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Binominal Changing Volatility Approach to Subprime Loan Problem

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Abstract

Recently, subprime loan problem is a hotly debated issue around the world. In this paper, binominal option approach is adopted to surmise the binominal option value of ABX.HE Index[1] by changing its volatility. A nonrecombining lattice is required to solve the option. Suppose that the current value of ABX.HE Index is US\$ α ,its annual volatility is β percent, risk free rate is γ percent. We can calculate the value of a three year option to abandon for a price of US\$ δ .

We simulate the change of solution when the annual volatility changes through time. We notice there is a tendency that higher credit ratings and worse scenarios have a leeway of more flexibility. Especially, we can find out the significant difference between second quarter and third quarter. Apparently, we can see the tendency that higher credit ratings indexes are highly correlated with lower credit ratings indexes.

Key Words : Subprime Loan Problem, Binominal Option, Volatility
JEL Classification : C-15, C-22, G-10

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[1] HE of ABX.HE Index means Home Equity. This Index is used as a hedge tool of RMBS(Residential Mortgage Backed Securities) which is related to subprime loan.

1. Introduction

The movement of subprime loan problem will affect not only U.S. economy, but also world economy. Many economists and analysts predict this precarious situation will continue for the time being. At first, many people believed that the loss was restrictive, because the related financial institutions can manage the losses.

However, when hedge fund companies involved these transactions and bankrupt due to high leverage, market environments suddenly became worse. Originally, they invested CDO (Collateralized Debt Obligation) which includes RMBS (Residential Mortgage Backed Securities). Here, we use the ABX.HE Index which is derived from CDS (Credit Default Swap) of RMBS. It is noteworthy that highly rated indexes such as AAA and AA are also decreasing the prices sharply since last July 2007. Thus, we simulate the future scenario of subprime loan problem by applying into the binominal option approach with changing index's volatility. The reason is that the volatility taken up in the binominal option is based on two methods. One method uses a historical volatility. The other method uses some cash flow scenarios. These volatilities are constant, however, the actual volatility usually changes through time. Thus, the scenario analysis of volatility is very important for the future prediction in the option value. Here, we use present value event tree, binominal option analysis decision tree and optimal execution tree.

2. Model

We adopt the binominal option approach to surmise the binominal option value of ABX.HE Index by changing its volatility. These indexes include five credit ratings from AAA, AA, A, BBB, BBB_. Here, we compare the affect of subprime loan problem in each credit rating. Suppose that the current value of ABX.HE Index is US$ α, its annual volatility is β percent, risk free rate is γ percent. We can calculate the value of a three year option to abandon for a price of US$ δ. We simulate the four scenarios in this analysis. The following Table-1 shows the details.

Table 1. Four Scenarios in the Volatility

Conditions[2]	First Period	Second Period	Third Period
Best	β	$(\beta-\theta_{d_1})$	$(\beta-\theta_{d_2})$
Better/Worse	β	$(\beta-\theta_{d_1})$	β
Worse/Better	β	$(\beta+\theta_{u_1})$	β
Worst	β	$(\beta+\theta_{u_1})$	$(\beta+\theta_{u_2})$

Namely, we simulate the change of solution when the annual volatility changes from β percent the first year to $(\beta-\theta_{d_1})$ $(\because \beta \geq \theta_{d_1})$ percent the second to $(\beta-\theta_{d_2})(\because \beta \geq \theta_{d_2})$ percent or β percent the third year[3]. We also simulate the reverse case. The annual volatility changes from β percent the first year to $(\beta+\theta_{u_1})$ percent the second to β or $(\beta+\theta_{u_2})$ percent the third year[4].

At first step, we calculate a present value tree for the ABX.HE Index in each credit ratings. While, the annual volatility is β percent and the beginning ABX.HE Index value[5] is US$ α. Next, we compute the BOA value by first calculating the optimal execution values for the end nodes of the decision tree. We use the replicating portfolio approach[6] to compute the value of each previous node with risk free rate γ percent. The option to abandon of it is US$ δ. In addition, when the volatility per period decreases from β percent to $(\beta-\theta_{d_1})$ in second period and to $(\beta-\theta_{d_2})$ or increase again to β percent in third period, and increases from β percent to $(\beta+\theta_{u_1})$ in second period and to $(\beta+\theta_{u_2})$ or decrease again to β percent in third period, the present value tree for the ABX.HE Index does not recombine in each case.

