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The synthesis of road vehicle vibrations based on the statistical distribution of segment lengths

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Abstract: This paper presents the development of a method for synthesizing non-stationary random vibrations generated by road transport vehicles. The paper builds upon the observation that non-stationary random vehicle vibrations are composed of a sequence of zero-mean random Gaussian processes of varying standard deviations. It shows how a change-point detection algorithm can be applied to the instantaneous magnitude of sample vibration records to identify the length of stationary segments within the record. The statistical distribution of the segment lengths is characterised with a hyperbolic trigonometric function. The paper shows how the segment length distribution model is used to form a sophisticated control strategy for synthesising non-stationary random vibrations. The paper explains how the new control system is incorporated into a standard random vibration simulation system and presents results which validate the effectiveness of the method.

Keywords: Non-Gaussian, nonstationarities, random vibrations, simulation, synthesis.

1 Introduction

It is self-evident that vehicle vibrations are caused, in the main, by uneven pavement surfaces. These have been found to be random in nature which, in turn, causes the vehicle vibrations to be random. Variations in pavement roughness and fluctuations in the vehicle's speed within a particular journey combine to produce variations in the overall energy levels of the vehicle vibrations. These variations make the process highly nonstationary and non-Gaussian, as illustrated in Fig. 1, and introduce a level of complexity that cannot be adequately dealt with using conventional methods that are used for Gaussian and stationary processes [1, 2]. The most commonly-used technique for laboratory synthesis of transport vibrations for testing purposes has been in place for some years [2]. The method assumes that the vibrations produced by wheeled vehicles can be approximated by a zero-mean, normally-distributed (Gaussian) random process. It is based on the overall root-mean-square (RMS) level of the process which results in a constant RMS level thus implying stationarity. It has been shown that vibration synthesis at a constant RMS level (Fig.2) fails to accurately reproduce the fluctuations in vibration levels that occur naturally during road transportation realizations [2].

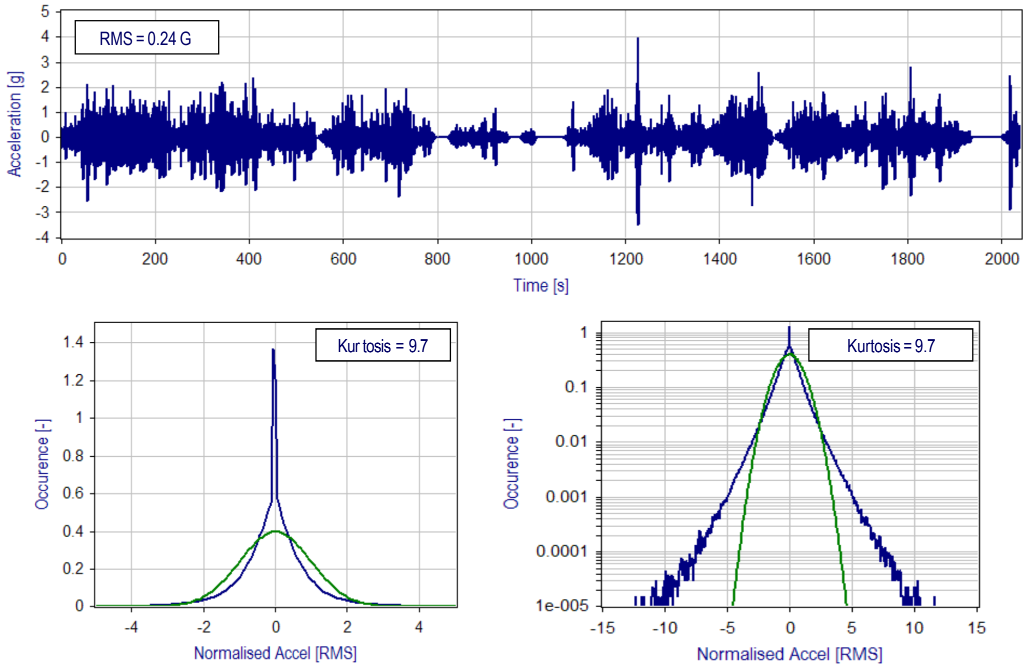

Figure 1. Example of the non-stationary, leptokurtic nature of road vehicle vibrations. (Green line: best fitting Gaussian distribution)

