

## THE DYNAMICAL DISTANCE TO M15: ESTIMATES OF THE CLUSTER'S AGE AND MASS AND OF THE ABSOLUTE MAGNITUDE OF ITS RR LYRAE STARS

B. J. McNAMARA AND T. E. HARRISON

Astronomy Department, New Mexico State University, Las Cruces, NM 88003

AND

H. BAUMGARDT<sup>1</sup>

Department of Astronomy, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

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### ABSTRACT

Newly determined high-precision relative proper motions determined from the *Hubble Space Telescope* Wide Field Planetary Camera 2 are used along with radial velocity measurements to determine the dynamical distance to the globular cluster M15. A comparison of the proper motion and radial velocity dispersions from a sample of 237 stars, located at an average radial distance of about  $10''$  from the cluster center, yields a cluster distance of  $9.98 \pm 0.47$  kpc. This distance agrees to within the stated errors to other distance estimates but places this object about 5% closer than the currently adopted value of 10.4 kpc. Using this new distance, we estimate that RR Lyrae stars having  $[\text{Fe}/\text{H}] = -2.15$  have a value of  $M_v(\text{RR}) = 0.51 \pm 0.11$ . We also estimate that M15 has an age of about 13.2 Gyr, which places it among the oldest of the Galactic globular clusters. From a comparison of the observed velocity dispersion with results from recent  $N$ -body calculations, we derive a total cluster mass for M15 of  $M_C = 4.5 \times 10^5 M_\odot$ .

*Subject headings:* distance scale — globular clusters: individual (M15)

*On-line material:* machine-readable table

### 1. INTRODUCTION

Globular cluster distances can be found using several different methods: main-sequence (MS) fitting (Grundahl, Vandenberg, & Andersen 1998), comparisons of the proper motion and radial velocity dispersions (Rees 1996, 1997), fitting the zero-age horizontal branch (HB) to theoretical models (Cassisi et al. 1998), the  $M_v$ - $[\text{Fe}/\text{H}]$  relation for RR Lyrae stars (Chaboyer 1999), stellar pulsation (Kaluzny et al. 1998), the top of the red giant branch (Madore & Freedman 1998), white dwarf fitting (Renzini et al. 1996), and eclipsing binaries (Guinan, Bradstreet, & DeWarf 1996). Unfortunately, with the exception of dynamical estimates, all of these methods rely on secondary calibrators. For instance, the MS-fitting technique relies on the absolute magnitudes of nearby metal-poor stars whose distances have been found through trigonometric parallaxes (Gratton et al. 1997; Reid 1998; Pont et al. 1998). The absolute magnitudes of the white dwarf cooling sequence are similarly based on trigonometric parallaxes (Gratton, Carretta, & Clementini 1999). Dynamical estimates are one of the few methods we possess that do not rely on a secondary calibrator. In this paper the proper motions computed by McNamara, Harrison, & Anderson (2003) are combined with the radial velocity study of Gebhardt et al. (2000) to determine a dynamical distance to M15.

Globular cluster distances have several useful purposes. They are needed to determine the age and mass of a cluster, to make comparisons between globular cluster color-magnitude diagrams and post-MS stellar models, and to test galaxy formation scenarios.

Globular cluster ages play an important role in constraining cosmological models. The oldest globular clusters are believed to have formed within 1 Gyr of the big bang, so their age distribution provides a stringent lower limit to the age of the universe (Sandage 1993). These age estimates, however, rely heavily on knowledge of a cluster's distance. A change of 0.1 mag in the distance modulus of a globular cluster changes its estimated age by about 10% (Chaboyer 1999). Since M15 is one of the oldest globular clusters in our Galaxy, it obviously plays an important role in establishing this constraint. Recent revisions in the globular cluster distance scale have already had an impact on cosmological research. The age of the oldest globular clusters are now consistent with an open universe or with a flat matter-dominated universe with  $H_0 \leq 68 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (Chaboyer 1999).

M15 has a metallicity of  $[\text{Fe}/\text{H}] = -2.15$  (Harris 1996). An accurate distance to this cluster therefore allows it to anchor the low-metallicity side of secondary distance calibrations that vary as a function of composition. The RR Lyrae  $M_v$  versus  $[\text{Fe}/\text{H}]$  relation is one such indicator. Models of post-MS stars are also frequently tested through comparisons of the theoretical and observed color-magnitude diagrams of globular clusters. Since stars in these systems are coeval and therefore presumably formed with the same chemical composition, the distribution of points in a distance-corrected cluster color-magnitude diagram can be compared with that expected from theory. These types of comparisons reveal deficiencies in the input physics used to construct stellar models. Globular clusters also are used to constrain galaxy formation models (Djorgovski & Meylan 1994; Van Den Bergh 2003). According to Gratton et al. (1999), the largest uncertainty in making these comparisons is the cluster distances.

