Numerical Simulation of Flow Past a Stationary and Rotating Sphere

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Abstract
Numerical simulations of the laminar flow past a rotating sphere are considered for Reynolds numbers $Re = 100$ and 250, which for a stationary sphere covers the steady axisymmetric and steady planar-symmetric regimes. The non-dimensional rotation speeds $\Omega^* = 0.05, 0.20, 0.50$ and $1.00$ are considered, where $\Omega^*$ is the maximum surface velocity normalized by the freestream velocity. The rotation axis of the sphere is oriented at various angles, $\alpha$, between the limits of streamwise rotation and transverse rotation. For each Reynolds number, the effects of $\Omega^*$ and $\alpha$ on the instantaneous wake structure and the forces acting on the sphere are presented.

Introduction
The flow past a sphere is of interest in a range of engineering applications involving the transport of particulates. These include chemical processing and combustion, where a sphere serves as a good model for more general bluff body particles. Particles in these environments are subjected to both translational and rotational velocities as a result of particle-particle and particle-wall collisions and the ambient shear in the flow field. Experimental and numerical studies of flow past a stationary sphere have been carried out by Taneda \cite{10}, and Johnson and Patel \cite{4}, among others. Studies of a streamwise and transversely rotating sphere at various Reynolds numbers have also been reported by Rubinnow and Keller \cite{9}, Kim and Choi \cite{5}, Kurose and Komori \cite{6}, and Giacobello \cite{3}. However, a particle traveling in free-space is usually subjected to forces in all directions. As a result, particles will rotate in all directions and the axis of rotation will not be perfectly aligned to the streamwise and transverse flow direction.

Numerical Formulation
The incompressible Navier-Stokes equations

\[
\frac{\partial \ddot{u}}{\partial t} + \ddot{u} \cdot \nabla \ddot{u} = -\nabla P + \frac{1}{Re} \nabla^2 \ddot{u} \quad (1)
\]

\[
\nabla \cdot \ddot{u} = 0 \quad (2)
\]

are solved directly for the primitive variables in a spherical polar coordinate system. The equations are spatially discretized using a Fourier-Chebyshev collocation method that employs a restricted double Fourier series in the wall-tangential directions and a Chebyshev discretization in the wall-normal direction. Time advancement is via a two-step fractional-step method that recovers a second-order accurate velocity and pressure field. The numerical scheme is based on that employed by Bagchi and Balachandar \cite{1}, and Mittal \cite{7} for the direct simulation of the flow past spheres and prolate spheroids, respectively.

\[
\Omega = \Omega (\cos \alpha \cos \theta + \sin \alpha \sin \theta \cos \phi) \hat{e}_r \\
+ (\sin \alpha \cos \theta \cos \phi - \cos \alpha \sin \theta) \hat{e}_\theta \\
- (\sin \alpha \sin \theta) \hat{e}_\phi \quad (3)
\]

Figure 1: Problem geometry and coordinate system. The freestream flow is along the $z$-axis and the sphere is constrained to rotate in between the $x$- and $z$-axis, where the rotation angle, $\alpha$, is measured from the positive $z$-axis.

The sphere is constrained to rotate at a constant angular velocity, $\Omega$, about an axis which is varied between the streamwise and transverse flow direction. The Reynolds number, $Re$, is defined based on the freestream velocity, $U_\infty$, the diameter of the sphere, $d$, and the kinematic viscosity, $v$. All length scales are therefore non-dimensionalized by the sphere diameter, $d$, all velocity scales by $U_\infty$ and all time scales by $d/U_\infty$. The other parameters important in this problem are the non-dimensional angular velocity of the rotating sphere, $\Omega^* = \Omega d/2U_\infty$, the unsteady vortex Strouhal number, $St_0 = f_s d/U_\infty$, the spinning sphere Strouhal number, $St_0 = f_s d/U_\infty$. Here $f_s$ is the unsteady vortex frequency (measured from the period between successive peaks in the lift force histories, $C_{z0}$), and $f_s$ is the rotational frequency of the sphere.

