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Counterintuitive problems in dynamics and vibration

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Abstract: Mechanical Vibration and Dynamics are taught to undergraduates as if they simple sciences. The mass-on-a-spring, uni-axial vibration of a rod, viscous damping, modal analysis - all these are the bread and butter of vibration science. As for rigid-body dynamics undergraduate courses remain fixed in 2-D planar motion. But real dynamic and vibrating systems just don't behave simply. There are pitfalls in even the most ordinary cases and some of these will be demonstrated: a tuning fork; a bottle of coke; a bending beam; a turbocharger wheel, a bouncing ball, a rolling ball and boomerangs. All of these things behave counter-intuitively.
The talk accompanying the paper will be filled with practical demonstrations - seeing is believing. Most are demonstrations that can be repeated at home.

acoustics, damping, dynamics, modal analysis, rigid body, vibration

1 Introduction

This paper is about simple problems in mechanics, but even the simplest problems can appear difficult. This is often because the observed behaviour is not what we are expecting – *i.e.* it is counterintuitive. There are several routes to counterintuitivity and something is counterintuitive if:

1) it requires advanced/specialist knowledge;
examples: that objects contract and get heavier at high speed due to relativitity
that alpha particles pass through gold leaf because they are mostly free space
2) it is obscure or difficult to observe
examples: that the apparent wobbling of the moon is due to its elliptical orbit (lunation)
that sea levels are rising due to global warming
3) it doesn't fit with our experience,
examples: what happens to an astronaut exposed unprotected to the vacuum of space?
topspin doesn't make tennis balls go faster
4) we've never noticed it before
examples: the longest day is longer than the longest night (the sun is a disc, not a point)
a photo taken of your shadow on grass has a halo around it (opposition effect)
5) we believed what our teachers said.
examples: lift on a wing is *not* explained by Bernoulli's principle, but rather the Coanda effect
there is such a thing as centrifugal force

In this paper, and more particularly in the delivered presentation, many counterintuitive problems will be discussed. All of them will be demonstrated "live" and the equipment required will be minimal – almost everything is available inside the average home, office or school. Most of these demonstrations have arisen from many years of lecturing, supervising and demonstrating in dynamics and vibration at the Department of Engineering in Cambridge University, and at Trinity College Cambridge. The importance of "seeing is believing" is one thing, but equally important in any lecture is the "theatre" of having things actually happening in front of the audience.

Mechanics comprises Dynamics and Statics. The problems discussed here are all within dynamics, but within dynamics is another subset, Vibration. It is curious that this subject area can yield counterintuitive problems even though it is almost entirely described by linear analysis.

2. Dynamics problems

2.1 Bouncing Balls

The first is a classic, where a rubber superball is thrown onto the floor under a table. The ball should if it were though to behave as a particle, bounce through as shown in Figure 1(a), and indeed this is what happens if the surfaces are wet or greasy. But it is almost always the case, even if the table rests on a carpeted floor, that the ball follows the path shown in Figure 1(b). Try it! The explanation is "simple" but it requires an understanding of how the spin of a ball changes after each of the three impacts.

