We consider the realization of a quantum Fredkin gate with only linear optics and single photons. First we construct a heralded Fredkin gate using four heralded controlled-NOT (CNOT) gates. Then we simplify this method to a post-selected one utilizing only two CNOT gates. We also give a possible realization of this method which is feasible with current experimental technology. Another post-selected scheme requires time entanglement of the input photons but needs no ancillary photons.

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I. INTRODUCTION

Quantum computing [1], due to its potential to solve problems far beyond classical computers, has attracted great attention in recent years. Many physical systems have been considered for a quantum computer [2]. One promising system is to use single photons, showing benefits such as low decoherence and easy single-qubit manipulation. However, such systems suffer a major disadvantage—the lack of interaction between individual photon qubits, which is needed for implementing nontrivial multiqubit gates. Surprisingly, Knill, Laflamme, and Milburn demonstrated that scalable quantum computing was possible using linear optical elements, single photons and photon detection [3]. Since then there has been considerable progress in improving the original scheme and demonstrating its basic elements [4].

Here we focus on the implementation of a linear optical Fredkin gate [5]. The Fredkin gate plays an important role in both classical computing and quantum computing [1]. The Fredkin gate is a three-qubit controlled-swap gate, that is, if the control qubit is in state $|1\rangle$, the two target qubits swap their states, otherwise they remain in their initial states. In the context of universal quantum computer, multiqubit gates are usually thought to be built by a combination of single- and two-qubit gates. Smolin and DiVincenzo have shown that five two-qubit gates [two controlled-NOT (CNOT) and three controlled-square-root-NOT (CSRNOT) gates] are sufficient to implement the Fredkin gate [6]. The most efficient known CNOT implementation requires two ancillary photons and has a probability of success of $1/4$ [7], while the CSRNOT also requires two ancillary photons but has a probability of success of $1/8$ [8]. Hence the Smolin, Divincenzo gate needs ten ancillary photons and the total success probability is $2^{-13}\approx 1.2 \times 10^{-4}$. Therefore, their scheme is too difficult to be realized with current experimental technology. Recently, another scheme was proposed in Ref. [9] by simulating the Kerr medium in Milburn’s optical Fredkin gate [10] with linear optical elements. It needs only six ancillary photons with the success probability $4.1 \times 10^{-3}$.

Recently, the complexity of the Toffoli gate was greatly reduced and the success probability was improved by exploiting additional photonic degrees of freedom [11]. In this paper, we wish to see if a similar effect can be achieved by applying those techniques to the Fredkin gate. We propose some methods for implementing the Fredkin gate with linear optics and single photons. The qubits in our schemes are all encoded in polarization states of single photons, so that $|0\rangle = |H\rangle$ and $|1\rangle = |V\rangle$, where $|H\rangle$ ($|V\rangle$) denotes the horizontal (vertical) polarization state. The rest of the paper is organized as follows. In the next section we propose a heralded Fredkin gate using four heralded CNOT gates. In Sec. III we give a post-selected Fredkin gate, i.e., working in the coincidence basis, and we also present a possible optical realization which is feasible with existing technology. In Sec. IV we replace the four heralded CNOT gates in the heralded scheme with four post-selected CNOT gates assisted by time entanglement but without ancillary photons. We conclude in Sec. V.

II. HERALDED FREDKIN GATE

Our heralded Fredkin gate is built up from four CNOT gates. The schematic structure is shown in Fig. 1. To show how the scheme works, we consider an arbitrary input state written as

$$a_1|H\rangle_{t1}|H\rangle_{t2}|H\rangle_{t3}|H\rangle_{t4}|V\rangle_{t5} + a_2|H\rangle_{t1}|H\rangle_{t2}|V\rangle_{t3}|V\rangle_{t4}|V\rangle_{t5} + \ldots$$

$$+ a_5|V\rangle_{t1}|H\rangle_{t2}|H\rangle_{t3}|V\rangle_{t4}|V\rangle_{t5} + a_6|V\rangle_{t1}|H\rangle_{t2}|V\rangle_{t3}|V\rangle_{t4}|V\rangle_{t5},$$

(1)

where $a_i(i=1,2,\ldots,8)$ is an arbitrary complex number satisfying normalization condition.

