{1}{\eta} \left[\left(\frac{\eta}{1 - \xi - \eta} \right) + \left(\frac{\eta}{1 - \xi} \right)^\tau \left(\frac{\eta}{\eta - 1 + \xi} \right) \right] \right\} \quad (\text{B.6})$$

⁶⁰See Ferson & Constantinides (1991, pp.202-203), Braun et al. (1993, pp. 900-901) and Ermini (1994, pp. 9-11).

$$\begin{aligned}
&= \xi(1-\xi)^\tau \left\{ 1 - h(1-\eta) \frac{1}{\eta} \left(\frac{\eta}{1-\xi-\eta} \right) - h(1-\eta) \frac{1}{\eta} \left(\frac{\eta}{1-\xi} \right)^\tau \left(\frac{\eta}{\eta-1+\xi} \right) \right\} \\
&= \xi(1-\xi)^\tau \left\{ 1 - h \left(\frac{1-\eta}{1-\xi-\eta} \right) \right\} + h\eta^\tau \xi \left(\frac{1-\eta}{1-\xi-\eta} \right).
\end{aligned}$$

which is precisely the equation (2.15) for the coefficient b_τ in the main text.

APPENDIX C

The periodical utility was defined in the equation (2.7):

$$U(c_t, C_{t-\varphi}) = (1-\gamma)^{-1} \left[\alpha(Z_t) + (1-\alpha) \left(\frac{c_t}{C_{t-\varphi}^\theta} \right) \right]^{1-\gamma},$$

$$\forall t \in [0, \infty), \quad \gamma > 1, \quad 0 \leq \theta \leq 1, \quad 0 \leq \alpha \leq 1, \quad \varphi > 0.$$

Case i : $\alpha = 0, \theta = 0.$

In the absence of internal and external effects, the results correspond to those of the standard von Neumann-Morgenstern model. The first-order conditions for two adjacent periods yield the familiar form of the Euler equation:

$$E_t \beta \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^i \right] = 1. \quad (\text{C.1})$$

Case ii : $\alpha = 0.$

The optimum for external effect is a straightforward application of the standard optimisation problem. The marginal utilities with respect to the consumption at periods t and $(t+1)$ are

$$\begin{aligned}
U'(c_t) &= \left(\frac{c_t}{C_{t-\varphi}^\theta} \right)^{-\gamma} \frac{1}{C_{t-\varphi}^\theta}, \\
U'(c_{t+1}) &= \left(\frac{c_{t+1}}{C_{t+1-\varphi}^\theta} \right)^{-\gamma} \frac{1}{C_{t+1-\varphi}^\theta}.
\end{aligned} \quad (\text{C.2})$$

After inserting these into the standard Euler equation, the expression for optimum becomes

$$E_t \left\{ \beta \left(\frac{\left(\frac{c_{t+1}}{C_{t+1-\varphi}^\theta} \right)^{-\gamma} \frac{1}{C_{t+1-\varphi}^\theta}}{\left(\frac{c_t}{C_{t-\varphi}^\theta} \right)^{-\gamma} \frac{1}{C_{t-\varphi}^\theta}} \right) R_{t+1}^i \right\} = 1 \quad (\text{C.3})$$

$$\begin{aligned}
E_t \left\{ \beta \left(\left(\frac{c_{t+1}}{c_t} \frac{C_{t-\varphi}^\theta}{C_{t+1-\varphi}^\theta} \right)^{-\gamma} \left(\frac{C_{t-\varphi}^\theta}{C_{t+1-\varphi}^\theta} \right) \right) R_{t+1}^i \right\} &= 1 \\
E_t \left\{ \beta \left(\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left(\frac{C_{t-\varphi}}{C_{t+1-\varphi}} \right)^{\theta(1-\gamma)} \right) R_{t+1}^i \right\} &= 1 \\
E_t \left\{ \beta \left(\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left(\frac{C_{t+1-\varphi}}{C_{t-\varphi}} \right)^{\theta(\gamma-1)} \right) R_{t+1}^i \right\} &= 1
\end{aligned}$$

Case iii) : $\alpha = 1$.

In the presence of internal effect, consider the case where a consumer reduces his consumption expenditures in period t from c_t to $c_t - \varepsilon$, where $\varepsilon > 0$ denotes a reduction in some of the elements $d_{i,t}$, $i \in [1, N]$.⁶¹ Investing this amount ε in an asset (risky or riskless) with return R_{t+1}^i increases his consumption expenditures in the next period from c_{t+1} to $c_{t+1} + \varepsilon R_{t+1}^i$. However, because of habit formation and durability, this decrease in consumption in period t will have a long-lasting effect on all the future periods through the equation (2.14)

$$Z_t = \sum_{\tau=0}^{\infty} b_\tau c_{t-\tau}, \quad (\text{C.4})$$

where the coefficient is parametrised as in the main text;

$$b_\tau = \delta_\tau - h \sum_{i=1}^{\tau} a_i \delta_{\tau-i}, \quad b_0 = 1, \quad \tau \geq 0. \quad (\text{C.5})$$

To illustrate, consider the effect of this infinitesimal decrease ε in consumption at period t on the term $Z_{t+\tau}$ (which was interpreted as a gap between the present and past flow of services) at all periods $t + \tau$, $\tau \geq -1$:

$$\begin{aligned}
Z_{t-1} &= b_0 c_{t-1} + b_1 c_{t-2} + b_2 c_{t-3} + b_3 c_{t-4} + \dots, \\
Z_t &= b_0 (c_t - \varepsilon) + b_1 c_{t-1} + b_2 c_{t-2} + b_3 c_{t-3} + \dots, \\
Z_{t+1} &= b_0 (c_{t+1} + \varepsilon R_{t+1}^i) + b_1 (c_t - \varepsilon) + b_2 c_{t-1} + b_3 c_{t-2} + \dots, \\
Z_{t+2} &= b_0 c_{t+2} + b_1 (c_{t+1} + \varepsilon R_{t+1}^i) + b_2 (c_t - \varepsilon) + b_3 c_{t-1} + \dots, \\
Z_{t+3} &= b_0 c_{t+3} + b_1 c_{t+2} + b_2 (c_{t+1} + \varepsilon R_{t+1}^i) + b_3 (c_t - \varepsilon) + b_4 c_{t-1} + \dots
\end{aligned} \quad (\text{C.6})$$

The derivatives with respect to ε are

$$\begin{aligned}
\frac{\partial Z_{t-\tau}}{\partial \varepsilon} &= 0, \\
\frac{\partial Z_t}{\partial \varepsilon} &= -b_0 = -1, \\
\frac{\partial Z_{t+\tau}}{\partial \varepsilon} &= (b_{\tau-1} R_{t+1}^i - b_\tau), \quad \tau \geq 1.
\end{aligned} \quad (\text{C.7})$$

⁶¹The following proof of the Euler equation for the internal effect follows the framework of Ferson & Constantinides (1991). The parametrisation is, however, slightly different. For convenience, we assume a single asset approach, that is, $\lambda_{i,t} = 1$.

The interpretation of the first equation in (C.7) is trivial. It tells that the infinitesimal decrease in consumption in period t has no effect on past consumption. The negative effect on current consumption stream is equal to unity, while the effect on future expenditures is two-fold. Because of the habit formation process, the lower subsistence level and habit formation decrease the need for future consumption stream, while, on the other hand, the return on investment will increase consumption. The final effect is ambiguous, depending on the relative size and sign of parameters $b_{\tau-i}$ and the rate of return. If $R_{t+1}^i > (<) b_\tau/b_{\tau-i}$, the sign is positive (negative). As an extreme case, if $r_{t+1}^i = (b_\tau/b_{\tau-i} - 1)$, there is no effect on future consumption.

After this intuition, a consumer's maximisation problem is

$$\max_{d_{i,t}} E_t \sum_{t=0}^{\infty} \beta^t U(Z_t) \quad (\text{C.8})$$

together with constraints

$$\begin{aligned} W_{t+1} &= (W_t + Y_t - c_t)R_{t+1}^i, \\ c_t &= \sum_{i=1}^N d_{i,t}. \end{aligned} \quad (\text{C.9})$$

The variables and their measurement units are denoted as in the main text. This optimisation problem can be solved by dynamic programming, but instead of that we apply here a more simple way.⁶² The discounted total lifetime utility can be re-written as

$$\begin{aligned} U &= U(Z_t) + \beta U(Z_{t+1}) + \beta^2 U(Z_{t+2}) + \beta^3 U(Z_{t+3}) + \dots \\ &= U(b_0 c_t + b_1 c_{t-1} + b_2 c_{t-2} + b_3 c_{t-3} + \dots) \\ &\quad + \beta U(b_0 c_{t+1} + b_1 c_t + b_2 c_{t-1} + b_3 c_{t-2} + \dots) \\ &\quad + \beta^2 U(b_0 c_{t+2} + b_1 c_{t+1} + b_2 c_t + b_3 c_{t-1} + \dots) \\ &\quad + \beta^3 U(b_0 c_{t+3} + b_1 c_{t+2} + b_2 c_{t+1} + b_3 c_t + b_4 c_{t-1} + \dots) + \dots \end{aligned} \quad (\text{C.10})$$

The marginal utilities for period t and $(t + 1)$ consumptions are

⁶²Note that although the maximisation is originally with respect to a single durable good $d_{i,t}$, the problem will generate the same results for maximisation with respect to c_t because $\partial c_t / \partial d_{i,t} = 1, \forall i \in [1, N]$.

$$\begin{aligned}
\frac{\partial U}{\partial c_t} &= U'(Z_t)b_0 + \beta U'(Z_{t+1})b_1 + \beta^2 U'(Z_{t+2})b_2 + & (C.11) \\
&\quad \beta^3 U'(Z_{t+3})b_3 + \dots \\
&= U'(Z_t)b_0 + \sum_{\tau=1}^{\infty} \beta^\tau b_\tau U'(Z_{t+\tau}) \\
\frac{\partial U}{\partial c_{t+1}} &= \beta U'(Z_{t+1})b_0 + \beta^2 U'(Z_{t+2})b_1 + \beta^3 U'(Z_{t+3})b_2 + \\
&\quad \beta^4 U'(Z_{t+4})b_3 + \dots \\
&= \sum_{\tau=1}^{\infty} \beta^\tau b_{\tau-1} U'(Z_{t+\tau})
\end{aligned}$$

Inserting these into the standard Euler equation and remembering $b_0 = 1$ gives

$$\begin{aligned}
U'(Z_t) + E_t \sum_{\tau=1}^{\infty} \beta^\tau b_\tau U'(Z_{t+1}) &= E_t \sum_{\tau=1}^{\infty} \beta^\tau b_{\tau-1} U'(Z_{t+1}) R_{t+1}^i. & (C.12) \\
U'(Z_t) &= E_t \sum_{\tau=1}^{\infty} \beta^\tau U'(Z_{t+1}) [b_{\tau-1} R_{t+1}^i - b_\tau] \\
1 &= E_t \sum_{\tau=1}^{\infty} \beta^\tau \left[\frac{U'(Z_{t+\tau})}{U'(Z_t)} (b_{\tau-1} R_{t+1}^i - b_\tau) \right].
\end{aligned}$$

The specific form of the utility function is $U = (1 - \gamma)^{-1} Z_{t+\tau}^{1-\gamma} \forall \tau \geq 0$, and the marginal utility with respect to $Z_{t+\tau}$ is $Z_{t+\tau}^{-\gamma} \forall \tau \geq 0$. Substituting these into the previous equation gives the recursive Euler equation for an internal optimum:

$$E_t \sum_{\tau=1}^{\infty} \beta^\tau \left[\left(\frac{Z_{t+\tau}}{Z_t} \right)^{-\gamma} (b_{\tau-1} R_{t+1}^i - b_\tau) \right] = 1. \quad (C.13)$$

For an empirical nonlinear estimation, this equation is too complex. Therefore, following Eichenbaum & Hansen (1990), Ferson & Constantinides (1991) and many others, we restrict only to a one-lag model, where $b_\tau = 0$, $\tau \geq 2$. The equations (C.10) and (C.11) are reduced to

$$\begin{aligned}
U &= (1 - \gamma)^{-1} (Z_t^{1-\gamma} + \beta Z_{t+1}^{1-\gamma} + \beta^2 Z_{t+2}^{1-\gamma}) & (C.14) \\
&= (1 - \gamma)^{-1} ((c_t + b_1 c_{t-1})^{1-\gamma} + \beta (c_{t+1} + b_1 c_t)^{1-\gamma} + \beta^2 (c_{t+2} + b_1 c_{t+1})^{1-\gamma}).
\end{aligned}$$

$$\begin{aligned}
\frac{\partial U}{\partial c_t} &= Z_t^{-\gamma} + \beta b_1 Z_{t+1}^{-\gamma}, & (C.15) \\
\frac{\partial U}{\partial c_{t+1}} &= \beta (Z_{t+1}^{-\gamma} + \beta b_1 Z_{t+2}^{-\gamma}).
\end{aligned}$$

Then, the Euler equation is

$$Z_t^{-\gamma} + \beta b_1 Z_{t+1}^{-\gamma} = E_t \left[\beta \left(Z_{t+1}^{-\gamma} + \beta b_1 Z_{t+2}^{-\gamma} \right) R_{t+1}^i \right] \quad (\text{C.16})$$

Case iv : $\alpha = 1, h = 0$.

The derivation follows the case i) with two exceptions. Firstly, the term Z_t reduces to $Z_t = S_t = \sum_{\tau=0}^{\infty} \delta_{\tau} c_{t-\tau}$, $0 \leq \delta_t \leq 1$. Secondly, the assumption of exponential decay $\delta_{\tau} = (1 - \delta)\delta^{\tau}$ implies the following total utility for the consumer (see appendix B):

$$\begin{aligned} U &= U(S_t) + \beta U(S_{t+1}) + \beta^2 U(S_{t+2}) + \beta^3 U(S_{t+3}) + \dots & (\text{C.17}) \\ &= U((1 - \delta)\delta^0 c_t + (1 - \delta)\delta^1 c_{t-1} + (1 - \delta)\delta^2 c_{t-2} + (1 - \delta)\delta^3 c_{t-3} + \dots) \\ &\quad + \beta U((1 - \delta)\delta^0 c_{t+1} + (1 - \delta)\delta^1 c_t + (1 - \delta)\delta^2 c_{t-1} + (1 - \delta)\delta^3 c_{t-2} + \dots) \\ &\quad + \beta^2 U((1 - \delta)\delta^0 c_{t+2} + (1 - \delta)\delta^1 c_{t+1} + (1 - \delta)\delta^2 c_t + (1 - \delta)\delta^3 c_{t-1} + \dots) \\ &\quad + \beta^3 U((1 - \delta)\delta^0 c_{t+3} + (1 - \delta)\delta^1 c_{t+2} + (1 - \delta)\delta^2 c_{t+1} + (1 - \delta)\delta^3 c_t + \dots) + \dots \end{aligned}$$

Repeating the steps as in the case i), the first-order conditions become

$$E_t \sum_{\tau=1}^{\infty} \beta^{\tau} \left[\left(\frac{S_{t+\tau}}{S_t} \right)^{-\gamma} \left(\delta^{\tau-1} R_{t+1}^i - \delta^{\tau} \right) \right] = 1. \quad (\text{C.18})$$

The corresponding Euler equation for a one-lag model is

$$S_t^{-\gamma} + \beta \delta S_{t+1}^{-\gamma} = E_t \left[\beta \left(S_{t+1}^{-\gamma} + \beta \delta S_{t+2}^{-\gamma} \right) R_{t+1}^i \right]. \quad (\text{C.19})$$

Although the equations (C.16) and (C.19) seem alike, the intuition behind them is not similar. The first-order condition (C.16) is based on the growth of Z_t , that is, the gap between the present flow of services and the subsistence level of the service flow attained at the previous periods. Coefficient b_{τ} is a function of all preference parameters δ_{τ} , a_s and h . In the equation (C.19), the growth is based only on service flow S_t and parameter δ measures only the durability of durables.⁶³

Case v : $\alpha = 1, h = 0, \delta_0 = 1$ and $\delta_{\tau} = 0$ if $\tau \geq 0$.

These parameter values imply that the flow of services is based only on the contemporaneous consumption expenditures, i.e. $S_t = c_t$. Since there is no recursive structure left, the first-order condition reduces back to the standard time-separable and time-additive Euler equation

$$E_t \beta \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R_{t+1}^i \right] = 1, \quad (\text{C.20})$$

where, however, consumption is measured as durables.

⁶³Mathematically a usual method for the evolution of the service flow processes in Z_t and S_t is to assume a steady-state equilibrium where consumption is a constant c at every period and/or the distributed lag coefficients are of the Koyck type. See Hayashi (1985) and Deaton (1992, pp.29-34), among others.

Chapter 3

Risk Aversion and Intertemporal Substitution in Aggregate Consumption: Finland 1975-2001

1. INTRODUCTION	107
2. MODEL	108
2.1. Background of Model	109
2.1.1. Ordinal Certainty Equivalence Hypothesis	109
2.1.2. Kreps-Porteus Preferences	111
2.2. Generalised Isoelastic Utility Approach	113
3. ESTIMATION METHODOLOGY	117
4. DATA	119
5. EMPIRICAL RESULTS	121
5.1. Estimation and Testing	122
5.2. Interpretation and Comparison of Results	127
6. CONCLUDING REMARKS	132
References	133
Appendix	137

Risk Aversion and Intertemporal Substitution in Aggregate Consumption: Finland 1975-2001

by

Jari Viitanen

Finnish Postgraduate Programme in Economics

University of Joensuu, Department of Business and Economics

Abstract

This study estimates the parameters of relative risk aversion and elasticity of intertemporal substitution in Finland by using a non-expected utility framework as originally proposed by Epstein & Zin (1989, 1991) and Weil (1990). Unlike the standard von Neumann-Morgenstern expected utility framework, non-expected utility representation allows to disentangle these preference parameters from each other, while maintaining the desired properties of utility functions. The GMM estimates based on the quarterly real aggregate consumption on nondurables and services, several real rates of return and an optimally constructed portfolio revealed that the elasticity of the intertemporal substitution has increased after the financial liberalisation of the mid-1980s, and is in the range 2-10. The estimates for the risk aversion parameter have been statistically insignificant under the regulation. After it, they are statistically between zero and one. Even though the standard expected utility approach cannot be rejected, the results revealed preference for an early resolution of uncertainty which indicates that consumers dislike risk more than intertemporal fluctuations in consumption.

Keywords: Risk aversion, intertemporal substitution, non-expected utility, GMM

1. INTRODUCTION

One important determinant, among others, in the efficiency of economic policy-making is how the expected real interest rate affects saving, consumption and investment decisions. Typically, an increase in expected real interest rate should delay investments and have an indirect negative lagged effect on consumption, and *vice versa*. Theoretically, however, the direct effect of an increase in interest rate on consumption is undetermined. On the one hand, consumption expenditures increase via the income effect; on the other hand, they decrease via the substitution effect. It is an empirical matter to find out which of these effects dominates. The direct effect of a change in interest rate on consumption expenditures can be studied by estimating a consumption function. Nevertheless, the observed interest rate elasticity only shows how much consumption in levels (or differences) will change with respect to the change in interest rate. It does not reveal how willing consumers are to substitute future for the present consumption or *vice versa*, except in the case of logarithmic variables.¹

Another important determinant besides the interest rate, which affects the lifetime consumption profile, is the consumers' attitude towards risk in consumption. Typically, a risk lover invests in risky assets with high expected rate of return. However, the realised yield of those assets may differ from that expected, and have a large effect on the future variability of consumption. A risk averse consumer may ensure a smooth lifetime consumption profile by selecting less risky assets in his portfolio.

In the literature, these factors, namely, risk aversion and intertemporal substitution in consumption, have been treated jointly such that a change in one parameter will affect also the other. For example, the conventional time-additive and time-separable von Neumann-Morgenstern expected utility preferences restrict the representative agent's risk aversion parameter equal to the reciprocal of his/her elasticity of intertemporal substitution parameter (see Grossman & Shiller (1981) and Hansen & Singleton (1983), among others). This connection between the parameters is, however, inconsistent. As noted by Hall (1988), there is no reason to believe that an economic agent's attitude toward risk is related to his preferences for substituting consumption between different time periods. Risk aversion describes a consumer's reluctance to substitute consumption across the states of the world, and is meaningful even in an atemporal setting, whereas the elasticity of intertemporal substitution describes the consumer's willingness to substitute consumption over time, and is meaningful even in a deterministic setting. Therefore, it is likely, that these parameters should be modelled independently.

¹Depending on the theoretical model, several variables, such as wages and salaries, can be used as explanatory candidates in a consumption function. In the case of logarithmic transformation, the estimated parameter for the interest rate can be interpreted as the elasticity of intertemporal substitution (see Hall (1988) for details).

In this study we estimate the parameters of relative risk aversion and elasticity of intertemporal substitution in consumption by using non-expected utility framework as originally proposed by Epstein & Zin (1989, 1991) and Weil (1990). This kind of utility representation enables disentangling the preference parameters from each other, while maintaining the desired properties of utility optimisation and consumers' expectations mechanism. Also, we study how these preference parameters have changed over time because of the financial liberalisation in Finland in the middle of 1980s. In the model, the risk aversion parameter determines how an agent divides current wealth across the assets in his portfolio at a particular point in time. The substitution parameter, on the other hand, governs the choice of how much to consume today versus in the future, which in turn dictates the amount of wealth in euros to be invested.

This study is motivated by the fact that not such research has been made with Finnish data. The knowledge of the magnitude of the consumers' preference parameters helps to understand the efficiency of monetary policy and anticipate business cycles. Also, the results help to prepare government budgeting, even though the freedom of the monetary policy interventions has decreased in Finland due to joining to the EMS. Given that the commercial banks are aware of the magnitude of the these preference parameters, they can use them for lending decisions. The works by Starck (1987) and Kostianen & Starck (1991) have studied the magnitude of the elasticity of intertemporal substitution, but their framework was based on the standard expected utility approach which does not allow to disentangle the parameters. Also, their research period covers the time before the financial liberalisation in Finland, and it is likely that the consumption behaviour has changed because of the changes in the economic environment.

The study is organised as follows. Section two begins by describing the history and background for the non-expected preferences and optimisation based on them. Then, the Euler equation for consumption is derived. Section three reviews the estimation method, and section four gives details of the construction of data. The estimation results and the interpretations are given in section five. Finally, the conclusion summarises the study.

2. MODEL

In order to understand the basis of the theoretical model to be presented in section 2.2., we briefly describe its background, that is, the principles and technical treatments of nonexpected certainty equivalence utility approach by Selden (1978) and preferences adapted by Kreps & Porteus (1978, 1979a,b). Based on these frameworks, the theoretical model in section 2.2 is presented in such a form that it is possible to obtain an empirically testable closed-form solution.

2.1. Background of Model

2.1.1. Ordinal Certainty Equivalence Hypothesis

In addition to the empirical failure (see Hansen & Singleton (1983) and Mehra & Prescott (1985), among others), the standard expected utility approach has certain theoretical weaknesses. Concerning the subject of this study, one crucial shortcoming of the expected utility approach together with time-additive and time-separable preferences is that it does not allow the separation between the parameters describing a consumer's attitude toward risk and the intertemporal substitution of consumption. This limitation of the VNM preferences has motivated researchers to look for alternative frameworks to analyse consumers' dynamic choices under uncertainty.

The problem of inverse relation between the relative risk aversion and the elasticity of intertemporal substitution was not solved until Selden (1978, 1979), who presented a non-expected utility maximising approach by proposing ordinal certainty equivalent (OCE) preferences in a two-period setting. First, an agent converts the uncertain future consumption into its certainty equivalent correspondent. Then, he divides his lifetime consumption between the present and future certainty equivalent consumption. While remaining the time-additive structure, this approach allows to distinguish between the parameters of risk aversion and intertemporal substitution by specifying distinct functions governing the intertemporal choice and atemporal utility provided by the certainty equivalent consumption. The technical treatment of the model can be expressed as follows.

Consider a standard two-period model of consumption where an agent has an amount of wealth W_t at the time t and has no other source of income in period $(t + 1)$. The saving in the first period equals to $S_t = W_t/\Delta t - c_t$, where S and c are saving and consumption at the time t , respectively. After investing in assets, the second period budget constraint becomes²

$$W_{t+1} = R_{t+1}^P(W_t - c_t\Delta t) = \tilde{c}_{t+1}, \quad (2.1)$$

where $R_{t+1}^P = (1 + r_{t+1}^P\Delta t)$ is a K -dimensional real return factor including risky and riskless assets, and r_{t+1}^P is a real rate of return on the portfolio between the periods t and $(t + 1)$. Under uncertainty, \tilde{c}_{t+1} is a random variable that depends on the realisation of the real return factor.³ According to Selden's (1978) ordinal certainty equivalent hypothesis, the agent maximises

$$U(c_t) + \beta U(\bar{c}_{t+1}), \quad (2.2)$$

²By $\Delta t = (t + 1) - t = 1$ we denote the length of one time period. Even though its numerical value is 1, one should not confuse it in the formulations with level quantities W_t measured in euros and flow quantities c_t with unit *euro*/ Δt .

³In this section, the notations \tilde{a} and \bar{a} denote a random future outcome of a variable and its certainty equivalence, respectively.

where $U(\cdot)$ is a concave utility function, $\beta = (1 + \rho\Delta t)^{-1}$ is the discount factor and ρ the subjective rate of time preference. \bar{c}_{t+1} is the nonstochastic certainty equivalent level of period $(t+1)$ consumption. It provides the same utility as the expected utility of the random consumption \tilde{c}_{t+1} , i.e.

$$V(\bar{c}_{t+1}) = E_t [V(\tilde{c}_{t+1})], \quad (2.3)$$

in which $V(\cdot)$ is a strictly concave function. The maximization problem of (2.2) subject to (2.1) and (2.3) results

$$\beta \frac{U'(\bar{c}_{t+1})}{U'(c_t)} \frac{E_t [V'(\tilde{c}_{t+1})]}{V'(\bar{c}_{t+1})} R_{t+1}^P - 1 = 0, \quad (2.4)$$

where $\bar{c}_{t+1} = V^{-1} E_t [V(\tilde{c}_{t+1})]$. This result differs from that of the standard Euler equation by including in the intertemporal marginal rate of substitution (IMRS) an adjusted risk preference term, that is, the ratio between the marginal utilities of random consumption and the certainty equivalent level. Specifying distinct utility functions as

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad \sigma > 0, \quad (2.5)$$

$$V(c) = \frac{c^{1-\gamma}}{1-\gamma}, \quad \gamma > 0, \quad (2.6)$$

will help to disentangle the risk aversion and intertemporal substitution parameters from each other. The curvature of the atemporal cardinal utility function $V(\cdot)$ governs risk aversion, while that of ordinal $U(\cdot)$ determines the willingness to substitute consumption over time. Specifically, σ is the inverse of the elasticity of intertemporal substitution for consumption and γ is the coefficient for relative risk aversion.⁴ When $\gamma = \sigma$, $U(c) = V(c)$ and the non-expected utility approach reduces to the standard VNM framework. Using (2.5), (2.6) and after some rearrangement, (2.4) can be expressed as

$$\psi_t = \frac{1}{1 + \beta^{\frac{1}{\sigma}} \bar{R}^{\frac{1-\sigma}{\sigma}}}, \quad (2.7)$$

$$\bar{R} = [E(R_{t+1}^P)^{1-\gamma}]^{\frac{1}{1-\gamma}},$$

where \bar{R} is the certainty equivalent rate of return and ψ_t is the marginal propensity to consume (MPC) at time t . The interpretation of these results is that the selection of the risky assets reflects a consumer's attitude toward risk, while the willingness to substitute consumption over time is reflected via the MPC by the choice between consumption at time t and the certainty equivalent consumption (which equals wealth) at time $(t+1)$.⁵

⁴According to the definitions, $-\frac{\partial(\frac{c_{t+1}}{c_t})}{\partial(\frac{U'(c_{t+1})}{U'(c_t)})} \times \frac{\frac{U'(c_{t+1})}{U'(c_t)}}{\frac{c_{t+1}}{c_t}} = \sigma^{-1}$ and $-\frac{V''(c)c}{V'(c)} = \gamma$.

⁵Barsky (1989) and Basu & Ghosh (1993) used this type of a model to study how an increase in uncertainty affects investment and saving decisions. Van der Ploeg (1993) examined precautionary saving in this framework.