2 Conditions mean the subprime loan problem's situations. For example, if the problem settle down smoothly by the policy of FRB, etc., the conditions become better with lower volatility. And if it continue two periods successively, the conditions become best with lowest volatility and vice versa.

3 The value of θ_{d_1} is smaller than θ_{d_2} in this case.

4 The value of θ_{u_1} is smaller than θ_{u_2} in this case.

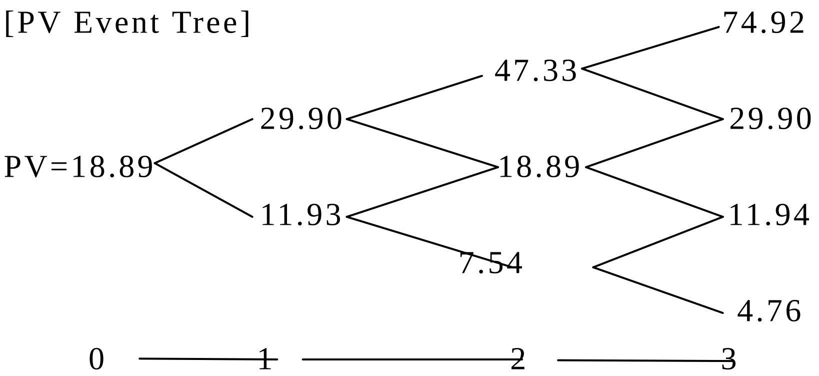
5 The price is based on at the end of Dec.31 2007.

6 Replicating portfolio is composed of m shares of the twin securities and B US $ of the risk free bond that present value is US$ 1 per bond.

3. Result of Data Analysis

We examine the case of 07-1[7]BBB_ in the ABX.HE Index. This rating is the highest credit risk and often referred to the symbol of subprime loan problem. The present value of BBB_ is α=18.89. Its annual volatility is β=45.93 percent. The PV event tree is illustrated in Figure-1.

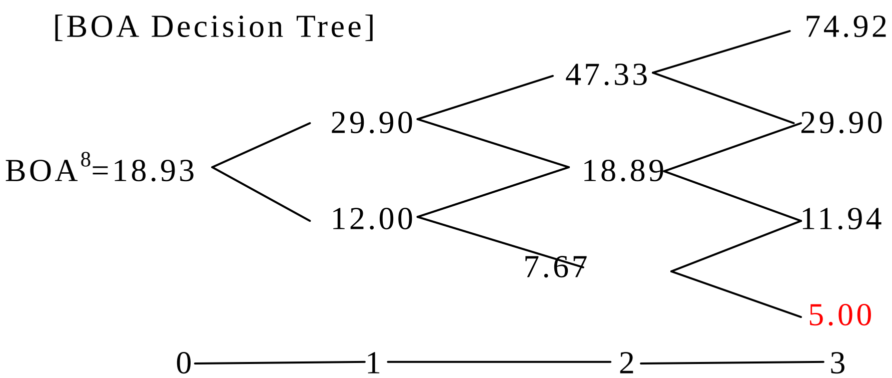
Figure-1



We calculate the BOA value by computing the optimal execution values for the end nodes of the decision tree. We use the replicating portfolio approach to get the value of each previous node in Figure 2.

Suppose that the abandonment value of it is δ=5.00.

Figure-2



7 There are two indexes in 2007. The one is 07-1 and the other is 07-2. 07-1 index starts from January 2007. But, 07-2 index starts from July 2007. Calculation is based on a daily data of the index.