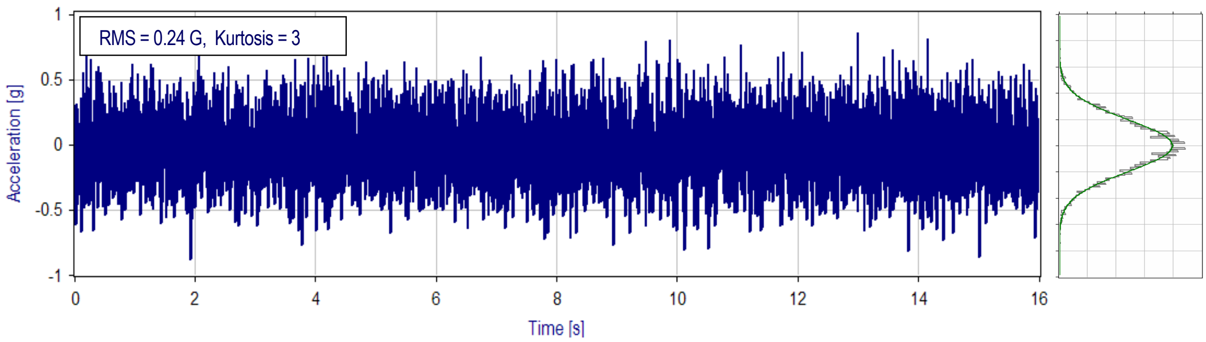

Figure 2. Example of the stationary, Gaussian vibrations produced by typical vibration controllers using the average PSD via the Inverse Fourier Transform.

Despite strong evidence that road vehicle vibrations are highly nonstationary and non-Gaussian, this method is still widely practiced my most laboratories.

The work presented herein addresses this severe shortcoming and is validated using a broad set of typical vibration records collected from a variety of vehicle types, routes and payload conditions [2]. In addition, the vertical acceleration responses of various linear quarter-car numerical models, made available in the literature, were computed for a range of pavement profiles [2] to supplement and complement the collection of measured vibration records.

2 Gaussian decomposition

In order to mimic nonstationarity, the process can be decomposed into a sequence of Gaussian components of varying RMS levels and relative duration (vibration dose) as illustrated in Fig. 3. This decomposition was first proposed by Charles [3] and applied and validated by Rouillard [2]. This showed that it is the nonstationarity of the process that is the cause of the non-Gaussian nature of road vehicle vibrations. Consequently, by synthesising a sequence of Gaussian vibrations of varying RMS levels, it should be possible to achieve the same non-Gaussian effect. However, the nature of the length and sequence of each constant RMS synthesized segment is not known.

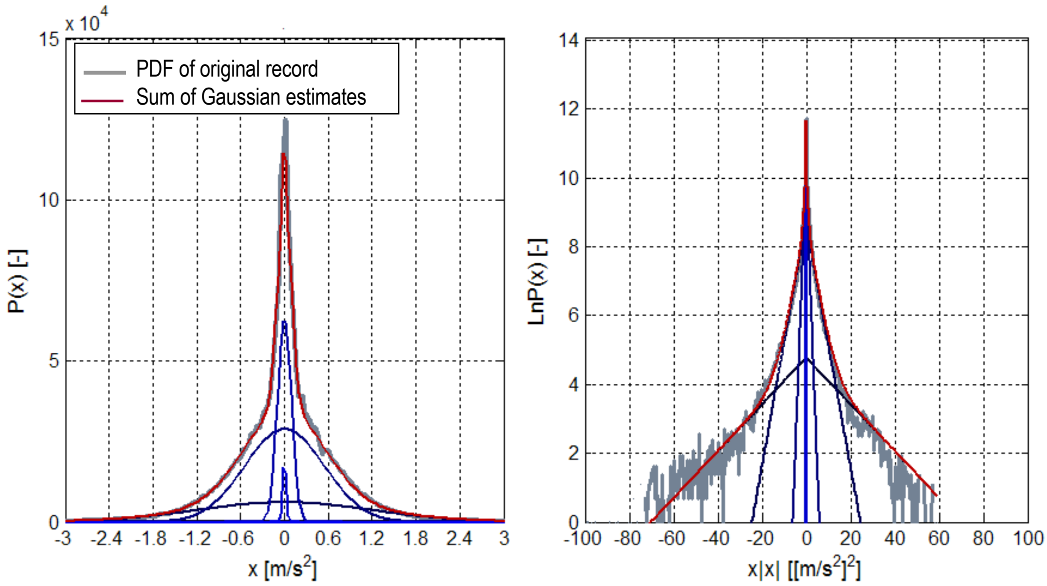

Figure 3. Typical plot of the decomposed Gaussian estimates (blue lines) along with the sum of the estimates (red line) and the Probability Density Function PDF of the original record (grey line).