2.1.2. Kreps-Porteus Preferences

Even though the separation between risk aversion and intertemporal substitution parameters is possible under OCE preferences, the generalisation of the approach to a multiperiod framework violates the time consistency. An agent behaving according to OCE preferences ignores the possibility that the consumption plans formulated at any given time will generally not be realised in the future.⁶ To overcome the problem of intertemporal consistency in the optimal consumption plans, several authors (Epstein (1988), Epstein & Zin (1989), Giovannini & Weil (1989), Farmer (1990), Weil (1990)) have independently adopted the preferences proposed by Kreps & Porteus (1978, 1979a,b). These allow us to separate preferences concerning risk aversion and intertemporal substitution while maintaining the desired properties of time consistency and stationarity of preferences.

Kreps-Porteus (KP) preferences generalise the standard VNM utility by relaxing the “axiom of reduction of compound lotteries”. According to this axiom, consumers are indifferent between the compound and reduced lotteries as long as the final outcome remains the same.⁷ When this axiom is imposed to the temporal games where the prices are tickets to the consumption bundles, VNM agents are indifferent to the way uncertainty is resolved over time as long as the compound probability of the final outcome remains the same. Under KP preferences, however, consumers are not indifferent to the timing of resolution of uncertainty over temporal consumption lotteries. Depending on the relative magnitudes of risk aversion and intertemporal substitution, consumers may prefer early or late resolution of uncertainty. The following description will help to understand the nonindifference between the timing of resolution. Consider the two temporal lotteries presented in Figure 2.1.⁸

In lottery A, an agent consumes a bundle c at times t and $(t + 1)$ with certainty. Then, with the probability p he consumes a bundle c thereafter, and with probability $(1 - p)$ a bundle $c' \neq c$.⁹ In lottery B, the agent consumes a bundle c at time t with certainty. Then, with probability p he consumes c forever, and with probability $(1 - p)$ a bundle c at the time $(t + 1)$ and c' thereafter. In both lotteries,

⁶Attanasio & Weber (1989) extended OCE preferences for a multiperiod case, but their work also suffers from time inconsistency. See Johnsen & Donaldson (1985) for a detailed discussion and technical treatment of the shortcomings of the OCE preferences.

⁷Kreps (1990, chapter 3) provides a thorough understanding of the nature of VNM preferences including the axiom of reduction of compound lotteries. The purpose of the following description is to give an intuition of the characteristic of the KP preferences. While the axiomatic foundations of the KP preferences are beyond the scope of this study, a reader can consult the original papers of Kreps & Porteus (1978, 1979a,b) or Farmer (1990) for details.

⁸This heuristic intuition is originally expressed in Giovannini & Weil (1989) and Weil (1990). It is important to recognise that the prizes of the lotteries are consumption bundles, not income. Even under VNM preferences an early resolution on income lotteries always reduces uncertainty and improves planning. See also Hassler (1996) for more discussion.

⁹For our illustrative purposes, there are only two possible consumption bundles. In general, there can be n possibilities with the corresponding probabilities.

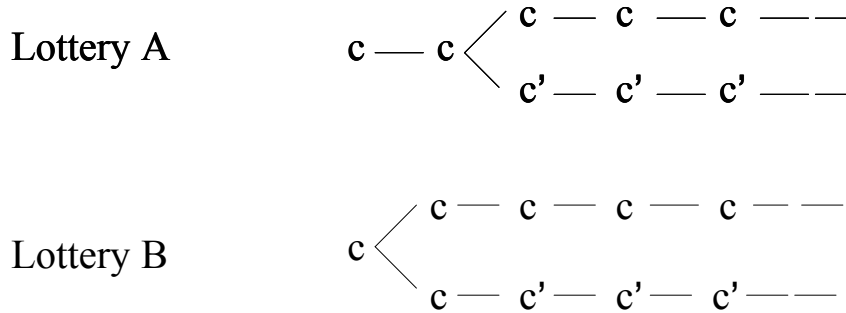


Figure 2.1: Timing of Resolution of Uncertainty

the compound probability of each consumption bundle is identical. Therefore, a VNM expected utility maximiser is indifferent between the lotteries. However, when a KP consumer ponders on the lotteries A and B, he recognises that the uncertainty in lottery B is resolved one period earlier than in lottery A. Then, depending on his preferences, he may prefer either lottery A or B.

The close connection between the concepts of preference for early and late resolution of uncertainty, risk aversion and intertemporal substitution can be understood as follows. In terms of utility, lotteries in which uncertainty is resolved earlier (lottery B) are less risky than lotteries in which uncertainty is resolved later (lottery A) even though the compound probability of the prize is identical. However, lotteries where uncertainty is resolved earlier feature certainty equivalent fluctuations of utility over time which are of larger amplitude. Therefore, there is a trade-off between safety and stability of utility. Agents, who dislike risk more than intertemporal fluctuations, prefer, *ceteris paribus*, early resolution. But consumers, who have stronger distaste for intertemporal fluctuation than for risk, prefer the late resolution.¹⁰

Technically, KP preferences can be presented recursively such that

$$V_t = F [c_t, E_t(V_{t+1})], \quad (2.8)$$

where V_t denotes the utility at time t . $F(.,.)$ is, in Koopmans (1960) terminology, an aggregator function through which current consumption c_t and expected future utility are aggregated. E_t is the expectation operator conditioned on the information available to the consumer at time t . Kreps and Porteus (1978) proved that agents prefer early or late resolution of uncertainty depending on the convexity or concavity of the aggregator function in its second argument. If the aggregator function is convex in its second argument, agents exhibit a preference for early resolution of uncertainty over temporal lotteries. Conversely, if the function is concave with respect to the second argument, agents prefer

¹⁰Weil (1990, pp.32-33).

late resolution. When the aggregator function is linear in its second argument, agents are indifferent towards the timing of resolution. Then, by recursion, one can obtain that the lifetime utility V_t is the expected sum of the discounted future subutilities, and the derivative of the aggregator function with respect to its second argument is interpreted as a subjective discount factor. Thus, this type of preferences generalises the standard VNM time- and state-separable utility representation.

2.2. Generalised Isoelastic Utility Approach

Consider an infinitely long-lived representative agent who derives utility either on a single commodity or on a consumption bundle in which the weights of the commodities remain the same over time.¹¹ Even though the current consumption level c_t is deterministic, the future consumption \tilde{c}_{t+1} and, therefore, future utility \tilde{V}_{t+1} , are uncertain. Following Selden (1978), Kreps & Porteus (1978, 1979a,b) and Epstein & Zin (1989, 1991), the intertemporal dynamic optimisation problem is two-phased.¹² First, a consumer is assumed to form a certainty equivalent of his random future utility such that

$$\mu_t = E_t [\bar{V}_{t+1} \mid I_t], \quad (2.9)$$

where I_t is the information set available for the agent at period t and μ_t is the certainty equivalent utility.¹³ Second, the agent is assumed to form his total lifetime utility by combining the current consumption c_t with the certainty equivalent of the random future utility via the aggregator function F :

$$V_t = F [c_t, \mu_t]. \quad (2.10)$$

In order to derive a closed-form decision rule, one has to specify the explicit forms for the aggregate and certainty equivalent functions. For the previous one, let us assume that it is of the CES form

$$F [c_t, \mu_t] = \begin{cases} [c_t^\sigma + \beta \mu_t^\sigma]^{\frac{1}{\sigma}}, & 0 \neq \sigma < 1, \\ \log(c_t) + \beta \log[\mu_t], & \sigma = 0, \end{cases} \quad (2.11)$$

where $0 \leq \beta = (1 - \rho \Delta t)^{-1} \leq 1$ is the discount factor and ρ the rate of the subjective time preference. The interpretation of the parameter σ can be understood

¹¹Expected utility models impose on several restrictions, which are summarized in Attanasio (1998, pp.18-19).

¹²In addition to the references below, this type of preferences is also applied by Svensson (1989), Kocherlakota (1990), Prasad (1991), Kandel & Stambaugh (1991), Wang (1993), Weil (1993), Obstfeld (1994), Chacko & Viceira (1999), Bansal & Yaron (2000) and Haliassos & Hassapis (2001), among others.

¹³It is important to recognize that the certainty equivalent is formulated from the random future *utility* and not from the random future *consumption* as in (2.3). The close connection to Selden's (1978) two-period OCE preferences can be understood as follows. Define \bar{V}_{t+1} and \tilde{V}_{t+1} as the certainty equivalent and random future utility, respectively. Then define the certainty equivalent utility by a utility function $G(\bar{V}_{t+1}) = E_t G(\tilde{V}_{t+1})$. After solving \bar{V}_{t+1} and using the terminology in (2.9), we get $\mu_t = E_t G^{-1} [G(\tilde{V}_{t+1})] = E_t(\bar{V}_{t+1})$.

as follows. In the absence of uncertainty and by holding the certainty equivalent of the next period's utility constant, the equation (2.11) is reduced to

$$V_t = \left[\sum_{t=1}^{\infty} \beta^{t-1} c_t^\sigma \right]^{\frac{1}{\sigma}}, \quad (2.12)$$

which is a standard intertemporal CES utility function where the magnitude of σ reflects the willingness to substitute consumption over time. Thus, following Epstein (1988), Epstein & Zin (1989) and many others, we interpret $\delta = (1 - \sigma)^{-1}$ as the constant elasticity of intertemporal substitution. Correspondingly, the certainty equivalent function (2.9) is specified as

$$\mu_t = \begin{cases} E_t(\bar{V}_{t+1}^\gamma)^{\frac{1}{\gamma}}, & 0 \neq \gamma < 1, \\ E_t[\log(\bar{V}_{t+1})], & \gamma = 0. \end{cases} \quad (2.13)$$

γ is the risk aversion parameter, and $(1 - \gamma)$ is the Arrow-Pratt coefficient of relative risk aversion. Thus, the smaller γ , the more a consumer dislikes risk.¹⁴ After the explicit functional forms of (2.11) and (2.13), the total utility (2.10) can be expressed recursively as¹⁵

$$V_t = \left\{ c_t^\sigma + \beta \left[(E_t(\bar{V}_{t+1}^\gamma)^{\frac{1}{\gamma}})^\sigma \right] \right\}^{\frac{1}{\sigma}}. \quad (2.14)$$

In the sense of Kreps & Porteus (1978), if σ is larger than γ , the aggregator function is convex in its second argument, and the representative consumer has preference for early resolution of uncertainty. Conversely, if σ is smaller than γ , the aggregator function is concave in its second argument, and he has a desire for late resolution. When $\gamma = \sigma$, equation (2.14) is linear in its second argument and, by recursion, it is reduced to the standard expected utility specification. Then, the consumer is indifferent between the late and early resolution of uncertainty.

The consumer's objective is to maximise his lifetime utility defined in (2.14). The budget constraint is

$$\begin{aligned} W_{t+1} &= R_{t+1}^P(W_t - c_t \Delta t), \\ R_{t+1}^P &= \sum_{i=1}^K \lambda_{i,t} R_{t+1}^i, \end{aligned} \quad (2.15)$$

where the symbols are denoted as in the previous section. Based on the information set available at time t , the weights of the single assets, $\lambda_{i,t}$, are optimally

¹⁴See Epstein (1988), Epstein & Zin (1989) and Weil (1990) for good illustrations of the properties of the functional forms and parameters σ and γ . See also Farmer (1990) who applies the special case of risk neutrality $\gamma = 1$.

¹⁵This formulation ensures that the current utility is increasing and concave with respect to the current consumption, and increasing in future utility.

chosen before the realisation of the state of the nature at period $(t + 1)$ such that $\sum_{i=1}^K \lambda_{i,t} = 1$. Thus, an agent is able to affect his lifetime consumption pattern by trading in the assets. The more risky assets he chooses in his portfolio, the more the future consumption may vary from planned. For convenience, we assume hereafter that the time periods are of equal length, and the term Δt can be scaled to one. Because of the recursive structure of the homogeneous preferences together with a linear budget constraint, the lifetime maximisation problem can be turned to the following dynamic programming problem where, in turn, the consumption-saving decision is separable from the portfolio optimisation. The optimal value function $J(W_t, I_t)$ is defined as the maximum utility achievable for the consumer over his future planning time horizon:

$$J(W_t, I_t) = \max_{c_t, \lambda_i} \left\{ c_t^\sigma + \beta [E_t J(W_{t+1}, I_{t+1})^\gamma]^\frac{\sigma}{\gamma} \right\}^\frac{1}{\sigma}. \quad (2.16)$$

The maximisation problem is restricted by (2.15). These types of recursive problems are typically solved by guess and verify -methods. Following Epstein & Zin (1989, 1991), guess that the value function can be written as¹⁶

$$J(W_t, I_t) = \phi(I_t)W_t = \phi_t W_t, \quad \forall t, \quad (2.17)$$

and that the consumption function is linear in wealth:

$$c_t = \psi(I_t)W_t = \psi_t W_t \quad \forall t. \quad (2.18)$$

For the time being, ϕ_t and ψ_t are undetermined coefficients. However, because of the non-negativity of the indirect utility, $\phi_t > 0$. The term ψ_t can be interpreted as a marginal propensity of consumption out of the current wealth such that $0 \leq \psi_t \leq 1$. In order to exploit the assumptions (2.17) and (2.18), one has to prove that they fulfill the maximisation problem (2.16).¹⁷ After some calculation, the coefficients can be expressed as

$$\psi_t = \frac{1}{1 + (\beta \xi^\sigma)^{\frac{1}{1-\sigma}}} \quad \forall t, \quad (2.19)$$

$$\phi_t = \psi_t^{\frac{\sigma-1}{\sigma}}, \quad (2.20)$$

where $\xi = E_t [(\phi_{t+1} R_{t+1}^P)^\gamma]^\frac{1}{\gamma}$ can be interpreted as a certainty equivalent rate of return. The optimal consumption-saving decision via the marginal propensity to consume ψ_t depends directly on the substitution parameter and indirectly on risk aversion.¹⁸ When σ decreases, δ decreases and the marginal propensity

¹⁶This assumption is justifiable because the value function can be understood as an indirect utility function. See also Giovannini & Weil (1989) and Jorion & Giovannini (1993) who assume a nonlinear value function which results to different Euler equation than that of following.

¹⁷In Appendix we give the details of the derivation for the undetermined coefficients and the closed-form decision rule (2.25).

¹⁸This parametrisation ensures that $0 \leq \psi_t \leq 1$. The shortcoming of the constant coefficient approach, such as in Weil (1990), is that it does not fulfill this restriction. Note also the close connection to the equation (2.7) and the interpretation of it.

to consume increases thus decreasing savings. Applying (2.17) and (2.18), the first-order condition of (2.16) with respect to the current consumption holds if

$$c_t^{\sigma-1} = \beta(W_t - c_t)^{\sigma-1}\xi. \quad (2.21)$$

Translating (2.20) one period ahead, using (2.18) and inserting into the equation (2.21), we get

$$\beta^{\frac{\gamma}{\sigma}} E_t \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma(\sigma-1)}{\sigma}} (R_{t+1}^P)^{\frac{\gamma}{\sigma}} = 1. \quad (2.22)$$

While this equation corresponds to the optimal consumption path, there is no guarantee that the portfolio is optimally chosen. To overcome this issue we take the first order-conditions of (2.22) with respect to the asset weights, $\lambda_{i,t}$, given the information set available at the time t . This results

$$\frac{\gamma}{\sigma} \beta^{\frac{\gamma}{\sigma}} E_t \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma(\sigma-1)}{\sigma}} (R_{t+1}^P)^{\frac{\gamma}{\sigma}-1} R_{t+1}^i = 0, \quad i = 1, \dots, K. \quad (2.23)$$

In equilibrium, the orthogonality condition (2.23) should hold for every single asset. Thus, for assets i and j :

$$\beta^{\frac{\gamma}{\sigma}} E_t \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma(\sigma-1)}{\sigma}} (R_{t+1}^P)^{\frac{\gamma}{\sigma}-1} [R_{t+1}^i - R_{t+1}^j] = 0, \quad \forall i, j \in K, i \neq j. \quad (2.24)$$

The interpretation of this equation is that in equilibrium the expected intertemporal marginal rate of substitution must be the same for any two separate assets. If this is not true, an agent should invest more in the asset which gives higher expected yield. Multiplying the previous equation by $\lambda_{j,t}$, summing over i and applying (2.22) gives

$$\beta^{\frac{\gamma}{\sigma}} E_t \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma(\sigma-1)}{\sigma}} (R_{t+1}^P)^{\frac{\gamma}{\sigma}-1} R_{t+1}^i = 1, \quad i = 1, \dots, K. \quad (2.25)$$

This Euler equation is the focal object for the empirical work. If $\gamma = \sigma$, this equation is analogous to the standard Euler equation of the expected VNM utility model.

Under the logarithmic preferences $\gamma = 0$ ($\sigma \neq 0$), equation (2.25) is reduced to

$$E_t \left(\frac{R_{t+1}^i}{R_{t+1}^P} \right) = 1, \quad i = 1, \dots, K. \quad (2.26)$$

which does not include the parameter governing intertemporal substitution. Following Epstein & Zin (1991), however, the elasticity of intertemporal substitution can be identified by using equation (2.22). Dividing both sides by the term $\eta = \gamma/\sigma$ gives

$$\frac{E_t \left[\beta \left(\frac{c_{t+1}}{c_t} \right)^{\sigma-1} (R_{t+1}^P) \right]^\eta - 1}{\eta} = 0. \quad (2.27)$$

As γ approaches one, η approaches zero and according to L'Hopital's rule, equation (2.27) converges to

$$-\rho + (\sigma - 1)E_t \log\left(\frac{c_{t+1}}{c_t}\right) + E_t \log(R_{t+1}^P) = 0. \quad (2.28)$$

This equation permits an independent identification of the parameter σ , and separates the logarithmic expected utility specification from that of the non-expected model with logarithmic risk preferences.

3. ESTIMATION METHODOLOGY

Hansen & Singleton's study (1982) was the first attempt to apply Hansen's (1982) Generalized Method of Moments (GMM) to estimate the structural parameters of nonlinear rational expectations closed-form models.¹⁹ Since, empirical work concerning intertemporal consumption decisions and asset pricing has been typically based on the GMM approach. This method has several advantages including only relatively modest requirements concerning the distribution the sample data is drawn. Rather than requiring an exact specification of a conditional distribution, GMM uses only moment functions. Another attractive property of GMM is that the disturbance terms of estimation can be interpreted as an additive forecast error of the Euler equations, and the disturbance terms are allowed to be heteroskedastic and serially correlated. Because the large sample properties of GMM estimators and tests have been thoroughly documented in Hansen (1982), Mátyás (1999), Hayashi (2000) or Ruud (2000), among others, this section presents only briefly the basics of the GMM method in order to understand the estimation of the preference parameters of the decision rule (2.25).

Suppose that the closed-form solution (2.25) can be described by a function

$$f(x_{t+1}, \theta), \quad (3.1)$$

where x_{t+1} is a strictly stationary k -dimensional vector of all the variables in the model observed at time $(t + 1)$, θ is a l -dimensional unobservable parameter

¹⁹In fact, they used generalized instrument variables method which is a special case of GMM.

vector to be estimated and $f(x_{t+1}, \theta)$ is a m -dimensional differentiable vector of functions $f_i(x_{t+1}, \theta)$.²⁰ For the estimation, we refer to

$$f(x_{t+1}, \theta) = u_{t+1} \quad (3.2)$$

as the disturbance term in the GMM estimation. If the parameter vector θ is true,

$$E_t [u_{t+1}] = E_t [f(x_{t+1}, \theta)] = 0. \quad (3.3)$$

Let z_t denote a q -dimensional vector of instrument variables observable to the econometrician at time t . Then, suppose that the orthogonality condition can be written as

$$h(x_{t+1}; \theta; z_t) = f(x_{t+1}; \theta) \otimes z_t, \quad (3.4)$$

where \otimes is the Kronecker product. The $(m \times q)$ -vector of orthogonality conditions $h(x_{t+1}; \theta; z_t)$ is obtained by multiplying the vector $f(x_{t+1}; \theta)$ by each element in the vector z_t . Since (3.3) holds, by iterative expectations follows that

$$E [f(x_{t+1}, \theta) \otimes z_t] = E [h(x_{t+1}; \theta; z_t)] = 0. \quad (3.5)$$

The method of moment estimator replaces $E [h(x_{t+1}; \theta; z_t)] = 0$ with its sample analog

$$g_n(\theta) = \frac{1}{n} \sum_{t=1}^n h(x_{t+1}; \theta; z_t) \quad (3.6)$$

which should be close to zero when evaluated at $\theta = \theta_0$ for large values of n . Hansen (1982) showed that the GMM estimator of θ_0 , θ_{GMM} , which makes $g_n(\theta_0)$ close to zero, can be accomplished by minimising the quadratic

$$J_n(\theta_0) = \min_{\theta_0} [g_n(\theta_0)' W_{GMM} g_n(\theta_0)] \quad (3.7)$$

with respect to θ_0 . The apostrophe denotes transposition and W_{GMM} is a $(r \times r)$ symmetric, positive semi-definite weighting matrix. Clearly, the asymptotic covariance matrix of the GMM estimator, θ_{GMM} , depends on the choice of the weighting matrix W_{GMM} . According to Hansen (1982), the smallest asymptotic covariance matrix of θ can be found by choosing the weighting matrix as $W = S^{-1}$ where S is a consistent estimate of the asymptotic covariance matrix such that

$$S = E [g_n(\theta) g_n(\theta)'] \quad (3.8)$$

In GMM estimation, it is assumed that the process $\{x_t\}$ is stationary and ergodic, the second moments of (3.1) and z_t exist and are finite, (3.6) is continuous and

²⁰In the model, $f(x_{t+1}, \theta) = \beta^n \left(\frac{c_{t+1}}{c_t} \right)^{\eta(\sigma-1)} (R_{t+1}^P)^{\eta-1} R_{t+1}^i - 1$, in which $\eta = \gamma\sigma^{-1}$, $x_{t+1} = \left(\frac{c_{t+1}}{c_t}, R_{t+1}^P, R_{t+1}^1, \dots, R_{t+1}^K \right)$ and $\theta = (\gamma, \beta, \sigma)$.

differentiable in θ , and that W_{GMM} converges to the true weighting matrix W for large values of n . Under these assumptions, Hansen (1982) proved that θ_{GMM} is a consistent estimator of the true θ and is asymptotically normally distributed with covariance matrix $\Lambda = (D'S^{-1}D)^{-1}$, where

$$D = E \left[\frac{\partial f}{\partial \theta}(x_{t+1}, \theta) \otimes z_t \right]. \quad (3.9)$$

In practice, because Λ is unknown, it must be replaced by a consistent sample estimate Λ_{GMM} , which is based on the sample counterparts D_{GMM} and S_{GMM} . In estimation, D_{GMM} and S_{GMM} can be computed by replacing the expectation operator in (3.8) and (3.9) with the sample average operator, and θ with θ_{GMM} .

When the GMM estimation procedure sets the l linear combinations of the r moment conditions to minimise the loss function (3.7) and when $r > l$, the system is overidentified and there remains $(r - l)$ linearly independent moment conditions which are not used to estimate θ . These overidentified restrictions may be used to test whether the model is correctly specified and/or the instrumental variables are valid. If the model is correct, the value

$$nJ_n(\theta_{GMM}) \rightarrow \chi^2(r - l). \quad (3.10)$$

Under the null hypothesis $E[h(x_{t+1}; \theta_{GMM}; z_t)] = 0$, the test statistic is asymptotically chi-square distributed with $(r - l)$ degrees of freedom. Thus, the statistic J_n provides a measure for the goodness-of-fit of the model. Subsequently, Hansen & Jagannathan (1989) suggested that the criteria should be based on r degrees of freedom.

To test the structural stability of the preference parameters, there exists a variety of tests for the models estimated through the GMM method. These test statistics vary depending on whether the breakpoint in the sample is known or unknown. We assume that the breakpoint in time-series is known, and test the structural stability of the parameter vector θ_{GMM} by using the Andrews & Fair (1988) (AF hereafter) test statistic. AF-test is a Wald type test where the whole research period is divided into two subsamples, that is, before and after the breakpoint. Then, the corresponding estimates and covariance matrices are calculated for each subperiod. The test statistic is

$$n(\theta_{GMM}^0 - \theta_{GMM}^1)' [\pi^{-1}\Lambda_{GMM}^0 + (1 - \pi)^{-1}\Lambda_{GMM}^1]^{-1} (\theta_{GMM}^0 - \theta_{GMM}^1) \sim \chi^2(l). \quad (3.11)$$

The null hypothesis is $\theta_{GMM}^0 = \theta_{GMM}^1$. In (3.11) π denotes the share of the first subsample to the total sample size and the superscripts refer to subsamples. This test statistic follows the χ^2 -distribution with the degrees of freedom equal to the number of estimated parameters.

4. DATA

To estimate the structural parameters of the Euler equation (2.25), we used aggregate Finnish quarterly consumption data from the time period 1975(I)-2001(II). Aggregate consumption is seasonally adjusted real aggregate consumption of non-durables and services. The data was chosen to justify the time-additive and time-separable structure of the theoretical model.²¹ To mimic a representative agent's behaviour and to reflect demographic changes, the data was converted into *per capita* form by dividing the total consumption by the total population at each quarter. The consumption data was taken from the database of The Research Institute of the Finnish Economy, and the population data was from the Statistics Finland.²²

One strong implication of the model is that the first-order condition (2.25) holds for any asset available to the consumer. For an econometrician, however, a number of important issues arise from the selection of adequate asset measures. First, most of the agents' wealth in Finland consist of (expected) human capital, which is an unobservable variable. Second, another important source of yield are real assets, such as real estates, housing, forest estates and summer cottages. Typically, the rates of return of these assets are distributed over several time periods, and it is difficult to measure such yields in practice. This holds true also for investments in foreign assets. Therefore, we used only the rates of return from financial assets. The third issue is that the closed-form Euler equation in (2.25) includes an optimally chosen portfolio, which is not directly observable. Thus, no matter how this optimal portfolio is constructed for the empirical purposes, Roll's (1977) critique for the CAP models is relevant.

We tested the model by using quarterly rates of return on government bonds, all bonds in the market (including corporations, commercial banks, local and state government), equities traded in the Helsinki Stock Exchange (HEX), and the average borrowing rate of commercial banks in Finland. The total rate of return for the stocks was measured by summing the average quarterly effective dividend yields and capital gains from the stocks. The capital gains were measured as quarterly percentage changes of the HEX index. All the nominal rates of return were converted to real by using an appropriate consumption price deflator. Thus, if we test the model with nondurables or services, the asset returns are deflated by the deflator of consumer nondurables or services, correspondingly. Before 1993, all the rates of return were converted to tax-free by using a personal marginal tax rate as reported in BOF4 model, and the prevailing capital income taxation rate after 1993.²³

²¹Theoretically, it is possible to test the model by using an imputed service flow from the stock of durables or semidurables. In practice, such measures are difficult to calculate, and no reliable method exists so far.

²²See chapter 2 for problems involved in the representative agent approach.

²³Negative rates of return are not taxed.

The model was tested using two different types of optimal portfolio. The first one was assumed to consist of financial wealth only, where the weights were the market value of a single asset with respect to the total market value of all assets in portfolio. This was motivated by the intuition that most of the expenditures on nondurables and services are financed by financial wealth only. The financial wealth consists of the rates of return from non-taxed bonds, taxed bonds, stocks (including the capital gain and effective dividend yields), markka deposits and value-weighted yields from mutual funds.²⁴ Figure 4.1 depicts the evolution of the weights of the financial wealth over time. Clearly, the markka deposits dominate over time even though the importance of stocks has increased, especially since the depression in the early 1990s. The share of the bond ownership has remained quite stable over time even though a declining trend seems to have occurred after the mid 1990s.