8 Calculation Formula : $BOA = Max[m * PV + B, \delta]$

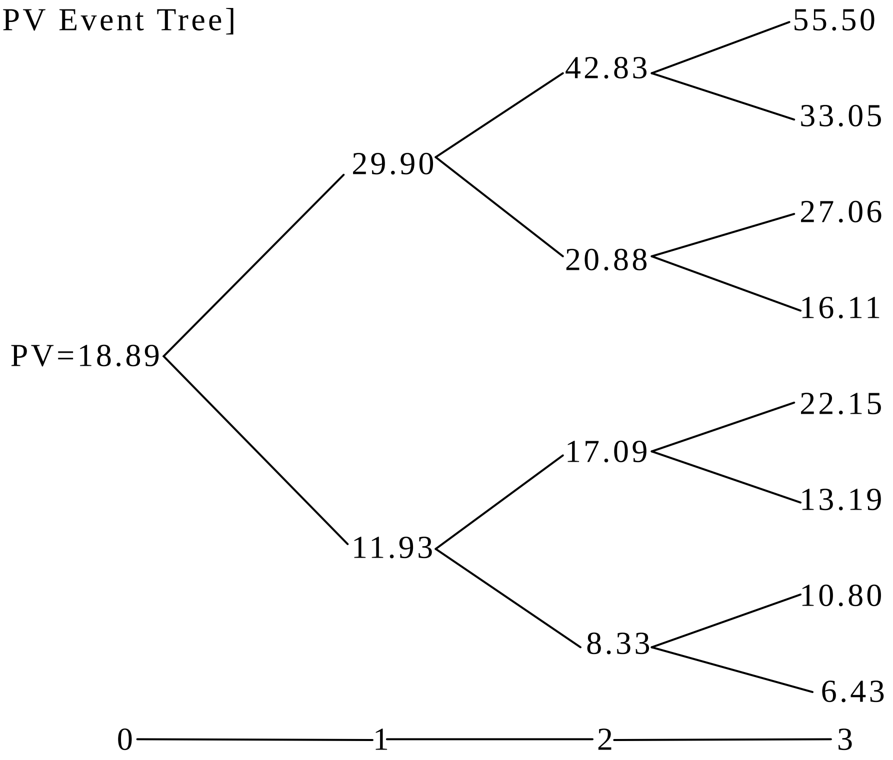
$$(\because m = \frac{(BOA_{up} - BOA_{down})}{(PV_{up} - PV_{down})},\quad B = \frac{(BOA_{up} - m * PV_{up})}{(1+\gamma)})$$

The result of BOA value (=18.93) is higher than the PV (=18.89). We can see that the option to abandon for US$5.00 is executed only when the value of it has declined continuously for three periods.

While, the volatility per period decreases from β=45.93 percent to $\left(\beta - \theta_{d_1}\right)$=35.93 percent in period 2 and to $\left(\beta - \theta_{d_2}\right)$=25.93 percent in period 3, the present value tree for it does not recombine[9].

Figure-3

[PV Event Tree]



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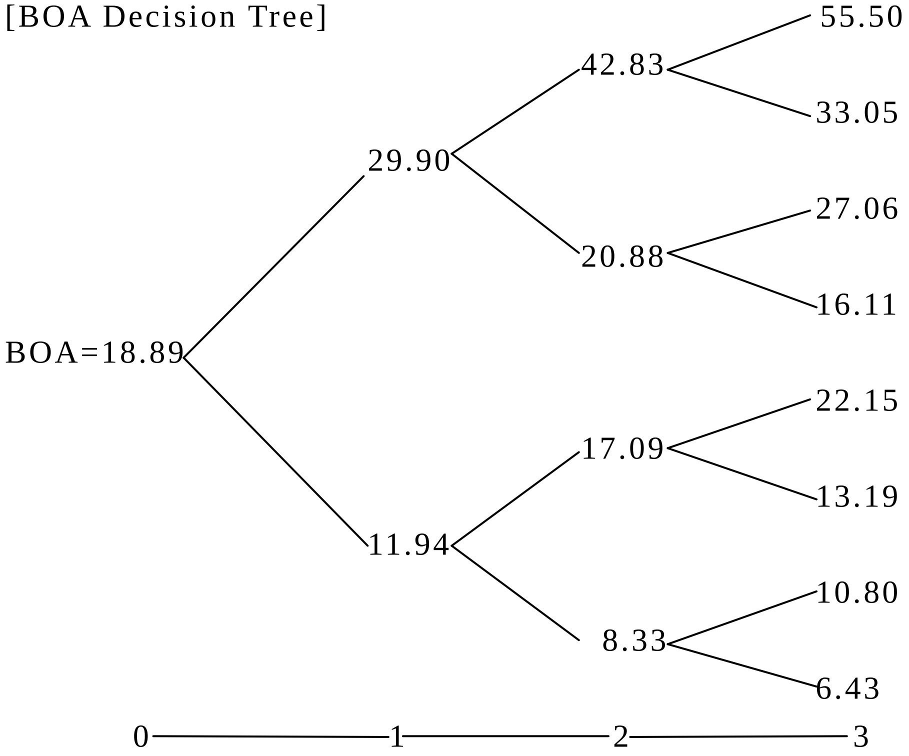
9 θ_{d_1} =10 percent and θ_{d_2} =20 percent in this case.