Finally, an accurate distance to M15 is needed to allow the dynamical state of this cluster to be more thoroughly

<sup>1</sup> Current address: Advanced Computer Center, RIKEN, 2-1 Hirosawa, Wako 351-0198, Japan.

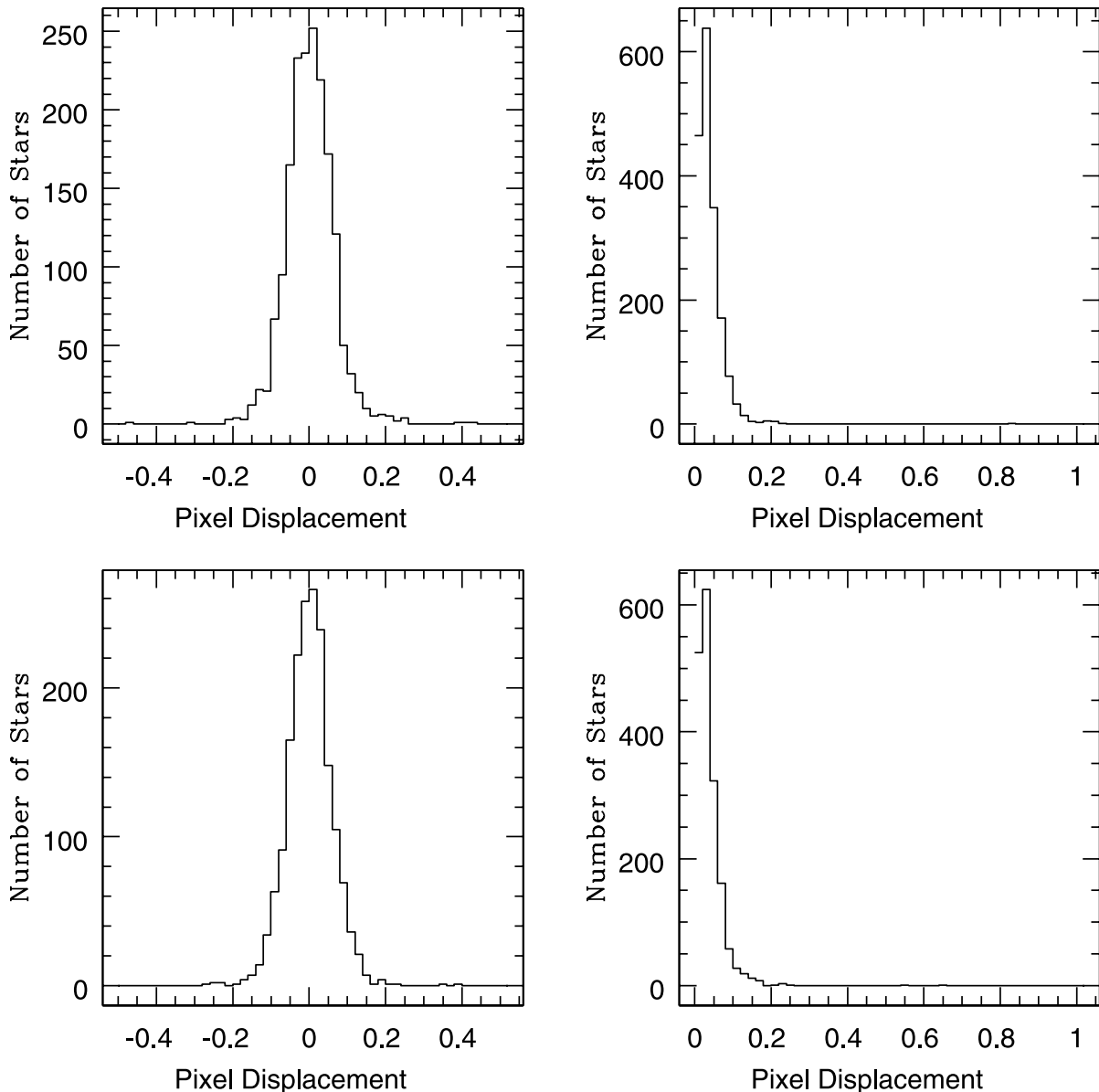


FIG. 1.—Distribution of the  $x$  and  $y$  proper motions and their errors for the 1764 M15 stars measured by McNamara et al. (2003) in units of pixel displacement over the 8 yr epoch difference available from their *HST* images. The top panels are for the  $x$  proper motions. The bottom panels are for the  $y$  proper motions. To convert the  $x$ -axis into units of arcseconds per century, multiply the numbers by 0.569.

investigated. M15 is a prominent member of the class of objects called core-collapsed globular clusters. These objects possess extremely dense nuclear regions whose surface density profiles are not easily fitted with normal King models. About 20% of all globular clusters fall into this category, yet their dynamics are still not fully understood. A distance to M15 will make it possible to convert proper motion data into physical velocities that can be compared to models. This velocity data can also be used to estimate the cluster's total (seen plus unseen) mass.