Since the hypothesis that the process can be modelled statistically as a sequence of segments, each belonging to a family of Gaussian processes with varying standard deviations has been shown to be valid [2], there must exist identifiable boundaries (change-points) at which the transition from one segment to another occurs. The detection of such change-points should make it possible to determine the segment lengths and their relationship with the segment standard deviation as well as their statistical characteristics.

3 Change-point detection

Change-point detection is a process that emerged out of a need to identify real changes in random processes such as economic indicators and the monitoring and control of quality in manufacturing processes. The cumulative sum (cum-sum) techniques are primarily geared toward detecting significant deviations in the mean of random processes and the procedure is greatly enhanced by including a bootstrapping algorithm which is used to provide an estimate of the significance of the change-point [2]. Rouillard [2] applied such a cum-sum change-point detection algorithm to the instantaneous magnitude of the sample vibration records. The argument for using the instantaneous magnitude is based on evidence which shows that level type non-stationarities in random signals are well manifested through changes in the instantaneous magnitude and is not reliant on subjective parameters such as the window width required to compute the moving RMS [2, 4]. The instantaneous magnitude accounts for deficiencies with the moving RMS method in detecting short duration fluctuations in the magnitude of vehicle vibrations which are typically induced when the vehicle encounters sudden changes in the roughness of pavements and when severe and localised pavement surface defects are present [5]. The method presented here makes use of the Hilbert transform to compute the instantaneous magnitude of the record. In its original form, the Hilbert Transform is used to produce the imaginary component, $\tilde{a}(t)$, of a measured, real signal, $a(t)$, thus enabling the creation of an analytical signal $\hat{a}(t)$:

$$\hat{a}(t) = a(t) + i\,\tilde{a}(t) \quad where\ \ \tilde{a}(t) = H\{a(t)\} = \frac{1}{\pi}\int_{-\infty}^{\infty} a(\tau)\left(\frac{1}{t-\tau}\right)d\tau = \frac{1}{\pi}\,a(t) * \left(\frac{1}{t}\right) \tag{1}$$

Once the analytical signal is created, the instantaneous magnitude of the vibration signal is easily computed. The change-point detection algorithm was applied to a number of typical non-stationary random vibration records obtained from a variety of vehicle types and routes to determine the length of stationary segments. The statistical distribution of segment lengths for each vibration record was computed and shown in Fig. 4. It is interesting to note that the shape of the segment length distributions are generally similar and exhibit an asymptotic like decrease in probability of occurrence as the segment length increases. This behaviour was found to be competently modelled with a hyperbolic function in the form

$$p(s) = C / sinh(ks) \tag{2}$$

where C » 4 and k » $¼$ are empirical constants obtained by non-linear regression. The fact that this function is easily integrated is advantageous as wiil be shown later.

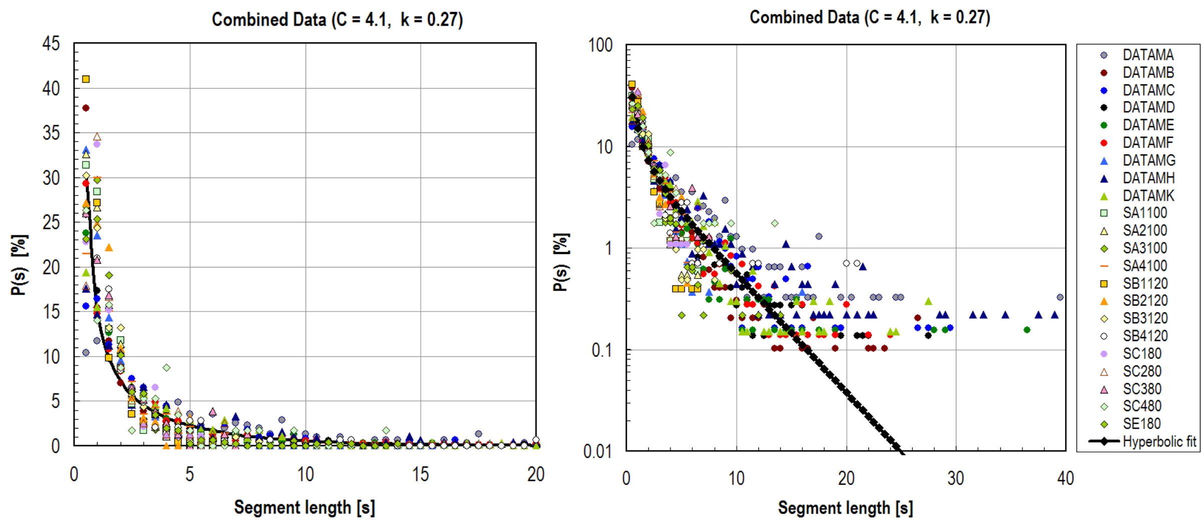

Figure 4. Statistical distributions of stationary segment lengths for all sample vibration records along with the hyperbolic curve of best fit. (Left: Linear scales, Right: semi-logarithmic scales for clarity).