Figure-4

[BOA Decision Tree]



The BOA value decreases with the decrease of volatility. It is same as the PV and then there is no flexibility. However, the decrease in the volatility of it also decreases the BOA value from 18.93 to 18.89. When the volatility per period decreases from β=45.93 percent to $(\beta - \theta_{d_1})$=35.93 percent in period 2 and increases to β=45.93 percent in period 3, the present value tree for it also does not recombine[10].

[10] θ_{d_1} =10 percent in this case.

Figure-5

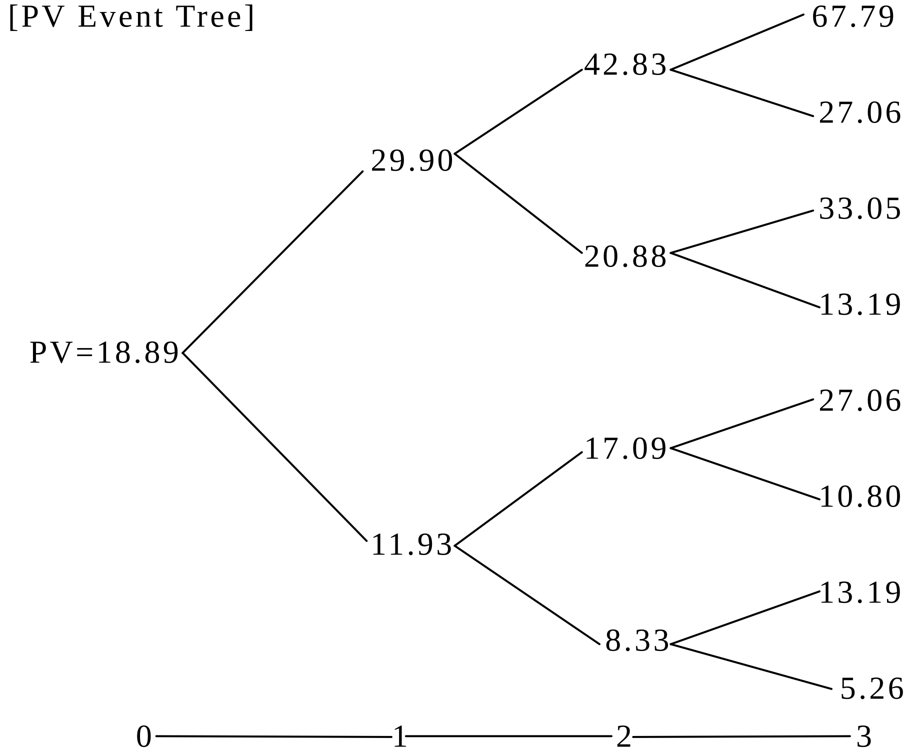
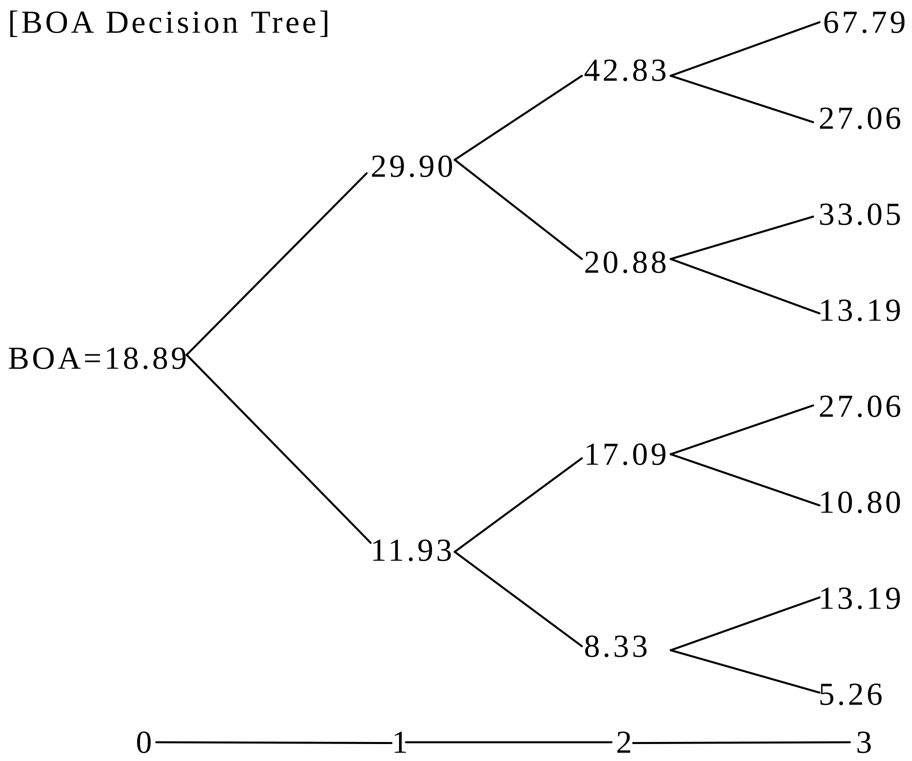