The goal of this paper is determine the distance to M15, the absolute magnitude of its RR Lyrae stars, the cluster's age, and its mass using proper motion and radial velocity data. The paper is organized in the following fashion. In § 2 we discuss the observational data used in this study. In § 3 we describe issues that can have a large influence on the computed dispersions. Section 4 describes our computational method.

Results are presented in § 5, and a brief concluding discussion is provided in § 6.

## 2. OBSERVATIONS

The proper motion and radial velocity measurements employed in this study are those published by McNamara et al. (2003) and Gebhardt et al. (2000), respectively. The former investigation relied on *Hubble Space Telescope* (*HST*) Wide Field Planetary Camera 2 (WFPC2) images taken in 1995 and 2003. The scale of these images is  $0''.0455 \text{ pixel}^{-1}$  and they cover an area of  $36'' \times 36''$  approximately centered on the cluster core. The relative proper motions of 1764 stars, having apparent visual magnitudes between 14.0 and 18.3, were measured to an accuracy of about  $0''.02 \text{ century}^{-1}$ .

The Gebhardt et al. (2000) investigation provided radial velocities for 1773 stars located within about  $10'$  of the cluster center. Of those stars, 1249 have radial velocity errors of less

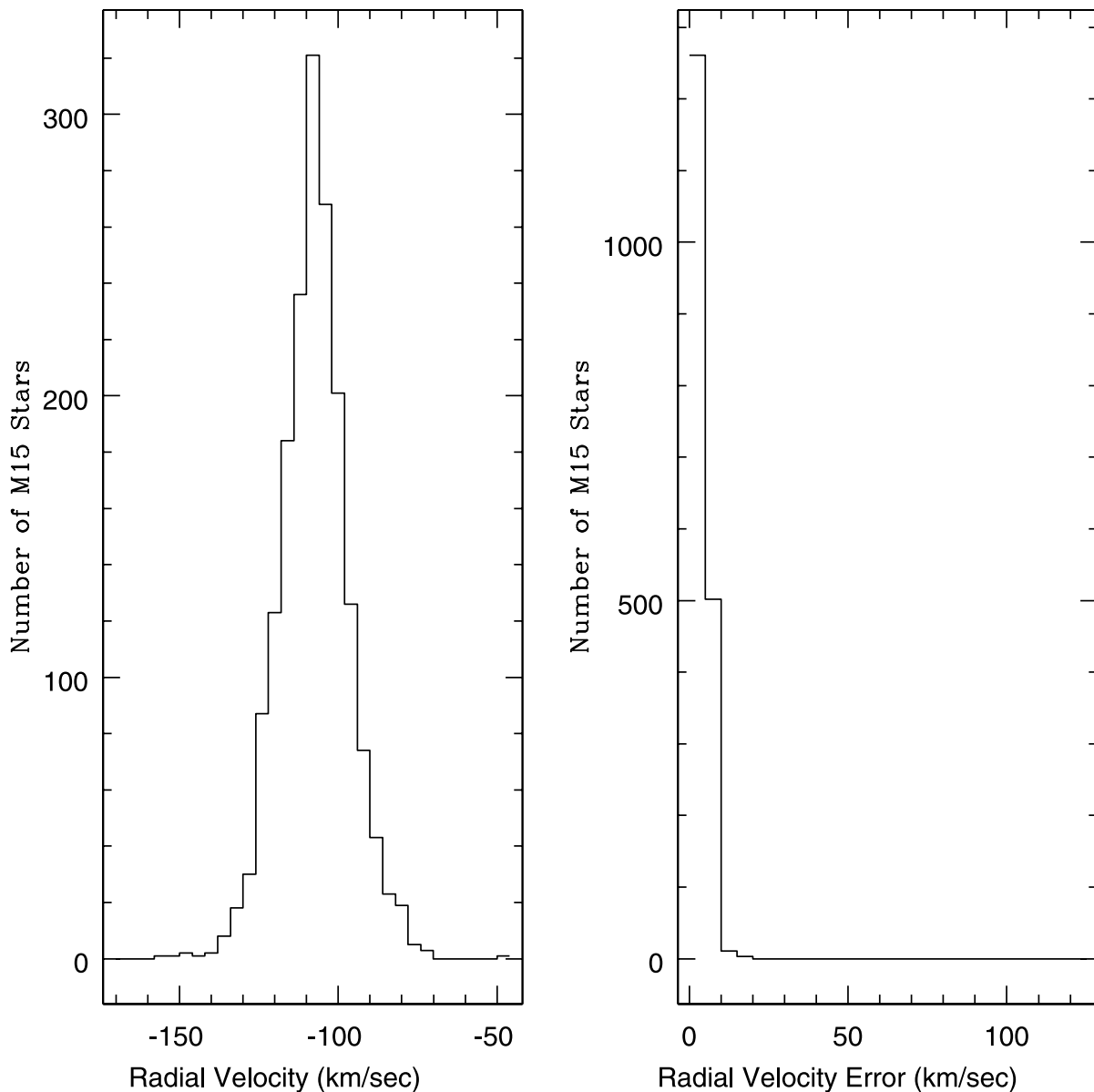


FIG. 2.—Distribution of the radial velocities and radial velocity errors measured for M15 stars by Gebhardt et al. (2000)

