Trapping Microscopic Particles with Singular Beams

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Abstract

It has been shown previously that it is possible to two-dimensionally trap a microscopic absorbing particle against a substrate using a focussed doughnut beam. Beam angular momentum associated with the phase singularity is transferred to the particle, causing it to rotate. A detailed consideration of the optical forces acting on a particle shows the importance of wavefront curvature for stable trapping and lead to a quantitative description of the motion of the particle in single and multiple beam traps.

1. Introduction

The irradiance available in a focussed laser beam of even modest power is sufficient to exert radiation forces capable of moving or trapping small particles. Since the ground breaking work of Ashkin[1] most work has been concentrated on optical forces on transparent particles and the many applications in biology where cells and bacteria and their component parts can be manipulated[2] .

Nearly all of this work has involved normal Gaussian beams but beams with phase singularities can also be used to advantage in some situations and produce some interesting results. In particular, doughnut beams have been used by several groups to trap highly absorbing particles which also take up the angular momentum of the light and are set into rotation. Here we consider a simple but quantitative model of the process.

2. Optical Trapping

The most commonly used configuration for trapping is the so-called single beam gradient trap or *optical tweezers*. A laser beam is focussed by a high numerical aperture lens such as a microscope objective to produce as large as possible an intensity gradient both radially and axially. Transparent particles with refractive index greater than their surroundings (typically immersed in water) experience a *gradient force* towards the focus as well as a *scattering force* along the beam direction[3] . When the axial component of the gradient force is greater than the scattering force plus the weight, the particle will be levitated by a downward propagating beam and can be manipulated in three dimensions. We call this *3-D trapping*. Typically, the particle is several times larger than the beam waist and if a singular beam like a TEM_{01}^{*} doughnut is substituted the particle will again centre itself on the beam.

Although, for the sake of simplicity, most workers use Gaussian beams, it has been recognized for some time[4, 5] that singular beams have some advantages, giving more efficient axial trapping because the dark central spot reduces the scattering forces due to back reflections from the top of the particle. This could be a real advantage in work with live biological specimens.

High refractive index particles much smaller than the beam waist will be attracted into the bright ring, while small particles with refractive index lower than their surroundings will suffer a negative gradient force, suggesting that they

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could be trapped in the dark central spot[5] .

Highly absorbing particles will always suffer too great an axial "scattering force" to allow 3-D trapping but can be trapped radially against a substrate which still allows useful manipulation. When a beam carrying angular momentum by virtue of elliptical polarization or the helical wavefronts associated with a phase singularity is used, the trapped particle experiences a torque and can be set into rotation[6, 10, 11] .

Estimates of the total angular momentum transfer and consequent rotation rates have been made and are consistent with experimental results[7] . Here we consider in some detail the forces and torques acting on an absorbing particle, as well as discuss configurations for 3-D trapping using multiple beams.

3. Calculation of Optical Forces

We consider a Laguerre-Gauss doughnut beam LG_{pl} with amplitude

$$E_0 = \sqrt{\frac{2}{c\varepsilon}}\sqrt{\frac{2p!P}{\pi\,(l+2+p)!}}\left(\frac{r\sqrt{2}}{w\,(z)}\right)^l L_p^l\left(\frac{2r^2}{w^2\,(z)}\right)\frac{1}{w\,(z)}e^{\frac{-r^2}{w^2(z)}} \tag{1}$$

where P is the beam power, L_p^l is the generalized Laguerre polynomial, c is the speed of light, ε is the permittivity of the medium and $w^2\,(z) = \frac{2(z_r^2+z^2)}{kz_r}$ is the beam width, where k is the wavenumber of the beam and z_r is the Rayleigh range.

We then consider a perfectly absorbing particle. The optical forces acting on it arise through transfer of momentum from the beam and will depend primarily on the beam geometry.

A small area element $\mathbf{dA}$ of such a particle experiences a force

$$\mathbf{dF} = \frac{1}{c}\mathbf{S}\cdot(-\mathbf{dA})\frac{\mathbf{S}}{|\mathbf{S}|} \tag{2}$$

where S is the Poynting vector given by

$$\mathbf{S} = \frac{c\varepsilon}{2}E_0^2\left(\frac{zr}{z_r^2+z^2}\widehat{\mathbf{r}} + \frac{l}{kr}\widehat{\boldsymbol{\phi}} + \widehat{\mathbf{z}}\right) \tag{3}$$

in a cylindrical coordinate system aligned with the beam axis.