4 Synthesis

Currently, vibration synthesis and control is achieved by defining of a Power Spectral Density (PSD) function that is coupled with a phase spectrum that is uniformly-distributed to form a complex frequency spectrum. The inverse Fourier Transform (IFT) is used to produce a normally-distributed synthetic random signal that is fed to a mechanical vibration generating device which, in turn, attempts to reproduce these vibrations. The synthesized and demand vibration spectra are repeatedly equalised via a feedback loop while truly random vibrations are achieved by continually regenerating a new uniformly-distributed phase spectrum for every control loop period. This well-established method, while suitable for synthesizing stationary random vibrations, is quite inadequate for dealing with non-stationary vibrations. The proposed method involves the inclusion of a modulation function to produce the random variations in magnitude necessary to yield nonstationary, and thus, non-Gaussian vibrations. The modulation function is synthesized from the RMS / vibration dose information and the segment length distribution function such that the length of each stationary segment conforms to the statistical distribution for segment lengths given in (2). This is achieved by transforming a sequence of uniformly-distributed random numbers using the cumulative distribution function obtained by integrating the segment length Probability Density function (2):

$$P(s)=\int p(s)\,ds \;=\int C\,csc\,h(ks)\,ds \;= \frac{C}{k}\,ln\left\{tanh\left(\frac{ks}{2}\right)\right\} \tag{3}$$

Fig. 5 (a) and (b) illustrate how a segment length vector was synthesized using this method. The random segment lengths are combined with the desired vibration dose table to produce the modulation vector (Fig. 5(c)). This is achieved by repeatedly selecting, at random, one of the RMS values from the vibration dose table and allocating it to the first segment length in the vector until

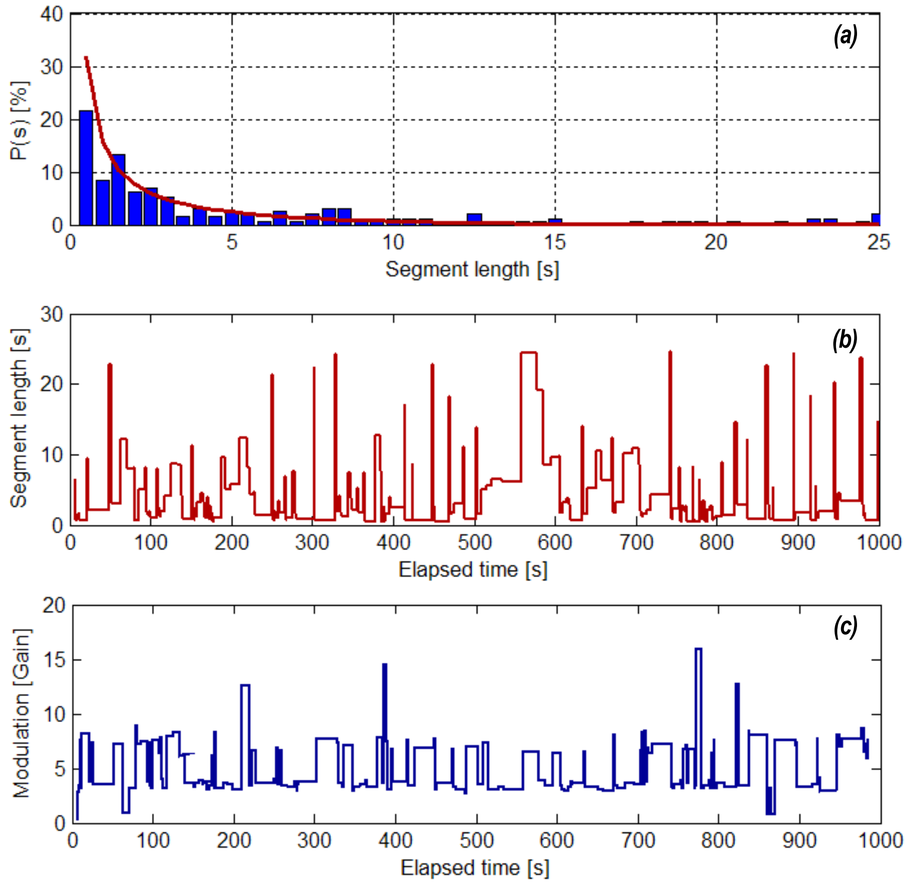

Figure 5. Example of a synthesized segment length vector (b) based on the hyperbolic probability density function (a); (c) shows a typical modulation vector generated by combining the segment lengths with the target vibration dose