than  $5 \text{ km s}^{-1}$ , or about half of the cluster's internal velocity dispersion. Additional radial velocities have been published by Gerssen et al. (2002). Even though these data are of comparable accuracy to the Gebhardt et al. (2000) data, they are not used in this investigation because only about two additional stars would have been added and we wished to avoid possible systematic errors between these two data sets. This omission has no effect on our final results.

Histograms of the above proper motion and radial velocity data are shown in Figures 1 and 2. These distributions have a Gaussian shape, indicating that relatively few stars possess large, nonrandom measurement errors or anomalous motions.

### 3. REDUCTION CONSIDERATIONS AND ANALYSIS

Equating the proper motion and radial velocity dispersions is a commonly used method of determining a cluster's distance. However, some difficulties exist with this procedure. Both the radial velocity and proper motion dispersions change with distance from the cluster center, so if the stars in these

samples come from different regions of the cluster, one cannot equate them. Second, the velocity dispersions might not be isotropic. In this case a dynamical model must be employed to guide the comparison between the proper motion and radial velocity dispersions. Third, if the cluster is rapidly rotating, the radial and tangential dispersions might be altered in different ways, and this could affect the computed distance. Finally, care must be taken to avoid including high-velocity stars or binaries in the computation of the velocity dispersions. The importance of each of these concerns to this study is discussed below.

#### 3.1. Spatial Selection of Program Stars

The Gebhardt et al. (2000) study extends out to a distance of about  $17'$  from the cluster center, whereas the McNamara et al. (2003) proper motion analysis is restricted to stars located within about  $0.3$  of the cluster center. Simply equating the global proper motion and radial velocity dispersions found from these data sets would therefore lead to an incorrect distance. One might have hoped that this problem would be

somewhat less severe for M15 since it is a core-collapsed system. In these systems the inner velocity dispersion profile varies slowly, as  $r^{-0.1}$ , so differences in the spatial distributions of their stars might not have been expected to significantly change the computed dispersions. Unfortunately, the actual M15 radial velocity dispersion profile measured by Gebhardt et al. is fairly complex, particularly within 0'.1 of the cluster center (see their Fig. 13). To avoid any spatially related problems, only those stars that have both a radial velocity and a proper motion are used in this study. This criterion limits program stars to the inner 0'.3 of M15. Stars in this region possess relatively large proper motions and radial velocities, but many of their images are blended with nearby stars, which increases their measurement errors.

### 3.2. Isotropy

If the motions within a globular cluster are not isotropic, one cannot simply equate the various dispersions. According to King & Anderson (2001), when a globular cluster forms, its stellar orbital motions are expected to be largely radial. Over a time period comparable to the cluster's relaxation time, these motions gradually become isotropic. This change occurs most rapidly in the inner cluster region and then progresses outward to the lower density envelope. Recent  $N$ -body simulations of the core-collapse evolution of star clusters confirm that the velocity profile in the inner region of a core-collapse cluster is nearly isotropic (Baumgardt et al. 2003a). Anisotropy is therefore not expected to be an important issue for M15 because it is an old system and only the inner cluster region is being examined. The nature of the orbital motions within this region have, in fact, been examined by McNamara et al. (2003). In agreement with the above expectations, they found that the motions are isotropic (see their Table 3). No adjustment for anisotropy is therefore applied in this study, and it is assumed that  $\sigma_{rv}^2 = \sigma_{\mu x}^2 = \sigma_{\mu y}^2$ .

### 3.3. Rotation

If a cluster has a measurable rotation, this motion will alter the computed dispersions. Gebhardt et al. (2000) claim to have detected a complex inner rotation profile in M15. However, this profile is based on relatively few stellar velocities, and the orientation of the rotation axis was allowed to vary with distance in their analysis. The changing rotation axis was interpreted by these investigators as evidence for a central black hole. The presence of this object has, however, been challenged. Baumgardt et al. (2003b) found that it was not necessary to invoke a black hole to fit the radial velocity dispersion profile measured by Gebhardt et al., and I. R. King (2003, private communication) has questioned whether a black hole could produce a changing cluster rotation axis. Finally, McNamara et al. (2003) were unable to detect this rotation profile in their proper motion data set. In view of these conflicting points of view, this study does not include the effect of rotation.