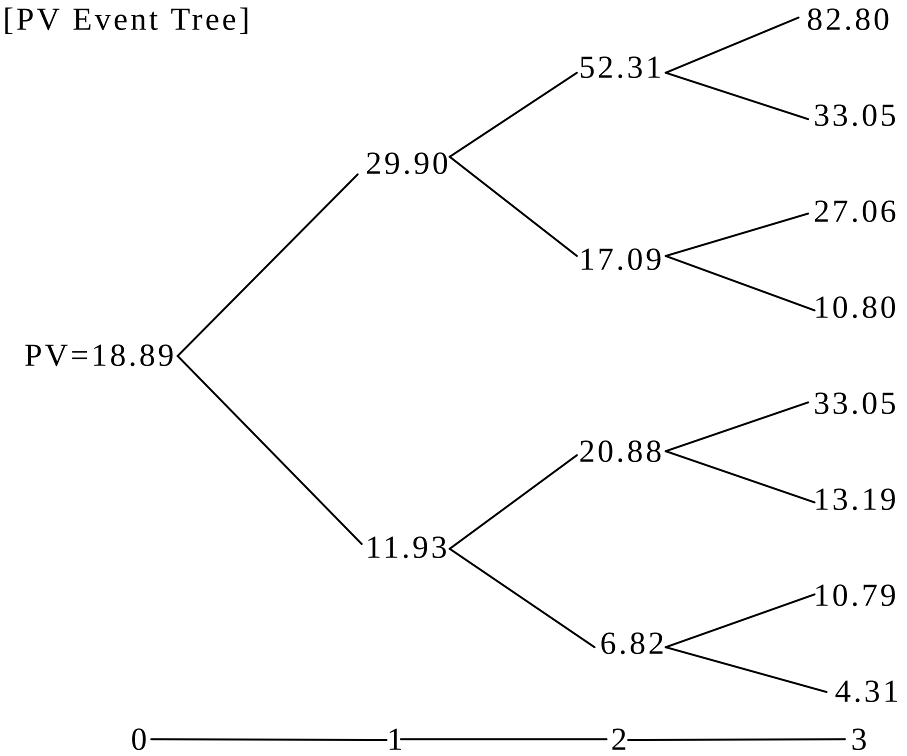
Figure-6



The BOA value decreases with the decrease of volatility. It is same as the PV and then there is no flexibility. The optimal execution of the abandonment option remains the same. However, we notice that the fluctuation of volatility decreases the BOA value from 18.93 to 18.89.