Figure 1 shows the spherical polar coordinate system and the angle of rotation of the sphere, $\alpha$. At the sphere surface, the no-slip and no-penetration boundary conditions are prescribed. For a rotating sphere, the surface velocity distribution is computed by taking the cross product of the angular velocity vector, $\Omega$, and the surface position vector, $\vec{r} = (d/2) \hat{e}_r$. For a sphere rotating in between the streamwise ($\alpha = 0$) and transverse flow direction ($\alpha = \pi/2$), at a constant angular speed, $\Omega$, the angular velocity vector is

\[
\ddot{u} = \ddot{u} (\cos \alpha \cos \theta + \sin \alpha \sin \theta \cos \phi) \hat{e}_r \\
+ (\sin \alpha \cos \theta \cos \phi - \cos \alpha \sin \theta) \hat{e}_\theta \\
- (\sin \alpha \sin \theta) \hat{e}_\phi \quad (3)
\]
Figure 2: Sphere rotating at $\Omega^* = 0.05$ for $Re = 100$.

therefore the velocity at the sphere surface is

$$\mathbf{u}|_{\partial \Gamma} = \Omega \times \hat{r}$$

$$= -\frac{\Omega d}{2} \left[ \sin \alpha \sin \phi \hat{e}_0 
- \left( \cos \alpha \sin \theta + \sin \alpha \cos \theta \cos \phi \right) \hat{e}_\theta \right]. \quad (4)$$

In this paper, simulations are performed at various angles, $\alpha$, for $\Omega^*$ up to 1.00. The Reynolds numbers $Re = 100$ and 250 are considered. Results are presented in terms of iso-surfaces of the wake structure and force coefficients. The wake structure is calculated using the $\lambda_2$ vortex identification method of Jeong and Hussain [3].

**Results and discussion**

Figures 2, 3, 4 and 5 show the vortical structures for the flow past a sphere at $Re = 100$ for different $\Omega^*$ and $\alpha$. A threshold value of $\lambda_2 = -8 \times 10^{-4}$ is presented. To indicate length scale, the wake cross-section is marked at increment of $\Delta \Omega^* = 1$, starting at $\zeta^* = 0$. Here $\zeta^*$ is the non-dimensionalized length in the streamwise direction. For all $\Omega^*$ and $\alpha$ considered, the wake structures are steady at $Re = 100$. At $\Omega^* = 0.05$, the wake structures are very weak and are not detected at the $\lambda_2$ threshold value presented. Despite this, the wake structures are qualitatively similar to those found at higher $\Omega^*$. Increasing $\Omega^*$ causes the streamwise threads in the wake to intensify and elongate. The effect of $\alpha$ is similar for all $\Omega^*$ presented, therefore only the cases at $Re = 100$ and $\Omega^* = 1.00$ are discussed in this paper.

Wake structures for $Re = 100$ and $\Omega^* = 1.00$ are presented in figure 5. For a sphere rotating in the streamwise direction ($\alpha = 0$), the wake structure consists of a single thread and is axysymmetric. For $\alpha = \pi/6$, two streamwise threads of different strength are formed. The thread with the same rotation direction as the sphere ($B$) is longer than the other one ($A$). The length of both threads increases at $\alpha = \pi/3$ with thread ($A$) doubling in length and the length of thread ($B$) increases marginally. At $\alpha = \pi/2$, two streamwise threads of opposite rotation and equal strength are formed, such that the wake is symmetric about the $y$-$z$ plane. The wake structures become increasing tilted towards the advancing side of the sphere as the rotation axis approaches $\alpha = \pi/2$.

Figure 6 shows the wake structures at $Re = 250$ and $\Omega^* = 0.05$. For streamwise rotation, two streamwise threads of different
strength are observed. This vortical structure is found to rotate about the streamwise axis at a constant angular velocity, with a period $T$. In figure 6(a) and subsequent figures which present an unsteady flow, the time instant when the lift force in $y$-direction, $C_{L_y}$, is at local minimum is shown. Figure 6(b) presents the evolution of the vortical structure at time intervals of $T/4$. Similar to the wake at $\alpha = \pi/2$, a double thread structure comprises threads of equal strength but opposite rotational sign is observed.