### 3.4. High-Velocity Objects

McNamara et al. (2003) searched for high-velocity stars in M15 as part of their proper motion study. However, with the possible exception of one star, all of the large proper motions were determined to be erroneous. In a crowded region, the images of neighboring stars can overlap and alter the measured stellar positions. When different epoch images are combined, this can produce large but spurious proper motions. Although

McNamara et al. strived to identify and delete these objects, it was decided to compare the histograms shown in Figure 1 to a Gaussian distribution to see if any anomalous motions remained. No such stars were found within the 0'.3 boundary of this study.

Image blending can also affect a star's measured radial velocity. Following the procedure outlined above, the radial velocity histograms (Fig. 2) were also reexamined. Five stars, within 0'.3 of the core and having a radial velocity error of 12 km s<sup>-1</sup> or higher, were deleted. Any star flagged by Gebhardt et al. (2000) as a possible binary was also dropped. Binary motion causes a star's radial velocity to change. Therefore, an incomplete sampling of this orbital motion produces an incorrect system velocity and measurement error.

After all of the above criteria were applied to the radial velocity and proper motion data sets, the final sample consisted of 237 stars. The great majority of the deleted stars were outside of the 0'.3 region defined by the proper motion study. A list of final program stars is given in Table 1. Columns (1) and (2) provide the star's  $x$  and  $y$  position in units of pixels on *HST* WFPC2 image U2AS0201T. Columns (3)–(6) give the star's proper motions and errors in units of arcseconds per century. The final three columns provide the identification number, radial velocity, and error assigned by Gebhardt et al. (2000). Figure 3 shows the radial distribution of these stars. Their average distance from the cluster center is 9'.1. The decrease at small radii is due to the paucity of stars measured by Gebhardt et al. and McNamara et al. in this region. At large radii, the density of cluster stars drops dramatically, and this decline is largely responsible for the decrease in the number of program stars.

## 4. THE DYNAMICAL DISTANCE TO M15

The proper motion and radial velocity dispersions were computed following the prescription given by Jones (1970) and described in detail by McNamara et al. (2003). The intrinsic velocity dispersion in one coordinate is given by the equation

$$\sigma_{\text{int}}^2 = \sigma_o^2 - (1/n) \sum \epsilon^2, \quad (1)$$

where  $\sigma_o$  is the observed dispersion,  $n$  is the sample size, and  $\epsilon$  is the rms uncertainty in the measured quantity. The cluster distance is then found by equating the proper motion and radial velocity dispersions using the equation

$$d \text{ (kpc)} = \frac{\sigma_{rv}}{47.35\sigma_{\mu}}, \quad (2)$$

where  $\sigma_{rv}$  is the radial velocity dispersion in units of kilometers per second and  $\sigma_{\mu}$  is the proper motion dispersion in units of arcseconds per century. Since the proper motion dispersions are available in two coordinates, distance estimates were obtained by equating each of these quantities to the radial velocity dispersion. The results of these computations are given in Table 2. The weighted mean distance to M15 is found to be  $9.98 \pm 0.47$  kpc.

## 5. DISCUSSION

The M15 distance of 9.98 kpc found in this study is close to that estimated by other investigators. Durrell & Harris (1993) evaluated four different methods for estimating globular cluster distances: MS-fitting to the subdwarf sequence, comparing the observed location of the cluster's HB to theoretical

TABLE 1  
PROPER MOTION AND RADIAL VELOCITY DATA

$x$ (pixels) (1)	$y$ (pixels) (2)	$\mu_x^a$ (3)	$\mu_y^a$ (4)	$\epsilon(\mu_x)^a$ (5)	$\epsilon(\mu_y)^a$ (6)	ID (7)	RV <sup>b</sup> (8)	$\epsilon(\text{RV})^b$ (9)
459.9	388.4	0.008	-0.030	0.006	0.009	5768	-108.2	3.2
460.5	397.2	0.038	-0.004	0.005	0.006	5785	-115.7	12.0
492.9	411.5	-0.036	0.040	0.010	0.005	6475	-107.3	7.6
496.1	378.5	-0.028	0.053	0.017	0.033	6541	-112.5	5.5
509.7	386.4	0.069	-0.023	0.013	0.004	6833	-122.5	4.1
440.5	386.5	-0.014	-0.019	0.014	0.010	5371	-107.9	8.6
507.2	390.6	0.052	0.003	0.007	0.006	6772	-109.3	2.6
440.2	397.5	0.035	0.001	0.001	0.003	5364	-101.3	2.7
447.3	419.4	-0.011	0.010	0.010	0.017	5515	-115.8	5.0
454.3	431.6	0.041	-0.001	0.015	0.013	5642	-105.0	3.6
442.1	331.8	0.007	0.057	0.009	0.016	5395	-114.2	2.9
441.7	336.9	-0.025	0.050	0.009	0.007	5389	-121.3	4.8
513.1	336.9	0.046	0.013	0.008	0.023	6904	-120.0	8.7
419.3	340.6	0.033	0.031	0.013	0.013	4968	-70.6	13.0
453.0	341.9	0.020	0.066	0.014	0.007	5619	-104.2	3.6
517.8	345.1	0.044	-0.054	0.010	0.032	7003	-120.1	5.8
510.1	348.4	0.008	0.002	0.009	0.011	6841	-131.0	7.0
418.3	352.6	-0.032	-0.022	0.010	0.005	4951	-104.9	2.6
428.5	352.8	-0.015	0.011	0.007	0.011	5132	-100.7	5.2
452.4	357.3	0.014	0.028	0.009	0.006	5610	-104.8	2.9