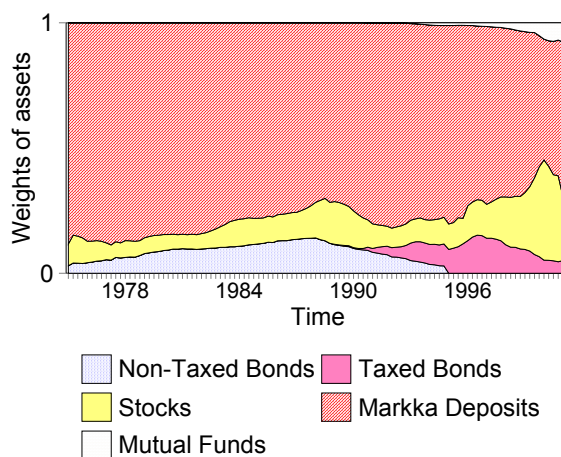


Figure 4.1: Weights of Financial Wealth

The second portfolio consisted of the first one plus the rate of return from housing wealth. This extension considers the possibility that housing wealth is sometimes used as a collateral when an agent is borrowing from financial markets to optimise his consumption behaviour.²⁵ Before 1985, the rates of return from housing wealth were the quarterly percent changes in middle prices (FIM/m²) from the old block of flats as reported by the Statistics Finland. Since 1985 the rates of return were percent changes of the housing price index. The rates of return were converted as real and after-taxed as before. In the second portfolio, the weight of

²⁴Of course, as noted by Takala (2001), the primary source of financing nondurable consumption is still current disposable net income, and a secondary source is previously accumulated financial savings. However, the current disposable income is not a rate of return as such. Even though the importance of life insurance and pension saving have increased in the late 1990s, they are not included in the financial portfolio - still, they are minor financial assets, and it is difficult to calculate aggregate rates of return for them.

²⁵Typically, housing wealth is used as a collateral when durable purchases are financed. Indirectly, however, the amount of housing wealth can reveal how liquidity constrained an agent is, and affect also his nondurable consumption expenditures.

the housing wealth dominated. In 1975 this weight was 0.71. During the research period the weight has smoothly decreased and in 2001 it was 0.65.

5. EMPIRICAL RESULTS

The main goal of the following empirical work was to obtain estimates for the structural parameters of the model (2.25), to test the model against the standard expected utility one and to test the restrictions imposed by the model on the comovements of consumption and asset returns with different portfolio constructions. Finally, we interpreted and compared the results against those from the other studies.

5.1. Estimation and Testing

The GMM estimation requires instrumental variables to construct the orthogonality conditions. Theoretically, it is possible to select any variables from the consumer's information set which are known at time t . We followed the suggestion by Ferson & Constantinides (1991) and used other variables than those involved in Euler equations as instruments to avoid spurious correlations between the regressors and instruments. This is also in accordance with Neely et al. (1999) who found that the lagged values of consumption growth and asset returns are not of much help in predicting either variable. Tauchen's (1986) simulation tests showed that when large instrument sets with high a number of lags are used, the bias of estimation increases and the model is rejected too frequently. Therefore, a small instrument set with recent lags are recommended. We followed this suggestion and used as small an instrument set as possible. Instead of recent lags, however, we used instruments which are lagged twice to allow, for example, slowness in reporting the economic fundamentals and inertia of new information to reach consumers' consciousness. Also, as noted by Hall (1988), this additional lag helps to eliminate bias of time aggregation. The instrument set is

$$Z_t = (\text{One}, GDP_{t-2}, WS_{t-2}, YD_{t-2}), \quad (5.1)$$

where One, GDP, WS and YD are the constant, growth rates of gross domestic product, wages and salaries and disposable income, respectively.²⁶

The estimation was performed in such a way that we attempted a number of starting values for numerical optimisation. If these converged to different parameter estimates, we selected the ones that produced the smallest value for the

²⁶Currently, there is a considerable amount of research on GMM that includes asymptotic properties of small samples, test statistics and moment restrictions imposed by different sets of instrument variables. However, the results and suggestions of these studies are not unique concerning, for example, the optimal set of instruments. See, for example, Sowell (1996), Hall & Sen (1999), Qian & Schmid (1999), Smith (1999), and the references cited. See also a special issue of *Journal of Business & Economic Statistics*, vol. 14, No.3, and chapter 2 of this thesis.

loss function. First, without paying too much attention to the magnitude of the relative risk aversion and elasticity of intertemporal substitution, we tested if the specification of the Euler equation differs statistically from those of the standard expected utility model or non-expected utility model with logarithmic preferences. That is, we tested whether $\gamma = \sigma$ and $\gamma = 0$ ($\sigma \neq 0$), correspondingly. Second, we attempted to reveal if there has been a structural change in preference parameters because of the financial deregulation in Finland, which culminated towards the end of 1986. Finally, we tested the model against two different constructions for household wealth portfolio.

An undesired feature of the model is the possibility of a trivial solution if a single rate of return follows too closely to that of the optimal portfolio. If $R_{t+1}^P \approx R_{t+1}^i$ and $\gamma = 0$, the error term becomes close to zero no matter of the magnitude of the other parameter values. Therefore, to avoid misspecification we also tested the model as a system. When multiple rates of return are used simultaneously, the whole system may satisfy the identification problem better, but the GMM

Table 1
GMM results for model $\beta \frac{\gamma}{\sigma} E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma}{\sigma}(\sigma-1)} (R_{t+1}^P)^{\frac{\gamma}{\sigma}-1} R_{t+1}^i \right\} = 1$,
nondurables, portfolio 1

Asset	Period	β	γ	σ	J^*	Wald	AF
GBOND	1975.2-1986.4	1.015*	-0.371	0.893	11.68	0.963	
		(0.012)	(0.763)	(0.937)			
	1987.1-2001.2	0.992*	0.676*	0.800*	54.70	1.695	3513.7
		(0.002)	(0.317)	(0.323)			
BOND	1975.2-1986.4	1.015*	-0.356	0.906	11.76	1.010	
		(0.013)	(0.736)	(0.965)			
	1987.1-2001.2	0.992*	0.704*	0.826*	55.23	1.614	3594.5
		(0.002)	(0.311)	(0.320)			
RRATE	1975.2-1986.4	1.009*	-0.242	0.726	15.97	0.465	
		(0.005)	(0.661)	(0.973)			
	1987.1-2001.2	0.993*	1.028*	1.144*	29.02	1.106	470.6
		(0.002)	(0.324)	(0.313)			
SHAR	1975.2-1986.4	1.006*	15.659	0.916	0.030	0.166	
		(0.006)	(36.28)	(0.550)			
	1987.1-2001.2	0.998*	-14.89*	2.633*	5.577	22.05	497.6
		(0.004)	(3.924)	(0.720)			
SYSTEM	1975.2-1986.4	1.000*	0.383	-0.282	high	0.020	
		(0.016)	(2.350)	(2.396)			
	1987.1-2001.2	0.994*	0.756*	1.422*	high	2.835	603.0
		(0.003)	(0.224)	(0.496)			

The asterisks denote that the parameter value differs statistically significantly from zero at 5% level of significance. For single models the critical values are $\chi^2(4) = 13.277$ for the HJ test and $\chi^2(1) = 6.635$ for the J test. For the system, the correspondent values are $\chi^2(12) = 27.688$ and $\chi^2(9) = 21.666$. The critical values for the Wald and AF tests are $\chi^2(1) = 6.635$ and $\chi^2(3) = 11.345$, respectively.

estimators are likely to have bad small sample properties.²⁷

Table 1 gives results for the nondurables when only financial wealth is used in representative agent's optimal portfolio. The first panel of Table 1 gives results for the government bonds, the second for all bonds in market, the third for the average borrowing rate of commercial banks, and the fourth for the stock returns. The last panel depicts the results for system estimation consisting of three different time-series; all bonds in the market, borrowing rates and shares. Before the breakpoint in 1987, the rates of return between government and all bonds in the market were closely related to each other because the government was almost the sole emitter in Finland during that time. Therefore, to avoid collinearity between the regressors and the consequent biased results, we excluded the rates of return of government bonds in the system estimation. The Newey-West autocorrelation and heteroskedasticity consistent standard errors of the parameters are given in parentheses.

The results reveal that during the period 1975-1986, the point estimates for the discount factor were statistically significant and slightly exceeded one implying that the subjective rate of time preference should have been negative. The parameters γ and σ did not significantly differ from zero.²⁸ According to the Wald statistics, the restriction $\gamma = \sigma$ cannot be rejected which implies that the model is in favour of the standard expected utility specification.²⁹ The J test rejected all other models except the one with shares. The HJ test accepted the models for government and all bonds in the market. However, this statistically approval of the models can be spurious and be due to the statistical insignificance of the preference parameters.³⁰ The correspondent results for the time period 1987-2001 revealed different pattern of co-movement between consumption and the rates of return. First, the discount factor was estimated between 0.992-0.998 implying that the subjective rate of time preference is between 1-8 per cent. Second, the point estimates for the parameters γ and σ were significantly different from zero. Even though the estimates for γ and σ in some cases did not satisfy the restrictions, statistically they did not differ from the values slightly less than one. These values are still according to the theory. Only the estimate for σ in the case of

²⁷Under the financial regulation, the correlations between the different portfolio constructions and rates of return for shares were in the range of 0.6-0.8. For the other rates of return they were in the range of 0.1-0.7. After the deregulation, the correlations were 0.7-0.95 and around -0.3, respectively. Thus, the misspecification problem is more likely to exist when shares are used as a rate of return. See Ogaki (1993) for more detailed discussion on the possibility of an identification failure.

²⁸Note that the coefficient for relative risk aversion was defined as $(1-\gamma)$, and the elasticity of intertemporal substitution was $(1-\sigma)^{-1}$.

²⁹While there are many candidates for the test for parameter restrictions, Smith (1999) has proved that the Wald test has greater power than other tests when applied to nonlinear Euler equations.

³⁰Without a few exceptions in Table 3, in all that follows, it seems that there is a trade-off between the goodness of the model and the significance of the parameter estimates. If the time-series are relatively smooth, the procedure automatically converges to parameters close to zero. Contemporaneously, the disturbance terms in the estimation become small, and the model is more easily accepted.

shares did not satisfy the restriction.³¹ Third, according to the goodness-of-fit tests, the models seemed not to fit the data the model for the shares being the only exception. In general, the models were more easily accepted for the time period under the financial regulation than after it. Finally, the AF test statistics rejects strongly the hypothesis of stable parameter values over the different time periods. Again, however, the reduction to the standard expected utility model cannot be rejected according to the Wald test, the model for the shares being the only exception.

Table 2 shows the results when services are used as a measure of consumption. In contrast to the results above, there are two exceptions that are noteworthy. First, in most cases the point estimates both for γ and σ were lower for services than for nondurables implying higher values for relative risk aversion and elasticity of intertemporal substitution. Second, the models were more easily accepted for the first and rejected for the second research period according to the test statistics.

Table 2
GMM results for model $\beta^{\frac{\gamma}{\sigma}} E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma}{\sigma}(\sigma-1)} (R_{t+1}^P)^{\frac{\gamma}{\sigma}-1} R_{t+1}^i \right\} = 1$,
services, portfolio 1

Asset	Period	β	γ	σ	J^*	<i>Wald</i>	<i>AF</i>
GBOND	1975.2-1986.4	1.014* (0.017)	-0.620 (7.712)	0.239 (1.053)	0.011	0.010	
	1987.1-2001.2	0.996* (0.001)	0.478* (0.119)	0.585* (0.181)	165.1	0.891	7320.0
BOND	1975.2-1986.4	1.014* (0.016)	-0.656 (7.812)	0.249 (0.973)	0.013	0.011	
	1987.1-2001.2	0.996* (0.001)	0.502* (0.120)	0.608* (0.180)	169.4	0.859	7454.6
RRATE	1975.2-1986.4	1.014* (0.020)	-0.069 (3.542)	0.066 (2.825)	0.314	0.078	
	1987.1-2001.2	0.996* (0.001)	0.789* (0.132)	0.924* (0.165)	143.6	0.789	5537.8
SHAR	1975.2-1986.4	1.009* (0.125)	-9.608 (10.27)	0.829 (1.245)	0.633	1.395	
	1987.1-2001.2	0.994* (0.006)	-8.074* (2.175)	2.390* (0.913)	61.57	50.33	152.4
SYSTEM	1975.2-1986.4	1.022* (0.004)	-0.130* (0.057)	-1.005 (0.549)	201.6	3.140	
	1987.1-2001.2	0.997* (0.002)	0.651* (0.134)	1.343* (0.258)	high	10.10	2338.1

See Table 1 for the critical values for the different test statistics.

³¹All the reported results are from unrestricted estimations. Typically, the inclusion of the parameter restrictions resulted in estimates close to the boundaries, and the models were highly rejected.

Otherwise, the interpretation of the results follows closely to those for non-durables. All the point estimates differed statistically significantly from zero in the second period. The standard errors for the estimates, which did not satisfy the parameter restrictions, implied that the parameters can indeed fulfill the restrictions. The standard expected utility model cannot be rejected, and the estimates were statistically significantly different between the two time periods indicating a structural change in the preference estimates.

Table 3
GMM results for model $\beta \frac{\gamma}{\sigma} E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma}{\sigma}(\sigma-1)} (R_{t+1}^P)^{\frac{\gamma}{\sigma}-1} R_{t+1}^i \right\} = 1$,
nondurables, portfolio 2

Asset	Period	β	γ	σ	J^*	<i>Wald</i>	<i>AF</i>
GBOND	1975.2-1986.4	1.007* (0.005)	1.188 (0.917)	0.808* (0.361)	13.75	0.232	
	1987.1-2001.2	0.990* (0.004)	0.925* (0.220)	1.059* (0.313)	1.673	1.629	1017.8
BOND	1975.2-1986.4	1.007* (0.005)	1.175 (0.909)	0.806* (0.362)	13.54	0.224	
	1987.1-2001.2	0.990* (0.003)	0.945* (0.215)	1.078* (0.304)	0.870	1.721	1007.5
RRATE	1975.2-1986.4	1.009* (0.004)	1.241 (0.951)	0.847* (0.356)	12.42	0.225	
	1987.1-2001.2	0.991* (0.002)	1.239* (0.195)	1.362* (0.266)	0.313	1.700	1080.3
SHAR	1975.2-1986.4	1.013* (0.002)	-2.640* (0.995)	0.976* (0.431)	23.07	10.77	
	1987.1-2001.2	1.021* (0.020)	-7.368* (2.902)	7.920 (6.774)	282.2	2.898	230.9
SYSTEM	1975.2-1986.4	0.959 (0.516)	-0.111 (0.321)	-5.155 (72.78)	high	0.005	
	1987.1-2001.2	0.993* (0.001)	2.084* (0.667)	2.379 (1.247)	high	0.170	647.3

See Table 1 for the critical values for the different test statistics.

Tables 3 and 4 repeat the results when the optimal portfolio was augmented to include housing wealth. While these results quantitatively slightly differed from those of the Tables 1 and 2, qualitatively the interpretations remained nearly the same. In Table 3 for the time period under the financial regulation, the discount factor was again higher than one implying that the subjective discount rate had been negative. The parameters for intertemporal substitution differed statistically significantly from zero, while the ones for the risk aversion did not

except the ones for shares. The Wald statistics indicated that the reduction to the standard expected utility specification cannot be rejected.³²

Again, the results for the time period 1987-2001 revealed a different pattern for the co-movement of consumption and asset returns. Except for the shares, the point estimates differed statistically significantly from zero and the subjective discount rate was positive. Contrary to the results in Tables 1 and 2, the models were more easily accepted according to the HJ and J test statistics for the second period than the first. The AF test statistics was in favour of the structural difference in preference parameters due to the financial liberalisation.

Table 4
GMM results for model $\beta_{\sigma}^{\frac{\gamma}{\sigma}} E_t \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma}{\sigma}(\sigma-1)} (R_{t+1}^P)^{\frac{\gamma}{\sigma}-1} R_{t+1}^i \right\} = 1,$
services, portfolio 2

Asset	Period	β	γ	σ	J^*	$Wald$	AF
GBOND	1975.2-1986.4	1.013* (0.017)	0.334 (2.054)	0.206 (1.562)	1.230	0.063	
	1987.1-2001.2	0.992* (0.006)	0.790* (0.373)	0.882 (0.579)	311.7	0.183	1051.2
BOND	1975.2-1986.4	1.013* (0.017)	0.310 (2.067)	0.190 (1.540)	1.254	0.049	
	1987.1-2001.2	0.992* (0.006)	0.789* (0.370)	0.868 (0.560)	307.4	0.150	585.6
RRATE	1975.2-1986.4	1.014* (0.016)	0.348 (1.986)	0.229 (1.649)	0.009	0.108	
	1987.1-2001.2	0.996* (0.006)	0.855* (0.288)	0.895* (0.405)	420.3	0.037	239.9
SHAR	1975.2-1986.4	1.013* (0.053)	-2.119 (8.669)	0.990 (7.367)	1.825	0.038	
	1987.1-2001.2	1.027* (0.007)	0.748 (3.147)	-0.287 (1.109)	396.3	0.059	297.8
SYSTEM	1975.2-1986.4	1.001* (0.014)	-0.135 (0.420)	0.409 (1.374)	201.4	0.092	
	1987.1-2001.2	0.998* (0.002)	1.593* (0.185)	1.468* (0.284)	high	0.709	92.8

See Table 1 for the critical values for the different test statistics.

Table 4 shows the results when services were used as a measure of consumption. In contrast to the previous results, there seems to be some differences. First,

³²Because of the significance of the parameter estimates indicates logarithmic risk preferences, we re-estimated the equation (2.28) with the same instrument set. The results, -0.012 (0.005) for the subjective time preference and 0.774 (0.336) for the intertemporal substitution, confirm the findings of Table 3.

except for that of the discount factor, the point estimates were typically not statistically significant. Only the estimates for risk aversion in the second research period had statistical power. Second, the estimates for γ were typically positive in all models and for both time periods. Third, according to the J and HJ tests, the models were highly rejected for the period after the financial deregulation. The Wald statistics were again in favour of the standard expected utility model, and the AF test was against the structural stability of the preference parameters.

5.2. Interpretation and Comparison of Results

Before turning to the interpretation of the reported results, it would be fertile to discuss some other issues and alternatives concerning the non-expected model and its estimation. In addition to the reported results above, we put a lot of effort to finding out whether the different parametrisation of the model (such as Giovannini & Weil (1989), Kandel & Stambaugh (1991), Campbell (1998), Bansal & Yaron (2000), among others), the optimal value functions (Abel (1990), Weil (1990), Jorion & Giovannini (1993)), the algorithms, and the instrument sets can improve the statistical performance of the estimates and the model. In spite of these experiments, we found that empirically the original model by Epstein & Zin (1989, 1991) performed best. The more complex parametrisation of the model typically hampered the estimation, and led to estimates that did not fulfil the restrictions. Also, these models performed poorly according to the goodness-of-fit statistics. We also found that the GMM estimates are somewhat sensitive to the instrument sets. The sets with own lags of the variables involved in the model sometimes resulted in estimates the magnitude of which is quite different from the reported ones, and which performed statistically “better”. However, as discussed earlier, because of the likely spuriousness, we do not report them³³.

Statistically, only in few cases we were able to accept the overidentifying restrictions imposed for the non-expected model. Rather, the tests were in a favour of reduction to the standard von Neumann-Morgenstern time and state separable expected utility specification, where the RRA coefficient equals the reciprocal of the IES. The rejection of the non-expected model (Epstein & Zin (1991), Jorion & Giovannini (1993)) seems to be a common phenomenon and is not restricted to the parametrisation of the model, different consumption measures, rates of return, instrument sets etc. In literature, several reasons have been presented to explain the poor performance of the model. For example, Bufman & Leiderman (1990) argued that the rejection may result from the relatively smooth time-series data, and the models fit empirically better when they are tested under a more volatile environment.³⁴ Wheatley (1988) argued that when the consumption is

³³In most cases, however, the parameter estimates in these experiments did not significantly differ from zero or the values reported.

³⁴Most of the empirical results are obtained from postwar U.S. economy. The time-series for this period, however, are relatively tranquil when measured by standard deviations. The results from other data sets are more encouraging: Bufman & Leiderman (1990) and Koskiewicz (1999) found statistical support for the model with Israelian and French data, respectively. We also found that the model performed best with share

measured with error, test statistics can be biased towards the rejection. Many simulation tests (Tauchen (1986) and Nelson & Startz (1990) among others) have performed that the standard errors of the parameters and goodness-of-fit tests of the model are sensitive to the small sample properties of the GMM estimation. When the sample size increases, the point estimates remain nearly the same, while their standard errors decrease together with smaller error in the estimation. Thus, the model is more easily accepted and the estimates have a greater statistical explanatory power.³⁵ Contradictory, Smith (1999) studied the small sample properties of Epstein-Zin type model and showed that the estimated asymptotic standard errors associated with the parameter estimates can be biased downward, and the tests of the model against an alternative one can be overrejected. Therefore, a researcher may incorrectly reject the expected utility based model in favor of the non-expected utility model.³⁶

To remind of the restrictions in estimation, the results can be cautiously interpreted and summarised as follows. First, despite of the different consumption measures, portfolio construction and the rates of return, the discount factor was above the restriction under the financial regulation, but after the deregulation it was slightly less than one implying an economically sensible behaviour. Second, under the financial regulation, the models with only financial wealth in consumer's portfolio typically resulted estimates for the risk coefficient which were less but did not statistically differ from zero. This implies that the RRA coefficient was close to one. After the deregulation, the coefficients turned out to be positive and differed statistically significantly from zero. This implies that the RRA has declined over time. Even though the point estimates for the risk aversion coefficient differed when the portfolio was augmented to include the housing wealth, their interpretation remained the same. Third, the coefficient for the RRA was considerably higher when shares were used as the rate of return. This held over different time periods and consumption measures. Fourth, under the regulation, the estimates for the substitution parameter did not differ statistically from zero, which implies that the IES was close to one. Only in the case of nondurables together with the augmented portfolio, the estimates indicated a higher IES. After the deregulation, the IES has increased and was in the range of 2–10. Fifth, under the financial regulation, the point estimates implied an early resolution of uncertainty when only financial wealth is used in portfolio, and late

yields, which are the most volatile time-series.

³⁵For example, consider that the model is “right” and $\gamma = -1.5$, $\sigma = 0.9$ provide a plausible combination of the parameters implying $RRA = 2.5$ and $IES = 10$. Because of the small sample properties, however, the large standard errors may lead to the wrong conclusion that both of the parameters do not statistically differ from zero and from each other. This leads to the rejection of the non-expected utility maximisation.

³⁶Smith (1999) showed also that the reason for the low power of the test statistics under the small sample is not the usage of “poor” instruments. Rather, he conjectured that the nature of the nonlinear Euler restrictions, the functional form, induces correlations among the columns of the Jacobian matrix. These dependencies degrade the preciseness of the parameter estimates much in the same way as collinear data in a linear model. In some experiments, we also found that the weighting matrix of GMM was estimated nearly singular and it was difficult to find suitable starting values. This collinearity between the columns of the matrix confirms Smith's conjecture.

resolution, when the portfolio was augmented to include housing wealth. For both portfolio constructions, however, an indifference towards the timing of resolution cannot be rejected because of the statistical significance of the parameters. After the deregulation, an early resolution of uncertainty was preferred.

To confirm, if the magnitude of the parameter describing intertemporal substitution is robust, we run a simple regression between the logarithmic growth rate of consumption and the rate of return together with several instruments.³⁷ The coefficient in this regression can be interpreted as the IES. The results of these experiments indicated a similar structural change in consumption as above. Under the financial regulation, the point estimates were in the range of -2-0.2 for nondurables and 0.1-2.8 for services, but did not statistically differ from zero despite of the consumption measures, rates of return and instrument variables. After the deregulation, the point estimates were in the range of 0.2-15 for nondurables and 0.2-6.5 for services. Most of these estimates turned out to be statistically significant. Thus, even though the results seem to be in accordance with those reported, the question whether the risk aversion parameter differs here statistically from that of the intertemporal substitution remains.

According to the results, it is evident that there has been a structural change in consumption and investment behaviour because of the financial liberalisation and the structural change in Finnish economy in the middle of 1980s. The point estimates for relative risk aversion (RRA) and intertemporal elasticity of substitution (IES) clearly reveal a different consumption pattern for these two time periods and different consumption measures. Necessarily, this does not prove that the attitude towards risk and the timing of consumption have changed. Rather, under the regulation the inability to follow an optimal intertemporal life-cycle consumption/saving behaviour was not possible because of the restrictions in loan markets. Also, the undeveloped investment markets restricted the investment possibilities. This can be seen from the point estimates of preference parameters which turned out to be statistically insignificant and, especially, from the estimates of discount factor which imply that the subjective rate of time preference has been negative. Economically, this result is unreasonable.

The results also confirmed the previous evidence with Finnish data by Takala (2001), Starck (1987), Kostiainen & Starck (1991) and Svento (1990). According to the excess sensitivity tests for consumption, Takala (2001) found that the proportion of liquidity constrained consumers in Finland has declined over time. However, it is noteworthy that the financial deregulation did not immediately decline this proportion until after the recession in the early 1990s. Before the recession, the proportion of liquidity constrained consumers was above 60%, but in the turn of the century it was about 30%. Based on the standard Euler equation approach, Starck (1987) and Kostiainen & Starck (1991) found that the elasticity of intertemporal substitution has been weak in Finland during

³⁷See Hall (1988) and Beaudry & van Wincoop (1996) for a theoretical justification of this.

the financial regulation. Svento (1991) observed that the yields from shares cannot explain the consumption behaviour in Finland. We also found that these observations hold true under the financial regulation. Recently, however, the development of investment markets and an increase in asset ownership in Finland seem to have an explanatory power in consumption to a certain extent, and enable a more efficient consumer's optimisation policy. This, for instance, is reflected by a higher values of IES. The results, on the other hand, contradict the findings in chapter 2 of this thesis, in which, using time-nonseparable preferences, we found that the relative risk aversion has slightly increased over time. While the magnitude of the RRA in that study is between zero and one after the financial liberalisation and coincides the results from the present study, it is likely to suspect that this difference in interpretation is due to the different parametrisation of the models or the inability of the Euler equation approach to explain the consumption behaviour under the financial market regulation.

The international evidence of the range of the preference parameters is ample.³⁸ With U.S. aggregate data, the estimates for the elasticity of intertemporal substitution typically extend from values close to zero (Hall (1988), Atkeson & Ogaki (1996), Vissing-Jorgensen (2002), some to mention) to values close to one (Epstein & Zin (1991)). With U.S. panel data, Beaudry & van Wincoop (1996) found evidence that the IES is significantly different from zero and probably close to one. Bufman & Leiderman (1990) found statistical evidence for the non-expected utility in Israel's economy with the IES ranging between 2-6. Koskiewicz (1999) included leisure into the model and found that the IES is about 3 with French data. Wirjanto (1995) studied the effect of liquidity constraints on consumption and found that IES is between 0.2-0.3 with Canadian aggregate data.

The international estimates for the RRA are even more volatile. Typically, the estimates for the RRA by a numerical simulations are higher than the estimates from time-series. For example, in their numerical simulations Kandel & Stambaugh (1991) found that the relative risk aversion in U.S. should be as high as 29 to match the observed asset returns and equity premiums. Attanasio & Weber (1989) estimated the RRA coefficient to be the same magnitude (5-30) for the UK economy. From the U.S. aggregate data, Epstein & Zin (1991) found values close to one while Jorion & Giovannini (1993) observed that RRA does not differ statistically from zero. Using consumption expenditures grouped by consumer income Smoluk & Neveu (2002) found even negative estimates for RRA. Constantinides et al. (1998) suggested that most probably the RRA is in the range of 2-5. Bufman & Leiderman (1990) found that in Israel the RRA is between 1.4-1.6 while Koskiewicz (1999) found that it is close to zero in France.

³⁸In literature, the number of empirical studies concerning the subject is extensive. For comparison, we review only few of them even though the comparison of the magnitude of the parameters is difficult. They vary in terms of parametrisation, restrictions in economic environment, data, econometric methodology etc. Currently, there exists a large debate if the magnitude of the preference parameters with US economy is high or low. See Neely et al. (1999) and Giuliano & Turnovsky (2000) for a review of this discussion.

While we find our results highly encouraging and in accordance with the international evidence, further research with Finnish data should be carried out along the lines we have indicated. Especially the small sample properties of the model should be studied. Also, the model ignores many important phenomena that affect consumption and investment behaviour. In particular, the consumption in excess of the subsistence of necessary goods may be less substitutable across time than is the consumption of other goods. Fauvel & Samson (1991) included durable goods into the consumer's choice problem, and found that this improves the performance of the model and gives more precise estimates. Alternatively, Vissing-Jorgensen (2002) found that the limits to the asset market participation are an important determinant for the IES. She found evidence that the IES between assetholders and non-assetholders are large: the larger the asset holdings, the higher the IES. This confirms the previous results by Atkeson & Ogaki (1996) who observed that the IES for rich consumers is higher than for poor consumers. Seckin (2000) combined the recursive preferences and habit formation. Unfortunately, his theoretical first-order condition for consumption and asset pricing seems to be too complex for empirical work. While such extensions, and many others, have not been carried out with Finnish data, one cannot give a clear answer for the precise estimates of the preference parameters unless a thorough research is conducted to evaluate these statements closely.