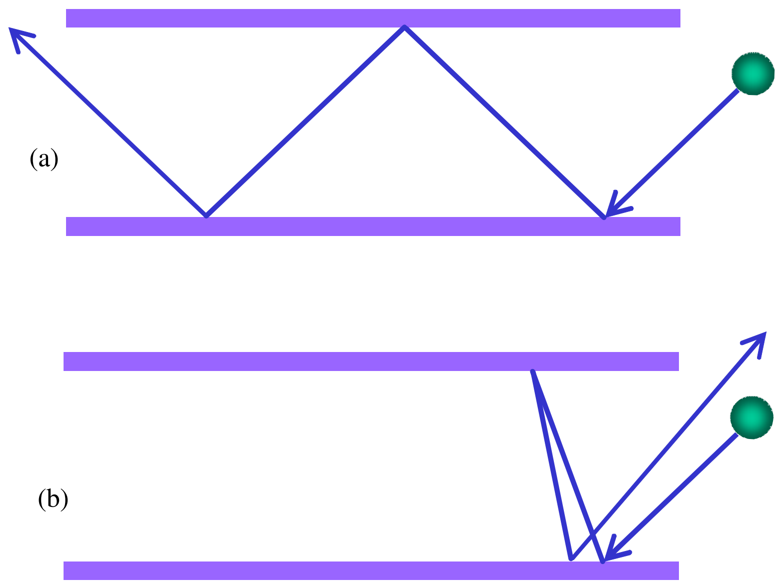


Figure 1 – a superball bouncing under a table. According to the particle theory of light we would expect to see (a) but the ball actually bounces back out as in (b).

Consider the three impacts A, B and C shown in Figure 2. The ball is thrown in towards A and it is not spinning, but the tangential component of the impact force at A causes the ball to start spinning and it moves on towards B. At B the tangential component of the impact force is large enough not only to reverse the direction of travel but also to reverse the direction of spin. This is easy to observe experimentally with a superball thrown to the ground with "backspin". Now the ball moves towards C and the collision at C is "topspin" and the ball is propelled out in the direction shown. It is possible to show mathematically that with two simple assumptions – conservation of energy and conservation of angular momentum – at each collision the ball's motion is exactly as described. It is also then possible to compute the ball's trajectory and to compare it with high-speed video footage. Such a comparison will be shown in the presentation at the conference, along with a live demonstration, but readers are invited to see these videos and animations for themselves on the website at www2.eng.cam.ac.uk/~hemh/movies.htmsuperballs.

On this website can also be found other counterintuitive problems, for instance does a ball bounce back faster when it is rolled towards a wall that is frictionless (*i.e.* a Teflon coated wall, or more easily a wall coated with a thin layer of oil) or when the wall is rough – *i.e.* a normal wall. There are lots of good bouncing-ball problems, and the website also shows the motion of a ball thrown at the ground near a wall – the game of "downball" – and that if the ball hits the wall first the motion of the ball is completely different. It's all about spin, and it soon becomes clear that the best counterintuive problems in dynamics involve spin.

The Bouncing Ball problem described here is essentially a planar 2D problem. Much more interesting things happen in 3D.

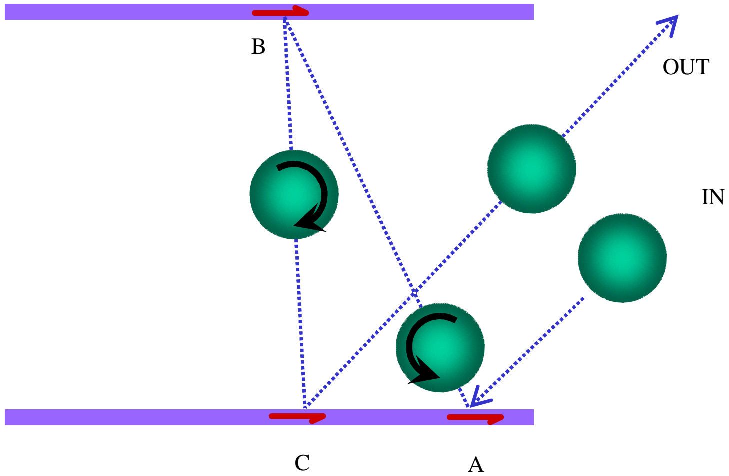


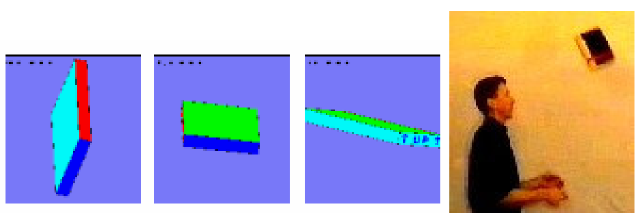
Figure 2 – Analysis of the motion. The ball makes three collisions at A, B and C. At A anticlockwise spin is created which acts as backspin at B. The spin direction reverses in the collision at B so that at C the effect of topspin shoots the ball out in the direction shown.