In figure 7, wake structures for $Re = 250$ and $\Omega^* = 0.20$ are presented. The wake remains “frozen” as it moves downstream. At $\alpha = \pi/6$, a double thread structure appears in the near wake region. The thread $(B)$ does not twist around the $z$-axis as for a streamwise rotating sphere. It oscillates slightly as it convects downstream. A steady solution is obtained for $\alpha = \pi/3$. The wake structure consists of two streamwise threads. The thread marked $(A)$ which rotates in a direction opposite to the sphere rotation direction has a slightly weaker strength than thread $(B)$. Thread $(A)$ increases in strength when $\alpha$ is increased to $\pi/2$. The threads are also tilted towards the advancing side of the sphere, as is also observed for $Re = 100$ and $\alpha = \pi/2$. 

The wake structures for $\Omega^* = 0.50$ and $Re = 250$ are shown in figure 8. At this higher rotation rate, the wake becomes unsteady under streamwise rotation. A single thread twists around the $z$-axis as it moves downstream. At $\alpha = \pi/6$, a double thread appears in the near wake region. The thread $(B)$ does not twist around the $z$-axis as for a streamwise rotating sphere. It oscillates slightly as it convects downstream. A steady solution is obtained for $\alpha = \pi/3$. The wake structure consists of two streamwise threads. The thread marked $(A)$ which rotates in a direction opposite to the sphere rotation direction has a slightly weaker strength than thread $(B)$. Thread $(A)$ increases in strength when $\alpha$ is increased to $\pi/2$. The threads are also tilted towards the advancing side of the sphere, as is also observed for $Re = 100$ and $\alpha = \pi/2$. 

Figure 6: Sphere rotating at $Re = 250$ and $\Omega^* = 0.05$. (a) Isosurfaces plot for all $\alpha$ considered. (b) Streamwise rotating sphere at $\Omega^* = 0.05$ for $Re = 250$. The evolution of the vortical structure is shown at interval of $T/4$, where $T$ is the period of rotation of the wake structure.

Figure 7: Sphere rotating at $\Omega^* = 0.20$ for $Re = 250$.

Figure 8: Sphere rotating at $\Omega^* = 0.50$ for $Re = 250$. 

and strength. The wake undergoes a transition to vortex shedding at $\alpha = \pi/3$. At this angle, a double thread structure is observed in the near wake region. The threads spiral around each other and form a bridge $(A)$. The wake at $\alpha = \pi/2$ consists of several $\Omega$-shape $(B)$ loops which increases in size as they convect downstream. 

872
α = 0
α = π
6
α = π
3
α = π
2
x
y
z
A
B
C
D
Induced thread
Figure 9: Sphere rotating at Ω∗ = 1.00 for Re = 250.

The wake structure for Ω∗ = 1.00 at Re = 250 is shown in figure 9. For α = 0, the wake twists around the z-axis forming helical wake structures. The threads start closely pack together in the near wake region. As the wake structures move downstream, the helical structures lose angular momentum and the spiral radius increases. Vortex threads are also induced in the wake as indicated in figure 9. The wake structure for α = π/6 is relatively complex. Three streamwise threads are observed in the near wake region. The threads spiral around the streamwise axis, and cross each other to form a hairpin wake structure (A). As they are convected downstream, the hairpin titles away from the streamwise direction and a Ω-shaped structure (B) is formed. A transition to a steady wake structure takes place between α = π/6 and π/3, as was also found for Ω∗ = 0.50. Streamwise threads of different sign and strength are observed α = π/3. The difference in strength for thread (C) and (D) are more pronounced at Ω∗ = 1.00. As α is increased to π/2, thread (C) increases in strength until it has the same strength as (D). The tilting of the threads also increases as Ω∗ increases.