NOTE.—Table 1 is published in its entirety in the electronic edition of the *Astrophysical Journal*. A portion is shown here for guidance regarding its form and content.

<sup>a</sup> Units of arcsec century<sup>-1</sup>.

<sup>b</sup> Units of km s<sup>-1</sup>.

HB models, using the  $M_v$ -[Fe/H] relation for RR Lyrae stars, and isochrone-fitting to the color-magnitude diagram. Applying these techniques to M15, they found that MS-fitting methods yielded distances between 9.6 and 11.1 kpc, HB fits produced distances between 9.7 and 10.4 kpc,  $M_v$ -[Fe/H] relations yielded distances between 9.6 and 10.5 kpc, and isochrone-fitting gave a distance of 10.4 kpc. On the basis of

an analysis of the errors in these values, these investigators concluded that the best distance to M15 was  $10.4 \pm 0.8$  kpc. Silbermann & Smith (1995) examined 44 RR Lyrae stars and one Cepheid in M15 and estimated that this cluster has a distance of  $9.5 \pm 0.6$  kpc. Kraft & Ivans (2003) have also recently determined this cluster's distance by comparing the colors and absolute magnitudes of low-metallicity field stars to similar composition M15 stars. They obtained a distance of 11.2 kpc, but this value was based on only four calibration stars, two of which were given low weight.

Below we discuss how our new distance affects prior results concerning M15, specifically the value of  $M_v(\text{RR})$  at [Fe/H] = -2.15, the cluster age, and its mass.

### 5.1. The Absolute Magnitude of M15 RR Lyrae Stars

Bingham et al. (1984) studied the light curves of 62 RR Lyrae stars in M15 and found that  $\langle V(\text{RR}) \rangle = 15.83 \pm 0.01$ . Adopting our dynamical distance of 9.98 kpc,  $E(B-V) = 0.10$  (Harris 1996; Durrell & Harris 1993; Kraft & Ivans 2003), and  $A_v = 0.32$ , we find that  $M_v(\text{RR}) = 0.51 \pm 0.11$ . This magnitude agrees with that obtained by other investigators. Cassisi et al. (1998) examined several relationships between  $M_v(\text{RR})$  and [Fe/H], and using [Fe/H] = -2.15, they found  $M_v(\text{RR})$  magnitudes between 0.30 and 0.69. Popowski & Gould (1999) reviewed various ways in which  $M_v(\text{RR})$  versus [Fe/H] is calibrated. For the composition of M15, these relations yielded  $M_v(\text{RR})$  magnitudes between 0.35 and 0.56. Chaboyer (1999) reexamined five different calibrations of the same relations used by Popowski & Gould and concluded that best overall equation for  $M_v(\text{RR})$  was

$$M_v(\text{RR}) = (0.23 \pm 0.04)([\text{Fe}/\text{H}] + 1.6) + (0.56 \pm 0.12). \quad (3)$$

Inserting [Fe/H] = -2.15 gives  $M_v(\text{RR}) = 0.43 \pm 0.12$ .

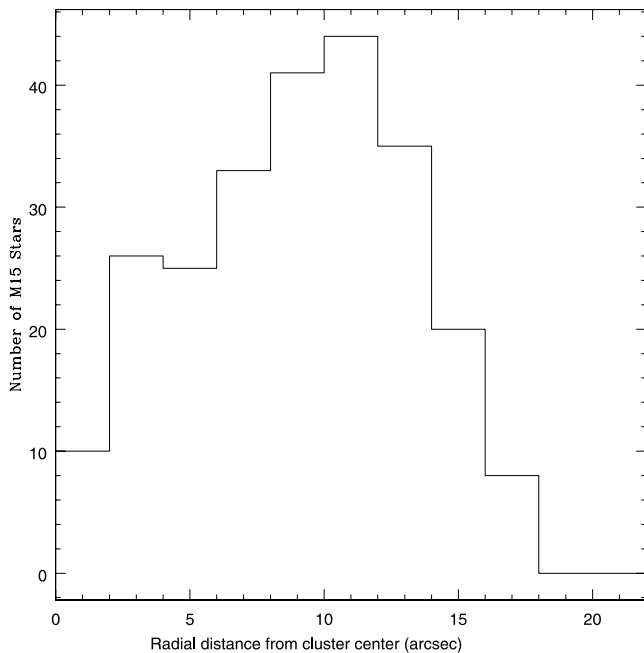


FIG. 3.—Radial distribution of the 237 stars used to compute the internal velocity dispersions. The distribution extends outward to about 18'' and is constrained by radial extent of the McNamara et al. (2003) study of M15.