2.2. Non-contact problems in 3D

Two problems will be discussed here:

1. Tossing a book up in the air

It is well known that a rigid body will spin stably if spun about its smallest or its largest moment of inertia, but that it does not spin stable about its intermediate moment of inertia, as shown in Figure 3. About this axis it will tumble. This will be demonstrated with several objects (a tennis racquet, a mobile phone and a book). The expolanation can be ound in standard dynamics texts [4, 5]



(a) (b) (c) (d)

Figure 3 – the instability of a rigid body spinning freely in space, (a) and (c) stable, (b) unstable. (d) the experiment is easy to do. For animations and explanations see www2.eng.cam.ac.uk/~hemh/movies.htmtumblebook

2. Pointing the Hubble telescope

When a spacecraft is floating in space there is no “terra firma” on which to push in order to point in any desired direction. But spinning discs (wrongly called “gyroscopes”, but more correctly called “reaction wheels”) can be used in most surprising ways to orient a spacecraft with great accuracy. This will be demonstrated using the experiment depicted in Figure 4.

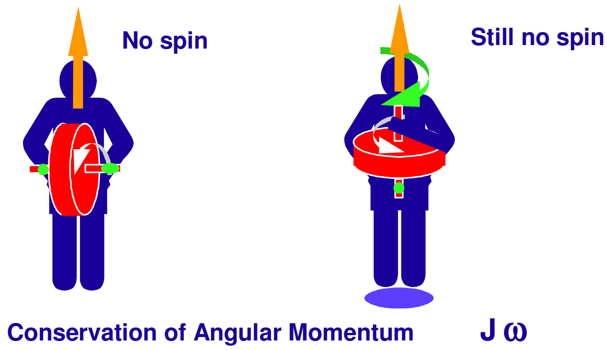


Figure 4 – orientation of a body in space by means of a reaction wheel.

3. Landing on your feet

A cat can do it, so why can't we? Well, we can – and the principles will be demonstrated live on stage. The principle involved is the same as that employed by ice skaters and ballet dancers as they vary their moment of inertia while conserving their angular momentum.

2.3. Contact problems in 3D

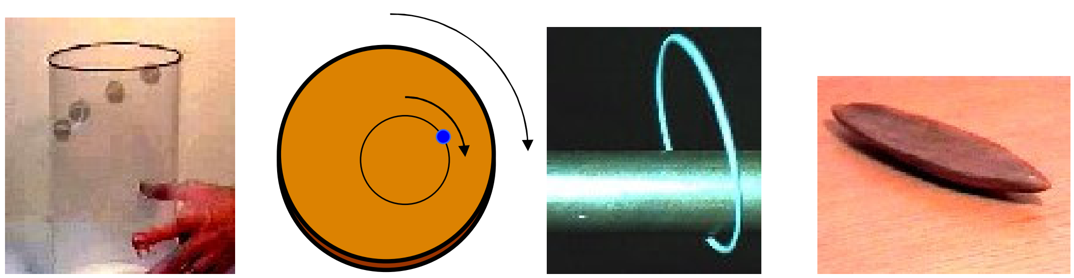
There are some extraordinarily counterintuitive things that can be observed when a body spins when in contact with a solid surface. There is not enough space in this short paper to give explanations (though they can be found at www2.eng.cam.ac.uk/~hemh/movies.htm) but some indication of the puzzling nature of the problems are given here.

1 Spinning Top

Why does a top stay up? And why is there a critical speed below which it will not stay up? The best way to think of this is by considering the consequences of the top falling over, and how it might violate principles of angular momentum and its conservation.

