CHANSON, H. (2013). "Explicit Equations for Critical Depth in Open Channels with Complex Compound Cross Sections. A Discussion." *Flow Measurement and Instrumentation*, Vol. 29, pp. 65-66 (DOI: 10.1016/j.flowmeasinst.2012.10.006) (ISSN 0955-5986).

EXPLICIT EQUATIONS FOR CRITICAL DEPTH IN OPEN CHANNELS WITH COMPLEX COMPOUND CROSS SECTIONS. A DISCUSSION

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Key words: Critical depth, Open channel flow, Critical flow conditions, Non hydrostatic pressure distribution.

The authors developed a series of expression for the critical depth in open channels with irregular channel cross-sections. It is believed that the article thrust and its conclusion missed a key point. The work is restricted to an open channel flow motion with hydrostatic pressure distributions although it was not stated explicitly. In turn the readers could be misled to assume that the results may apply to a wide range of open channel situations incl. weirs, spillway crests, gates, ... Figure 1 illustrates some flow situations in which the flow is critical but the pressure distributions are not hydrostatic. It is shown herein that the critical depth may be derived more broadly for flow situations with non-hydrostatic pressure distributions.

At critical flow conditions, the specific energy is minimum (Bakhmeteff 1912, 1932, Liggett 1993). The cross-sectional averaged specific energy H is commonly expressed following Chanson (2006):

$$H = \frac{1}{A} \times \iint \left(\frac{v_x^2}{2 \times g} + z + \frac{P}{\rho \times g} \right) \times dA = \beta \times \frac{V^2}{2 \times g} + \Lambda \times y \qquad (1)$$

where A is the wetted cross-section area, y the flow depth, P the pressure, V the depth-averaged velocity, v_x the longitudinal velocity component, z the vertical elevation above the crest, g the gravity constant, ρ the water density, β the Boussinesq momentum correction coefficient, and Λ a pressure correction coefficient:

$$\Lambda = \frac{1}{2} + \frac{1}{A} \times \iint \frac{P}{\rho \times g \times y} \times dA \qquad (2)$$

For an uniform flow above a flat rectangular invert with streamlines parallel to the crest, the velocity distribution is uniform ($\beta = 1$), the pressure is hydrostatic ($\Lambda = 1$), and Equation (1) equals the classical result: $H = 1.5 \times y_c$ where y_c is the critical depth. For an irregular channel cross-section with uniform velocity distribution ($\beta = 1$) and hydrostatic pressure ($\Lambda = 1$), the differentiation of Equation (1) with respect of the flow depth gives:

$$\frac{Q^2}{g \times \frac{A^3}{B}} = 1 \qquad \text{Hydrostatic pressure distribution} \quad (3)$$

at critical flow conditions (Henderson 1966, Chanson 2004). In many practical applications, the velocity distributions are not uniform, the streamlines were not parallel to the invert everywhere (Fig. 1) and the pressure gradient is not hydrostatic. In turn Equation (3) becomes inapplicable.

In the general case, the specific energy is minimum at critical flow conditions (Henderson 1966, Liggett 1993). For a wide channel, the flow depth y must satisfy one of four physical solutions (Chanson 2006):

$$\frac{y}{H} \times \Lambda = \sqrt[3]{\frac{1 - 2 \times \beta \times {C_D}^2 \times \Lambda^2}{27} + \Lambda^3 \times \sqrt{\Delta}} + \sqrt[3]{\frac{1 - 2 \times \beta \times {C_D}^2 \times \Lambda^2}{27} - \Lambda^3 \times \sqrt{\Delta}} + \frac{1}{3} \qquad \Delta > 0 \quad (4a)$$

$$\frac{y}{H} \times \Lambda = \frac{2}{3} \qquad \Delta = 0 \quad (4b)$$

$$\frac{y}{H} \times \Lambda = \frac{2}{3} \times \left(\frac{1}{2} + \cos\frac{\varepsilon}{3} \right) \qquad \Delta < 0, \text{ Solution S1} \quad (4c)$$

$$\frac{y}{H} \times \Lambda = \frac{2}{3} \times \frac{1 - \cos\frac{\varepsilon}{3} + \sqrt{3 \times \left(1 - \left(\cos\frac{\varepsilon}{3} \right)^2 \right)}}{2} \qquad \Delta < 0, \text{ Solution S3} \quad (4d)$$

where C_D is a dimensionless discharge, $\cos\varepsilon = 1 - 2 \times \beta \times {C_D}^2 \times \Lambda^2$ and the discriminant Δ equals:

$$\Delta = \frac{4 \times \beta \times {C_D}^2 \times \Lambda^2}{(3 \times \Lambda)^6} \times (\beta \times {C_D}^2 \times \Lambda^2 - 1) \qquad (5)$$

Equation (4) expresses the flow depth at critical flow conditions in the general case when $\beta > 1$ and $\Lambda \neq 1$. Equation (4) is tested against a series of experimental data in Figure 2 with the dimensionless water depth $y \times \Lambda / H_1$ at critical flow conditions being a function of the dimensionless discharge $\beta \times {C_D}^2 \times \Lambda^2$, where β and Λ were calculated based upon the pressure and velocity distribution data and C_D was calculated:

$$C_D = \frac{\int_0^y v_x \times dz}{\sqrt{g \times \left(\frac{2}{3} \times H\right)^3}} \qquad (6)$$

In Figure 2, the physical data showed a good agreement with the theory, in particular with the solutions S1 and S3 ($\Delta < 0$) (Fig. 2). The agreement between Equation (4) and data highlighted that the notion of critical flow conditions may be applied broadly to open channel flows with non-uniform velocity and non-hydrostatic pressure distributions.

In summary, the notion of critical flow conditions and critical depth are not restricted to open channel flows with hydrostatic pressure distributions. This discussion showed an extension of the concept of critical flow conditions linked with the minimum specific energy, as introduced by Bakhmeteff (1912). It demonstrated that the critical depth may be defined more broadly including when the pressure field is not hydrostatic.

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LIST OF FIGURE CAPTIONS

Fig. 1 - Critical flow conditions in open channels

(A) Overflow above the Little Nerang dam spillway crest on 28 December 2010 - Head above crest: 0.4 m, q = 0.43 m^2/s, Q = 14 m^3/s

(B) Undular flow in a Venturi flume along an irrigation canal near Hualien on 10 November 2010 - Flow from foreground to background

Fig. 2 - Dimensionless critical depth $y \times \Lambda/H$ as a function of the dimensional discharge $\beta \times C_D^2 \times \Lambda^2$ - Comparison between analytical solutions (Eq. (4)), broad-crested weir data (Felder and Chanson 2012), circular crested weir data (Fawer 1937, Vo 1992) and undular flow data (Chanson 2005)

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