In order to estimate the flight path of a particle, it is necessary to investigate the forces acting on it. Forces are non-dimensionalized and reported in terms of force coefficients for consistency. Table 1 compares the numerical results of Cp and St reported by Kim and Choi [5] for streamwise rotation and Niaznamin and Renksizbulut [8] for transverse rotation at Re = 250 and at different Ω∗. The results presented are in good agreement with these earlier studies except for a difference in St for sphere at transverse rotation. The present study indicates

Table 1: Comparison of Cp and St with earlier numerical studies for streamwise and transverse sphere rotation at Re = 250 and various Ω∗

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<th>α</th>
<th>Ω∗</th>
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<th>St</th>
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<td>0.712</td>
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<td>Niaznamin and Renksizbulut [8]</td>
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Figure 10: Average force coefficients versus Ω∗ for various α. (a) Cp. (b) CLx. (c) CLz. For Re = 100: — α = 0; — α = π/6; — α = π/3; — α = π/2. For Re = 250: — α = 0; — α = π/6; — α = π/3; — α = π/2.
a transition from a steady to unsteady flow between $\Omega^*$ = 0.05 and 0.20, and the solution becomes steady again in between $\Omega^*$ = 0.20 and 0.50. In contrast, the study carried out by Niazmand and Renkizibutul [8] reports unsteady solutions over the complete range of $\Omega^*$ = 0.05 to 0.50.

Figure 10 shows the time averaged drag coefficients and lift coefficients in the $x$- and $y$-directions. For all rotation angles and rotation rates considered, the average drag coefficient, $C_D$, increases as $\alpha$ increases as shown in figure 10(a). $C_D$ increases as both $\Omega^*$ and $\alpha$ increases. At low $\Omega^*$, the effect of $\alpha$ is less pronounced. The increment of $C_D$ is not linear across the range of $\alpha$ considered. $C_D$ increases slightly when $\alpha$ moves from 0 to $\pi/6$. A comparatively large increment for $C_D$ is observed when sphere is rotating at $\alpha = \pi/3$. The increment of $C_D$ is reduced as $\alpha$ approaches transverse flow direction $\alpha = \pi/2$.

Figure 10(b) presents $C_{Lx}$ versus $\Omega^*$. For a sphere rotating about the streamwise axis, no lift force is observed for $Re = 100$ due to the wake being axisymmetric. When $Re = 250$ and $\Omega^* \leq 0.20$, two streamwise threads of different strength and rotational sign are observed for rotation about streamwise axis. The shape and strength of the threads remain unchanged at all time (see figure 6(b)). This is confirmed by the time history plots of $C_{Lx}$ and $C_{Ly}$ presented in figure 11, which oscillate sinusoidally with a constant amplitude and period, $T$, but are out of phase by $T/4$. Therefore, the resultant lift force coefficient, $C_L$, remains constant at all time. At $\Omega^* \geq 0.50$, although the instantaneous wake structure is asymmetric, the strength of the wake structure remains constant and the direction of the resultant lift force rotates about the freestream axis. As a result, $C_{Lx} = 0$. At $\alpha = \pi/6$, $C_{Lx}$ is recorded to be negative. For both $Re = 100$ and 250, $C_{Lx}$ decreases with increasing $\Omega^*$ considered. For $\Omega^* = 0.20$, a higher $C_{Lx}$ is reported for sphere rotating at $\alpha = \pi/6$ than at $\alpha = \pi/3$. In between $\Omega^* = 0.20$ and 0.50, a crossover of $C_{Lx}$ for $\alpha = \pi/6$ and $\pi/3$ takes place, with $C_{Lx}$ becoming higher for $\alpha = \pi/3$ and $\Omega^* \geq 0.50$. The angle at which this crossover takes place is yet to be determined. When the sphere is rotating in transverse flow direction, the wake exhibits a planar-symmetry about the $y$-$z$ plane, and therefore experiences no lift force in the $x$-direction. The sphere only experiences a net lift force in $y$-direction. It is also observed that at a higher Reynolds number, the magnitude of $C_{Lx}$ is greater.