2 Ball rolling in a cylinder

Take a rubber ball – or even better, a mouse ball (the rubber-coated ball that you might find in a computer mouse) – and throw it tangentially into a cylindrical container (a vase, or a rubbish bin, or a pipe) and you'll find that the ball rolls down around the side and then back up and out again, see Figure 5(a). Look on the website for a video of this. The dynamics are fascinating [1], and has to be seen to be believed.



(a) (b) (c) (d)

Figure 5 – Contact problems in dynamics. (a) a ball rolling in a cylinder. (b) a ball rolling on a rotating turntable. (c) a ring spinning on a rod (d) a wobblestone.

3 Ball rolling on a rotating turntable

Like the ball in a cylinder, the dynamics of a ball rolling on a rotating turntable are extraordinary, see Figure 5(b). . Videos can be seen on the website, but what is most extraordinary is that the motion is *always* a circle *fixed in space* and the period of motion around this circle is *always* 2/7 of the rotating period of the table. This is independent of the size or mass of the ball, and it doesn't matter where the ball is thrown onto the table. The mathematics are not all that difficult [1], but it is amazing to watch.

4 Ring spinning on a rod

Another class of counterintuitive problems is that of a ring spinning on a rod – just like a hula hoop, see Figure 5(c). But simplify the hula hoop problem as far as it will go – a tinn ring spinning on a rigid circular rod – and it does amazing things. It can bounce up and down like a mass on a spring, as it spins, and more.

5 Rattleback, Tippetop, Hard-boiled egg

The final class of problems presented here are fiendishly complex to analyse, but extremely simple to observe. The rattleback (also known as a wobblestone, or a celt, see Figure 5(d)) is a solid object with a curved surface, sort of like an upside-down VW beetle. When it spins one way nothing much happens [2]. But the other way it wobbles, stops and spins in the opposite direction. It seems only to be happy spinning one way. A Tippetop is a small toy that spins like a top, but wait a few seconds and you'll see it slowly turn over and stand upside down on its handle [3]. Then the simples, a hardboilded egg, if spun on a table will stand up on its point end. Why? It's all rather complicated, but beautiful to see.

3. Vibration problems

> *"In problems relating to vibrations, nature has provided us with a range of mysteries which for their elucidation require the exercise of a certain amount of mathematical dexterity.*
>
> *In many directions of engineering practice, that vague commodity known as common sense will carry one a long way, but no ordinary mortal is endowed with an inborn instinct for vibrations; mechanical vibrations in general are too rapid for the utilization of our sense of sight, and common sense applied to these phenomena is too common to be other than a source of danger."*
>
> *Professor C E Inglis, FRS,*
> *James Forrest Lecture, 1944*

In this quotation Inglis illustrates clearly just how counterintuive problems in vibration can be. Inglis was an engineer and these days we might translate his thinking in terms of the design process which is illustrated in Figure 6. The difficulty with dealing with vibration problems is that they are indeed mysterious and it is usual for vibration problems to be dealt with when they occur, rather than as part of the iterative design process. This keeps vibration consultants in business.

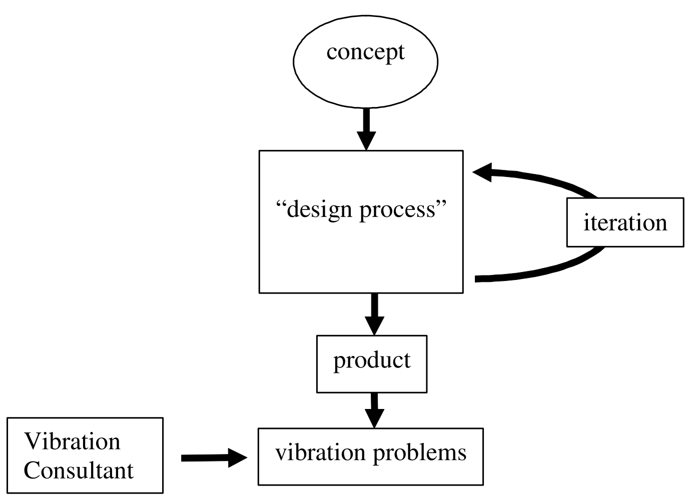


Figure 6 - The design process, where vibration is usually dealt with as an afterthought.