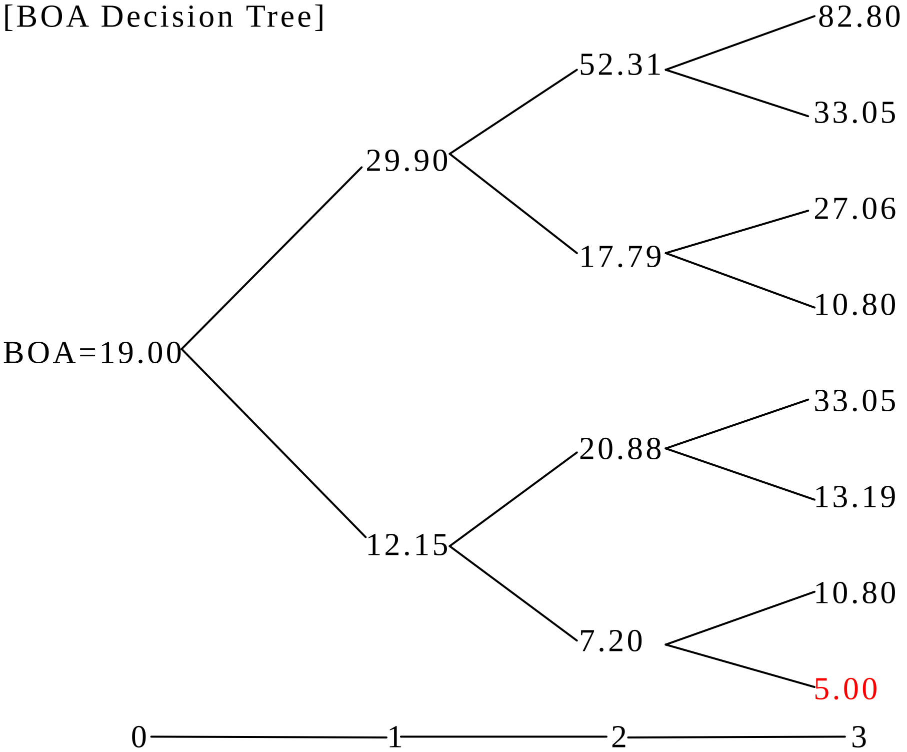
When the volatility per period increases from β=45.93 percent to $(\beta + \theta_{u_1})$=55.93 percent in period 2 and decreases to β=45.93 percent in period 3, the present value tree for it also does not recombine[11]

Figure-7



[11] θ_{u_1} =10 percent in this case.

Figure-8



The BOA value is higher than the PV and then the flexibility is valuable. The optimal execution of the abandonment option remains the same. However, we notice that the fluctuation of volatility increases the BOA value from 18.93 to 19.00.

Furthermore, the volatility per period increases from β=45.93 percent to $(\beta + \theta_{u_1})$=55.93 percent in period 2 and to $(\beta + \theta_{u_2})$=65.93 percent in period 3, the present value tree for it also does not recombine[12].

[12] θ_{u_1} =10 percent and θ_{u_2} =20 percent in this case.