6. CONCLUDING REMARKS

The aim of this study was to provide empirical evidence of the magnitude of the relative risk aversion and elasticity of intertemporal substitution in Finland, and to study how these preference parameters have changed over time because of the financial liberalisation in the middle of 1980s. The theoretical model was based on the non-expected utility optimization as originally expressed by Epstein & Zin (1989, 1991) and Weil (1990). Unlike the standard von Neumann-Morgenstern expected utility framework, non-expected utility representation enables us to disentangle independently the parameters representing risk aversion and willingness to substitute consumption over time while maintaining the desired properties of utility maximisation. The knowledge of the magnitude of these parameters plays a key role in many policy evaluations, and it helps to understand the effects of monetary policy for consumption decisions and business cycles.

Even though the statistical performance of the models were poor, the GMM estimates based on the aggregate *per capita* consumption growth for nondurables and services, different real rates of return and optimal portfolio constructions reveal a structural change in consumption and investment decisions. This has occurred because of the financial liberalisation and change in economic environment in Finland. The results indicate that after the financial liberalisation the relative risk aversion is between zero and one. The results also confirm the suspicion that an investment in shares has been the most risky investment decision across time

and policy regimes. The intertemporal elasticity of substitution has increased over time, and is in the range of 2-10. However, we were not able to reject the hypothesis that the model differs from that of the standard utility maximisation in which the parameters covering the willingness to substitute consumption over time and risk aversion are inversely related. Also, the results revealed that on average the consumers in Finland prefer early resolution of uncertainty. Thus, they dislike the risk in consumption more than the intertemporal fluctuations of consumption.

Evidently, the reason for the structural change in preference parameters is the consumers' better ability to follow their optimal consumption and investment behaviour over the life-cycle. Thus, it is possible to save and defer consumption over time by saving for a rainy day. This implies that there seems to be some efficiency of monetary policy transmission mechanism at making consumers in aggregate to defer consumption by raising their expectations of real rate of returns.

REFERENCES

- Abel, A.B. (1990): "Consumption and Investment." in *Handbook of Monetary Economics*, Vol II, eds. by Friedman, B.M. and F.H. Hahn. Elsevier Science Publishers, B.V. The Netherlands.
- Andrews, D.W.K. & Fair, R. (1988): "Inference in Econometric Models with Structural Change." *Review of Economic Studies* 55, 615-640.
- Atkeson, A. & Ogaki, M. (1996): "Wealth-Varying Intertemporal Elasticities of Substitution: Evidence from Panel and Aggregate Data." *Journal of Monetary Economics* 38, 507-534.
- Attanasio, O.P. (1998): "Consumption Demand." *NBER Working Paper No. 6466*.
- Attanasio, O.P. & Weber, G (1989): "Intertemporal Substitution, Risk Aversion, and the Euler Equation for Consumption." *Economic Journal* 60, 119-127.
- Bansal, R. & Yaron, A. (2000): "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles." *NBER Working Paper No. 8059*.
- Barsky, R.E. (1989): "Why Don't the Prices of Stocks and Bonds Move Together?" *The American Economic Review* 79, 1132-1145.
- Basu, P. & Ghosh, S. (1993): "Optimal Saving under General Changes in Uncertainty: A Nonexpected Utility Maximizing Approach." *Southern Economic Journal* 60, 119-127.
- Beaudry, P. & van Wincoop, E. (1996): "The Intertemporal Substitution: An Exploration Using a US Panel of State Data." *Economica* 63, 495-512.

- Bufman, G. & Leiderman, L. (1990): "Consumption and Asset Returns under Non-Expected Utility." *Economics Letters* 34, 231-235.
- Campbell, J.Y. (1998): "Asset Prices, Consumption, and the Business Cycle." *NBER Working Paper No. 6485*.
- Chacho, G. & Viceira, L.M (1999): "Dynamic Consumption and Portfolio Choice with Stochastic Volatility in Incomplete Markets." *NBER Working Paper No. 7377*.
- Constantinides, G.M., Donaldson, J.B. & Mehra, R. (1998): "Junior Can't Borrow: A New Perspective on the Equity Premium Puzzle." *NBER Working Paper No. 6617*.
- Epstein, L.G. (1988): "Risk Aversion and Asset Prices." *Journal of Monetary Economics* 22, 179-192.
- Epstein, L.G. & Zin, S.E. (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework." *Econometrica* 57, 937-969.
- Epstein, L.G. & Zin, S.E. (1991): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis." *Journal of Political Economy* 99, 263-286.
- Farmer, R.E.A. (1990): "RINCE Preferences." *The Quarterly Journal of Economics* 105, 43-60.
- Fauvel, Y. & Samson, L. (1991): "Intertemporal Substitution and Durable Goods: An Empirical Analysis." *Canadian Journal of Economics* 23, 192-205.
- Ferson, W.E. & Constantinides, G.M. (1991): "Habit Persistence and Durability of Aggregate Consumption." *Journal of Financial Economics* 29, 199-240.
- Giovannini, A. & Weil, P. (1989): "Risk Aversion and Intertemporal Substitution in the Capital Asset Pricing Model." *NBER Working Paper No. 2824*.
- Giuliano, P. & Turnovsky, S.J. (2000): "Intertemporal Substitution, Risk Aversion, and Economic Performance in a Stochastically Growing Open Economy ." Discussion Paper No. 2/2000. Department of Economics, University of Washington.
- Grossman, S.J. & Shiller, R.J. (1981): "The Determinants of the Variability of Stock Market Prices." *American Economic Review* 71, 222-227.
- Haliassos, M. & Hassapis, C. (2001): "Non-Expected Utility, Saving and Portfolios." *The Economic Journal* 111, 69-102 Harvester Wheatsheaf, UK.
- Hall, R.E. (1988): "Intertemporal Substitution in Consumption." *Journal of Political Economy* 96, 339-357.
- Hall, A.R. & Sen, A. (1999): "Structural Stability Testing in Models Estimated by Generalized Method of Moments." *Journal of Business & Economic Statistics* 17, 335-348.

- Hansen, L.P. (1982): "Large Sample Properties of Generalized Method of Moments Estimators." *Econometrica* 50, 1029-1054.
- Hansen, L.P. & Jagannathan, R. (1989): "Restrictions on Intertemporal Marginal Rates of Substitution Implied by Asset Returns." Northwestern University, Department of Finance. Working Paper 12.
- Hansen, L.P. & Singleton, K. (1982): "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." *Econometrica* 50, 1269-1286.
- Hansen, L.P. & Singleton, K. (1983): "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns." *Journal of Political Economy* 91, 249-265.
- Hassler, J. (1996): "Risk and Consumption." *Swedish Economic Policy Review* 2, 303-337.
- Hayashi, F. (2000): *Econometrics*. Princeton University Press. Princeton, New Jersey.
- Johnsen, T.H. & Donaldson, J.B. (1985): "The Structure of Intertemporal Preferences under Uncertainty and Time Consistent Plans." *Econometrica* 53, 1451-1458.
- Jorion, P. & Giovannini, A. (1993): "Time-Series Tests of a Non-Expected-Utility Model of Asset Prices." *European Economic Review* 37, 1083-1100.
- Kandel, S. & Stambaugh, R.F. (1991): "Asset Returns and Intertemporal Preferences." *Journal of Monetary Economics* 27, 39-71.
- Kocherlakota, N.R. (1990): "Disentangling the Coefficient of Relative Risk Aversion from the Elasticity of Intertemporal Substitution: An Irrelevant Result." *The Journal of Finance*, 175-190.
- Koopmans, T. (1960): "Stationary Ordinal Utility and Impatience." *Econometrica* 28, 287-309.
- Koskiewicz, Jean-Max. (1999): "An Intertemporal Consumption-Leisure Model with Non-Expected Utility." *Economics Letters* 64, 285-289.
- Kostiainen, S. & Starck, C. (1991): "Does Government and Corporate Saving Influence Household Saving in Finland?" *Liiketaloudellinen aikakauskirja* 2/91, 128-144.
- Kreps, D.M. (1990): *A Course in Microeconomic Theory*. Harvester Wheatsheaf, UK.
- Kreps, D.M. & Porteus, E. (1978): "Temporal Resolution of Uncertainty and Dynamic Choice Theory." *Econometrica* 46, 185-200.
- Kreps, D.M. & Porteus, E. (1979a): "Dynamic Choice Theory and Dynamic Programming." *Econometrica* 47, 91-100.

- Kreps, D.M. & Porteus, E. (1979b): "Temporal Von Neumann-Morgenstern and Induced Preferences." *Journal of Economic Theory* 20, 81-109.
- Mátyás, L. (1999): *Generalized Method of Moments Estimation*. Cambridge University Press, Cambridge, UK.
- Mehra, R & Prescott, E. (1985): "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15, 867-887.
- Neely, C., Roy, A. & Whiteman, C. (1999): "Risk Aversion vs. Intertemporal Substitution: Identification Failure in the Intertemporal Consumption CAPM." Federal Reserve Bank of St. Louis, Discussion Paper 95-002C.
- Nelson, C. & Startz, R. (1990): "The Distribution of Instrumental Variables Estimator and its t-ratio when the Instrument is a Poor One." *Journal of Business* 63, 125-140.
- Obstfeld, M. (1994): "Evaluating Risky Consumption Paths: The Role of Intertemporal Substitutability." *European Economic Review* 38, 1471-1486.
- Ogaki, M. (1993): "Generalized Method of Moments: Econometric Applications." in *Handbook of Statistics* 11 eds. by Maddala, G.S., Rao, C.R. and H.D. Vinod. Elsevier Science Publishers.
- Prasad, K. (1991): "Consumption, Asset Returns and Taxes in a Nonexpected Utility Model." *Economic Letters* 37, 433-437.
- Qian, H. & Schmidt, P. (1999): "Improved Instrumental Variables and Generalized Methods of Moments Estimators." *Journal of Econometrics* 91, 145-169.
- Roll, R. (1977): "A Critique of the Asset Pricing Theory's Tests: Part I: On Past and Potential Testability of the Theory." *Journal of Financial Economics* 4, 129-176.
- Ruud, P.A. (2000): *An Introduction to Classical Econometric Theory*. Oxford University Press. New York.
- Seckin, A. (2000): "Habit Formation with Recursive Preferences." CIRANO Working Paper 2000s-43.
- Selden, L. (1978): "A New Representation of Preferences over 'Certain \times Uncertain' Consumption Pairs: The Ordinal Certainty Equivalent Hypothesis." *Econometrica* 46, 1045-1060.
- Selden, L. (1979): "An OCE Analysis of the Effect of Uncertainty on Saving under Risk Independence." *Review of Economic Studies* 46, 73-82.
- Smith, D.C. (1999): "Finite Sample Properties of Tests of the Epstein-Zin Asset Pricing Model." *Journal of Econometrics* 93, 113-148.
- Smoluk, H.J. & Neveu, R.P. (2002): "Consumption and Asset Prices. An Analysis across Income Groups." *Review of Financial Economics* 11, 47-62.
- Sowell, F (1996): "Optimal Tests for Parameters Instability in the Generalized Method of Moments Framework." *Econometrica* 64, 1085-1107.

- Starck, C. (1987): "Intertemporal Substitution in Consumption: Some Empirical Evidence from Finnish Data." *Liiketaloudellinen aikakauskirja* 2/87, 128-143.
- Svensson, L.E.O. (1989): "Portfolio Choice with Non-Expected Utility in Continuous Time." *Economic Letters* 30, 313-317.
- Svento, R. (1990): "Tutkimus kulutusfunktion tulokäsitteestä." Research Reports No. 34. University of Oulu, Department of Economics. (In Finnish)
- Takala, K. (2001): "Studies in Time Series Analysis of Consumption, Asset Prices and Forecasting." Bank of Finland Studies E:22.
- Tauchen, G. (1986): "A Note on the Asymptotic Lower Bound for the Covariance Matrix of the GMM Estimator of the Parameters of Agents' Utility Functions." *Economics Letters* 20, 151-155.
- Van Der Ploeg, F. (1993): "A Closed-Form Solution for a Model of Precautionary Saving." *Review of Economic Studies* 60, 385-395.
- Vissing-Jorgensen, A. (2002): "Limited Asset Market Participation and the Elasticity of Intertemporal Substitution." *NBER Working Paper* No. 8896.
- Wang, S. (1993): "The Local Recoverability of Risk Aversion and Intertemporal Substitution." *Journal of Economic Theory* 59, 333-363.
- Weil, P. (1990): "Nonexpected Utility in Macroeconomics." *The Quarterly Journal of Economics* 105, 29-42.
- Weil, P. (1993): "Precautionary Savings and the Permanent Income Hypothesis." *Review of Economic Studies* 60, 367-383.
- Wheatley, S. (1988): "Some Tests of the Consumption-Based Asset Pricing Model." *Journal of Monetary Economics* 22, 193-215.
- Wirjanto, T.S. (1995): "Aggregate Consumption Behaviour and Liquidity Constraints: The Canadian Evidence." *Canadian Journal of Economics* 28, 1135-1152.

APPENDIX

This Appendix proves that the undetermined constants are given as in equations (2.19) and (2.20), and that the closed-form solution is given as in (2.25). The maximisation problem (2.16) subject to (2.15) and (2.17) is reduced to

$$\max_{c_t} \left\{ c_t^\sigma + \beta(W_t - c_t)^\sigma E_t \left[(\phi_{t+1} R_{t+1}^P)^\gamma \right]^{\frac{\sigma}{\gamma}} \right\}^{\frac{1}{\sigma}} \quad (\text{A.1})$$

or

$$\max_{c_t} \left\{ c_t^\sigma + \beta(W_t - c_t)^\sigma \xi_t \right\}^{\frac{1}{\sigma}}, \quad (\text{A.2})$$

in which $\xi_t = E_t [(\phi_{t+1} R_{t+1}^P)^\gamma]^\frac{\sigma}{\gamma}$. Because of the structure of the model, consumption-saving decision can be separated from that of the portfolio optimisation. The first-order condition for the optimal consumption for the period t is given by

$$\frac{\{c_t^\sigma + \beta(W_t - c_t)^\sigma \xi_t\}^\frac{1}{\sigma-1}}{\sigma} \{\sigma c_t^{\sigma-1} - \beta \sigma (W_t - c_t)^{\sigma-1} \xi_t\} = 0. \quad (\text{A.3})$$

This equality holds true if

$$c_t^{\sigma-1} = \beta (W_t - c_t)^{\sigma-1} \xi_t. \quad (\text{A.4})$$

According to the assumption, $c_t = \psi_t W_t$, the equation (A.4) can be rewritten as

$$\begin{aligned} \psi_t^{\sigma-1} W_t^{\sigma-1} &= \beta (1 - \psi_t)^{\sigma-1} W_t^{\sigma-1} \xi_t, \\ \psi_t^{\sigma-1} &= \beta (1 - \psi_t)^{\sigma-1} \xi_t \end{aligned} \quad (\text{A.5})$$

After some rearrangements, this expression can be defined as in (2.19) in the main text. According to the envelope theorem, the indirect utility function should fulfil the maximisation problem.

$$\begin{aligned} \phi_t W_t &= \max \{c_t^\sigma + \beta(W_t - c_t)^\sigma \xi_t\}^\frac{1}{\sigma} \\ &= \left\{ \psi_t^\sigma W_t^\sigma + \beta(1 - \psi_t)^\sigma W_t^\sigma \frac{\psi_t^{\sigma-1}}{\beta(1 - \psi_t)^{\sigma-1}} \right\}^\frac{1}{\sigma} \\ &= \{\psi_t^\sigma W_t^\sigma + (1 - \psi_t) W_t^\sigma \psi_t^{\sigma-1}\}^\frac{1}{\sigma} \\ &= \{\psi_t^\sigma W_t^\sigma + \psi_t^{\sigma-1} W_t^\sigma - \psi_t^\sigma W_t^\sigma\}^\frac{1}{\sigma} \\ &= \psi_t^\frac{\sigma-1}{\sigma} W_t. \end{aligned} \quad (\text{A.6})$$

which leads to the equation (2.20).

Next we give the detailed derivation of the closed-form Euler equation (2.25) in the main text. Because of the recursive and stationary structure of the problem, the results above hold also for the period $(t + 1)$. Thus,

$$\phi_{t+1} = \psi_{t+1}^\frac{\sigma-1}{\sigma} = \left(\frac{c_{t+1}}{W_{t+1}} \right)^\frac{\sigma-1}{\sigma} = \left(\frac{c_{t+1}}{R_{t+1}^P} \right)^\frac{\sigma-1}{\sigma} (W_t - c_t)^\frac{1-\sigma}{\sigma}, \quad (\text{A.7})$$

where $R_{t+1}^P = \sum_{i=1}^K \lambda_{i,t} R_{t+1}^i$. Inserting (A.7) into the definition of ξ_t and this to the equation (A.5) results

$$\begin{aligned} \psi^{\sigma-1} &= \beta(1 - \psi)^{\sigma-1} E_t \left[\left(\left(\frac{c_{t+1}}{R_{t+1}^P} \right)^\frac{\sigma-1}{\sigma} (W_t - c_t)^\frac{1-\sigma}{\sigma} R_{t+1}^P \right)^\gamma \right]^\frac{\sigma}{\gamma} \\ \left(\frac{c_t}{W_t} \right)^\sigma &= \beta \left(1 - \frac{c_t}{W_t} \right)^\sigma E_t \left[\left(\left(\frac{c_{t+1}}{R_{t+1}^P} \right)^\frac{\sigma-1}{\sigma} (W_t - c_t)^\frac{1-\sigma}{\sigma} R_{t+1}^P \right)^\gamma \right]^\frac{\sigma}{\gamma} \\ \left(\frac{c_t}{W_t} \frac{W_t}{W_t - c_t} \right)^\frac{\gamma(\sigma-1)}{\sigma} &= \beta^\frac{\gamma}{\sigma} E_t \left(\frac{c_{t+1}}{W_t - c_t} \right)^\frac{\gamma(\sigma-1)}{\sigma} (R_{t+1}^P)^\frac{\gamma}{\sigma}. \end{aligned} \quad (\text{A.8})$$

After some rearrangement this is reduced to

$$\beta^{\frac{\gamma}{\sigma}} E_t \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma(\sigma-1)}{\sigma}} (R_{t+1}^P)^{\frac{\gamma}{\sigma}} = 1, \quad (\text{A.9})$$

which corresponds the equation (2.22) in the main text. The optimality of the portfolio decision can be found by taking the first-order conditions of (A.9) with respect to the single weights $\lambda_{i,t}$:

$$\frac{\gamma}{\sigma} \beta^{\frac{\gamma}{\sigma}} E_t \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma(\sigma-1)}{\sigma}} (R_{t+1}^P)^{\frac{\gamma}{\sigma}-1} R_{t+1}^i = 0, \quad i = 1, \dots, K. \quad (\text{A.10})$$

In equilibrium, this condition should hold for every asset. Thus, for assets i and j :

$$\beta^{\frac{\gamma}{\sigma}} E_t \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma(\sigma-1)}{\sigma}} (R_{t+1}^P)^{\frac{\gamma}{\sigma}-1} [R_{t+1}^i - R_{t+1}^j] = 0, \quad \forall i, j \in K, i \neq j. \quad (\text{A.11})$$

Multiplying this equation by $\lambda_{j,t}$, summing over j , applying (A.9) and the definition $\sum_{j=1}^K \lambda_{j,t} = 1$ gives

$$\beta^{\frac{\gamma}{\sigma}} E_t \left(\frac{c_{t+1}}{c_t} \right)^{\frac{\gamma(\sigma-1)}{\sigma}} (R_{t+1}^P)^{\frac{\gamma}{\sigma}-1} R_{t+1}^i = 1, \quad i = 1, \dots, K, \quad (\text{A.12})$$

which is the closed-form solution (2.25).

Chapter 4

Costs, Uncertainty and Durable Adjustments - Finnish Cross-Sectional Evidence from Automobile Purchases

1. INTRODUCTION	143
2. (S,s) MODEL	145
2.1. Introduction to (S,s) Rule	145
2.2. Review of Irreversibility Literature	147
2.3. Model	150
2.4. Increase in Risk and Timing of Purchases	156
3. DATA DESCRIPTION	157
4. RESULTS	158
4.1. Estimation Methods	158
4.2. Estimation Results	162
4.3. Discussion and Evaluation of Results	166
5. CONCLUSION	170
References	171
Appendix A	174
Appendix B	175
Appendix C	181
Appendix D	183

Costs, Uncertainty and Durable Adjustments - Finnish Cross-Sectional Evidence from Automobile Purchases

by

Jari Viitanen

Government Institute for Economic Research

University of Joensuu, Department of Business and Economics

Abstract

Based on the Cobb-Douglas preferences and standard (S,s) model this study extends the previous theoretical frameworks by deriving explicitly the optimal consumption rule for durables. The model states that an agent has a desire to keep a fraction of his wealth to be invested in a durable. Because of the depreciation of the durable together with stochastic movements in prices and agent's wealth, the actual fraction deviates from that of the target. Including the possibility of uncertainty, the model shows that it is optimal for the agent to allow an inaction (S,s) band around the target level of durable to avoid irreversible investment costs, and not to adjust until the critical band trigger is reached. The implications of the model - the width of the inaction (S,s) band is positively related to an increase in uncertainty, while a higher depreciation rate leads to more frequent adjustment - were tested using four Household Budget Surveys from Finland. The results on automobile purchases did not reject the (S,s) model. Higher income uncertainty widens the inaction band and decreases the probability of adjustment, while an increase in repair costs increases the probability of adjustment.

Keywords: Uncertainty, irreversible investment, durable, (S,s) rule, idiosyncratic and systematic risk

1. INTRODUCTION

It is a well-known fact that uncertainty about future consumption possibilities determines also today's saving and consumption purchases. This has a direct link to business cycles. An increase in uncertainty contributes a fall in consumption and may lead the economy to a depression. In literature, large empirical and theoretical work has been carried out to understand the link between uncertainty and consumption. Especially two mechanisms have been found to be important in determining this link. The first one, originally introduced by Leland (1968), is known as a precautionary saving motive. An increase in uncertainty creates an incentive for an agent to increase the precautionary savings and, in Deaton's (1991) terminology, to exhibit buffer-stocks for a rainy day. This reduces today's consumption purchases.

The second mechanism which affects the timing of consumption purchases is the irreversibility mechanism which - besides on uncertainty - builds on the existence of transaction costs for durables. Typically, these costs should be paid every time when durables are purchased or sold, including searching and information cost (lemons problem), sales taxes and commissions to brokers, among others.¹ While the investments are often at least partially irreversible, it can sometimes be better to postpone the purchases until the consumer obtains more information about the future. As McDonald & Siegel (1986), Pindyck (1991) and Dixit & Pindyck (1994) have shown, uncertainty over income, asset and commodity prices, costs and other market conditions create an option value of waiting for new information to arise before adjusting consumption to the desired level. Therefore, there is often an incentive to delay the purchase/selling decisions to the future until new information arrives.²

At the individual level, uncertainty over future consumption possibilities may be divided into idiosyncratic and systematic risk. The first means a possibility that a consumer faces an unexpected shock in his nominal wealth due to, for example, accident or illness. The second means that he is conscious of the uncertainty which has an influence on the whole economy, and which may affect his wages and wealth position. An adverse supply shock which affects prices is a good example. As one or both of the uncertainty components increase, consumers postpone their decision-making more easily to the future and the stronger is the incentive to wait for new information. Clearly, if this increase in uncertainty affects many agents at the same time, the theory provides a link between individual purchases, aggregate consumption and business cycles.

¹Bernanke (1985) included the consumer's distaste for shopping and learning how to use new durables. Lam (1989) argued that there exist no perfect resale market for durables which creates additional costs for consumers.

²High technology such as mobile phones and computers provide other examples. Typically, the newest versions of those goods provide features that the older ones do not. Thus, there is always a temptation to postpone purchases until the latest version arrives, which carries an option value to wait.

Recent consumption research has considered irreversible investment, transaction costs and uncertainty to be crucial reasons in explaining individual investments in durables. Pindyck (1991) and Dixit & Pindyck (1994) have argued that the level of risk may be even more important than taxes and interest rates for investment decisions. The irreversible investment decision under uncertainty can be featured by the (S,s) model. According to this model, the durable is adjusted to a target level when the state variable crosses the critical lower or upper band trigger. When the state variable is inside the inaction band, the optimal policy is not to adjust the durable. The attractiveness of the (S,s) model is in its implications such as, in most of the times, consumers do not adjust their stock of the durables, and when they do, the adjustments are substantial and lumpy. However, the former (S,s) models (see e.g. Grossman & Laroque (1990) and Lam (1991)) explaining the effect of uncertainty suffer from the fact that risk is typically defined as a constant parameter. In the series of papers, Hassler (1994, 1996a,b,c) has combined the preliminary work of option value of waiting by McDonald & Siegel (1986) and the standard (S,s) model of inventory. His contribution to the model is that the risk level is in itself a stationary stochastic process over time. An increase in risk increases the value of waiting and the purchases are more easily postponed to the future. The shorter the high-risk periods are expected to be, the stronger is the incentive to wait for new information.

This paper concentrates on the irreversibility mechanism.³ The theoretical model extends the framework of Hassler (1994, 1996a) by deriving the intertemporal consumption rule explicitly. The foundation of the model comes from the famous optimisation results of the static Cobb-Douglas preferences which state that the optimum is a function of (relative) commodity prices, wealth and the (constant) parameter reflecting a fraction of total wealth spent on the good. This result is extended to the dynamic context so that a consumer has a desire to keep a fraction of his wealth close to a constant target value. Since the durable depreciates over time together with the stochastic movements in prices and consumer's wealth, the actual level of the fraction deviates from that of desired. While the transaction costs prevent an agent to adjust the stock of a durable continuously, the adjustment is not made until a critical lower (or upper) bound is reached. The model includes uncertainty over the future consumption possibilities by assuming that an agent faces the two types of risk as discussed above. First, he faces idiosyncratic risk. To model this, we follow Hassler (1994) and use standard Poisson process for unexpected jumps in prices and consumer's real wealth and show how a change in the expected time until the jump occurs will have an effect on the durable purchases. Second, he faces systematic risk which is defined as a switch between two states of the economy, namely, low and high risk state. The model shows how an increase in systematic risk will cause the inaction range to increase. Then, the purchases are postponed until the critical bounds are reached or there is a switch back to the low risk state.

³For the importance of precautionary saving motive and its empirical relevance, see Caballero (1990), Hubbard et al. (1995) and Carroll & Samwick (1996, 1998) and the referenced cited there.

The theoretical model implicates that an increase in uncertainty and adjustment costs should have a positive effect on the width of the (S,s) inaction bands while a greater depreciation should lead to more frequent adjustment. The validity of implications of the model is then tested using three distinct methods and data on automobile purchases from four cross-sectional Finnish Household Budget Surveys conducted by Statistics Finland. First, the width of the inaction (S,s) band is tested against the household-level uncertainty which is measured by the difference between the actual and predicted disposable income of the household, housing debt and expenditures on health. The repair costs are included to test the effect of depreciation rate on automobile purchases. The second method builds on the multiplicative heteroskedasticity approach to test a household-level variance of income. The third method is based on heteroskedastic probit analysis in order to identify the effect of the variables above on the probability of transaction.

This study is organised as follows. Section two illustrates the basic (S,s) rule and the augmented (S,s) model in which the uncertainty is included and divided into the components discussed above. Section three describes the data used in the study. In section four, we report the main results, evaluate them with respect to the international evidence, and discuss the problems concerning the model's goodness-of-fit and the measurement of household-level uncertainty. Finally, section five concludes the paper.

2. (S,s) MODEL

In this section we introduce the theoretical (S,s) model used in the study. First, in section 2.1 we introduce the standard (S,s) rule in order to get an intuition of discontinuous investment decisions. Then, section 2.2 reviews the irreversibility literature. In section 2.3 we include individual and systematic risk in the model and show how an increase in one of the risk components will affect the timing of the durable purchase.