The element will also experience a torque due to absorption of "spin" angular momentum from a circularly polarized beam given by

$$\mathbf{d\tau} = \frac{\sigma_z}{k}\mathbf{S}\cdot(-\mathbf{dA})\frac{\mathbf{S}}{|\mathbf{S}|} \tag{4}$$

where $\sigma_z = \pm 1$ for left and right-circular polarization and $\sigma_z = 0$ for linearly polarized light.

To calculate the total force on a particle we need to integrate equation 2 over its illuminated surface. A simple approximation is to represent the particle as a disk, essentially extending the paraxial approximation involved in using equation 1 to the illumination of the particle[8] . The total torque can be found by integrating the spin torque from equation 4 and the moment of the force about the centre of the particle over the illuminated surface.

Since the elemental forces (equation 2) are normal to the wavefronts, when the wavefronts are converging, radial forces will act inwards, and while they are diverging, no radial trapping can occur. Thus we see qualitatively that particles can be trapped two dimensionally above the beam waist as shown in figure 1. We can also see that the presence or absence of a dark central spot is not the fundamental consideration.

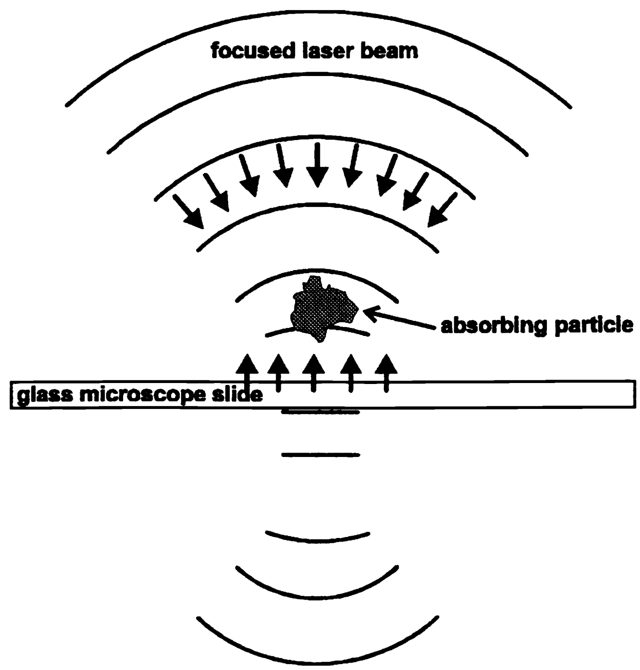

Figure 1. A diagram of an absorbing particle in a focussed laser beam showing two-dimensional trapping in the converging region.

This simple picture of the forces is borne out by numerical integration of equations 2 and 4 for a disk of radius 1 μm in water, with a linearly polarized LG_{03} beam of 20 mW power, a 633 nm wavelength, and waist width 1 μm as shown in figure 2. A TEM_{00} beam gives a comparable radial force, but considerably higher axial force which means more frictional drag during manipulation.

Figure 2 also shows axial torque and an azimuthal force on a de-centered particle. These arise because the helical structure of the wavefronts associated with the phase singularity leads to an azimuthal component of S as shown in equation 3 and are considered further below.

4. Multibeam Traps

A single laser beam can trap absorbing particles only in two dimensions so it is of interest to determine if 3-D trapping could be achieved using multiple beams. We ignore interference effects here, as in principle, the beams could be frequency shifted using acousto-optic modulators.

A simple case similar to some of Ashkin's first experiments with Gaussian beams[9] is to use two identical coaxial counterpropagating phase-singular beams. If the beams share the same focal plane the nett radial and axial absorption forces will be zero everywhere.

If the focal planes are distinct, and there is a region where both beams are converging, radial trapping could be achieved there. The axial force on a particle in the centre of the trap will be zero but unfortunately this equilibrium