Figure-9

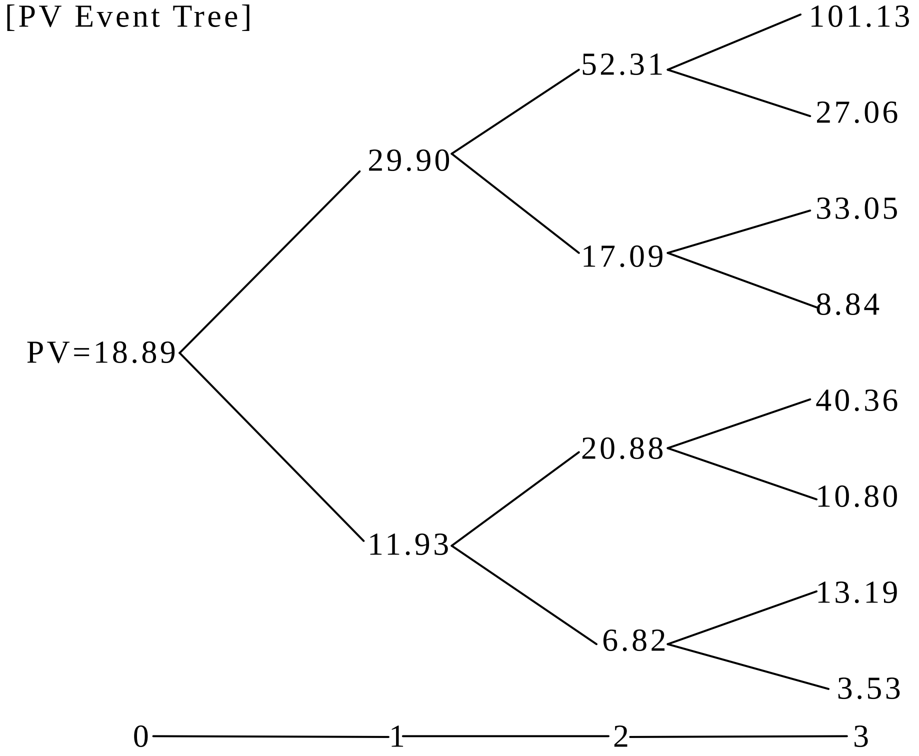
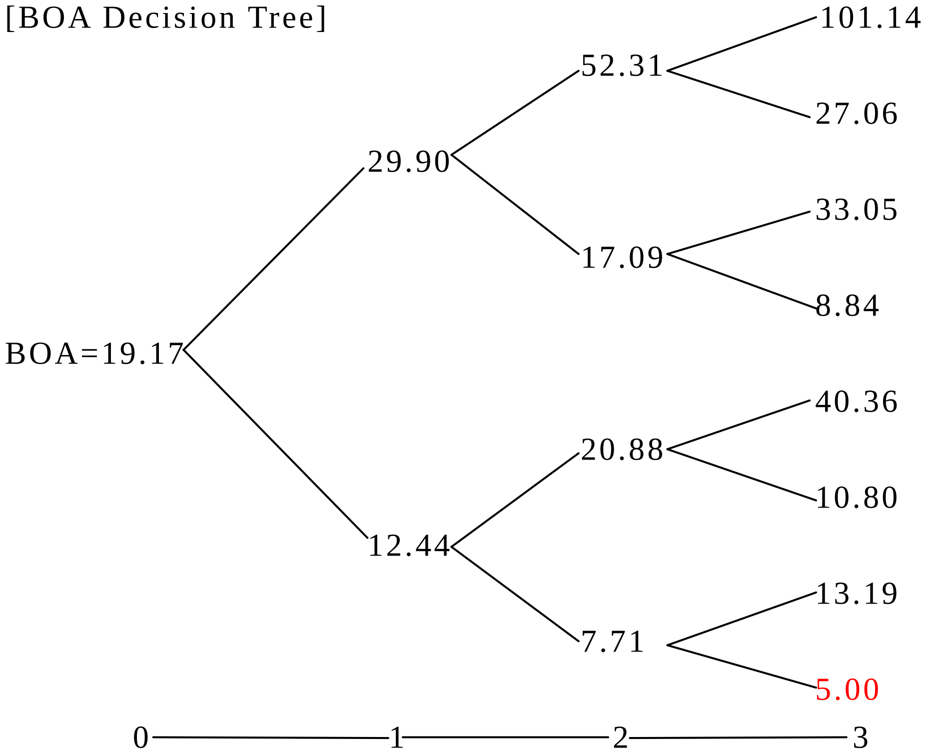


Figure-10



The BOA value increases with the increase of volatility. It is higher than the PV and then the flexibility is valuable. The optimal execution of the abandonment option remains the same. However, we notice that the fluctuation of volatility increases the BOA value from 18.93 to 19.17. Likewise, we execute the scenario analyses of other credit ratings such as 07-1BBB, 07-1A, 07-1AA, 07-1AAA respectively. Judging from the Table-2, we notice there is a tendency that higher credit ratings and worse scenarios have a leeway of more flexibility.