TABLE 2  
M15 DISTANCE ESTIMATES

Value	Dispersion	Distance (kpc)
$\sigma_{rv}$ .....	$10.71 \pm 0.57^a$	
$\sigma_{\mu x}$ .....	$0.0213 \pm 0.0010^b$	$10.52 \pm 0.69$
$\sigma_{\mu y}$ .....	$0.0237 \pm 0.0011^b$	$9.50 \pm 0.63$

<sup>a</sup> Units of  $\text{km s}^{-1}$ .

<sup>b</sup> Units of  $\text{arcsec century}^{-1}$ .

A major uncertainty in the computation of the M15  $M_v(\text{RR})$  magnitude is the cluster's reddening, or  $E(B-V)$ . These values vary from 0.05 (Bica & Pastoriza 1983) to 0.12 (Sandage 1969). A difference in  $E(B-V)$  of 0.03 mag changes  $M_v(\text{RR})$  by almost 0.10 mag. Since the number of similar composition calibration stars that have known absolute magnitudes and colors is small, it is understandable why it is difficult to obtain a more precise value of  $E(B-V)$ . As additional calibration stars become available, this situation is expected to improve.

### 5.2. The Age of M15

The low metallicity of M15 suggests that this cluster is very old. Chaboyer et al. (1998) have found that a linear relation exists between the logarithm of the age of a globular cluster and  $M_v(\text{RR})$ . This relation is

$$\log(t_9) = 0.888 + 0.454M_v(\text{RR}). \quad (4)$$

Using  $M_v(\text{RR}) = 0.51 \pm 0.11$  yields an estimated age for M15 of  $13.2 \pm 1.5$  Gyr. This age places M15 among the oldest globular clusters in our Galaxy.

### 5.3. Cluster Mass

In order to determine the cluster mass, we have compared the measured radial velocity and proper motion dispersion profiles with the results of  $N$ -body calculations reported in Baumgardt et al. (2003b). Their simulations followed the evolution of multimass star clusters, starting with a Kroupa (2001) initial mass function and  $N = 131,072$  stars initially, through core collapse, and up to complete dissolution, using the GRAPE-6 special purpose hardware (J. Makino, T. Fukushige, & K. Namura 2003, in preparation). They simulated two initially identical clusters, incorporating the effects of stellar evolution, relaxation, and an external tidal field, and assuming a 100% and a 0% neutron star (NS) retention fraction for the two cases. The clusters went into core collapse at  $T = 12.6$  and 14.3 Gyr, respectively, and we compare the combined data from 10 snapshots following core collapse with the observations. We note that their core-collapse times are in agreement with the age that we derive for M15. More details of the simulations can be found in Baumgardt & Makino (2003) and Baumgardt et al. (2003b).

In order to compare the simulations with M15, the simulated clusters have to be scaled radially to fit the surface density profile of M15, and their masses have to be adjusted to the mass of M15. Hence, the velocities of the cluster stars have to be changed according to

$$v' = v \left( \frac{r'_h}{r_h} \right)^{-1/2} \left( \frac{M'_C}{M_C} \right)^{1/2}, \quad (5)$$

where unprimed and primed values denote quantities before and after rescaling. The mass of M15 is obtained by matching

the velocity dispersion of the model clusters with the observed velocity dispersion of M15.

Figure 4 compares the surface brightness profile of M15 with that of the cluster with a 100% NS retention fraction. The observed profile is based on the ground-based data of Trager, King, & Djorgovski (1995), replaced in the inner  $\sim 10''$  by the star count data from Sosin & King (1997) to avoid seeing problems. The surface density in the  $N$ -body run is taken to be the number density of all stars with magnitudes brighter than  $M_v = 22$  at the distance of M15, which corresponds to the magnitude limit in the Sosin & King study. The  $N$ -body data was rescaled to have the same projected half-light radius as M15,  $\log r_h = 1.78$  (Trager et al. 1995). It can be seen that in the central parts, where the surface density profile is created by the core collapse of the cluster, a very good agreement is obtained between both profiles. Outside  $r \sim 20''$ , the  $N$ -body profile differs from that of M15. Since the density distribution influences the kinematics, we limit our analysis to stars that lie within this radius. This restriction is important only for a comparison with the radial velocities since the proper motions go out to only  $15''$ . Tests show that the differences in the profiles outside  $20''$  influence the mass estimates inside this region by less than 5%.