every vibration dose table element is full. In order to ensure that each element of the vibration dose table is filled at a uniform pace (hence producing a truly random sequence), the random allocation process was modified from a simple uniformly distributed random variable to a weighted allocation process that favours each Gaussian element in proportion to its corresponding vibration dose value.

5 Validation experiments

The non-stationary random vibration synthesis method proposed above was experimentally validated using a typical laboratory-based random vibration synthesis system fitted with a purpose-built variable gain amplifier/attenuator and a PC to generate the synthesized modulation function as illustrated in Fig. 6.

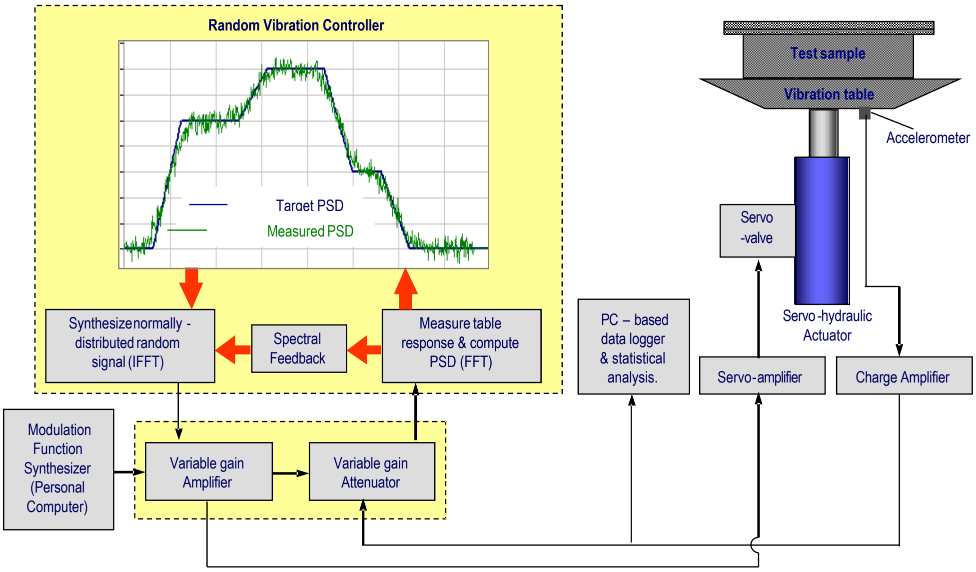

Figure 6. Experimental configuration for validation experiments.

Numerous validation experiments were carried out using the statistical and spectral models generated from the vibration samples. Fig. 7 shows the signals for both the original vibration record and the synthesized vibrations for a typical, representative case. The non-stationary character of the synthesized vibrations is clearly evident and, although quite different from the original signal (as expected), possess the same spectral and statistical characteristics as those of the original record used to generate the statistical models as shown in Fig 8.