2.1. Introduction to (S,s) Rule

Individual investment decisions together with transaction costs can be described by using a (S,s) model originally introduced by Arrow et al. (1951) for the study of inventories. According to the model, the state variable evolves stochastically over time and is allowed to deviate from the optimal target which can be interpreted as a frictionless target. The more it deviates, the more disutility the consumer obtains, and the temptation to readjust it back to the target level increases. However, every time when this readjustment is made, the consumer has to pay a lump sum adjustment cost which prevents him from updating continuously. Under uncertainty, consumer is unaware of his future wealth, prices and other market conditions. Thus, by postponing purchase decision to the future and waiting for new information to arrive, he can possibly make a better deal. This creates an option value of waiting. The state variable continues to deviate

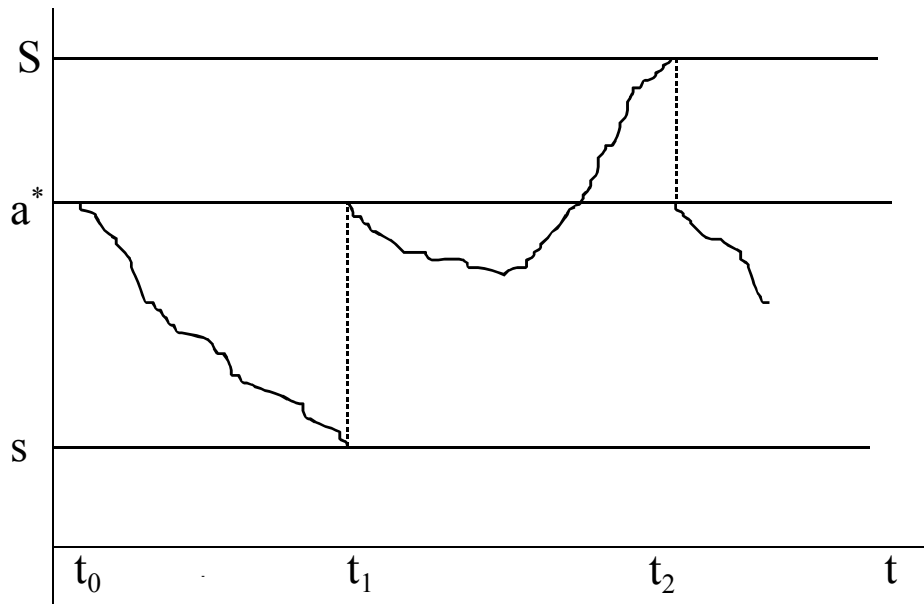


Figure 2.1: Standard (S,s) Model.

from the target until the lower/upper threshold/trigger is reached. Then it is readjusted back to the optimal level. The triggers can be seen as limits in which the temptation to adjust (utility gain) equals the adjustment costs plus the value of waiting. Inside the band the costs are higher than the utility gain from the updating, and no adjustment is made. Clearly, there is always a trade-off between adjusting now or waiting for new information before making the purchase decision.

Figure 2.1 illustrates the optimal (S,s) rule. For convenience, let us consider that the state variable is a stock of durable which depreciates over time.⁴ Also, assume that the consumption flow is proportional to the stock of the good. The vertical axis represents the deviation of the current stock from some target level a^* . This frictionless target would be chosen if no transaction costs existed, that is, the stock of a durable would be continuously updated. However, continuous adjustment would entail infinite transaction costs and, therefore, cannot be optimal. After the purchase at the time t_0 , the state variable is most of the time inside the inaction band (S,s) and the readjustment is not made until it reaches one of the triggers S or s . When a falls below some lower bound s , consumer pays a lumpy adjustment cost and the stock is readjusted back to a target size a^* . In figure 2.1, this readjustment is not made until at time t_1 . Similarly, when the state variable increases above the upper trigger S , it is readjusted back to the target level a^* . This happens at time t_2 . As long as the stock of the durable good remains inside the band, no action takes place.

⁴In this illustrative example, the stock of a durable can also be interpreted as a control variable.

Often, it is unrealistic to define the state variable only as a stock of durable which depreciates over time. Instead, consumer's wealth can be seen as one of the important variables determining the level of the stock. Typically, consumer's wealth grows over time and reflects the changes in consumer's spending. Even though the amount of the stock itself can remain unchanged, the quality of this stock may depend on the wealth level. Also, the relative prices of durables and nondurables are important determinants for consumption.⁵ On the other hand, the optimal target level may depend on the quality and relative prices of goods, consumer's wealth, seasonal dummies and various demographic factors⁶, while the size of the (S,s) band can be related, among others, to the size of the transaction cost, changes in household size, depreciation rate and to factors affecting the opportunity cost of deviating from the optimal level of durables. The use and decay characteristics of capital goods also differ because of the intensity of use, or the different extraction rates once the good has been purchased and installed.

Unfortunately, the theoretical model which covers all those notions is technically complex. Therefore, the construction of the model must include compromises which are often unrealistic but necessary for analytical or numerical solutions. First, the stochastic process of the state variable must be characterised. Second, the target level and the trigger points have to be characterised. Third, one have to characterise how an increase in uncertainty will effect on the timing of the purchases. Finally, one has to characterise the source of uncertainty. This motivates the theoretical model in the section 2.3.

2.2. Review of Irreversibility Literature

Recently, irreversibility and uncertainty over future have been considered important factors in determining the timing of the durable purchases. Even though the link between uncertainty and consumption has been understood for a long time, the theoretical models concentrating on uncertainty, irreversibility mechanism and consumption did not appear until the mid-1980s.⁷ In our opinion, there are three main reasons for this appearance. First, the dynamic nature of durable consumption seemed much more complex than that of the other components of consumption, and the former consumption models, including the standard permanent income hypothesis, were incapable on explaining the large variations in consumption over the business cycles, especially for durables purchases.⁸ Second,

⁵In literature, the state variable has been modelled as the ratio of the consumption good to wealth (Grossman & Laroque (1990) and Eberly (1994)), the ratio of the durable stock to nondurables (Attanasio (1995, 2000)) and the ratio of the durable to permanent income (Dunn (1998)), among others.

⁶In addition, the target level does not necessarily coincide with the optimal frictionless level without transaction costs. For such models, see Attanasio (1995, 2000) and Hassler (1996a).

⁷This review concentrates only on literature concerning uncertainty, irreversibility and consumption. See Hassler (1996c) for a general discussion of the connection between risk and consumption. Carruth et al. (2000) and the references cited there give a thorough survey on the literature of industrial investment decisions under uncertainty.

⁸Typically, new models appeared because the old ones were not able to describe the U.S. economy.

the irreversibility models were developed to test if the results from the successors of Hall (1978), such as Mankiw (1982), hold also under liquidity constraints and other market incompletenesses.⁹ Finally, the framework of McDonald & Siegel (1986), Pindyck (1991) and Dixit & Pindyck (1994) enabled us to understand better the significant effect which uncertainty and expectations have on irreversible investment decisions. Instead of using these categories in the following, we, however, used the classification between microeconomic and aggregate level studies. The following survey by no means covers the whole literature. Rather, our purpose is to show the importance of the irreversibility mechanism and the role of uncertainty on the timing of consumption decision and to sketch the evolution of irreversibility theory.

Typically, the literature involving aggregation first studied convex (quadratic) costs of adjustment. For example, Bernanke (1985) were among the first ones to include adjustment costs in a partial-adjustment model to study the separability in utility between durables and nondurables and the persistence of aggregate durables expenditures. He concluded that the quadratic adjustment costs are not sufficient in explaining the excess sensitivity to transitory income in the aggregate time-series. Subsequently, many other studies based on the convex adjustment costs turned out to be unsuccessful. The main reason for this poor performance is that the convex costs approach predict a smooth adjustment towards an equilibrium. Also, to avoid increasing costs, agents will adjust their stocks infrequently and by small amounts. This, of course, contradicts common observations that durables are typically purchased in lumpy increments and updated only infrequently.¹⁰

A direction of improvement was then to consider nonconvex costs of adjustment (or “kinked” adjustment costs as described by Bertola & Caballero (1990)), such as fixed or proportional costs. The model by Bar-Ilan & Blinder (1987, 1992) can be seen as a preliminary work towards inertial models of consumer expenditures and an extension of the standard LC/PIH framework to include consumer durables. In their model it was sometimes optimal for the agents “to do nothing” if the transactions involve lumpy costs and to choose a finite range rather than a single level for their durables. The study of Grossman & Laroque (1990) was also based on the idea of nonconvex costs. Their theoretical model extended an inventory model of Arrow et al. (1951) to study the portfolio choice and a single illiquid durable purchase. It was proved that the optimal strategy can be modeled as an (S,s) rule as in Figure 2.1 above. The attractiveness of this approach was that it supports the common observations, that is, in most periods consumers do not adjust their stock of durables and when they finally do, the adjustments

⁹Mankiw (1982) showed that under the rational expectations augmented permanent income hypothesis, consumer durables expenditures should follow an ARMA(1,1) process. However, this implication was strongly rejected by the aggregate data (see e.g. Attanasio (1998)) and new models were needed to understand why it did not work.

¹⁰See also Bar-Ilan & Blinder (1992).

are usually substantial and lumpy. A closely related approach to this was employed by Lam (1989) who used irreversibility mechanism and imperfect resale market approach, and showed that these cause substantial serial correlation on aggregate durable expenditures.¹¹ His interpretation was that when resales are costly, consumers are reluctant to adjust their stocks downward, and, because of the possibility of having to resell, hesitant in adjusting their stocks upward. Thus, consumers tolerate their actual stocks to deviate from their desired target levels over the business cycles.

As noted above, one purpose of the adaption of the micro-level models together with the assumption of incomplete markets was that they helped to understand why the time-series of aggregate expenditures behave unlike the prevailing theories (such as Mankiw (1982)) predicted. Based on the individual behaviour several studies have concentrated on the aggregate effects and the business-cycle dynamics. In his slow adjustment models Caballero (1990, 1993) showed how the lumpy purchases in microeconomic level can explain different features of the aggregate time-series behaviour of durable goods, and how shocks can have persistent effects when individuals follow (S,s) policies. Caballero & Engel (1993) used a model in which the probability that an agent adjusts his durable stock is increasing to the deviation of the state variable from its moving target. Leahy & Zeira (2000) used shocks in individual wealth and decline in productivity to show that the timing decision can serve as a mechanism for the amplification and propagation of aggregate shocks. A decline in wealth causes individuals to rebuild their wealth position and during this time they delay durable purchases, which reduces the total demand dramatically for some time.

Even though a common feature of all these studies is that they showed clearly the important connection between uncertainty, individual purchases and business cycles, the weakness of the theoretical presentation is that risk was typically defined as a constant parameter. To overcome this issue Hassler (1994, 1996a,b,c) presented a model where the risk level is in itself a stationary stochastic process over time. Based on the preliminary work of option value of waiting by McDonald & Siegel (1986) and the standard (S,s) model of inventory, he showed how an increase in risk increases the value of waiting, and the purchases are more easily postponed to the future. The shorter the high-risk periods are expected to be, the stronger is the incentive to wait for new information. Hassler also showed how an increase in uncertainty affects the dynamics of aggregate consumption.

One shortcoming of the theoretical (S,s) models based on the individual behaviour is that they are restrictive in assumptions and the characterisation of an individual behaviour is possible only under very special circumstances.¹² Also,

¹¹See also House & Leahy (2000) who used adverse selection and lemons problem to study the effect of resale market imperfections.

¹²Bar-Ilan & Blinder (1992) pointed out that even simple generalisations of the (S,s) models are extremely difficult to analyse because one loses the possibility of having a single state variable.

these models typically do not have a closed-form solutions and they must be employed numerically. Nevertheless, several studies (see Lam (1989) and Carroll & Dunn (1997), among others) use calibration and simulation tests to show that the (S,s) models can explain the empirical data better than the previous theoretical models. To summarise, both theoretical and empirical studies based on (S,s) model have shown the importance of uncertainty and irreversibility mechanism on the timing of individual consumption purchases and, thus, aggregate dynamics of durables consumption.¹³

The following augmented (S,s) model introduces the effect of an increase in uncertainty on the timing of durable purchases. While the foundation of the model builds on the framework of Hassler (1994, 1996a), it derives the consumption rule more explicitly by using Dixit & Pindyck's (1994) methodology and by dynamising the famous optimisation rule of the static Cobb-Douglas preferences.

2.3. Model

In a static context, the maximisation of the standard Cobb-Douglas preferences subject to the linear budget constraint produces an optimal amount of the commodity. This optimum is a function of prices, consumer's wealth and the constant fraction of wealth spent on that good. Thus, the fraction of the wealth can be expressed as

$$a_t(P_t, C_t, W_t) = \frac{P_t C_t}{W_t}, \quad (2.1)$$

where C_t is the stock of durable, P_t is the (relative) price of the durable and W_t is consumer's total wealth including income, real and financial assets. Consider an agent who continuously faces the problem when to update his stock of a durable in response to the stochastic movements of the variables on the right-hand side of the equation (2.1). Assume that he wants to follow an optimal rule where the stock of a durable is kept in a level where it costs a certain constant fraction of his total wealth. Typically, durables are expensive and their purchases include lumpy costs that are at least partially irreversible. This feature prevents him from continuous updating. In the absence of adjustment costs, the consumer is willing to update continuously his stock of a durable and keep a equal to the frictionless target a^* .¹⁴ Together with the depreciation of the durable each of the variables in the right-hand side in (2.1) evolves stochastically according to the following geometric Brownian motions

$$\begin{aligned} dW &= \alpha_W W dt + \delta_W W dz_W, \\ dP &= \alpha_P P dt + \delta_P P dz_P, \end{aligned} \quad (2.2)$$

¹³In section 4.3 we discuss more on the empirical microeconomic evidence of the (S,s) model.

¹⁴It could be more appropriate to model consumer's behaviour such that a certain fraction of wealth is spent on consumption categories. This includes transportation, electronics, clothes etc. rather than a single commodity such as a car or a computer. However, the technical treatment, then, becomes more difficult. See Estola & Hokkanen (1999) for a more detailed discussion on this subject.

$$dC = -\delta_C C dt.$$

Typically, both consumer's wealth and the price of the durable are increasing over time. Thus, the parameters α_W and α_P can be interpreted as drift components. δ_W and δ_P are the standard deviations of the processes reflecting the uncertainty over future. The terms dz_k are the increments of the Brownian motion capturing the idea that the variables satisfy the Markov property so that their next period probability distributions are functions of their current stage only. Also, the variance of the prediction error grows linearly with the time horizon.¹⁵ δ_C is the rate of the depreciation. In Appendix A we show that the fraction of wealth a evolves as well according to the geometric Brownian motion

$$da = \alpha_a a dt + \delta_a a dz_a. \quad (2.3)$$

Infrequently, the consumer faces unexpected idiosyncratic shocks in his total wealth such as unemployment or illness.¹⁶ Also, an accident can cause an immediate depreciation of the commodity. We will use a Poisson process to capture the idea of unexpected jumps in the ratio $\frac{PC}{W}$. Letting λ denote the mean arrival rate of an event, during a time interval of (infinitesimal) length dt , the probability that a negative or positive shock will appear is given by $\frac{\lambda}{2} dt$. Thus, $\frac{PC}{W}$ shifts an amount $\pm\xi$ with equal probability. The probability that an event will not occur is given by $1 - \lambda dt$. The combined Poisson and Ito processes are given as

$$da = \alpha_a a dt + \delta_a a dz_a + a dq \quad (2.4)$$

in which

$$dq = \begin{cases} \xi & \text{with probability } \frac{\lambda}{2} dt \\ 0 & \text{with probability } 1 - \lambda dt \\ -\xi & \text{with probability } \frac{\lambda}{2} dt \end{cases}. \quad (2.5)$$

The increments dz and dq are assumed to be independent such that $E[dz dq] = 0$ and $E[dz dz] = dt$. The expected length of time until the shock appears is λ^{-1} .

Correspondingly, the consumer faces the systematic risk concerning the economy as a whole. The risk comes, for instance, from the threat of war, stock markets or an adverse supply shock.¹⁷ Hereafter, risk is defined to be synonymous with uncertainty regarding future events that are relevant for the agent's decision-making. Following Hassler (1994, 1996a, 2001) the systematic risk is defined such that the consumer expects the state of the economy to switch stochastically

¹⁵Typically, in the long run the trend component is the dominant determinant of the Brownian motion, whereas in the short run the volatility component of the process dominates. See Dixit (1993), Dixit & Pindyck (1994) and Merton (1999) for an introduction and for mathematical properties of Brownian motion.

¹⁶Bernanke (1985) noted that at the family level, the most important influences on income are non-systematic factors such as ability, education and inheritance, among others.

¹⁷Bonoma & Garcia (1997) included infrequent information arrivals such as periodic releases of macroeconomic statistics and dividend announcements.

between two levels.¹⁸ The state 0 is defined as a low risk state, while 1 is high risk state. The state u is assumed to follow a first-order Markov process with a transition matrix

$$\begin{bmatrix} 1 - \gamma_0 & \gamma_0 \\ \gamma_1 & 1 - \gamma_1 \end{bmatrix}. \quad (2.6)$$

At time t , if the economy is in state $u_t = 0, 1$, it is assumed to switch to the state \bar{u}_t with probability γ_u during the time dt . The probability that there does not exist a switch is $1 - \gamma_u$. The expected length until the risk switches is γ_u^{-1} .

The augmented (S,s) model in figure 2.2 illustrates the evolution of the variable a and the adjustment process. The vertical axis is defined as in figure 2.1. Assume that $S_1 > S_0 > a^* > s_0 > s_1$. At time t_0 the fraction of the wealth a is at the frictionless optimum a^* . Typically, consumer's real wealth is increasing over time (the ratio $\frac{P}{W}$ is decreasing) while the depreciation decreases the stock of the durable. This means that a decreases over time until the lower trigger s_0 is reached at time t_1 . Then the consumer pays the adjustment costs and the stock is adjusted back to the optimal level a^* . Sometimes the opposite happens, so that the real wealth is decreasing over time and the depreciation of the durable is not high enough to force the variable a to decrease.¹⁹ At time t_2 the upper trigger S_0 is hit and the stock of durable is adjusted back to the frictionless optimum.²⁰

Occasionally, a consumer faces unexpected changes in his real wealth (idiosyncratic risk). For example, at time t_3 he confronts a reduction in real wealth which causes an upward jump on a variable a . However, this jump is not high enough for an adjustment to be made. At time t_4 a positive shock to real wealth occurs (for instance, a win in a lottery or a bequest) and shifts the variable below the lower trigger. This causes an upward adjustment of a durable back to the target level a^* . For convenience, if the shock occurs, it is hereafter assumed large enough to be optimal to adjust the variable immediately.²¹ This assumption together with depreciation and positive growth of the real wealth imply that the variable is nearly always in the region between the lower trigger and target level. Thus, we will concentrate on the range $[s_1, a^*]$ and we will not formulate the situation such that between the times t_1 and t_2 . At time t_5 an uncertainty concerning the systematic risk increases widening the inaction band to (S_1, s_1) . Clearly, a con-

¹⁸The assumption of bivariate risk in economy is unrealistic but necessary for the technical treatment. To allow more states results in the multivariate systems of differential equations which are difficult to solve.

¹⁹Technically, depending on the sign of the drift parameter α_a , a is increasing or decreasing over time. See equation (A.6) in Appendix A.

²⁰Hassler (1994,1996a) assumed that there exist upper and lower targets \bar{a} and \underline{a} such that if the state variable is hit by the upper trigger S_u , then it is readjusted to the upper target \bar{a}_u . Similarly, if the lower bound s_u is reached then the variable is readjusted to the lower target \underline{a}_u . For the technical treatment to become easier we assume that $\bar{a}_u = \underline{a}_u = a^*$.

²¹Of course, after the shock occurs the adjustment of a durable does not take place immediately. Instead, there is an "adjustment period" when, for instance, the consumer is collecting information about the prices and properties of durables available in the market. Even though it is possible to build models with deliberating time and jumps inside the bands without adjustment, they are difficult to treat technically.

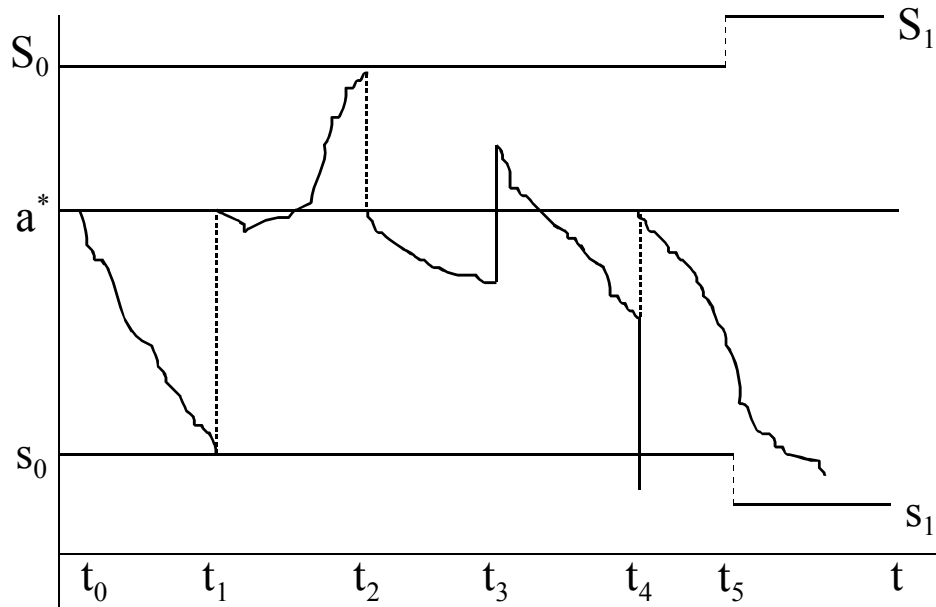


Figure 2.2: Augmented (S,s) Model.

sumer allows larger deviation from the target level before adjusting. The larger the uncertainty, the wider is the inaction range and the larger is the deviation between actual and target level.

The problem is to determine the critical points, triggers, at which it is optimal to pay the adjustment costs and adjust the commodity back to the optimum level. Since a evolves stochastically, we are not able to determine an explicit time when the investment is to be made. Instead, the investment rule which follows will take the form of a critical values (s_1, s_0) such that it is optimal to invest once the variable a goes below one of these band triggers.

As shown above, an agent obtains disutility if the state variable deviates from its frictionless optimum which is, for convenience, assumed to be a constant.²² Let $x_t = a_t - a^*$ be the state variable denoting the gap between the actual and the target level, and let the loss of deviation be a quadratic distance from the target level.²³ If the total costs are the sum of expected discounted values of present- and future-period costs, then the optimisation problem for an infinitely

²²This assumption is made to simplify the mathematical treatment of the model. However, the assumption is in harmony with the standard Cobb-Douglas preferences where the fraction of the wealth is a constant number. See Niemeläinen (1995) for details and other preferences where this fraction can be shown to depend on prices, wealth or both. See also Hassler (1996a) for a model which allows the target level to depend on wealth and the state of the economy.

²³The quadratic loss function approach means that a consumer's disutility is symmetric around the target level. While this assumption can be questionable, the technical reasons prevent us to use asymmetric disutility approach.

long-lived consumer can be written as

$$\min \left[E_t \int_t^\infty e^{-rt} \left(\frac{x_t^2}{2} + I_t c \right) dt \right], \quad (2.6)$$

subject to the equations (2.4) and (2.5). E_t denotes the expectation operator conditional on the information set available to consumer at time t and r is the subjective discount rate. For convenience, the consumption flow is assumed to be proportional to the stock of the durable. I_t is a bivariate variable such that

$$I_t = \begin{cases} 1, & \text{if } a \text{ is adjusted} \\ 0 & \text{otherwise} \end{cases}. \quad (2.7)$$

It denotes that an adjustment cost c should be paid only when the adjustment is made. The optimal value function $V(x_t, u_t)$ is defined as the minimum of discounted expected total costs over the infinite future time horizon when the economy is in state u_t . In Appendix B we show that if the consumer is following the optimal policy, the Bellman equation

$$V(x_t, u_t) = \frac{x_t^2}{2} dt + e^{-rdt} E_t V(x_{t+dt}, u_{t+dt}) \quad (2.8)$$

can be rewritten as

$$\begin{aligned} rV(x_t, u_t) = & \frac{x_t^2}{2} + \left[\alpha_x x_t V'(x_t, u_t) + \frac{1}{2} \sigma^2 x_t^2 V''(x_t, u_t) \right] \\ & + \lambda [(V(a^*, u_t) + c) - V(x_t, u_t)] + \gamma_u [V(x_t, \bar{u}_t) - V(x_t, u_t)]. \end{aligned} \quad (2.9)$$

The left-hand side is the value of the cost function multiplied by the discount rate. The first term on the right-hand side is the utility loss during the time dt . The second term is the expected change in total costs if no shock nor state shift occur. The third term captures the idea that there exists a wealth shock during the time period dt . Then, by assumption, the durable is purchased and the state variable is adjusted to the target level a^* . The last term comes from the possibility of a state shift.²⁴ This causes the expected total costs to shift from $V(x_t, u_t)$ to $V(x_t, \bar{u}_t)$ where $u_t \neq \bar{u}_t$.

While $u_t = 0, 1$, the equation (2.9) results in the following system of second-order differential equations

$$\begin{aligned} & \frac{1}{2} \sigma^2 x_t^2 V''(x_t, 0) + \alpha_x x_t V'(x_t, 0) - (\lambda + r)V(x_t, 0) + \gamma_0 [V(x_t, 1) - V(x_t, 0)] \\ = & -\frac{x_t^2}{2} - \lambda(V(a^*, 0) + c), \end{aligned} \quad (2.10)$$

²⁴The optimisation problem could be stated and solved by two different techniques: contingent claims analysis or stochastic dynamic programming. Even though in most applications they both give identical decision rules, their assumptions are different concerning discount rates and financial markets. Also, the interpretation of the equation (2.9) slightly differs. See Dixit & Pindyck (1994) or Pietola (1997) for more details.

$$\begin{aligned}
& \frac{1}{2}\sigma^2 x_t^2 V''(x_t, 1) + \alpha x_t V'(x_t, 1) - (\lambda + r)V(x_t, 1) + \gamma_1 [V(x_t, 0) - V(x_t, 1)] \\
&= -\frac{x_t^2}{2} - \lambda(V(a^*, 1) + c).
\end{aligned}$$

In Appendix B we show that the algebraic solution for (2.10) is

$$\begin{aligned}
V(x_t, 0) &= \frac{\gamma_0 \gamma_1}{\gamma_0 + \gamma_1} A_1 x^{\beta_1} - \frac{\gamma_0}{\gamma_0 + \gamma_1} C_1 x^{\theta_1} \\
&\quad - \frac{1}{2(\sigma^2 + 2\alpha - (\lambda + r))} x^2 + \frac{\lambda c}{r}, \\
V(x_t, 1) &= \frac{\gamma_0 \gamma_1}{\gamma_0 + \gamma_1} A_1 x^{\beta_1} + \frac{\gamma_1}{\gamma_0 + \gamma_1} C_1 x^{\theta_1} \\
&\quad - \frac{1}{2(\sigma^2 + 2\alpha - (\lambda + r))} x^2 + \frac{\lambda c}{r},
\end{aligned} \tag{2.11}$$

in which the roots β_1 and θ_1 are defined as

$$\begin{aligned}
\beta_1 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda + r)}{\sigma^2}} > 1, \\
\theta_1 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda + r + \gamma_0 + \gamma_1)}{\sigma^2}} > \beta_1.
\end{aligned} \tag{2.12}$$

The total cost functions in (2.11) are valid only for $x \in [s_0, a^*]$. For some values of $V(x_t, u_t)$, a switch of the state will lead to an immediate adjustment. For example, if x_t is in the region $[s_0, s_1]$, a switch from the high risk state to the low risk state causes an adjustment. Thus, in the region between s_1 and s_0 the cost function $V(x_t, 0)$ is a constant and equals $V(s_0, 0)$ because $x_t < s_0$. In the range $[s_1, s_0]$ the system of differential equations degenerates to

$$\begin{aligned}
V(x_t, 0) &= V(s_0, 0), \\
V(x_t, 1) &= D_1 x^{\mu_1} - \frac{1}{2(\sigma^2 + 2\alpha - (\lambda + r + \gamma_1))} x^2 + \frac{\gamma_1 V(a^*, 0) + (\gamma_1 + \lambda)c}{r + \gamma_1},
\end{aligned} \tag{2.13}$$

in which the root μ_1 is defined as

$$\beta_1 < \mu_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda + r + \gamma_1)}{\sigma^2}} < \theta_1. \tag{2.14}$$

Economically, the system of cost functions in (2.13) shows the effect of the change in uncertainty on durable purchases. It also reveals how the consumer will benefit of the new information and do what is optimal then. If the state of the world is risky and the state variable is close to the lower band trigger s_1 , a shift to low risk will cause an immediate adjustment. If a high number of consumers are close to this lower band trigger and the state of the economy is high risk, a switch to the low risk will cause an aggregate investment boom.