The “bread and butter” of vibration analysis is the mass-on-a-spring. This is depicted in Figure 7. It is easy to understand, even without any understanding of the underlying mathematics. All an engineer needs to know is that vibration problems occur at resonance, *i.e.* when the frequency of excitation coincides with the natural frequency = $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (even the factor of 2π is something we have to live with – the mathematics behind it is not intuitive). So it is usual to consider raising or lowering the natural frequency by adjusting the system mass or stiffness. And if it is not possible to shift a resonance away from the frequency range of interest then it is sensible to add some damping. But there are many puzzling things that can happen, and these require an understanding of vibration modes and nodal points, and a bit about non-linearity. And as for damping, the pitfalls are legion. The real problem is that we are lured into a false sense of security, just because we understand how the mass-on-spring system works.

- Stiffness
  Frequency = $\sqrt{\frac{k}{m}}$
- Mass

- Nodal points
- Vibration modes
- Non-linearity
- Damping

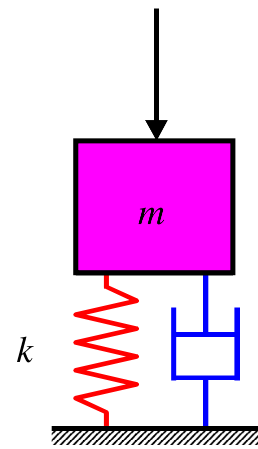


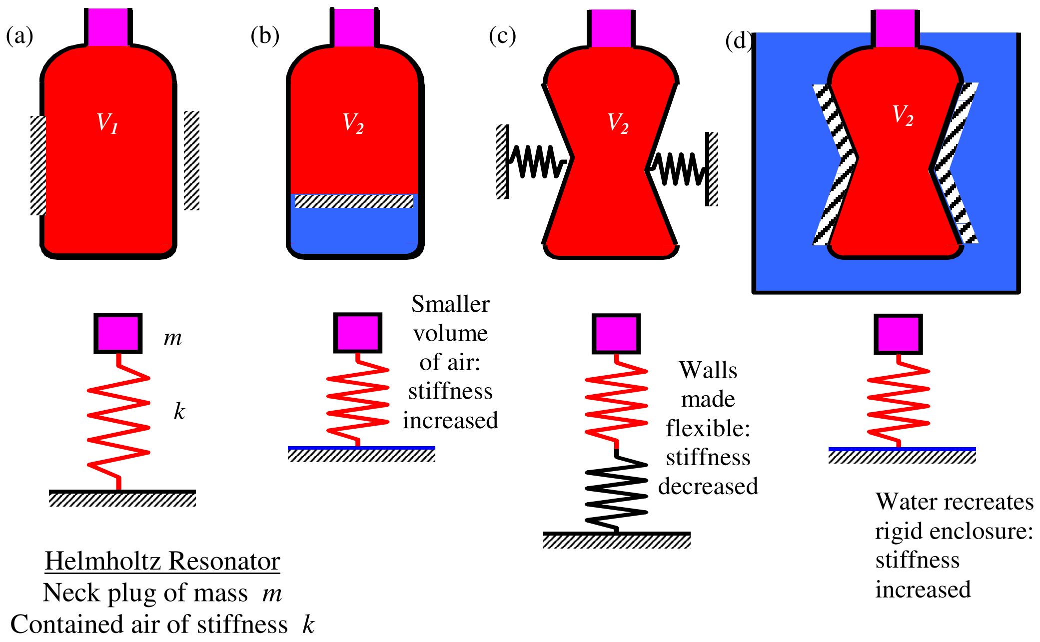
Figure 7 – the mass-on-spring model, the workhorse of vibration analysis