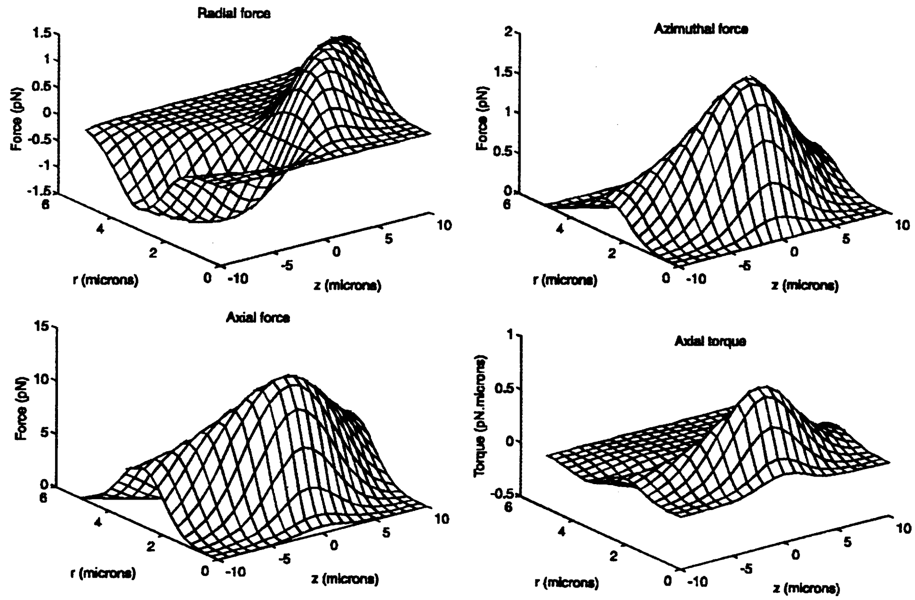

Figure 2. Force and torque due to a laser beam. The forces and torques acting on an absorbing particle of radius 1 μm trapped in water by a plane polarized LG_{03} 20 mW power laser beam of wavelength 632.8 nm and waist width 1 μm. The focal plane of the beam is at $z = 0$ with the beam propagating in the $+z$ direction. The inwards (ie negative) radial force in the region before the beam waist is responsible for trapping absorbing particles.

point is unstable. Figure 3 shows the force field for this configuration.

The situation is improved by crossing such a beam pair with another at right angles as the radial forces of one can overcome the axial force in the other if the particle is of a suitable size (see figure 4). Six beams would give a better result still although the difficulty of forming such configurations on a microscopic scale should not be underestimated.

5. Rotational Motion

Azimuthal forces due to beam helicity will exert a torque about the beam axis of $\mathbf{r} \times \mathbf{dF}$ which gives a constant contribution of $l\hbar$ per photon energy. For a particle of radius a trapped on the beam axis, the torque will be proportional to the power absorbed, giving

$$\tau_o = \frac{lP}{\omega}\left(1 - e^{\frac{-2a^2}{w^2(z)}} \sum_{k=0}^{l+2} \frac{1}{k!}\left(\frac{2a^2}{w^2(z)}\right)^k\right) \tag{5}$$

In a viscous medium, a drag torque will limit the rotation speed. For a smooth spherical particle in a medium of viscosity μ, ignoring any effects of the substrate, this will be given by

$$\tau_d = -8\pi\mu a^3 \Omega \tag{6}$$

and will equal the optical torque when the angular velocity Ω reaches a terminal value.

Figure 5 shows spin rates for some typical cases, corresponding to experiments carried out. For mW power levels they are a few Hz and easily observable. The rate is maximum when the particle is large enough to intercept most of the beam but then drops off again rapidly as the drag torque increases as a^3.

Figure 5 also includes the effect of circular polarization, most easily calculated by allowing one unit of angular mo-

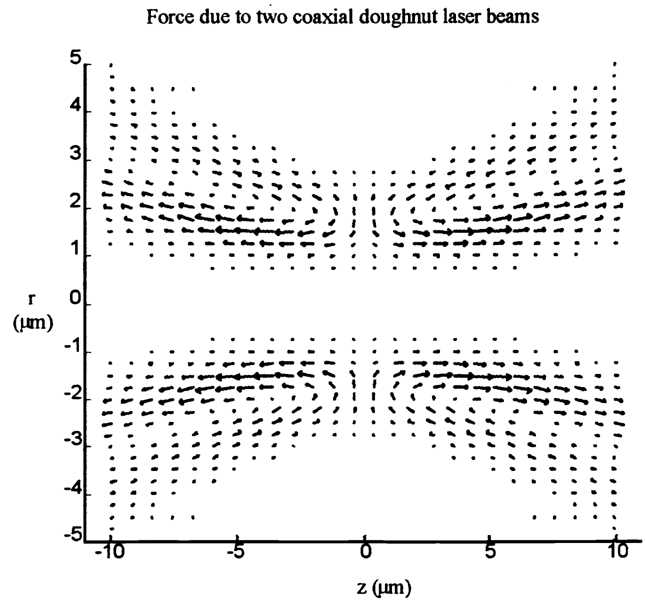

Figure 3. Forces due to two counter-propagating coaxial beams. In the central region, both beams are converging and strong radial trapping is achieved, but the central equilibrium point is axially unstable. The lengths of the arrows are proportional to the force.