Stars with measured proper motions and radial velocities in M15 have mostly  $V < 19$ , so we use only stars brighter than this limit in our  $N$ -body clusters to determine the velocity dispersion in the simulations. Table 3 of McNamara et al. (2003) shows the dispersion in proper motions as a function of distance from the center of M15. We have put the stars in the simulated clusters into similar bins and determined the best-fitting cluster mass by a  $\chi^2$  test against the observational data. For the cluster with a 100% NS retention fraction, the best fit for  $d = 9.98$  kpc gives  $M_C = (4.18 \pm 0.37) \times 10^5 M_\odot$ . A similar analysis for the radial velocities of Gebhardt et al. (2000) gives  $M_C = (4.60 \pm 0.44) \times 10^5 M_\odot$ . Combining both estimates, we obtain

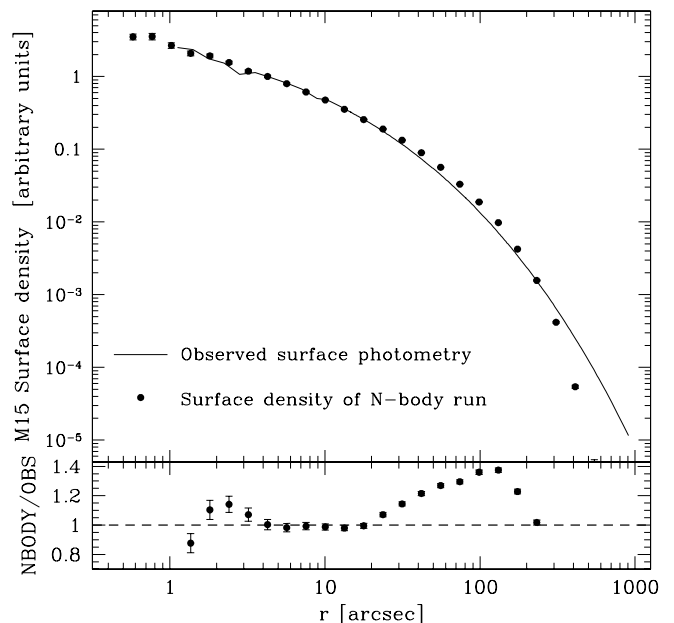


FIG. 4.—Comparison of the observed surface brightness profile of M15 with the surface number density of bright stars in a simulation with 100% NS retention rate. Observational data is from Trager et al. (1995) and Sosin & King (1997). The bottom panel shows the ratio of the surface densities. There is good agreement in the inner parts out to  $r = 20''$ , where the density profile is created by the core collapse of the cluster.

a mass of  $M_C = (4.35 \pm 0.28) \times 10^5 M_\odot$  for M15. Repeating the same calculations for the 0% NS retention model gives a best-fitting cluster mass of  $M_C = (4.54 \pm 0.30) \times 10^5 M_\odot$ . The estimated cluster mass is, therefore, relatively insensitive to the assumed NS retention rate.

Our best-fitting models depend rather sensitively on the assumed cluster distance. Repeating the calculations with the distance increased by  $1 \sigma$  to 10.45 kpc would for example give a value of  $M_C = (4.81 \pm 0.31) \times 10^5 M_\odot$  for the mass in case of the 100% NS retention model. In view of this, and since additional systematic errors might be present, we suggest that the mass of  $M_C$  is approximately  $(4.5 \pm 0.5) \times 10^5 M_\odot$ . For comparison, Dull et al. (1997) obtained a value of  $M_C = 4.9 \times 10^5 M_\odot$  for the mass of M15 based on fits to Fokker-Planck calculations. Sosin & King (1997) found a mass of only  $M_C = 3.4 \times 10^5 M_\odot$  from fits of static King-Michie models to the surface brightness profile in the center. However, this mass estimate was not well constrained since they did not use velocity information,

## 6. CONCLUSIONS

In this paper we have presented a dynamical distance to M15 based on newly computed proper motions of the inner

region of M15 using *HST* WFPC2 images and radial velocities published by Gebhardt et al. (2000). Our main conclusions are as follows:

1. The distance to M15 obtained by equating the proper motion and radial velocity dispersions over an identical spatial region is  $9.98 \pm 0.47$  kpc.
2. The M15 RR Lyrae stars have an absolute magnitude of  $M_v(\text{RR}) = 0.51 \pm 0.11$ .
3. On the basis of the Chaboyer age versus RR Lyrae absolute magnitude, M15 has an age of  $13.2 \pm 1.5$  Gyr.
4. On the basis of *N*-body simulations, surface density profiles, and velocity profiles, we estimate that M15 has a total mass of  $(4.5 \pm 0.5) \times 10^5 M_\odot$ .

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