Table-2. Scenario Analyses in Each Credit Ratings

	Best	Better/Worse	Worse/Better	Worst
07-1BBB_*	Not Flexible	Not Flexible	Flexible	Flexible
BOA-PV(=18.89)	0	0	0.11	0.28
07-1BBB**	Flexible	Flexible	Flexible	Flexible
BOA-PV(=19.87)	0.12	0.32	0.53	0.56
07-1A***	Flexible	Flexible	Flexible	Flexible
BOA-PV(=25.9)	0.41	0.69	0.97	1.25
07-1AA****	Flexible	Flexible	Flexible	Flexible
BOA-PV(=45.53)	0.33	0.83	1.46	1.89
07-1AAA*****	Not Flexible	Flexible	Flexible	Flexible
BOA-PV(=75.64)	0	0.24	1.12	2.68

*Annual Volatility=0.4593, Abandonment=US$5.00

**Annual Volatility=0.4669, Abandonment=US$7.50

***Annual Volatility=0.5232, Abandonment=US$10.00

****Annual Volatility=0.4243, Abandonment=US$20.00

*****Annual Volatility=0.2018, Abandonment=US$50.00

Finally, we can confirm the diffusion of bad affection of subprime loan problem from Table-3 to Table-6.The correlated coefficients in each quarter shows the difference. Especially, we can find out the significant difference between second quarter and third quarter. Apparently, we can see the tendency that higher credit ratings indexes are highly correlated with lower credit ratings indexes.

Table-3. Correlated Coefficients in Each Credit Ratings (Jan.2007 - Mar.2007)

	07-1BBB_	07-1BBB	07-1A	07-1AA	07-1AAA
07-1BBB_	1				
07-1BBB	0.9298	1			
07-1A	0.8172	0.8237	1		
07-1AA	0.4554	0.4563	0.3528	1	
07-1AAA	0.5005	0.3971	0.4734	0.7075906	1

Table-4. Correlated Coefficients in Each Credit Ratings (Apr.2007 - Jun.2007)

	07-1BBB_	07-1BBB	07-1A	07-1AA	07-1AAA
07-1BBB_	1				
07-1BBB	0.8960	1			
07-1A	0.6671	0.7140	1		
07-1AA	0.1733	0.2543	0.3800	1	
07-1AAA	0.1745	0.1690	0.1645	0.2240	1

Table-5. Correlated Coefficients in Each Credit Ratings (Jul.2007 - Sep.2007)

	07-1BBB_	07-1BBB	07-1A	07-1AA	07-1AAA
07-1BBB_	1				
07-1BBB	0.8318	1			
07-1A	0.6943	0.7476	1		
07-1AA	0.6874	0.7370	0.8537	1	
07-1AAA	0.6026	0.5890	0.6405	0.746798	1

Table-6. Correlated Coefficients in Each Credit Ratings (Oct.2007 - Dec.2007)

	07-1BBB_	07-1BBB	07-1A	07-1AA	07-1AAA
07-1BBB_	1				
07-1BBB	0.8910	1			
07-1A	0.6862	0.7383	1		
07-1AA	0.6283	0.6623	0.8458	1	
07-1AAA	0.5894	0.5694	0.7419	0.8620666	1

4. Conclusion

When we simulate the change of volatility in ABX.HE Index, we notice in most cases that BOA value is higher than the PV and the flexibility is valuable irrespective of the change of volatility. We use a nonrecombining lattice method to solve this kind of option problem. In addition, we notice that if the volatility increases in the second period and/or in the third period, the BOA value increases. If not, the BOA value decreases. We can confirm the tendency that higher credit ratings indexes are highly correlated with lower credit ratings indexes during the period between second quarter and third quarter. Because subprime loan problem quivered the economy not only U.S. but the rest of the world. Namely, investor’s psychological factors spur the vile situations. It is necessary that U.S. government sets up the public company to purchase all the structured bonds related to subprime loan to settle down this problem smoothly.

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