Even though we are not able to find an algebraic solution for the integration constants in (2.11), (2.13) and band triggers (S_u, s_u) , it is possible to find numerical solution using the following conditions

$$\begin{aligned} \text{Smooth - pasting} & : \begin{cases} V'(s_0, 0) = 0 \\ V'(s_1, 1) = 0 \\ V'(a^*, 0) = 0 \\ V'(a^*, 1) = 0 \end{cases}, & (2.15) \\ \text{Value - matching} & : \begin{cases} V(s_0, 0) = V(a^*, 0) + c \\ V(s_1, 1) = V(a^*, 1) + c \end{cases}. \end{aligned}$$

The smooth-pasting conditions require that the derivatives or slopes of the functions meet tangentially at the boundaries and target level under the states $u_t = 0, 1$. The value-matching conditions match the values of the unknown function $V(x_t, u)$ to those of the known values if the adjustment is made.²⁵

2.4. Increase in Risk and Timing of Purchases

Technically, the smooth-pasting and value-matching conditions enable us to find numerical solutions for the total cost functions in (2.11) and (2.13). Economically, the value-matching conditions imply that at the band triggers the temptation to adjust equals the temptation to wait an instant. This interpretation becomes more clear if we evaluate the equation (2.9) at the target point a^* and at the band trigger s_u . Following Hassler (1996a) it can be shown (see Appendix C) that the indifference condition becomes

$$\frac{(a_u - a^*)^2}{2} = \underbrace{rc}_1 + \underbrace{\lambda c}_2 + \underbrace{\gamma_u [c + V(a^*, \bar{u}) - V(s_u, \bar{u})]}_3 \quad (2.16)$$

where $a_u - a^*$ means the deviation of the state variable from the target level evaluated at the band trigger point s_u . The left-hand side is the immediate temptation to adjust. An adjustment decreases the utility loss from deviating from the target point. The right-hand side is the temptation to postpone the purchase an instant dt .²⁶ It has three different parts. If a consumer does not adjust but invests the amount c in a safe asset, he receives an interest rate yield during the time dt . This is the first part of the right-hand side. The second one reflects the possibility of a wealth shock. Delaying the purchase will save one adjustment if the shock occurs after the time dt . The third term denotes the possibility of a state shift in economy. If it switches during the instant dt and a consumer waits until after that instant he can do what is optimal then. This creates an option value of waiting which is non-negative (see Appendix C).

It is straightforward to analyse the effects of the determinants on the size of the inaction band $[s_u, a^*]$. From (2.16) it is evident that if the adjustment costs

²⁵See Dixit (1993) for a thorough discussion.

²⁶The left-hand side is interpreted as an instantaneous cost of waiting in Hassler (1996a). The right-hand side is a value of waiting in Dixit & Pindyck's (1994) terminology.

increase, the consumer tolerates a larger deviation from the target before adjustment. The same happens if the probability λ of idiosyncratic risk increases or the expected time decreases until the personal shock will appear. Even though the state shift does not occur during the time dt , the decrease in the expected time until it may appear (or an increase in probability γ_u) will lead to the wider inaction band.

3. DATA DESCRIPTION

Because of the lack of micro-level data, the formal tests of the (S,s) model are rare. Ideally, the panel of household data should be long enough to track the stock of the durables over time so that one can identify times of adjustment, targets and adjustment triggers together with the information on income, wealth and heterogeneous characteristics of the households. Also, as pointed out by Attanasio (1998), it is desirable to follow households over some time to bound the inaction range by the households that are observed not to adjust. Such data is difficult to find.²⁷ Also, another reason for the lack of econometric analysis is the difficulty in defining irreversibility and uncertainty. Concerning the irreversible mechanism, the only studies that have utilised panel data are Lam (1991), Eberly (1994), Attanasio (1995) and Foote et al. (2000). In all these studies, however, there are problems in accounting properly the variables involved in the model.

In this study we used Household Budget Surveys conducted by Statistics Finland. The data is drawn from four cross-section Surveys made in Finland in the years 1985, 1990, 1994-96 and 1998. The data from 1994-96 is combined from three annual Surveys and processed such that it can be used as a one cross-section. The number of households in the Surveys are 8200 in 1985, 8258 in 1990, 6743 in 1994-96 and 4087 in 1998. The respondents in the Surveys were asked several questions concerning the characteristics of the household, income, liabilities, education, and detailed expenditures in different consumption categories.²⁸ Also, the respondents were asked if they owned a certain durable. Unfortunately, the Surveys did not follow the same households over time, and it is not possible to construct a panel tracking the stock of the durable and evaluate the depreciation rate together with the wealth position of the households over time.

The information on car ownership is best documented in the data because the respondents were asked the gross and net markka-value of the acquisition of the cars as well as the exchange value of the used cars. Also, the information of expenditures on repairs as well as other charges and costs are available. Therefore, we used the data from the car acquisitions to identify the (S,s) triggers and the

²⁷In fact, the only appropriate large microeconomic data sets are the Consumer Expenditure Survey (CEX) from U.S. and Family Expenditure Survey (FES) from U.K.

²⁸Suoniemi & Sullström (1995) provide a thorough description of the Surveys and of the change of the consumption structure in Finland.

target, and to evaluate the effects of the determinants on them. According to the data, the households can be categorised as follows.

1. Those who upgrade by buying a used car and give a car in exchange.
2. Those who upgrade by buying a used car without a trade-in car.
3. Those who upgrade by buying a new car and give a car in exchange.
4. Those who upgrade by buying a new car without a trade-in car.
5. Those who upgrade by buying both a used and a new car and give a car in exchange.
6. Those who downgrade to zero by selling a car.
7. Those who downgrade by selling a car and buy a cheaper one.
8. Those who do not engage in transaction.

For households engaging in a transaction so that they either buy a new or a used car and give a car in exchange, the width of the lower inaction band is observable. Correspondingly, the width of the upper band is observable only for those who downgrade by selling a car and buy a cheaper one. For households that upgrade by buying either a used or new car without a trade-in car, only the target level can be identified. For those who downgrade by selling a car, only the upper trigger can be identified. Due to the limited time-series information for those not engaging in a transaction, neither the triggers nor the target is observable. In the following section, these categories are referred to when analysing the data.

4. RESULTS

4.1. Estimation Methods

As a measure of the state variable, we used a ratio of the value of the car to the disposable income of the household. Even though this measure of wealth does not properly account for the total lifetime wealth, it still implicitly includes the unobservable human wealth, the existence of liquidity constraints, and gives the wealth position of the household. The triggers were calculated by dividing either the value of the trade-in cars with respect to the households disposable income depending whether the household updates its durables stock up (lower trigger, categories 1 and 3) or down (upper trigger, category 7), or with respect to the selling value of the car (upper trigger, category 6)²⁹. The gross value of the purchased car with respect to the disposable income gives the target value for the state variable. Table 1 depicts the means and standard deviations for the triggers and target, and the number of observations in each categories. The last panel in Table 1 gives the results for the total number of entries of the triggers and target. The state variable values higher than one were omitted from the analysis for practical reasons.³⁰

²⁹In category 6, the low values of the state variable indicate perhaps a sort of a scrap value of the cars rather than (S,s) behaviour. Then, it is a matter of taste if this trigger should be treated as S or s.

³⁰Typically, each category contained few values which were more than one. In the categories less than 100 observations, these outliers had a substantial effect on mean and standard error (the highest value for

Table 1
(S,s) target and triggers for state variable calculated
as means in each category

		1985	1990	1994-96	1998
Category 1	a*	0.31 (0.21)	0.34 (0.21)	0.31 (0.21)	0.33 (0.21)
	s	0.12 (0.12)	0.13 (0.13)	0.13 (0.13)	0.13 (0.12)
	n	886	610	438	285
Category 2	a*	0.14 (0.15)	0.17 (0.18)	0.15 (0.15)	0.16 (0.16)
	n	583	606	396	357
Category 3	a*	0.53 (0.17)	0.52 (0.21)	0.59 (0.19)	0.60 (0.19)
	s	0.25 (0.15)	0.24 (0.16)	0.26 (0.15)	0.27 (0.14)
	n	297	227	114	124
Category 4	a*	0.43 (0.17)	0.43 (0.23)	0.43 (0.23)	0.49 (0.22)
	n	48	67	30	33
Category 6	S	0.09 (0.10)	0.11 (0.16)	0.10 (0.14)	0.14 (0.16)
	n	78	82	123	36
Category 7	S	0.21 (0.19)	0.23 (0.18)	0.19 (0.20)	0.18 (0.15)
	a*	0.10 (0.11)	0.14 (0.13)	0.10 (0.12)	0.09 (0.08)
	n	74	68	78	34
	S	0.15 (0.16)	0.17 (0.18)	0.14 (0.17)	0.16 (0.16)
Total	n	152	150	201	70
	a*	0.29 (0.23)	0.30 (0.23)	0.27 (0.23)	0.30 (0.24)
	n	1888	1578	1056	833
	s	0.15 (0.14)	0.16 (0.15)	0.16 (0.14)	0.17 (0.14)
	n	1183	837	552	409

The state variable is the value of the car/disposable income. a, S, s and n are the target, upper and lower triggers and number of observations, correspondingly. Categories are as in Chapter 2. The standard deviations are given in parentheses.*

The results from the categories that reveal the width of the inaction band are the state variable was 60), and some of them may be a consequence of data processing. Therefore, to make results comparable over time, the state variable is restricted between zero and one.

the most interesting. For example, the results from the category 1 imply that on the average the value of the trade-in cars is allowed to drift down to 13 percent of the disposable income before adjustment. Then, it is adjusted slightly above 30 percent. This behaviour clearly differs from that of buying a new car and giving a car in exchange (category 3). These households adjust when the value of the trade-in car is about one fourth of the disposable income, while the target is slightly above half. In both cases, the width of the inaction band slightly changes over time. Households that downgrade by selling a car and buy a cheaper one (category 7) tolerate the value of the car to increase to one fifth of the disposable income before adjusting it back to about 10 percent. However, it is noteworthy that the high standard errors indicate a high cross-sectional heterogeneity of behaviour.

The common feature across the categories is that, in aggregate, both the triggers and target seem to be quite stable over time implying only a small variation in the width of the inaction band. Somehow this is surprising since the Surveys are from the years of different economic circumstances. In Finland, in 1985 financial markets were regulated, in 1990 there was a boom and overheating of the economy, while the years 1994-96 and 1998 were times of economic recovery along with a high rate of unemployment. Casually, however, only the year 1990 seems to be an exception. The lower band width for the category 1 is slightly higher, and the upper trigger and target for the category 7 is somewhat higher compared to other years.

After identifying the upper and lower band width, we focused on the implication of the (S,s) model. That is, an increase in the adjustment costs and uncertainty leads households to purchase a durable less frequently. While the heterogeneous adjustment costs including searching and information costs, commissions to brokers etc. cannot be observed from the cross-sectional data, we use instead pure cost measures which are available in the Surveys, and which are assumed to have an effect on the timing of the purchases. These repair costs include expenditures on repair pairs, accessories, maintenance and other repairing. A higher rate of depreciation will indicate more need for repairing, which should lead to more frequent adjustment.

A problem for the applied econometrician is the identification and integration of uncertainty into the theory. Even though the theoretical model assumed idiosyncratic risk to follow Poisson process with immediate adjustment after the shock occurs, such behaviour is difficult to capture empirically. Instead, we study how idiosyncratic risk effects on the width of the inaction band. To measure the household-level uncertainty we used two distinct methods.³¹ Following Eberly (1994) the first method builds on the difference between actual and predicted disposable income. First, we regressed disposable income of each household on

³¹See Carruth et al. (2000) who give a survey of uncertainty proxies in irreversible investment research. In time-series and panel data the conditional variances of the underlying variables (such as the growth rates of income, stock prices and inflation) are typical proxies.

a number of household characteristics including socio-economic status, province, local authority, total months of unemployed, type of the household, educational attainment and gender of the household head.³² Then, we used these coefficients to impute a predicted income for each household in the Surveys. Finally, we calculated the difference between the actual and predicted disposable income, and used it as a measure of uncertainty. The advantage of this method is that it takes into account the households' heterogeneous characteristics. For example, if some of the household members are unemployed, it is reasonable to assume that the households' income is less than that of reference income, thus, affecting on the willingness to adjust the durable back to the target level.

The shortcoming of this method, however, is that the measured uncertainty has an asymmetric effect on the purchases. If the disposable income of the household is higher than that of predicted (that is, the residual is positive), the household can be assumed to be better-off than the average and it may be more willingness to update the durable back to the target sooner than those of the worse-off households. Thus, it is reasonable to assume that the coefficients for the worse-off households should be positive and statistically significant indicating a wider inaction band, while the coefficients for the better-off households are assumed to be statistically negative or insignificant.

While this measure of household-level uncertainty based on the residual method above may be questioned for many reasons³³, we added other cross-sectional factors that may be related to the households uncertainty concerning the future. These are housing debt and expenditures on health.³⁴ The magnitude of both of these measures implicitly include a sort of uncertainty.

The second method is based on the Harvey's (1976) multiplicative heteroskedasticity, which can be seen as an stochastic volatility type method without the time-dimension structure in error terms. Analysis of the OLS residuals of the first method reveals that depending on the Survey the error variance is mostly related to the education of the household head and/or socio-economic status of the household. Therefore, the skedastic function is

$$\sigma_{M,i}^2 = \exp(\beta_0 + \beta_1 \text{Dummy}(\text{Education})_i + \beta_2 \text{Dummy}(\text{Status})_i), \quad (4.1)$$

where i denotes each household in the Survey. The maximum likelihood estimation procedure involves deriving first derivatives of the log-likelihood function

³²The regression results from these dummy variables are available from the author by request. See Appendix D for the description of the variables. We also tried other candidates which may affect the disposable income such as education of the spouse. For all of these, however, the coefficients turned out to be statistically insignificant.

³³For example, Pagan (1986) has shown that in time-series analysis these two-stage/step regressions with expectations provide consistent parameter estimates but the covariance matrix of the parameter estimates is usually inconsistent. To correct the estimation, one should use instrument variables. Even when such expectations of the future variables do not exist explicitly in our cross-sectional regressions, it is likely to assume that there may exist a sort of inconsistency in the variances of the parameter values as well.

³⁴Instead of housing debt in 1985 we use total debt because of the lack of data.

with respect to mean equation parameters and skedastic function parameters. The resulting conditional variance estimates ($\hat{\sigma}_i^2$) were used as a proxy of the household-level uncertainty and entered as regressors in the band estimation.

The analysis above utilises only a subset of the Surveys disregarding the categories 2, 4, 6 and 8, that is, the households who only sell or buy a car, or do not engage in transaction at all. Thus, it is fertile to include these observations into the analysis to identify the effect of uncertainty and repair costs on the probability of transaction. To evaluate this behaviour, the third method is based on a probit analysis to get the adjustment probits. However, while the Surveys include households which are heterogeneous in their characteristics, the standard probit estimation generates parameter coefficients which are both biased and inconsistent. To improve the statistical performance of the estimates, we used heteroskedastic probit model, where the skedastic function is

$$\sigma_{P,i}^2 = \exp(\beta_1 DI_i)^2. \quad (4.2)$$

DI_i is the disposable income of the household explaining the variation in the error terms.³⁵

4.2. Estimation Results

Table 2 presents the results of the determinants on the lower band width based on the residual method. While it is difficult to interpret quantitatively the standard linear regression coefficients, we regressed log-linearized versions which give the direct percent changes in the band width. All the independent variables in the model were divided by the disposable income to scale the variables and to get consistent measurement units in regression. To control the asymmetric behaviour between the worse-off and better-off households, we added dummies for the constant and uncertainty (Dconstant and Dincome) for the better-off households.

According to the results, all the intercepts were statistically significant for the worse-off households. The intercepts for the better-off households differed from those of the worse-off only for the years 1985 and 1990 indicating a narrower inaction band.³⁶ The measured income uncertainty effect was statistically significant for both household types only for the years 1985 and 1990. In 1985 an increase of one percent in the income uncertainty increased the lower band width for 6.3 percent for the worse-off households but decreased it for 6.9 percent for the better-off households. These results did not reject the (S,s) model. The other

³⁵See Harvey (1976) or Greene (2003) for a theoretical justification of the methods.

³⁶The exclusion of some of the regressors did not change the statistical interpretations for the remaining coefficients. The sum of the constant and Dconstant, and the sum of Income and Dincome are the intercept and the coefficient of the measured income uncertainty for the better-off households, respectively. The antilogs of the intercepts give the standard constant terms.

uncertainty variables (housing debt and health) seemed not to perform well statistically, except for the years 1994-96, which seemed to generate reverse results in general. An increase in housing debt with respect to the disposable income seemed to decrease the band width, but statistically this was significant only in 1994-1996. As expected, throughout the Surveys an increase in repair costs typically seemed to decrease the inaction band for few percents, but these coefficients did not either differ statistically from zero. The coefficient of determination (R^2) was typically less than 0.10.

Table 2
Determinants of lower band width, residual approach

	Dependent variable is $\log(a^* - s)$			
	1985	1990	1994-96	1998
Constant	-1.741** (0.129)	-1.568** (0.156)	-1.835** (0.188)	-1.833** (0.243)
Dconstant	-0.333** (0.103)	-0.406** (0.127)	0.209 0.167	-0.096 (0.220)
\widehat{Income}	0.063* (0.032)	0.096** (0.040)	-0.079* (0.048)	0.022 (0.065)
$\widehat{Dincome}$	-0.132** (0.043)	-0.159** (0.057)	0.087 (0.076)	-0.042 (0.105)
Debt	-0.001 (0.007)	-0.011* (0.007)	-0.024** (0.009)	-0.011 (0.010)
Health	-0.010 (0.015)	0.028 (0.024)	0.044* (0.023)	0.029 (0.022)
Repair	-0.002 (0.009)	-0.009 (0.011)	0.026** (0.013)	-0.025 (0.019)
n_1	542	399	266	184
n_2	641	438	286	225

*Income is the absolute value of the difference between predicted and observed disposable income. All the regressors are divided by the disposable income and are in logarithms. n_1 and n_2 denote the number of the worse-off and better-off households, respectively. The asterisks * and ** denote that the coefficients differ statistically from zero at 10% and 5% levels of significance, respectively. The standard errors in parantheses are heteroskedasticity corrected.*

Table 3 presents the corresponding results for the upper band width. The common feature of these results is that they performed poorly statistically. In most cases we even cannot reject the hypothesis that all the coefficients are zero (Survey 1990). Except of the intercepts, only the income uncertainty for the worse-off households in years 1985 and 1994-96 were statistically significant. The magnitude of these coefficients, however, was unconvincing. Even though the coefficients of the other uncertainty measures typically had the right sign, statistically they were irrelevant for the width of the upper band. Also, the coefficients for the repair costs had mostly the expected sign, but they did not differ statistically from zero. The coefficients of determination for the regressions were low.

Nevertheless, the number of observations is small and one should avoid making too strict interpretations of the results.

Table 3
Determinants of upper band width, residual approach

Dependent variable is $\log(S - a^*)$				
	1985	1990	1994-96	1998
Constant	-5.673** (1.651)	-2.540 (1.878)	-2.667** (1.187)	-5.572* (3.360)
Dconstant	-0.047 (1.700)	1.098 (2.116)	-0.579 (1.393)	1.342 (2.583)
\widehat{Income}	-0.542** (0.261)	-0.069 (0.667)	0.920** (0.325)	0.014 (1.121)
$\widehat{Dincome}$	-0.280 (0.651)	0.846 (1.036)	-0.264 (0.703)	-0.267 (1.201)
Debt	0.170 (0.135)	-0.015 (0.087)	0.048 (0.068)	0.175 (0.130)
Health	0.072 (0.240)	0.281 (0.324)	0.033 (0.166)	-0.218 (0.192)
Repair	-0.074 (0.152)	0.054 (0.128)	-0.065 (0.102)	-0.028 (0.191)
n_1	43	41	43	15
n_2	31	27	35	19

See Table 2.

Based on the multiplicative heteroskedasticity approach, Table 4 gives the results both on the upper and lower band widths. Again, the independent variables were scaled by the disposable income, and the log-linearised version of the models were estimated.³⁷ The results seemed highly consistent with those in Tables 2 and 3 with one exception: the model for the lower band width in 1994-96 seemed to fit the data well. The other coefficients and their statistical interpretations were closely related to those of Tables 2 and 3. Again, the magnitude and the statistical relevance of the coefficients for the upper band width were dubious because of the small sample properties in estimation. An interesting feature was revealed by the coefficients for the housing debt ratio: an increase in this ratio seemed to increase the upper inaction band while decreasing the lower one. Typically, these coefficients, however, did not statistically differ from zero. Also, the coefficients for repairing costs were generally of the expected sign, but were statistically insignificant. Even though not reported, the coefficients of the

³⁷Instead of $\log(\hat{\sigma}_i^2)$ the uncertainty measure is also scaled like the other explanatory variables, and is $\log(\hat{\sigma}_i^2 / DI_i)$, where DI_i is the disposable income of the household. When regressing the model without scaling the variables involved in the model, the results were in accordance with those reported in Table 4. Thus, the scaling does not distort the statistical significance and interpretations of the parameters.

skedastic equation (4.1) turned out to be statistically significant at 5% level of significance.

Table 4
Determinants of band width when uncertainty is based on
multiplicative heteroskedasticity approach

		Regressors (in logs)				
	Band	Constant	$\log(\hat{\sigma}_{M,i}^2/DI_i)$	Debt	Health	Repair
1985	Upper	5.424 (7.796)	-1.027 (0.836)	0.186 (0.129)	0.126 (0.239)	-0.046 (0.142)
	Lower	-4.319** (0.503)	0.267** (0.054)	0.004 (0.007)	-0.012 (0.015)	-0.002 (0.009)
1990	Upper	-2.996 (4.685)	-0.022 (0.411)	-0.022 (0.084)	0.236 (0.319)	0.015 (0.122)
	Lower	-2.941** (0.431)	0.123** (0.043)	-0.011* (0.006)	0.036 (0.024)	-0.008 (0.011)
1994-96	Upper	1.976 (5.482)	-0.678 (0.584)	0.008 (0.072)	0.079 (0.184)	-0.096 (0.107)
	Lower	-3.149** (0.648)	0.156** (0.066)	-0.018** (0.009)	0.041* (0.023)	-0.027** (0.013)
1998	Upper	0.593 (6.799)	-0.484 (0.642)	0.215* (0.120)	-0.190 (0.204)	0.020 (0.214)
	Lower	-2.722** (0.547)	0.080* (0.048)	-0.012 (0.010)	0.027 (0.022)	-0.026 (0.019)

See Table 2.

Table 5 presents the results from the heteroskedastic probit analysis. The uncertainty measure was calculated as in equation (4.2). The first row corresponding to each Surveys gives the standard probit estimates. The second row reports the marginal effects around the means of the independent variables.

With a few exceptions, the coefficients were statistically significant either in 10 percent or 5 percent level of significance. Except for the year 1990, the effect of an increase in uncertainty on the probability of adjusting was negative, as predicted by the (S,s) model. An increase in housing debt, on the other hand, had a statistically significant positive effect on the probability of purchase, which seems to contradict the theory. The health effect had negative effect while an increase in repair costs affected positively on the probability to adjust. Both of these were in accordance with the theory. The likelihood-ratio (LR) test of heteroskedasticity which tests the model with heteroskedasticity against the model without it was highly significant in all cases. Even though the coefficients for the uncertainty term differed statistically from zero, their magnitude on the probability of adjustment was only few percents.

Table 5
Maximum likelihood estimates from heteroskedastic probit model

Regressors						
	Constant	$\hat{\sigma}_{P_i}^2$	Debt	Health	Repair	LR
1985	-2.056* (1.223)	-0.013 (0.115)	0.304** (0.045)	-2.509** (0.921)	0.191 (0.379)	121.62
		-0.001 (0.011)	0.030** (0.004)	-0.245** (0.090)	0.019 (0.037)	
1990	-2.887** (0.639)	0.092 (0.057)	0.133** (0.030)	-1.034 (0.813)	4.184** (0.655)	165.45
		0.011* (0.006)	0.015** (0.004)	-0.123 (0.098)	0.498** (0.075)	
1994-96	1.073** (0.356)	-0.234** (0.034)	0.053* (0.028)	-0.849 (0.728)	0.886** (0.356)	39.05
		-0.044** (0.007)	0.010* (0.005)	-0.159 (0.136)	0.166** (0.059)	
1998	-0.140 (0.541)	-0.139** (0.051)	0.177** (0.055)	-4.614** (1.161)	1.043** (0.477)	91.67
		-0.019** (0.007)	0.024** (0.008)	-0.630** (0.155)	0.142** (0.065)	

See Table 2. The 5% critical value for the LR test is $\chi^2(1) = 3.84$.

This held true also for the housing debt. Instead, the measures of health and repair costs generated probabilities, which seem somewhat unreasonably large: the estimates indicate even as large as 60 percent effect on the probability of adjustment.

4.3. Discussion and Evaluation of Results

The interpretation of the above results is not straightforward and requires discussion. A major failure of the results concerning the estimates on the upper band width are most likely related to the small number of observations and, therefore, the estimates cannot give a reliable picture of the adjustment behaviour and they should be interpreted as preliminary rather than strictly concluding.³⁸ On the other hand, the estimates for the lower band width as well as the estimates for the probability of adjustment give more plausible explanation between the uncertainty and on the frequency of adjustment. Even though the residual method dividing the households into two groups - worse-off and better-off households - was able to found statistically significant evidence only for the years 1985 and 1990, the other methods found more systematic significance. According to these results, a percent increase in uncertainty increases the inaction band more than

³⁸As mentioned earlier, the low values of the state variable may indicate a scrap value of the vehicle rather than voluntary adjustment.

10 percent while an unit increase in uncertainty decreases the probability of adjustment for few percents. Even though the magnitude of the latter result seems to be more realistic than the former, both are in favour of the (S,s) model.

The other uncertainty measures generated ambiguous results. The coefficients of the housing debt on the band width and on the adjustment probabilities were typically either statistically insignificant or of the wrong sign. This is hard to interpret. We also tried total debt as a regressor, but the results were parallel. One explanation for the latter is that in Finland the expensive net purchases (such as cars) are typically financed by taking out a loan, thus, generating a positive correlation between the borrowings and net purchases. The regression results based on the amount of total consumer credit support this insight. Also, while the expenditures on health certainly describe a kind of individual uncertainty and even though the estimates are of the right sign and in accordance with the (S,s) model, statistically they cannot explain the behaviour in automobile market.

The negative sign of the coefficients for the repair cost indicating a sort of depreciation of the cars turned out to be in favour of the infrequent adjustment theory. However, only in few cases the effect on the inaction band width was statistically significant. Instead, an increase in repair costs had a statistically significant positive effect on the probability of adjustment. When adding other user costs (automobile tax, inspection fee, traffic insurance charge and other costs including expenditures on gasoline) to explain the effect on the inaction band width, the coefficients turned out to be positive but statistically insignificant. On the other hand, the effect on the adjustment probabilities was even more statistically consequential than the pure repair cost effect.

One explanation for the poor performance of the regressors on the width of the inaction bands is the possibility that the regressors have a parallel effect both on the triggers and target which remains the band width unchanged, but changes the location of the whole (S,s) band. To control this possibility we run separate regressions for the triggers and target (results are not reported). According to these results, however, the different uncertainty measures as well as the repair costs seem not to have a statistically significant effect on the location of the (S,s) band. Only on few cases, the coefficients became statistically significant, but not systematically.