3.1 The Helmholtz resonator

A very nice experiment can be performed with an empty plastic drinks bottle, as illustrated in Figure 8.. It is widely presumed that the note obtained when blowing over the open neck of a bottle is due to the “organ-pipe” resonance of the column of air within the bottle, and this explains why the not rises when water is added to the bottle. But why does the note not change if the bottle is tilted – the water moves and the air-column length changes. A little knowledge is a dangerous ting, because it is not the organ-pipe resonance at all, and in fact the simple mass-on-spring is the best explanation. The air in the neck (known as the “neck plug”) acts as a mass which oscillates on the volume of air in the bottle which acts as a spring. Adding water reduces the volume of air and so stiffens the spring. This effect can be illustrated in a most interesting way by squeezing the bottle so that the air volume reduces – but very oddly the pitch of the note falls. This is because the walls of the bottle are no longer in the form of a cylinder (known for its stiffness) and the flat sides are free to vibrate. The effective stiffness of the contained air is reduced, and so the note goes down – as would be expected from $\sqrt{\frac{k}{m}}$. If the whole bottle is submerged in water, so as to stiffen the walls, the note goes up to where it might be expected to be. Throughout this entire experiment the neck-plug mass remains constant and it is only the stiffness of the contained air that changes. If the neck is lengthened then the note will go down on account of the increased neck-plug mass.

3.2 Nodal points, non-linearities and damping

It is important to understand that very few problems can be fully understood based on the single mass on a single spring depicted in Figure 7. The following examples illustrate this, and they introduce the concepts of vibration modes, modeshapes and nodal points.



$$\omega = \sqrt{\frac{k}{m}}$$

Figure 8 – an empty – and partly-empty plastic drinks bottle makes an excellent demonstrator of a Helmholtz Resonator, but it is a source of great confusion. (a) Blow a note across the neck of an empty bottle. (b) add some water to reduce the bottle volume and the note goes up. (c) instead of adding water try reducing volume by squeezing the bottle – this time the note goes down (counterintuitively). (d) immerse the bottle in water and the note goes up again. All very unexpected.

1. a coffee cup with a handle (nodal points)

In figure 9 is shown a coffee mug. If it is tapped with a teaspoon then a note is heard. But listen carefully – the note is different depenting on where on the rim of the cup the cup is tapped. This is because there are two possible modes that can be excited, both elliptical in shape, but one of these modes requires that the handle moves and the other has the handle at a nodal point. The moving mass will be different for the two modes and so according to $\sqrt{\frac{k}{m}}$ the frequencies will be different. This is exactly as expected but the concept of modes and nodal points has to be understood. These modes and nodal points are invisible, so as Inglis said, they are difficult to understand.

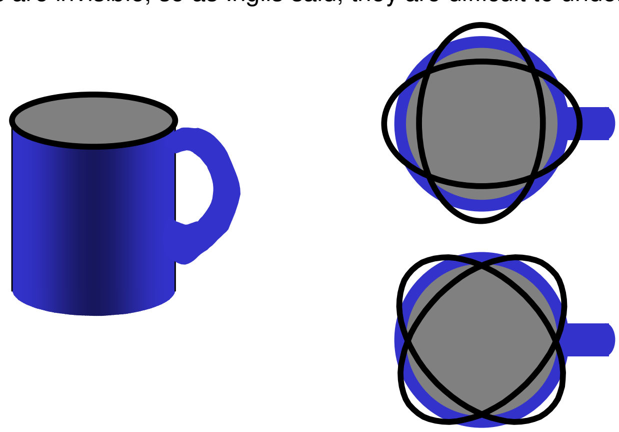
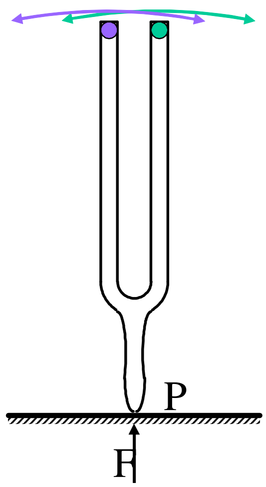


Figure 9 – A coffee cup has two natural frequencies owing to the mass of the handle, so it depends where you hit it with your teaspoon.