mentum per photon absorbed, equivalent to substituting $l + \sigma_z$ for l in equation 5. Measurements of this case, showing $\pm 33\%$ changes in rotation speed when a quarter wave plate was used to circularly polarize the helical LG_{03} beam have been reported[7] as well as the case where the "orbital" angular momentum associated with the phase singularity of an LG_{01} beam was cancelled by circularly polarizing it in the opposite sense so that rotation of a trapped particle ceased[10] . To complete the picture, we have recently trapped similar particles in elliptically polarized Gaussian beams and were able to smoothly vary the angular velocity as the ellipticity was varied from circular to linear polarization[11] . Although it is more difficult to trap absorbing particles in a Gaussian beam, it can be done above the beam waist, confirming our understanding of the forces responsible for trapping and rotating particles.

6. Orbital Motion

We have seen that an absorbing particle trapped on axis will spin in a singular beam. If the particle is off-axis, azimuthal forces will act, tending to throw the particle out of the trap. We now consider under what conditions the particle will escape, or move to the centre, or take up a stable orbit.

We consider a sphere of radius a experiencing a Stokes' law drag force

$$D = -6\pi\mu a v \tag{7}$$

when moving with velocity v. In a steady circular orbit the optical forces will balance the drag force and centrifugal force, so that

$$F_r(r, \phi, z) = -\frac{m}{r}\left(\frac{F_\phi(r, \phi, z)}{6\pi\eta a}\right)^2 . \tag{8}$$

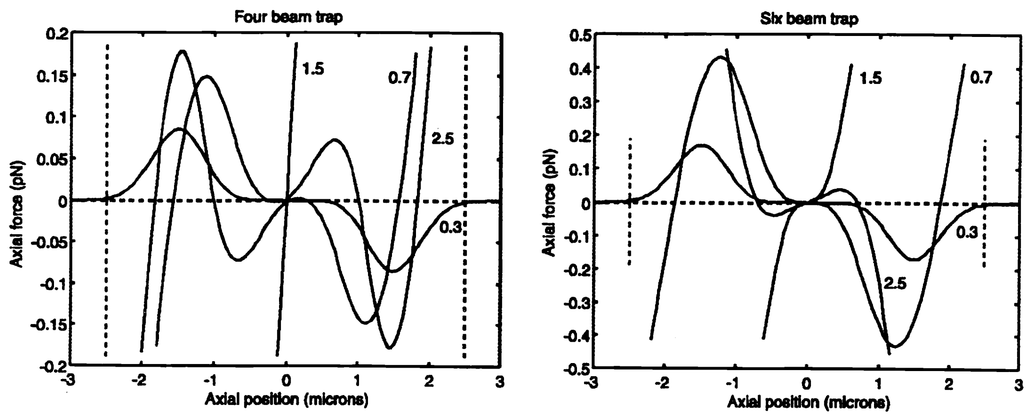

Figure 4. The axial forces acting on particles in four and six beam traps are shown. The particle radius in microns is shown in the figure. Small particles are readily trapped. Particle of an intermediate size cannot be trapped in either the four or six beam trap, but sufficiently large particles can be trapped near the trap centre (although not in the centre). The range of particle sizes which cannot be trapped is smaller for the six beam trap. All the beams here are LG_{03} beams of power 1 mW and waist size 0.5 μm.

For doughnut modes

$$F_r = \frac{2Pa^2}{c(l+2)!}\left(\frac{2r^2}{w^2}\right)^{l+2}\frac{1}{w^2}e^{\frac{-2r^2}{w^2}}\frac{zr}{z_r^2+z^2} \tag{9}$$

$$F_\phi = \frac{2Pa^2}{c(l+2)!}\left(\frac{2r^2}{w^2}\right)^{l+2}\frac{1}{w^2}e^{\frac{-2r^2}{w^2}}\frac{l}{kr} \tag{10}$$