So far we have not discussed anything concerning the effect of general economic situation in Finland on the estimation results. The Household Budget Surveys are collected under different economic circumstances and it is reasonable to assume that the economic environment matters on the intertemporal consumption decisions. To evaluate the effect of the general economic confidence, Figure 4.1 presents two different indicators concerning the expectations of future in Finland. The first is Finnish industrial confidence indicator (FICI) collected by the Confederation of Finnish Industry and Employers, and the second is consumer

confidence indicator (CCI) supplied by Statistics Finland³⁹. Both indicators reveal that years 1985 and 1990 contained more systematic risk concerning the future than the later years. Especially, the end of year 1990 generated negative expectations reflecting the forthcoming deep depression in Finland. The expectations were most positive in 1994 according to the economic outlook. CCI considers year 1998 to contain least uncertainty with respect to the other years.

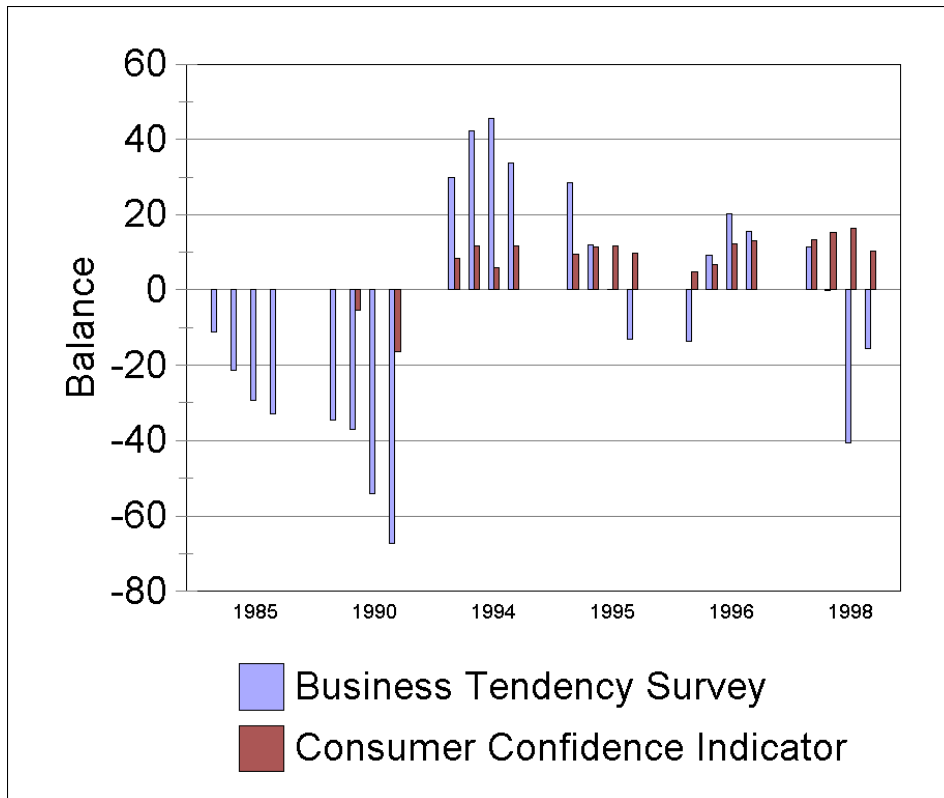


Figure 4.1: Economic Outlook and Consumer Confidence Survey

Although it is suspected that the investment decisions will be more sensitive to variations in household-level uncertainty than to increases in risk which affects all households in general, the occasional dominance of the latter may result to behaviour which cannot be revealed from the estimation. This insight can explain some of the statistical performance of the income uncertainty on the inaction band width. The income uncertainty was statistically significant in years 1985 and 1990 (high risk according to the indicators) while in 1998 (low risk according to the indicators) it was hard to find connection between income uncertainty and frequency of adjustment. This observation justifies also the assumption that the household-level uncertainty and the general economic situation may be highly correlated. Thus, while no household-level uncertainty exists as such, the general negative expectations of the future may induce precautionary saving behaviour which the cross-sectional data cannot reveal.

³⁹Consumer confidence indicator was collected semiannually since 1987. In 1991-1995 it was collected quarterly and monthly thereafter.

Most earlier empirical studies of durable purchases were based on the aggregate consumption data (see Bertola & Caballero (1990), Caballero (1990, 1993) and Hassler (2001), among others). Evidently, this was because of the lack of appropriate micro-level data. However, there are few studies which have utilized individual data. Lam (1991) used threshold adjustment model and panel data to study the consumption behaviour in an automobile market. He found that the resale market imperfections and liquidity constraints have important effects on automobile expenditures. Also, the upward adjustments are substantially quicker than downward adjustments. His interpretation of this asymmetry in differences between the upper and lower bands from the desired level implies that the efficiency of policy depends on its direction. A policy change that increases the desired stock can be expected to be more effective than a policy that reduces the desired stock by the same magnitude. Using U.S. panel data on automobile purchases, Eberly (1994) conducted similar results regarding an increase in uncertainty. One of her findings was that the width of the inaction band is positively related to the income variability. Carroll & Dunn (1997) studied the effect of an unemployment risk on durable and nondurable spending and household balance sheets. They found that the durable expenditures are very robustly correlated with lagged unemployed expectations. Dunn (1998) used household level data from 1983 and 1992 and found similar results to that of Eberly (1994): households with a higher probability of becoming unemployed are less likely to have recently purchased home or an automobile. Thus, the inaction range will be wider for those who face greater unemployment risk.

Using Finnish quarterly data from 1979 to 1992 and the conditional variance of the innovations in the aggregate income and the change in unemployment rate as a source of systematic risk, Koivumäki (1999) found statistical evidence that increased income uncertainty has suppressed consumption growth in Finland. Also, he found a negative relationship between consumption and unemployment rate. Correspondingly, Foote et al. (2000), using adjustment probits and panel of U.S. automobile holdings, found that more variable income leads to less frequent adjustment while more miles driving indicating a greater rate of depreciation leads households to adjust more often. All of these findings are in accordance to our findings and support the (S,s) behaviour. The only exception to these mainstream conclusions is Attanasio (2000) who showed that it is difficult to characterise the time-series properties of aggregate expenditure from the estimated (S,s) rules.

Even though we found some evidence of the importance of uncertainty to postpone automobile purchases, it is likely to assume that all the identified uncertainty measures and repair costs are not adequate proxies to emulate the real uncertainty and depreciation rate, respectively. To obtain more reliable results, one should improve the data by bringing time structure into the empirical analysis. While such microeconomic data does not exist, one fertile approach may be to construct an artificial panel by dividing each Survey into the groups, say, according to the income deciles, and then follow each group over time to study if

the income variance of the groups have any effect on adjustment. Evidently, this approach needs restrictive assumptions of preferences and may lead to the further problems, for example, because of the households movement between income deciles. Also, the other long-lived durables should be used to test the validity of the (S,s) model. These are, however, left for the further research.

5. CONCLUSION

This study investigated implications of uncertainty, depreciation and adjustment costs on the timing of adjustment and purchases of durable goods. The model based on the (S,s) rule extended the theoretical framework by Hassler (1994, 1996a) by deriving an (S,s) rule explicitly from the Cobb-Douglas preferences. The model states that an agent has a desire to keep a certain fraction of his wealth to be invested in one (expensive) durable. Because of the depreciation of the good and stochastic movements in prices and agent's wealth over time, the actual level of fraction deviates from that of target. While the continuous updating is costly and the purchases include costs that are at least partially irreversible, it is optimal for an agent to allow an inaction band around the frictionless target and not to adjust until the actual fraction goes outside the band. Including the possibility of idiosyncratic and systematic risk, the model states that the width of the inaction band is positively related to the systematic risk. An increase in risk increases the option value of waiting, and the purchases are postponed to the future. On the other hand, a greater depreciation should lead to more frequent adjustment.

Using four different cross-sectional Household Budget Surveys from the years 1985, 1990, 1994-96 and 1998, the empirical consumption behaviour based on the Finnish automobile purchases was in most cases in favour of the (S,s) rule. A percent increase in household-level income uncertainty increases the inaction band more than 10 percent, while an unit increase in uncertainty decreases the probability of adjustment with few percents. The other uncertainty measures - housing debt and expenditures on health - did not perform well statistically, and typically did not affect on the width of the inaction band. An increase in depreciation of automobiles measured by repair costs increases the probability of adjustment, which is consistent with the infrequent adjustment theory.

The finding that income uncertainty has a large role in household's decision-making and affects intertemporal consumption behaviour is not surprising, but the results help to understand better the effect of uncertainty on the magnitude of saving and business cycles. While the lack of data prevented us to study other uncertainty measures, durable goods and the dynamic nature of the purchases, it is likely to assume that including these elements into the study would even strengthen the importance of uncertainty on the timing of durable purchases.

REFERENCES

- Arrow, K.J., Harris, T. & Marschak, J. (1951): "Optimal Inventory Policy." *Econometrica* 19, 250-272.
- Attanasio, O.P. (1995): "Consumer Durables and Inertial Behavior: Estimation and Aggregation of (S,s) Rules." NBER Working Paper No. 5282.
- Attanasio, O.P. (1998): "Consumption Demand." NBER Working Paper No. 6466.
- Attanasio, O.P. (2000): "Consumer Durables and Inertial Behaviour: Estimation and Aggregation of (S,s) Rules for Automobile Purchases." *Review of Economic Studies* 67, 667-696.
- Bar-Ilan, A. & Blinder, A.S. (1987): "Life-Cycle Permanent-Income Model and Consumer Durables." NBER Working Paper No. 2149.
- Bar-Ilan, A. & Blinder, A.S. (1992): "Consumer Durables and the Optimality of Usually Doing Nothing." *Journal of Money, Credit and Banking* 24, 258-272.
- Bernanke, B.S. (1985): "Adjustment Costs, Durables, and Aggregate Consumption." *Journal of Monetary Economics* 15, 41-68.
- Bertola, G. & Caballero, R.J. (1990): "Kinked Adjustment Costs and Aggregate Dynamics." *NBER Macroeconomic Annual*, Vol 5, edited by Olivier Blanchard and Stanley Fischer. Cambridge, MIT Press 1990.
- Bonomo, M. & Garcia, R. (1997): "The Macroeconomic Effects of Infrequent Information with Adjustment Costs." University of Montreal. Cahiers de Recherche du C.R.D.E. Working Papers No. 20.
- Caballero, R.J. (1990): "Expenditure on Durable Goods: A Case for Slow Adjustment." *The Quarterly Journal of Economics* 105, 727-743.
- Caballero, R.J. (1993): "Durable Goods: An Explanation for Their Slow Adjustment." *Journal of Political Economy* 101, 351-743.
- Caballero, R.J. & Engel, E.M.R.A. (1991): "Dynamic (S,s) Economies." *Econometrica* 59, 1659-1686.
- Caballero, R.J. & Engel, E.M.R.A. (1993): "Microeconomic Adjustment Hazards and Aggregate Dynamics." *The Quarterly Journal of Economics* 108, 359-383.
- Carroll, C.D. & Dunn, W.E. (1997): "Unemployment Expectations, Jumping (S,s) Triggers, and Household Balance Sheets." *NBER Macroeconomic Annual* 1997, 165-217.
- Carroll, C.D. & Samwich A.A. (1996): "The Nature of Precautionary Wealth." Johns Hopkins University, Department of Economics. Working Paper No. 007.
- Carroll, C.D. & Samwich A.A. (1998): "How Important is Precautionary Saving?" *Review of Economics and Statistics* 80, 410-

- Carruth, A., Dickerson, A. & Henley, A. (2000): "What We Know about Investment under Uncertainty?" *Journal of Economic Surveys* 14, 119-153.
- Cochrane, J.H. (2000): *Asset Pricing*. Unpublished manuscript.
- Deaton, A. (1991): "Saving and Liquidity Constraints." *Econometrica* 59, 1221-1248.
- Dixit, A.K. (1993): "The Art of Smooth Pasting." Vol. 55 in *Fundamentals of Pure and Applied Economics*, ed. by J. Lesourne and H. Sonnenschein. Harwood Academic Publishers.
- Dixit, A.K. & Pindyck, R.S. (1994): *Investment under Uncertainty*. Princeton University Press. Princeton, New Jersey.
- Dunn, W.E. (1998): "Unemployment Risk, Precautionary Saving, and Durable Goods Purchase Decisions." Board of Governors of the Federal Reserve System. Finance and Economics Series No. 49.
- Eberly, J.C. (1994): "Adjustment of Consumers' Durables Stocks: Evidence from Automobile Purchases." *Journal of Political Economy* 102, 403-436.
- Estola, M. & Hokkanen, V-M. (1999): "A Dynamic Theory of Consumer Behaviour." *The Current State of Economic Science*, Vol 2. ed. by S.B. Dahiya, 691-710.
- Foote, C., Hurst, E. & Leahy, J. (2000): "Testing the (S,s) Model." *American Economic Review*, Papers and Proceedings 90, 116-119.
- Greene, W.H. (2003): *Econometric Analysis, 5th edition*. Prentice Hall.
- Grossman, S.J. & Laroque, G. (1990): "Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable." *Econometrica* 58, 25-51.
- Hall, R.E. (1978): "Stochastic Implications of the Cycle-Permanent Income Hypothesis: Theory and Evidence." *Journal of Political Economy* 86, 971-987.
- Harvey, A.C. (1976): "Estimating Regression Models with Multiplicative Heteroskedasticity." *Econometrica* 44, 461-465.
- Hassler, J.A.A. (1994): "Effects of Variations in Risk on Demand and Measures of Business Cycle Comovements." Institute for International Studies, Stockholm University. Monograph Series 24.
- Hassler, J.A.A. (1996a): "Variations in Risk and Fluctuations in Demand: A Theoretical Model." *Journal of Economic Dynamics and Control* 20, 1115-1143.
- Hassler, J.A.A. (1996b): "Fluctuating Risk in an Aggregated Ss-Model." Federal Reserve Bank of Minneapolis. Discussion Paper 113.
- Hassler, J.A.A. (1996c): "Risk and Consumption." *Swedish Economic Policy Review* 2, 303-337.
- Hassler, J.A.A. (2001): "Uncertainty and the Timing of Automobile Purchases." *Scandinavian Journal of Economics* 103, 351-366.

- House, C.L. & Leahy, J.V. (2000): "An sS Model with Adverse Selection." NBER Working Paper No. 8030.
- Hubbard, R.G., Skinner, J. & Zeldes, S.P. (1995): "Precautionary Saving and Social Insurance." *Journal of Political Economy* 103, 360-399.
- Koivumäki, T. (1999): "Uncertainty and the Dynamics of Demand." Ph.D. Thesis, University of Oulu, Serie G5.
- Lam, Pok-sang (1989): "Irreversibility and Consumer Durables Expenditures." *Journal of Monetary Economics* 23, 135-150.
- Lam, Pok-sang (1991): "Permanent Income, Liquidity, and Adjustments of Automobile Stocks: Evidence from Panel Data." *Quarterly Journal of Economics* 106, 203-230.
- Leahy, J.V. & Zeira, J. (2000): "The Timing of Purchases and Aggregate Fluctuations." NBER Working Paper No. 7672.
- Leland, H. (1968): "Saving and Uncertainty: The Precautionary Demand for Saving." *Quarterly Journal of Economics* 82, 465-473.
- Malliari, A.G. & Brock W.A. (1982): *Stochastic Methods in Economics and Finance*. Advanced Textbooks in Economics, Vol. 17 ed. by Bliss, C.J. & Intriligator, M.D. North-Holland.
- Mankiw, G.N. (1982): "Hall's Consumption Hypothesis and Durable Goods." *Journal of Monetary Economics* 10, 417-425..
- McDonald, R. & Siegel, D. (1986): "The Value of Waiting to Invest." *Quarterly Journal of Economics* 101, 707-727.
- Merton, R.C. (1999): *Continuous-Time Finance*. Blackwell Publishers Ltd. Oxford, UK.
- Niemeläinen, H. (1995): "The Theory of Consumer's Optimum and Utility Weights with an Application to the Endogenous Trade Union Fee." In *Essays on Labor Supply, Wage Formation and the Trade Union Fee*. University of Joensuu, Publications in Social Sciences N:o 22.
- Pagan, A. (1986): "Two Stage and Related Estimators and Their Applications." *Review of Economic Studies* 53, 517-538.
- Pietola, K. (1997): "A Generalized Model of Investment with an Application to Finnish Hog Farms." Agricultural Economics Research Institute, Finland. Publications 84.
- Pindyck, R.S. (1991): "Irreversibility, Uncertainty, and Investment." *Journal of Economic Literature* 29, 1110-1148.
- Suoniemi, I. & Sullström, R. (1995): "The Structure of Household Consumption in Finland, 1966 - 1990." Government Institute for Economic Research. Research Reports 27.

APPENDIX A

Proposition:

If the variables P , W and C are evolving over time according to the following geometric Brownian motions

$$\begin{aligned}\frac{dW}{W} &= \alpha_W dt + \delta_w dz_W, \\ \frac{dP}{P} &= \alpha_P dt + \delta_P dz_P, \\ \frac{dC}{C} &= -\alpha_C dt,\end{aligned}\tag{A.1}$$

then the function $a_t(P_t, C_t, W_t) = \frac{P_t C_t}{W_t}$ follows geometric Brownian motion

$$\frac{da}{a} = \alpha_a dt + \delta_a dz_a.\tag{A.2}$$

The terms α and δ may be interpreted as drift and variance parameters of the processes, respectively. Especially, the term α_C is the rate of depreciation. The terms dz are the increments of a Wiener process such that $dz_k = \varepsilon_k \sqrt{dt}$. While $\varepsilon_k \sim N(0, 1)$, $E(dz_k) = 0$ and $Var(dz_k) = E[(dz_k)^2] - [E(dz_k)]^2 = dt$.

Proof:

In this proof we apply the Fundamental Theorem of stochastic calculus which is expanded to functions of several Ito processes.⁴⁰ In general, in the presence of several Wiener processes the differential dF for a function $F(t, x_1, \dots, x_m)$ is given as

$$dF = \frac{\partial F}{\partial t} dt + \sum_i \frac{\partial F}{\partial x_i} dx_i + \frac{1}{2} \sum_i \sum_j \frac{\partial^2 F}{\partial x_i \partial x_j} dx_i dx_j\tag{A.3}$$

where dx_i and dx_j , $i, j = 1, \dots, m$; $i \neq j$, are independent Ito processes. Inserting the derivatives and noting that there does not exist time explicitly in the function $a(P, C, W)$, the expression (A.3) becomes

$$\begin{aligned}da &= \left(\frac{C}{W} dP + \frac{P}{W} dC - \frac{PC}{W^2} dW \right) + \frac{1}{2} \left\{ \frac{1}{W} (dP)(dC) \right. \\ &\quad \left. - \frac{C}{W^2} (dP)(dW) + \frac{1}{W} (dC)(dP) - \frac{P}{W^2} (dC)(dW) + \frac{2PC}{W^3} (dW)^2 \right. \\ &\quad \left. - \frac{C}{W^2} (dW)(dP) - \frac{P}{W^2} (dW)(dC) \right\} \\ &= \frac{C}{W} dP + \frac{P}{W} dC - \frac{PC}{W^2} dW + \frac{1}{W} (dP)(dC) - \frac{C}{W^2} (dP)(dW) \\ &\quad - \frac{P}{W^2} (dC)(dW) + \frac{2PC}{W^3} (dW)^2.\end{aligned}\tag{A.4}$$

⁴⁰See Malliaris & Brock (1982) and Dixit & Pindyck (1994) who give more background for the stochastic calculus and describe the properties of the Ito processes in more detail.

After substituting the Ito processes from (A.1) the expanded form of the equation becomes

$$\begin{aligned}
da &= \frac{C}{W} (\alpha_P P dt + \delta_P P dz_P) - \frac{P}{W} (\alpha_C C dt) \\
&\quad - \frac{PC}{W^2} (\alpha_W W dt + \delta_W W dz_W) - \frac{1}{W} (\alpha_P P dt + \delta_P P dz_P) (\alpha_C C dt) \\
&\quad - \frac{C}{W^2} (\alpha_P P dt + \delta_P P dz_P) (\alpha_W W dt + \delta_W W dz_W) \\
&\quad + \frac{P}{W^2} (\alpha_C C dt) (\alpha_W W dt + \delta_W W dz_W) + \frac{2PC}{W^3} (\alpha_W W dt + \delta_W W dz_W)^2.
\end{aligned} \tag{A.5}$$

All the terms $(dt)^{\frac{3}{2}}$ and $(dt)^2$ go to zero faster than dt as time increments become infinitesimal small, so these terms are ignored. Noting that the term $E [dz_i dz_j] = \rho_{ij} dt$ is the coefficient of correlation⁴¹ between the two processes the expression can be rewritten as

$$\begin{aligned}
da &= \frac{PC}{W} (\alpha_P - \alpha_C - \alpha_W) dt + \frac{PC}{W} (\delta_P dz_P - \delta_W dz_W) \\
&\quad - \frac{PC}{W} \delta_P \delta_W \rho_{PW} dt + \frac{2PC}{W} \delta_W^2 dt \\
&= \left[\alpha_P - \alpha_C - \alpha_W - \delta_P \delta_W \rho_{PW} + 2\delta_W^2 \right] a dt + [\delta_P dz_P - \delta_W dz_W] a \\
&= \alpha_a a dt + \delta_a a dz_a.
\end{aligned} \tag{A.6}$$

This is the equation (2.3) in the main text. It is easy to show that the mean and the variance of this process are

$$\begin{aligned}
E \left(\frac{da}{a} \right) &= \alpha_a dt, \\
Var \left(\frac{da}{a} \right) &= E [(da)^2] - E [(da)]^2 = \delta_a^2 dt.
\end{aligned} \tag{A.7}$$

APPENDIX B

Suppose that each time increment is of length Δt and denote $x_t = a_t - a^*$, then $\Delta x = \Delta a$. The Bellman equation for the problem is

$$V(x_t, u_t) = \frac{x_t^2}{2} \Delta t + e^{-r\Delta t} E_t [V(x_{t+\Delta t}, u_{t+\Delta t})], \tag{B.1}$$

in which $V(x_t, u_t)$ denotes the total cost function and $V(x_{t+\Delta t}, u_{t+\Delta t}) = V(x_t + \Delta x, t + \Delta t, u + \Delta u)$. Using the approximation $e^{-r\Delta t} \approx (1 + r\Delta t)^{-1}$ and multiplying

⁴¹Note that because Wiener processes have variances and standard deviations per unit of time equal to one, ρ_{ij} is also the covariance per unit of time between the processes.

(B.1) with the term $(1 + r\Delta t)$ gives

$$\begin{aligned} rV(x_t, u_t)\Delta t &= \frac{x_t^2}{2}\Delta t(1 + r\Delta t) + E_t[V(x_{t+\Delta t}, u_{t+\Delta t}) - V(x_t, u_t)] \quad (\text{B.2}) \\ &= \frac{x_t^2}{2}\Delta t(1 + r\Delta t) + E_t[dV]. \end{aligned}$$

Dividing by Δt and letting it approach zero we get

$$rV(x_t, u_t) = \frac{x_t^2}{2} + \frac{1}{dt}E_t[dV]. \quad (\text{B.3})$$

The right-hand side of the equation can be interpreted as a current flow of disutility plus the expected rate of change of the total cost function. Using the version of Ito's lemma for combined Brownian and Poisson processes⁴², the expectation of the differential V is given by

$$\begin{aligned} E[dV] &= \left[\alpha x_t V'(x_t, u_t) + \frac{1}{2}\sigma^2 x_t^2 V''(x_t, u_t) + h.o.t \right] dt \quad (\text{B.4}) \\ &\quad + \lambda [(V(a^*, u_t) + c) - V(x_t, u_t)] dt + \gamma_u [V(x_t, \bar{u}_t) - V(x_t, u_t)] dt, \end{aligned}$$

where *h.o.t* means higher order terms which approach zero faster than dt as $dt \rightarrow 0$. These terms are omitted. The second term on the right-hand side captures the idea that a Poisson shock occurs with probability λdt . Then, by assumption, the variable a is adjusted back to the target level a^* after the lumpy sum cost is paid. Note that while the immediate utility loss is zero at the target level, the term $V(a^*, u_t) \neq 0$ because of the expectation of the future deviations from the target. The term γ_u is the probability of the switch of the economy. If the switch occurs during the time increment dt , then the expected total costs shift from $V(x_t, u_t)$ to $V(x_t, \bar{u}_t)$.⁴³ Inserting the previous equation to (B.3) we get

$$\begin{aligned} &\frac{1}{2}\sigma^2 x_t^2 V''(x_t, u_t) + \alpha x_t V'(x_t, u_t) - (\lambda + r)V(x_t, u_t) + \gamma_u [V(x_t, \bar{u}_t) - V(x_t, u_t)] \\ &= -\frac{x_t^2}{2} - \lambda (V(a^*, u_t) + c). \quad (\text{B.5}) \end{aligned}$$

While $u_t = 0, 1$, (B.5) constitutes a system of two second-order differential equations with two unknown functions. Even though this system is quite complex its set of solutions can be found using the following procedure. The system of the second-order differential equations can be rewritten as

$$\frac{1}{2}\sigma^2 x_t^2 V''(x_t, 0) + \alpha x_t V'(x_t, 0) - (\lambda + r)V(x_t, 0) + \gamma_0 [V(x_t, 1) - V(x_t, 0)]$$

⁴²See Dixit & Pindyck (1994), Merton (1999) or Cochrane (2000) for a thorough mathematical treatment.

⁴³To be precise, the terms $V(a^*, u_t)$ and $V(x_t, \bar{u}_t)$ should be written as $V(0, u_{t+1})$ and $V(x_t, \bar{u}_{t+1})$ to capture the idea of a jump or a switch between the times t and $t + 1$. However, in an infinite context these two are equal. Also, at the target level the state variable is $x_t = a^* - a^* = 0$. To avoid confusion later on, however, we use the notation $V(a^*, u_t)$ to describe the (expected) total costs at the target level when the state of the economy is u_t .

$$\begin{aligned}
&= -\frac{x_t^2}{2} - \lambda(V(a^*, 0) + c), \tag{B.6} \\
&\quad \frac{1}{2}\sigma^2 x_t^2 V''(x_t, 1) + \alpha x_t V'(x_t, 1) - (\lambda + r)V(x_t, 1) + \gamma_1 [V(x_t, 0) - V(x_t, 1)] \\
&= -\frac{x_t^2}{2} - \lambda(V(a^*, 1) + c).
\end{aligned}$$

This is the equation (2.10) in the main text. Define two new functions (without the subscripts) such that

$$\begin{aligned}
K(x) &= V(x_t, 0) - V(x_t, 1), \tag{B.7} \\
J(x) &= \frac{V(x_t, 1)}{\gamma_1} + \frac{V(x_t, 0)}{\gamma_0}.
\end{aligned}$$

Then

$$\begin{aligned}
&\frac{1}{2}\sigma^2 x^2 K''(x) + \alpha x K'(x) - (\lambda + r + \gamma_0 + \gamma_1)K(x) \tag{B.8} \\
&= \lambda(V(a^*, 0) - V(a^*, 1)), \\
&\quad \frac{1}{2}\sigma^2 x^2 J''(x) + \alpha x J'(x) - (\lambda + r)J(x) \\
&= -\frac{\gamma_0 + \gamma_1}{2\gamma_0\gamma_1}x^2 - \lambda\left(\frac{1}{\gamma_0}(V(a^*, 0) + c) + \frac{1}{\gamma_1}(V(a^*, 1) + c)\right).
\end{aligned}$$

The first equation comes from subtracting the first equation from the second in (B.6) and using (B.7). Adding up the equations in (B.6) and using (B.7) results the second equation in (B.8). Each of the equations in (B.8) yields an 'independent' solution. Consider the second equation in (B.8) and guess that the solution for the homogeneous part is of the general form

$$J(x) = Ax^\beta, \tag{B.9}$$

where A is a constant to be determined. Then, the homogeneous part can be rewritten as

$$Ax^\beta \underbrace{\left(\frac{1}{2}\sigma^2\beta^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\beta - (\lambda + r)\right)}_{Q_\beta(\beta)} = 0. \tag{B.10}$$

The roots of the fundamental quadratic $Q_\beta(\beta)$ are

$$\begin{aligned}
\beta_1 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda + r)}{\sigma^2}} > 1, \tag{B.11} \\
\beta_2 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda + r)}{\sigma^2}} < 0.
\end{aligned}$$

Thus, the general solution of the homogeneous part is

$$J_H(x) = A_1 x^{\beta_1} + A_2 x^{\beta_2}. \tag{B.12}$$

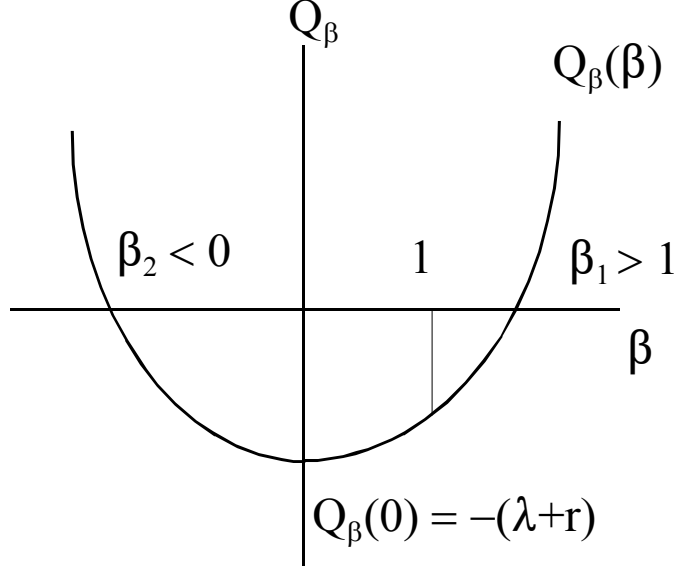


Figure B1: Fundamental Quadratic

The parameter A_2 can be determined as follows. Evaluating the fundamental quadratic at points $(0,1)$ results $Q_\beta(0) = -(\lambda+r) < 0$ and $Q_\beta(1) = \alpha - (\lambda+r) < 0$.⁴⁴ Thus, $\beta_1 > 1$ and $\beta_2 < 0$. This result can be understood from the figure B1. The limiting behaviour near zero gives one condition. When a is expected to remain at its target value, there is no utility and adjustment costs. This gives the condition $J(0) = 0$. However, when $a \rightarrow a^*$, that is, when $x \rightarrow 0$ and $\beta_2 < 0$, the term $A_2 x^{\beta_2} \rightarrow \infty$. To ensure that $J(x)$ goes to zero as $x \rightarrow 0$, we set the coefficient of the negative power of x equal to zero, that is, $A_2 = 0$.