2. a tuning fork (nodal points and non-linearities)

In figure 10 is shown a tuning fork. It is well known that the mode of a tuning fork is that when the two prongs move in antiphase so that the handle os nodal. This is perfect, because the handle can be help without any danger of dissipating vibrational energy. But the conundrum is this: most musicians know that a tuning fork is too soft to be heard clearly, but that if its end is placed on a table, or on a piano lid, then the sound is much louder. But why? If the handle is nodal then surely there is no vibration there to amplify? It turns out that while the tuning fork itself is very well described by lnear vibration theory the vibration at the handle is not exactly zero. It is necessary to include the small non-linear effect due to the small arc of travel of the tips of the tuning fork. The centrifugal force causes a vertical motion and this is transmitted to the table. It is clear that this frequency must be at double the tuning fork frequency, and measurements show that it is. Our ear doesn't seem to mind an octave discrepancy in frequency – the ear seems to know how to compensate.



The tips of the tuning fork move on the arcs of circles and centrifugal inertia forces are generated, twice per cycle.

Suppose tip amplitude is 0.2mm,
oscillating frequency is 440Hz,
moving mass is 20% of the fork mass, then
the 880Hz component of tip force F
is about 10% of the weight of the fork.

Figure 10 – Tuning Fork: "P" is a nodal point, so why do we get more sound when "P" is put on a table? It all comes down to a small non-linear effect.

3. a bending beam (nodal points and mode shapes)

In figure 11 is shown a vibrating beam. There is nothing new here [6] but there are some very nice things to observe experimentally: 1) the beam held at different nodes produces different notes; 2) a given mode cannot be excited or damped at a nodal point; 3) all even modes are excited at the centre. The lessons from this are important: a) many vibration problems can be solved by moving the source of excitation, or the observer, to a nodal point. Also, there is no point in applying vibration countermeasures to a system near a nodal point of the mode that is causing problems. All engineers should see this demonstration