The radial velocity will be zero when

$$r = \exp\left(-\frac{lW\left(\frac{-h}{l}\exp\left(\frac{\ln g}{l}\right)\right) - \ln g}{2l}\right) \tag{11}$$

where W is Lambert's W function and

$$g = \frac{-54\pi\mu^2 c\,(l+2)!z}{a^3\rho P l^2 w_0^2}\left(\frac{w^2}{2}\right)^{l+2} \tag{12}$$

and

$$h = \frac{2}{w^2} \tag{13}$$

For typical conditions we find $W(x) \approx x$ so that

$$r = g^{\frac{1}{2l}}. \tag{14}$$

This gives the radius of an unstable orbit. Inside this radius, the radial trapping force is large enough to overcome the centrifugal force due to the azimuthal velocity caused by the azimuthal optical force. For typical values of $P = 10^{-3}$ W, $l = 3$, $\mu = 10^{-3}$ Nsm, $c = 2.25 \times 10^8$ ms^{-1}, $z = 10^{-6}$ m, $w = 10^{-6}$ m, $w_0 = 0.5 \times 10^{-6}$ m, $a = 0.1 \times 10^{-6}$ m and $\rho = 2000$ kgm^{-3}, the radius of the unstable orbit is 17.8 μm. With fluids of much lower viscosity (such as gases), or less tightly focussed beams, this radius can be comparable to the beam radius, providing a limit to the effective size of the trap.

Figure 6 shows a numerical solution of the equations of motion for a particle in a singular trap under conditions where it is trapped.

Motion of this kind is frequently observed in experiments and on some occasions the converse behavior of a very rapidly rotating particle being flung out of the trap is also seen.

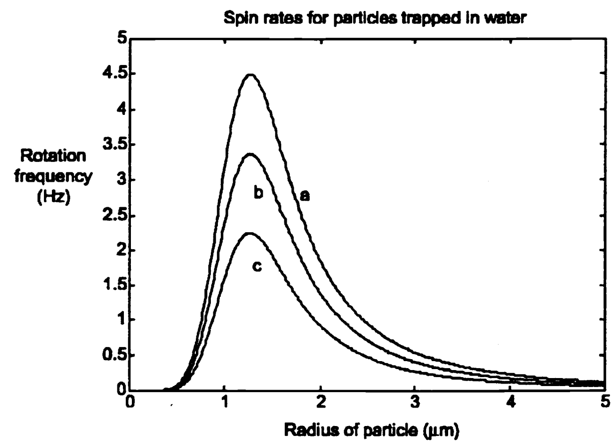

Figure 5. Spin rates of trapped particles. The variation of spin rate of a trapped particle with size and beam polarization is shown, with case (a) for a beam with circular polarization in the same direction as the orbital motion, (b) for a linearly polarized beam, and (c) for the case when the circular polarization and orbital motion are in opposite directions. Here, the particles are trapped in water by a 1 mW LG_{03} laser beam with a width of 1 μm.

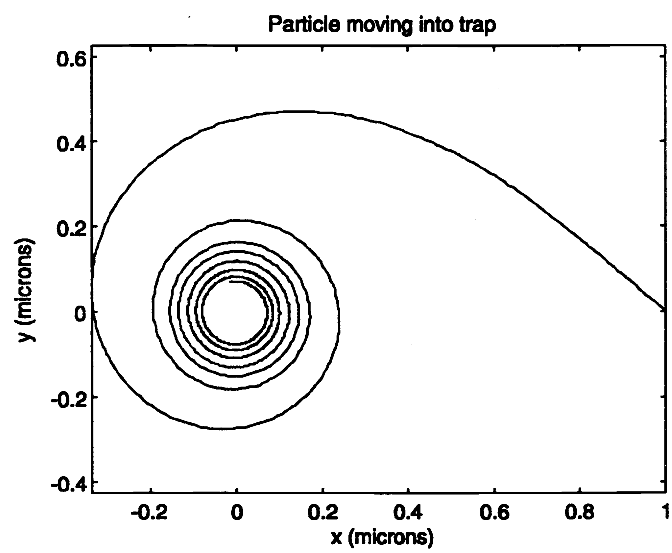

Figure 6. Trapping of an absorbing particle. A particle is shown moving in a spiral path into the centre of an optical trap. The particle has a radius of 0.1 μm and the beam is a 1 mW LG_{03} beam with a waist width of 0.5 μm. The particle is trapped in water 1 μm before the beam waist.

7. Conclusions

Singular beams can be used in optical trapping experiments in several interesting ways. Doughnut beams provide more efficient 3-D trapping of transparent particles which could be important for biological applications due to reduced heating. When used in 2-D traps of absorbing particles angular momentum associated with the helical structure of the beam can be transferred to the particles causing them to rotate. A very simple model based on radiation pressure forces acting in the direction of the local Poynting vector explains observations very well. With a single doughnut beam, 2-D trapping can be achieved with absorbing particles. It appears that there are some stable multiple beam 3-D trapping configurations but it is not clear whether they can be realized on a microscopic scale.

8. References

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