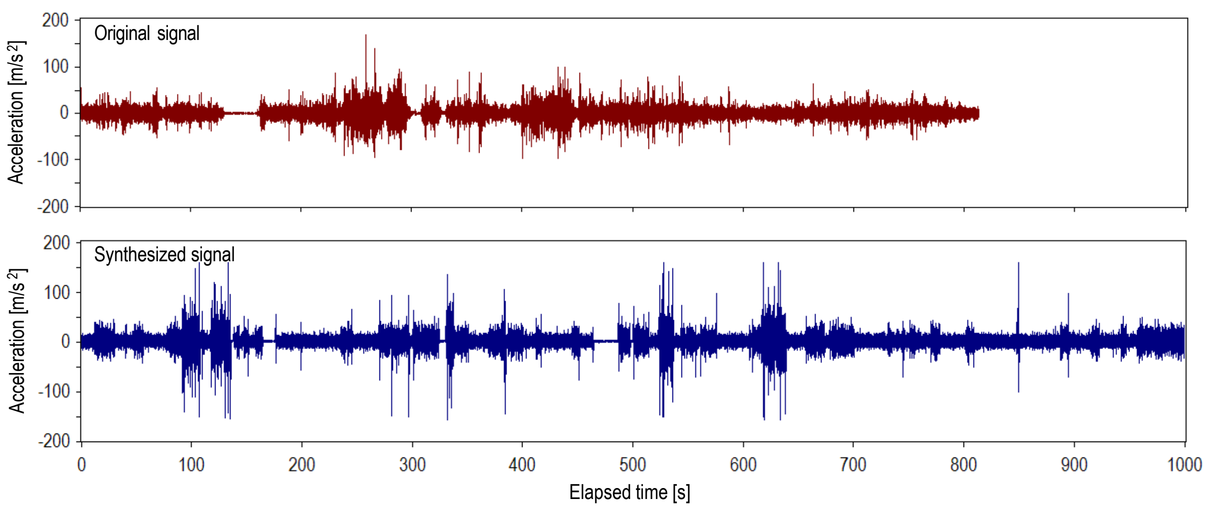

Figure 7. Original and synthesized vibration records for a typical case.

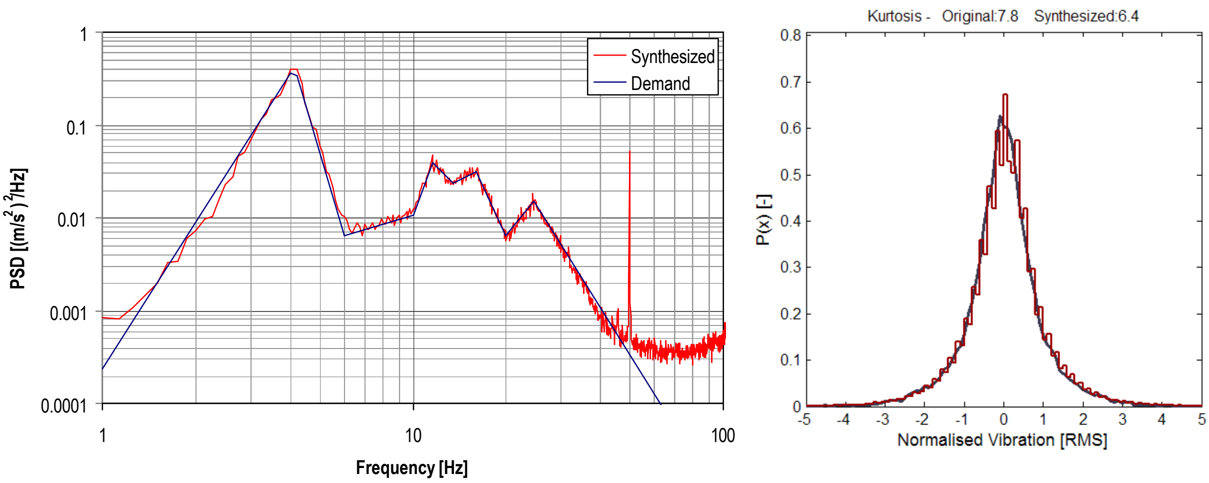

Figure 8. Demand (target) and synthesized spectra (left) and PDF of the original and synthesized records (right) for a typical case.

Overwhelmingly, results generated by the many validation experiments carried out and represented here have clearly demonstrated that the proposed method affords a practical solution for synthesizing non-stationary random vibrations in the laboratory.

6 Conclusions

The paper has demonstrated that the proposed method offers an increased level of sophistication for synthesizing non-stationary random vibrations in the laboratory. vibrations records can be used to create both spectral and statistical signatures of the process which can be used to physically synthesize non-stationary random vibrations of the same statistical character in the laboratory.

A Random Gaussian Sequence Decomposition algorithm which automatically extracts the parameters of each constituent Gaussian process, namely the RMS level and the Vibration Dose, was successfully developed and was found to be very effective in characterising non-stationary random vehicle vibrations in such a way as to facilitate laboratory synthesis.

A statistical change-point detection technique based on a cumulative sum – bootstrapping approach applied to the instantaneous vibration magnitude signals was developed. The results show that this makes for a useful technique for identifying the length of stationary segments within the process which is essential for the realistic and accurate synthesis of non-stationary random vibrations. The statistical distribution of the segment lengths for a wide range of representative vibration signals were found to conform to a simple hyperbolic model which can be used to synthesize the random modulation function necessary to reproduce non-stationary random vibrations in the laboratory.

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