First the polarizing beam splitter PBS1 (PBS2) transmits the horizontally polarized photons to beam 1 (4) and vertically polarized photons to beam 2 (3). Then the photons in each of the beams 1, 2, 3, and 4 undergo a CNOT gate controlled by the control state. Therefore, the input state becomes

$$a_1|H\rangle_{1}|H\rangle_{2}|H\rangle_{3} + a_2|H\rangle_{1}|H\rangle_{2}|V\rangle_{3} + a_3|H\rangle_{1}|V\rangle_{2}|V\rangle_{4} + a_4|H\rangle_{1}|V\rangle_{2}|H\rangle_{3}$$

$$+ a_5|V\rangle_{1}|V\rangle_{2}|V\rangle_{4} + a_6|V\rangle_{1}|V\rangle_{2}|H\rangle_{3},$$

(2)
The success probability for vacuum detections at detectors $D_1$ and $D_2$ is $1/4$. If we use the heralded CNOT gate proposed by Pittman et al. [7], we need eight ancillary photons and the success probability is $4^{-5} \approx 1.0 \times 10^{-3}$. Compared with the scheme by Smolin and DiVincenzo [6], our scheme has higher success probability and needs less ancillary photons. However, our scheme is not as efficient as Fiurášek’s scheme [9], as we need more ancillary photons and have lower success probability. However, as we shall see, our scheme can be simplified to a post-selected gate using only two ancillary photons with higher success probability, which may be feasible with existing experimental technology.

III. POST-SELECTED FREDKIN GATE USING TWO CNOT GATES

We now consider the construction of a post-selected gate. By this we mean that a gate succeeds conditioned on simultaneous successful detection of exactly one photon for each qubit, so-called coincidence detection. Figure 2 is the schematic of a post-selected Fredkin gate. Comparing this scheme with the heralded one shown in Fig. 1, we can see that the simplification is replacing the two CNOT gates implementing on the photons in beams 3 and 4 controlled by the photon in beam $c$ by HWPs $(67.5°)$ and HWPs $(22.5°)$ with the transformations given by Eqs. (4) and (5), respectively.

Therefore, for the input state given by Eq. (1), the state before PBS3 and PBS4 is

\[
(a_1|H⟩|H⟩|H⟩|V⟩ + a_3|H⟩|H⟩|V⟩|H⟩ + a_5|V⟩|V⟩|V⟩|H⟩ + a_7|V⟩|V⟩|H⟩|V⟩) \times \frac{1}{\sqrt{2}}.
\]

Then through the analogous analysis in Sec. II and in the case of coincidence detection of the output modes $c$, $t_1$, and $t_2$, we can obtain the success output state the same as Eq. (7).
in our scheme the target state of the second CNOT gate is known (|V⟩ or vacuum), it turns out that the gate can be optimized for maximum success probability 1/6 [11,13] (see the gate in the dashed box). Therefore in the case of fivefold coincidence, i.e., detection of exactly one photon in each of the output modes c, t₁, and t₂ and successful detection at D₁ and D₂, the gate succeeds with a total probability of success 1/4 × 1/6 × 1/8 = 1/192 ≈ 5.2 × 10⁻⁵. Note that the success probability for coincidence detection in Figs. 2 and 3 is reduced to 1/8 compared with that of 1/4 in Fig. 1 due to HWP5 and HWP6. However, this is more than offset by the halving of the number of CNOTs required. As an ancillary Bell state is needed, to implement this scheme requires at least a five-photon source, which is available at present [14–16], and therefore our scheme is feasible with current technology. However, the low success probability of our scheme would make the experiment more difficult and longer time detection would be needed.