Figure 11: Lift coefficient for a streamwise spinning sphere at $Re = 250$ and $\Omega^* = 0.05$. The resultant lift force coefficient is $C_L = \sqrt{C_{Lx}^2 + C_{Ly}^2}$.

Figure 12: Unsteady Strouhal number $St$ versus $\Omega^*$ for various $\alpha$. For $Re = 100$, $\Omega^* = 0.00$; $\Delta$ $\Omega^* = 0.20$; $\circ$ $\Omega^* = 0.50$; $\ast$ $\Omega^* = 1.00$. For $Re = 250$, $\Omega^* = 0.00$; $\ast$ $\Omega^* = 0.20$; $\circ$ $\Omega^* = 0.50$; $\ast$ $\Omega^* = 1.00$.

Figure 10(c) shows that $C_{Lx}$ increases as both $\alpha$ and $\Omega^*$ increase. At $\alpha = \pi/6$, it is observed that $C_{Lx}$ is higher for $Re = 100$ for all $\Omega^*$ except $\Omega^* = 0.05$. At $\alpha = \pi/3$, $C_{Lx}$ for $Re = 100$ is lower than for $Re = 250$ when $\Omega^* \leq 0.20$. Similar behavior is also observed for sphere rotating at $\alpha = \pi/2$. $C_{Lx}$ tends to approach a constant value as $\Omega^*$ approaches 1.00 for all $\alpha$ considered.

For all simulations considered, the unsteady vortex Strouhal number, $St_v$, is plotted versus $\alpha$ in figure 12. $St_v = 0$ is obtained for all steady solutions. At $Re = 100$, the wake is steady for all $\alpha$ and $\Omega^*$ considered. At $Re = 250$ and $\Omega^* = 0.05$, the wake is steady for all $\alpha$ except $\alpha = 0$, where the wake is in a “frozen” regime. Similar behavior is observed for $\Omega^* = 0.20$ and $\alpha = 0$. In this “frozen” regime, an increase in $\Omega^*$ increases the unsteady vortex Strouhal number, $St_v$. This is in agreement with Kim and Choi [5] who found that, in general, the unsteady vortex shedding Strouhal number, $St_v$, is different from the spinning sphere Strouhal number, $St_s$. When $\alpha = \pi/6$ and $\Omega^* = 0.20$, the wake undergoes a transition to steady, and it becomes unsteady again at $\alpha = \pi/3$. With further increase in $\alpha$, a larger $St_v$ is obtained. For $\Omega^* > 0.20$, $St_v$ increases in the range of $\alpha = 0$ to $\pi/6$, and the flow becomes steady with a further increase in $\alpha$. Future work needs to be undertaken to determine the angle, $\alpha_{crit}$, at which the flow undergoes a transition from unsteady to steady.

Conclusion

Flow past a spinning sphere was simulated. At $Re = 100$, the wake structure loses its axisymmetric properties as the rotation axis of the sphere is oriented at $\alpha > 0$. The wake structure becomes planar-symmetric when rotating about the transverse flow direction. For $Re = 250$ and $\alpha = 0$, the vortical structure is “frozen” in the range $\Omega^* = 0.05$ to 0.20, and asymmetric at $\Omega^* \geq 0.50$. For $\alpha > 0$, the wake becomes asymmetric for all $\Omega^*$. For transverse rotation, the wake structures are planar-symmetric as in $Re = 100$. For both Reynolds numbers considered, $C_{Lx} = 0$ for streamwise rotation ($\alpha = 0$), $C_{Lx}$ decreases and $C_{Ly}$ increases as the sphere rotation axis is varied from the streamwise to the transverse direction. For transverse rotation,
the sphere only experiences a net lift force in y-direction. It is also observed that $C_D$ increases as $\alpha$ and $\Omega^*$ increase for both Reynolds numbers. $St_c$ also increases with increasing $\alpha$ for all unsteady solutions considered.

References