The particular solution can be found by using the method of undetermined coefficients. Guess that the solution is of the form

$$J(x) = B_2 x^2 + B_1 x + B_0. \quad (\text{B.13})$$

Inserting the correspondent derivatives to (B.8) and comparing the coefficients result

$$\begin{aligned} B_2 &= -\frac{\gamma_0 + \gamma_1}{2\gamma_0\gamma_1(\sigma^2 + 2\alpha - (\lambda+r))}, \\ B_1 &= 0, \\ B_0 &= \frac{\lambda}{\lambda+r} \left(\frac{1}{\gamma_0}(V(a^*, 0) + c) + \frac{1}{\gamma_1}(V(a^*, 1) + c) \right). \end{aligned} \quad (\text{B.14})$$

⁴⁴The assumption $\alpha < \lambda+r$ ensures that there exists finite time when it is optimal to adjust. Otherwise, waiting longer would always be a better policy, and the optimum would not exist. See Dixit & Pindyck (1994, pp.137-138, pp.171-173) for illustrative calculations.

The general solution for $J(x)$ is

$$J(x) = A_1 x^{\beta_1} - \frac{\gamma_0 + \gamma_1}{2\gamma_0\gamma_1(\sigma^2 + 2\alpha - (\lambda + r))} x^2 + \frac{\lambda}{\lambda + r} \left(\frac{1}{\gamma_0} (V(a^*, 0) + c) + \frac{1}{\gamma_1} (V(a^*, 1) + c) \right). \quad (\text{B.15})$$

Consider next the first equation in (B.8). Following the same steps as above the homogeneous part can be rewritten as

$$C x^\theta \underbrace{\left(\frac{1}{2} \sigma^2 \theta^2 + (\alpha - \frac{1}{2} \sigma^2) \theta - \eta \right)}_{Q_\theta(\theta)} = 0, \quad (\text{B.16})$$

where $\eta = \lambda + r + \gamma_0 + \gamma_1$. Thus, the general solution of the homogeneous part is

$$K_H(x) = C_1 x^{\theta_1} + C_2 x^{\theta_2}, \quad (\text{B.17})$$

where the roots are

$$\begin{aligned} \theta_1 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2(\lambda + r + \gamma_0 + \gamma_1)}{\sigma^2}} > \beta_1, \\ \theta_2 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2(\lambda + r + \gamma_0 + \gamma_1)}{\sigma^2}} < \beta_2. \end{aligned} \quad (\text{B.18})$$

It is easy to show that $Q_\theta(0) < Q_\beta(0) < 0$ and $Q_\theta(1) < Q_\beta(1) < 0$. This results $\theta_1 > \beta_1 > 1$ and $\theta_2 < \beta_2 < 0$. The coefficients C_1 and C_2 can be determined as above, leaving only C_1 to be determined. The particular solution is easy to find. It is

$$K_P(x) = -\frac{\lambda [V(a^*, 1) - V(a^*, 0)]}{\eta}. \quad (\text{B.19})$$

The general solution for $K(x)$ is

$$K(x) = C_1 x^{\theta_1} - \frac{\lambda [V(a^*, 1) - V(a^*, 0)]}{\eta}. \quad (\text{B.20})$$

The solutions for $V(x_t, 0)$ and $V(x_t, 1)$ can be found by using (B.7), (B.15) and (B.20). Thus,

$$\begin{aligned} V(x_t, 0) &= \frac{\gamma_0\gamma_1}{\gamma_0 + \gamma_1} \left\{ A_1 x^{\beta_1} - \frac{\gamma_0 + \gamma_1}{2\gamma_0\gamma_1(\sigma^2 + 2\alpha - (\lambda + r))} x^2 \right. \\ &\quad \left. + \frac{\lambda}{\lambda + r} \left(\frac{1}{\gamma_0} (V(a^*, 0) + c) + \frac{1}{\gamma_1} (V(a^*, 1) + c) \right) \right\} \\ &\quad - \frac{\gamma_0}{\gamma_0 + \gamma_1} \left\{ C_1 x^{\theta_1} - \frac{\lambda [V(a^*, 1) - V(a^*, 0)]}{\eta} \right\}, \\ V(x_t, 1) &= \frac{\gamma_0\gamma_1}{\gamma_0 + \gamma_1} \left\{ A_1 x^{\beta_1} - \frac{\gamma_0 + \gamma_1}{2\gamma_0\gamma_1(\sigma^2 + 2\alpha - (\lambda + r))} x^2 \right. \end{aligned} \quad (\text{B.21})$$

$$\begin{aligned}
& + \frac{\lambda}{\lambda + r} \left(\frac{1}{\gamma_0} (V(a^*, 0) + c) + \frac{1}{\gamma_1} (V(a^*, 1) + c) \right) \Big\} \\
& + \frac{\gamma_1}{\gamma_0 + \gamma_1} \left\{ C_1 x^{\theta_1} - \frac{\lambda [V(a^*, 1) - V(a^*, 0)]}{\eta} \right\}.
\end{aligned}$$

The functions $V(a^*, 0)$ and $V(a^*, 1)$ can be found by evaluating (B.21) at the point a^* (then $x = 0$). After some rigorous calculus⁴⁵, the solution becomes

$$\begin{aligned}
V(x_t, 0) &= \frac{\gamma_0 \gamma_1}{\gamma_0 + \gamma_1} A_1 x^{\beta_1} - \frac{1}{2(\sigma^2 + 2\alpha - (\lambda + r))} x^2 & (B.22) \\
&\quad - \frac{\gamma_0}{\gamma_0 + \gamma_1} C_1 x^{\theta_1} + \frac{\lambda c}{r}, \\
V(x_t, 1) &= \frac{\gamma_0 \gamma_1}{\gamma_0 + \gamma_1} A_1 x^{\beta_1} - \frac{1}{2(\sigma^2 + 2\alpha - (\lambda + r))} x^2 \\
&\quad + \frac{\gamma_1}{\gamma_0 + \gamma_1} C_1 x^{\theta_1} + \frac{\lambda c}{r}.
\end{aligned}$$

This is the equation (2.11) in the main text. However, the equation (B.22) is valid only in the range $x_t \in [s_0, a^*]$. If the state of the economy is low ($u_t = 0$) and $x_t \leq s_0$, the durable is immediately adjusted. Also, if the state variable is in the range $[s_1, s_0]$, a switch from the high risk state to the low risk state will cause an immediate adjustment. Thus, $V(x_t, 0)$ is a constant for $x_t \in [s_1, s_0]$. The system of differential equations degenerates to

$$\begin{aligned}
V(x_t, 0) &= V(s_0, 0), & (B.23) \\
V(x_t, 1) &= \frac{x_t^2}{2} \Delta t + e^{-r\Delta t} E_t [V(x_{t+\Delta t}, u_{t+\Delta t})].
\end{aligned}$$

Following the same steps as above the solution for the function $V(x_t, 1)$ is

$$\begin{aligned}
V(x_t, 1) &= D_1 x^{\mu_1} - \frac{1}{2(\sigma^2 + 2\alpha - (\lambda + r + \gamma_1))} x^2 & (B.24) \\
&\quad - \frac{\gamma_1 V(a^*, 0) + (\gamma_1 - \lambda)c}{(r + \gamma_1)},
\end{aligned}$$

in which the root μ_1 is defined as

$$\beta_1 < \mu_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda + r + \gamma_1)}{\sigma^2}} < \theta_1. \quad (B.25)$$

To summarise:

- 1⁰** If $x_t \in [s_0, a^*]$ and the state of the economy is $u_t = 0$, $V(x_t, 0)$ is given by the equation (B.22).
- 2⁰** If $x_t < s_0$ and the state of the economy is $u_t = 0$, $V(x_t, 0)$ is constant and

⁴⁵We have benefited from Scientific Workplace in calculus.

equals $V(s_0, 0)$.

3⁰ If $x_t \in [s_0, a^*]$ and the state of the economy is $u_t = 1$, $V(x_t, 1)$ is given by the equation (B.22).

4⁰ If $x_t \in [s_1, s_0]$ and the state of the economy is $u_t = 1$, $V(x_t, 1)$ is given by the equation (B.24).

5⁰ If $x_t < s_1$ and the state of the economy is $u_t = 1$, $V(x_t, 1)$ is constant and equals $V(s_1, 1)$.

Analytically, we are not able to find the unknown integration constants and band limits in (B.22) and (B.24). However, using the following smooth-pasting and value-matching conditions we are able to find the solutions numerically.

$$\begin{aligned}
 \text{Smooth - pasting} & : \begin{cases} V'(s_0, 0) = 0 \\ V'(s_1, 1) = 0 \\ V'(a^*, 0) = 0 \\ V'(a^*, 1) = 0 \end{cases}, & \text{(B.26)} \\
 \text{Value - matching} & : \begin{cases} V(s_0, 0) = V(a^*, 0) + c \\ V(s_1, 1) = V(a^*, 1) + c \end{cases}.
 \end{aligned}$$

APPENDIX C

From Appendix B the equations (B.3) and (B.4) result

$$\begin{aligned}
 rV(x_t, u_t) & = \frac{x_t^2}{2} + \left[\alpha x_t V'(x_t, u_t) + \frac{1}{2} \sigma^2 x_t^2 V''(x_t, u_t) \right] \\
 & \quad + \lambda [(V(a^*, u_t) + c) - V(x_t, u_t)] + \gamma_u [(V(x_t, \bar{u}_t)) - V(x_t, u_t)].
 \end{aligned} \tag{C.1}$$

Evaluating this equation at point a^* and trigger bands s_u , $u = 0, 1$, gives

$$\begin{aligned}
 rV(a^*, u_t) & = 0 + 0 + 0 + \lambda [(V(a^*, u_t) + c) - V(a^*, u_t)] \\
 & \quad + \gamma_u [(V(a^*, \bar{u}_t)) - V(a^*, u_t)],
 \end{aligned} \tag{C.2}$$

$$\begin{aligned}
 rV(s_u, u_t) & = \frac{s_u^2}{2} + \alpha s_u V'(s_u, u_t) + \frac{1}{2} \sigma^2 s_u^2 V''(s_u, u_t) \\
 & \quad + \lambda [(V(a^*, u_t) + c) - V(s_u, u_t)] + \gamma_u [(V(s_u, \bar{u}_t)) - V(s_u, u_t)].
 \end{aligned}$$

The first three terms in the first equation are zero because at the target level a^* the state variable is $x_t = a^* - a^* = 0$. Subtracting the first equation from the second, using second-order Taylor approximation for the term $V''(s_u, u_t)$ and the smooth-pasting and value-matching conditions (B.26), and after some rearrangement gives

$$\frac{(a_u - a^*)^2}{2} = \underbrace{>0}_{rc} + \underbrace{>0}_{\lambda c} + \gamma_u [c + V(a^*, \bar{u}_t) - V(s_u, \bar{u}_t)] \tag{C.3}$$

On the left-hand side we have utilised the information that at the trigger band point the state variable becomes $(x_t) |_{s_u} = a_u - a^*$ denoting the largest deviation from the target level. The left-hand side is the temptation to adjust, and the right-hand side is the value of waiting still an instant dt . The first two terms on the right-hand side are positive. The third term can be rewritten as

$$\underbrace{\gamma_u}_{>0} \overbrace{[c - (V(s_u, \bar{u}_t) - V(a^*, \bar{u}_t))]}^{\geq 0} \geq 0. \quad (\text{C.4})$$

This inequality can be understood as follows: If $x_t < s_u$, then $c \geq V(x_t, \bar{u}_t) - V(a^*, \bar{u}_t)$. If $x_t = s_u$, then $c = V(x_t, \bar{u}_t) - V(a^*, \bar{u}_t)$ according to the value-matching condition.

APPENDIX D

Table D1
Dummy variables used in estimation

Constant	
Province	Southern Finland Western Finland Eastern Finland Oulu Lapland Åland
Local Authority	City Commune
Gender of household head	Male Female
Type of household	Family without children One-parent family Family with children Aged family Other families
Educational attainment	Basic Lower middle-level Higher middle-level Lowest high-level Lower candidate-level Higher candidate-level Researcher or similar Unknown Employee
Socio-economic status	Subordinate official Superior official Entrepreneur Agricultural entrepreneur Student Retired Long-term unemployed Others
Months of unemployed	1-3 months 4-6 months 7-9 months 10-12 months More than 12 months

**JOENSUUN YLIOPISTON YHTEISKUNTATIETEELLISIÄ JULKAISUJA
UNIVERSITY OF JOENSUU, PUBLICATIONS IN SOCIAL SCIENCES**

1. Hautamäki, Jarkko. Peruskoululaisten loogisen ajattelun mittaamisesta ja esiintymisestä. English abstract: The Measurement and Distribution of Piagetian Stages of Thinking in Finnish Comprehensive School. Joensuu 1984. 157 s.
2. Nyblom, Jukka. Comparisons of Tests for the Presence of Random Walk Coefficients in Two Simple Linear Models. Joensuu 1984. 18 p.
3. Vartiainen, Perttu. Maantieteen konstituoitumisesta ihmistieteenä. English abstract: On the Constitution of Geography as a Human Science. Joensuu 1984. 590 s.
4. Saariluoma, Pertti. Essays on the Foundations of Cognitive Psychology. Joensuu 1985. 129 p.
5. Eskelinen, Heikki. Sijainti ja tuotantorakenne riippuvuuksineen. Tutkimuksia alue-taloudellisesta integraatiosta. English abstract: Interdependencies in Location and Production Structure. Joensuu 1985. 73 s.
6. Hilpelä, Jyrki. Filosofia kritiikkinä. English abstract: Philosophy as Criticism. Joensuu 1986. 205 s.
7. Karjalainen, Pauli. Geodiversity as a Lived World: On the Geography of Existence. Joensuu 1986. 190 p.
8. Paasi, Anssi. Neljä maakuntaa. Maantieteellinen tutkimus aluetietoisuuden kehittämisestä. English abstract: Four Provinces in Finland. A Geographical Study of the Development of Regional Consciousness. Joensuu 1986. 345 s.
9. Paasi, Anssi. The Institutionalization of Regions. Theory and Comparative Case Studies. Joensuu 1986. 36 p.
10. Rätty, Hannu. Uhka vai uhri. Tutkimus mielisairauteen asennoitumisesta. English abstract: Threat or Victim. A Study of Attitudes toward Mental Illness. Joensuu 1987. 142 s.
11. Sabour, M'hammed. Homo Academicus Arabicus. Joensuu 1988. 329 p.
12. Rannikko, Pertti. Metsätyö-pienviljelykylä. Tutkimus erään yhdyskuntatyyppin noususta ja tuhosta. Joensuu 1989. 114 s.
13. Ahponen, Pirkkoliisa. Kulttuuripolitiikka ja sen representaatiot. Tutkimus ylevän maallistumisesta. Joensuu 1991. 311 s.
14. Sabour, M'hammed (ed.). Liberté, Egalité, Fraternité. Bicentenaire de la Grande Révolution Française. Joensuu 1992. 246 s.
15. Häyrynen, Yrjö-Paavo & Perho, Hannu & Kuittinen, Matti & Silvonen, Jussi. Ilmapiirit, kentät ja kulttuurit. Suomen korkeakoulutus 1973B1989. Joensuu 1992. 151 s.
16. Turunen, Panu. Der unbefangene Blick: Mach, Husserl und die ursprüngliche Anschauung. Joensuu 1993. 169 s.
17. Koski, Leena. Tieteen tahtomana, yliopiston tekemänä. Yliopiston sisäiset symboliset järjestykset. Joensuu 1993. 201 s.
18. Pirttilä, Ilkka. Me ja maailman mallit. Tiedonsosiologian ydintä etsimässä. Joensuu 1993. 244 s.
19. Mjelde, Liv. Apprenticeship. From Practice to Theory and Back Again. Joensuu 1993. 308 p.
20. Leiman, Mikael. Integrating the Vygotskian Theory of Sign-mediated Activity and the British Object Relations Theory. Joensuu 1994. 70 p.

21. Puuronen, Vesa. Nuoret maailmansa tuottajina? Tutkimus nuorisososiologian metodologiasta ja nuorten keskusteluista. Joensuu 1995. 186 s.
22. Niemeläinen, Heikki. Essays on Labor Supply, Wage Formation and the Trade Union Fee. Joensuu 1995. 97 p.
23. Hölttä, Seppo. Towards Self-Regulative University. Joensuu 1995. 278 p.
24. Kortelainen, Jarmo. Tehdasyhdyskunta talouden ja ympäristötietoisuuden murrosvaiheissa. Joensuu 1996. 155 s.
25. Hong, Jianzhong. Development of Verbal and Non-verbal Thinking. A Review and Re-analysis of Studies in Children of Six Nationalities on the Qinghai Plateau. Joensuu 1996. 159 s.
26. Kekäle, Jouni. Leadership Cultures in Academic Department. Joensuu 1997. 253 p.
27. Tammi, Timo. Essays on the Rationality of Experimentation in Economics. The Case of Preference Reversal Research. Joensuu 1997. 194 p.
28. Moisseinen, Eija. Contingent Valuation. The Case of the Saimaa Seal. Joensuu 1997. 208 p.
29. Jolkkonen, Arja. Paikan vaihto. Tutkimus irtisanomisuhan alaisten naisten työmarkkinastrategioista ja paikallisista työmarkkinoista. Joensuu 1998. 328 s.
30. Oksa, Jukka. Syrjäkylä muutoksessa. Joensuu 1998. 300 s.
31. Rätty, Tarmo. Efficient Facets, Influential Observations and Revisions of the Best Practice Boundary for DEA. Joensuu 1998. 141 p.
32. Ilmonen, Kari. Kulttuuri alueen käyttövarana. Kulttuuripolitiikan hallitsevat ja marginaaliset diskurssit Keski-Pohjanmaalla. Joensuu 1998. 287 s.
33. Varis, Eira. Syrjäkylä murros Venäjän Karjalassa ja Unkarissa. Tutkimus post-sosialistisen maaseudun restruktuuraatiosta ja resurssiyhdyskuntien selviytymisestä. Joensuu 1998. 286 s.
34. Vähämöttönen, Timo. Reframing Career Counselling in Terms of Counsellor-Client Negotiations. An Interpretive Study of Career Counselling Concepts and Practice. Joensuu 1998. 183 p.
35. Komulainen, Katri. Kotihiiriä ja ihmisiä. Retorinen minä naisten koulutusta koskevissa elämäkertomuksissa. Joensuu 1998. 312 s.
36. Pyy, Ilkka. Paikallisvaltion loppu? Hyvinvointivaltion murros syrjäisessä kunnassa. Joensuu 1998. 287 s.
37. Susag, Chris. Finnish American Ethnicity as Measured by Collective Self Esteem. Joensuu 1999. 182 p.
38. Aaltonen, Elli. Sosiaalitoimen johtaminen - byrokratiaa vai tulosjohtamista? Tutkimus sosiaalitoimen johtamisesta Suomessa 1950-luvulta 1990-luvun puoliväliin. English abstract: Management within Municipal Social Administration - Bureaucracy or Managing for Results? A Study of the Management of Municipal Social Administration in Finland from the 1950's to the mid 1990's. Joensuu 1999. 201 s.
39. Korhonen, Merja. Isyyden muutos. Keski-ikäisten miesten lapsuuskokemukset ja oma vanhemmuus. The Change in Fatherhood. The Childhood Experiences of Middle-aged Men and Their Own Parenthood. Joensuu 1999. 292 s.
40. Tedre, Silva. Hoivan sanattomat sopimukset. Tutkimus vanhusten kotipalvelun työntekijöiden työstä. English abstract: The Unspoken Contracts in Social Care. A Case Study of Paid Workers in Finnish Municipal Home Help Services for the Elderly. Joensuu 1999. 188 s.

41. Kurvinen, Arja. Tilinteon aika. Tutkimus pankista työnsä menettävien naisten identiteetin uudelleen arvioinneista. Joensuu 1999. 357 s.
42. Koivuluhta, Merja. Ammatti-intressit ja ura. Pohjoiskarjalaisen ammattikoulutetun ikäluokan seuranta 1975 – 1991. English summary: Vocational Interests and Career. A Follow-up of a Vocationally Trained North Karelian Age Cohort from 1975 – 1991. Joensuu 1999. 381 s.
43. Westman, Anna Liisa. Under the Northern Lights. The Reflection of Gender on the Career of Women Managers in Finnish Municipalities. Tiivistelmä/Finnish abstract: Revontulten alla. Sukupuolen heijastumia naisten uraan ja työhön Suomen kunnissa. Tutkimus naisten urakehityksestä ja työstä johtajina Suomen kunnissa. Joensuu 2000. 460 p.
44. Puhakka, Antero. Hoitoonohjaus hoitoon ohjatun silmin – subjektiivinen totuus ja sen tulkinta. English abstract: Referral to Treatment from the Point of View of the Referred: Interpreting Subjective Truth. Joensuu 2000. 264 s.
45. Puustinen, Raimo. Sign, Symptom and Interpretation. Medical Consultation in the Light of Mikhail Bakhtin's Semiotics. Tiivistelmä/Finnish Abstract: Merkki, oire ja tulkinta – lääkärin vastaanotto Mihail Bahtinin semiotiikan valossa. Joensuu 2000. 79 p.
46. Silvonen, Jussi. "Tässä muuten hienossa näytelmässä." – Työhön-ryhmät työttömän roolim muutoksen tukijoina laadullisen analyysin valossa. English abstract: "In this otherwise so great play ...". JOBS groups in Finland as supporter of the change in the job-seeker role of the unemployed. Joensuu 2000. 267 s.
47. Komonen, Katja. Koulutusyhteiskunnan marginaalissa? Ammatillisen koulutuksen keskeyttäneiden nuorten yhteiskunnallinen osallisuus. English summary: In the margin of an education society? the social inclusion of adolescents who drop out of vocational education. Joensuu 2001. 304 s.
48. Leskinen, Pekka. Statistical methods for measuring preferences. Joensuu 2001. 113 p.
49. Motola Michael. Social Habitus and The Impact of Socio-Cultural Factors on Vocational Behavior of Adolescents. A cross-national comparison of French and Finnish students. Tiivistelmä/Finnish abstract: Sosiaalinen habitus ja sosiokulttuuristen tekijöiden vaikutus nuorten ammatilliseen käyttäytymiseen. Suomalaisten ja ranskalaisten opiskelijoiden vertailu. Joensuu 2001. 327 p.
50. Lehtola, Ilkka: Palvelujen muutos ja syrjäkylälaisten arki. English abstract: Change of Services and Everyday Life in a Remote Village. Joensuu 2001. 238 s.
51. Vapaavuori, Jukka. Dialoginen muutos ryhmäkeskusteluissa: työyhteisön jumi konsultaation kohteena. English abstract: Dialogical change in staff group discussions: the 'snag' as target of consultation. Joensuu 2001. 214 s.
52. Kuittinen, Matti. Defensiivinen käyttäytyminen yhteistyön ja kommunikaation esteenä. Pienyrityksen tapaustutkimus. English abstract: Organisational defensive routines as obstacles of learning and communication: Case of a Family Company in Crisis. Joensuu 2001. 185 s.
53. Karttunen, Sari: Taiteilijan määrittely: refleksiivisyyden esteitä ja edellytyksiä taidepoliittisessa tutkimuksessa. English abstract: Defining an Artist: Obstacles and Requisites for Reflexivity in Art-Political Research. Joensuu 2002. 105 s.
54. Saqib, Omar Farooq. Currency Crises in Emerging Economies: Essays in International Macroeconomics. Joensuu 2002. 145 + VIII p.

55. Oker-Blom, Max. Bakom rätten. En rättsekonomisk argumentationsmodell. Joensuu 2002. 311 s.
56. Sireni, Maarit. Tilansa tekijät. Tutkimus emännyydestä ja maatilasta naisen paikkana. English abstract: Gendered Identities and Place. A Study of Female Farmers. Joensuu 2002. 320 s.
57. Häyrynen, Simo. Kansakunnan ja maakunnan välissä. Pohjois-Karjalan alueyhteisön rakentuminen kulttuuripoliittisen edunvalvonnan keskus/periferia – suhteessa. English abstract: Between the Nation and Province. Constructing the regional community of North Karelia in the centre/periphery relation of cultural policy. Joensuu 2002. 314 s.
58. Kasanen, Kati. Lasten kyvykäsitteet koulussa. English abstract: School Children's Notions of Ability. Joensuu 2003. 165 s.
59. Gasso, Chaltu Deressa. Breaking the Silence: Conceptions of Adolescents on Sexual Health Issues in Modernising Development Society: The Case of Nakamte High School, Ethiopia. Tiivistelmä: Hiljaisuutta murtaamassa. Nuorten käsitykset seksuaalisuudesta ja ehkäisystä modernisoituvassa kehitysmaassa: tapaustutkimus Nakamten lukiosta, Etiopiasta. Joensuu 2003, 188 s.
60. Huusko, Liisa. Sopiva tiimi. English Abstract: A Suitable Team. Joensuu 2003. 302 s.
61. Moore, Erja. Pitkä opintie. Aikuisiällä suoritettu yliopistotutkinto ja koulutuksellisen elämäntulon muutos. English abstract: Long Study Paths. Studying for a Degree at Adult Age and Change in Educational Life Course. Joensuu 2003. 212 s.
62. Kerkkänen, Paavo. Huumorintaju ja terveys itäsuomalaisten poliisien työssä 1995 - 1998. English abstract: Sense of Humor and Health of Finnish Policemen 1995 -1998. Joensuu 2003, 190 s.
63. Hurskainen, Aimo. Hallintopäätöksen perustelemisesta. English summary: On the importance of the presentation of reasons. Joensuu 2003, 374 s.
64. Lin, Chijien, Generating Forest Stands with Spatio-Temporal Dependencies. Tiivistelmä: Tila-aika –riippuvien metsiköiden generointi. 中文摘要: 用时空相关性模拟森林林相. Joensuu 2003. 119 p.
65. Muurinen, Raimo. Yksilöllisten kiinnostusten suora ja epäsuora mittaaminen GRIDillä. Kellyn teorian sovellus ammatinvalinnanohjauksessa. English Abstract: Direct and indirect measurement of individual vocational interests using GRID: Applying Kelly's theory to career counseling. Joensuu 2004. 146 s.
66. Viitanen, Jari. Essays on Intertemporal Consumption Behaviour in Finland. Tiivistelmä: Esseitä intertemporaalisesta kulutuskäyttäytymisestä Suomessa. Joensuu 2004, 184 s.