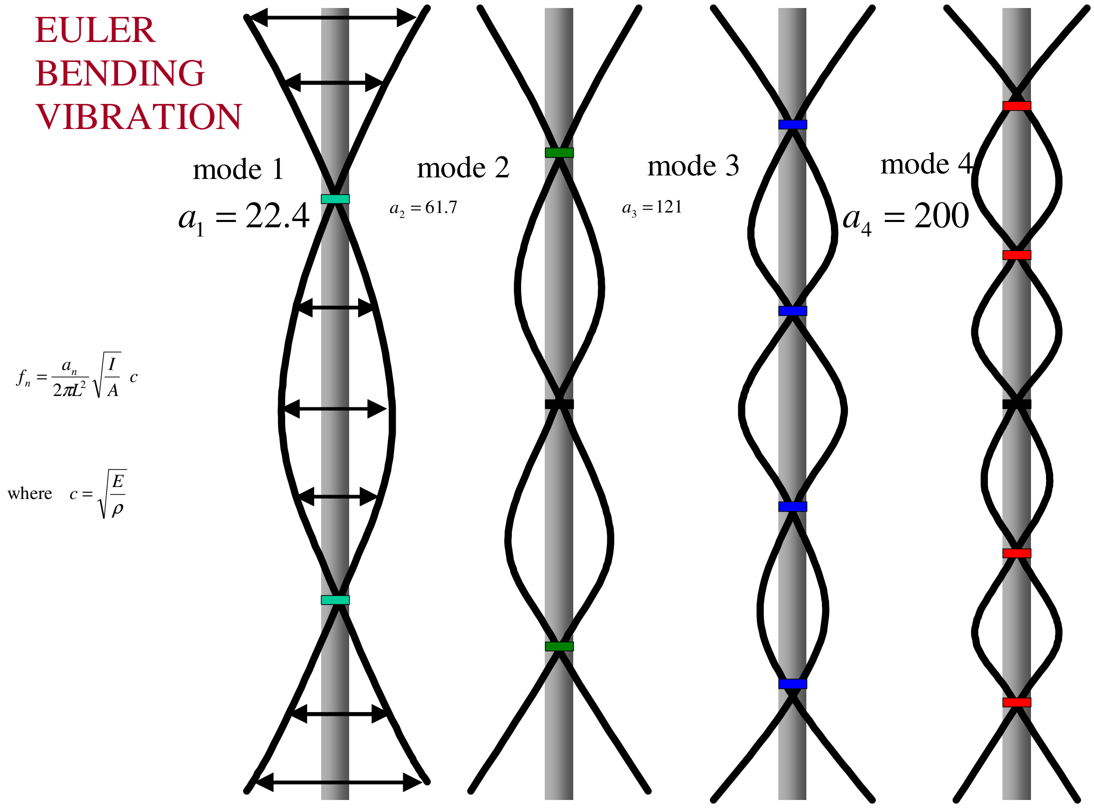
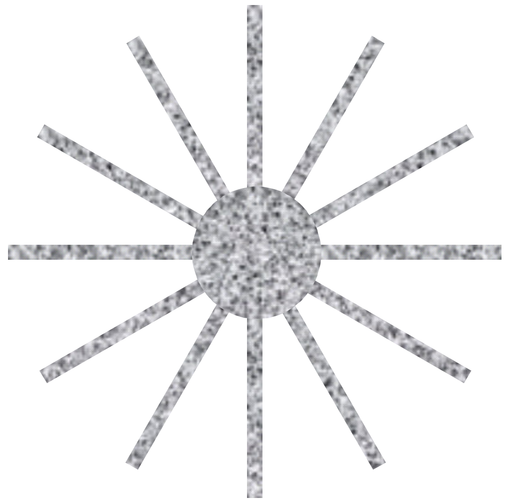


Figure 11 – a vibrating beam for illustrating vibration modes and nodal points.

4. a turbocharger (nodal points, mode shapes, axisymmetry, damping)

To end with a most extraordinary demonstration, consider a simple model for a turbocharger, as shown in Figure 12. According to modal analysis as observed on the bending beam of Figure 11, it would make sense to suppose that a blade that is excited by tapping it with a hammer can be silenced by holding it at the same point as it was excited. But this is not what happens. It will be shown that each of the blades can be “stopped” in turn and the turbocharger keeps on vibrating. The reason for this is that there is coupling between the various blades and that this coupling is through damping. Damping is so poorly understood at the best of times and it would be beyond the experience of almost any engneer to expect that damping is responsible for this totally counterintuitive phenomenon. It goes some way to explain why turbocharger vibration is so difficult to mdel and to understand.



Axisymmetric bodies

Turbocharger blade vibration

Questions:

- Do the blades fatigue less rapidly if they are perfectly tuned, or is it better to mistune them?
- Can vibration measurements made on a rotor be used to estimate its fatigue life?

Figure 12 – vibration of a model turbocharger.

Conclusions and recommendations

The bottom line of all this is that dynamics and vibration are both very interesting subjects of study, and they impinge on many engineering problems. But without seeing first hand how some really simple problems can be totally baffling one might be led to believe that all problems in dynamics and vibration have simple solutions. This is certainly not the case. I recommend that all engineers try some of these simple experiments for themselves – it is a humbling experience.

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