IV. POST-SELECTED FREDKIN GATE ASSISTED BY TIME ENTANGLEMENT

In this section we introduce another post-selected Fredkin gate assisted by time entanglement. Let us first remind the reader of the CNOT gate presented by Sanaka et al. [17] (see Fig. 4).

The control and target photons are a photon pair generated by spontaneous parametric down-conversion pumped by a continuous wave laser. Such a source is said to be time-energy entangled [18] as the photon pair is in a superposition of many possible emission times. The control photon is split along the short (c₅) or long (c₃) path at PBS1 and combined again in the same path at PBS2. The target photon is split along the short (t₈) or long (t₅) path at the first beam splitter BS1 and combined again in the same path at BS2. A HWP oriented at 45° rotates the polarization state of the photon taking the long path by 90°. The path-length difference ΔL of c₂ and c₅ is the same as that of t₅ and t₈ and satisfies the condition

\[ l_{\text{SPDC}} \leq ΔL \leq l_{\text{pump}}, \]

where \( l_{\text{SPDC}} \) is the coherence length of the down-converted photon and \( l_{\text{pump}} \) is the spectral width of the pump laser. Conditioned on coincidence of detection with the time window of the coincidence counter satisfying \( ΔT < ΔL/c \), we can write the evolution of an arbitrary input state as

\[
\begin{align*}
&b₁[H]_{\text{in}}[H]_{\text{in}} + b₂[H]_{\text{in}}[V]_{\text{in}} + b₃[V]_{\text{in}}[H]_{\text{in}} + b₄[V]_{\text{in}}[V]_{\text{in}} \\
&\quad \rightarrow b₁[H^3]_{\text{out}}[H]_{\text{out}} + b₂[H^3]_{\text{out}}[V]_{\text{out}} + b₃[V]_{\text{out}}[V]_{\text{out}} + b₄[V^3]_{\text{out}}[V^3]_{\text{out}} \\
&\quad + b₅[V^3]_{\text{out}}[H]_{\text{out}}',
\end{align*}
\]

where \( b₁, b₂, b₃, b₄, b₅ \) is an arbitrary complex number satisfying normalization condition, and the superscript \( S(L) \) denotes the photon passing the short (long) path. Here the coincidence counting has post-selected out unwanted state components in which the control and target photons followed paths of different lengths. Because of the time-energy entanglement, paths of the same length are indistinguishable and so add coherently. The success probability is 1/4.

Figure 5 shows an optical realization of a post-selected Fredkin gate by replacing the four CNOT gates in Fig. 1 with the CNOT gates we have just introduced. Based on the analysis above, in the case of the input state given by Eq. (1), the successful output state can be found to be
From Eqs. (10) and (11), we can see that to make the output state entangled the three input photons need to be time-entangled in the two time bins. "S" and "L." This scheme needs no ancillary photons and the probability of success is 1/64. Three-qubit time entangled states of the type required, i.e., in which a triple coincidence is in a superposition of many times, have been described in Refs. [19–21], however, an experimental demonstration of such states has not yet been made.

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The success probability of our scheme is $4^{-5} \approx 1.0 \times 10^{-3}$, which is higher than that of $1.2 \times 10^{-4}$ in Ref. [6], but is less than that of $4.1 \times 10^{-3}$ in Ref. [9]. We have also simplified the heralded scheme to a post-selected one by replacing two CNOT gates with two HWP s. This scheme needs only two ancillary photons, and therefore is feasible with existing technology. However, the low success probability of $1/192 \approx 5.2 \times 10^{-3}$ would make the experiment very difficult. The other post-selected Fredkin gate we have proposed is assisted by time entanglement. Although this scheme needs no ancillary photons and has higher success probability of 1/64, the three-photon time entangled source required is not available at present.

It should be noted that since the post-selected schemes work in the coincidence basis, such schemes could not be scalable without photon-number quantum nondemolition (QND) detectors were added to each output beam, nevertheless they open the door to experimental tests of an optical Fredkin gate and would make its application possible. We hope our proposals will stimulate such investigations of the Fredkin gate.

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