

DSMC Calculations of Shock Structure with Various Viscosity Laws

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Abstract. It has long been known that: the thickness Δ of a plane 1D shock, expressed in terms of the mean free path in the upstream (pre-shock) flow, is a strong function of shock Mach number; and that the form of this function is sensitive to the form of the viscosity law $\mu = \mu(T)$ of the gas. On the other hand, the approximate kinetic theory method of Mott-Smith [1] shows that for many different assumed molecular models and viscosity laws, the average number of collisions suffered by a typical molecule as it traverses the shock quickly approaches a limit as the Mach number increases [2]. This suggests that a mean free path based on a collision cross-section characteristic of the high speed collisions between up and downstream molecules is the appropriate length scale for normalising shock thickness results. One such length scale is a kinetic length scale L^* defined for the sonic conditions of the flow [3], and another is the mean free path in the downstream flow λ_2 , where the average collision energy approaches the stagnation energy. Here we use DSMC results, for many different collision models, (and hence viscosity laws), to show that Δ/λ_2 is virtually constant for $M_1 \gtrsim 3$, with little variation shown between different forms of the viscosity law. We also show that Δ/L^* displays similar behaviour for $M_1 \gtrsim 10$. Experimental measurements of shock thickness in argon [4], up to downstream temperatures of $\sim 2,000$ K, for which the viscosity is known from experiments, have been used to estimate a limiting value of Δ/λ_2 for argon. Similarly, a limiting value of Δ/L^* has been estimated using results for which the sonic temperature is $\lesssim 2,000$ K. These limiting values are then used to deduce the viscosity of argon at temperatures far exceeding 2,000 K, for which there appears to be no reliable experimental data.

INTRODUCTION

The internal structure of one dimensional shocks has been studied extensively, both experimentally and theoretically, in order to determine the nature of intermolecular forces in gases. Argon, a monatomic gas for which there are no extra complications arising from energy exchange between rotational, vibrational and translational modes, is particularly useful for this purpose. Mott-Smith [1] found analytical solutions for shock structure based on an assumed bimodal velocity distribution within the shock. Gilbarg and Paolucci [5] solved the Navier-Stokes equations numerically to obtain shock thickness as a function of Mach number M_1 , for an assumed viscosity relation $\mu \propto T^\omega$, and showed how the results were sensitive to the value of ω . Although the Navier-Stokes solutions consistently underestimated the shock thickness compared to the Mott-Smith kinetic theory, the variation with M_1 was similar.

Bird [6, 7] used the direct simulation Monte Carlo (DSMC) method to show that the density profile obtained with an inverse 11th power repulsive potential, for which $\omega \approx 0.68$, gave good agreement with the experimentally measured profile obtained by Schmidt [8] for $M_1 = 8$ in argon. Macrossan [9] also showed generally good agreement with the same measured profile using a variable hard sphere collision model with $\omega = 0.68$. Alsmeyer [4] repeated and extended Schmidt's experiments and found better agreement with Bird's DSMC results [6] using an inverse 9th power repulsive potential, for which $\omega \approx 0.72$.

Muckenfuss [2] used the Mott-Smith method with various intermolecular potentials, and found that the average number of collisions suffered by a molecule as it traverses the shock thickness quickly approaches a limit for high M_1 . Macrossan [9] defined an approximate mean free path λ_{12} for collisions between upstream and downstream molecules, which depended on the viscosity law, and showed that DSMC simulations with various collision models gave a shock thickness $\Delta \approx 4\lambda_{12}$, with a standard deviation of 6%. Macrossan also showed that the experimental measurements of Schmidt and Alsmeyer could be approximated by $\Delta = 4\lambda_{12}$ to the same accuracy, provided it was assumed that argon displayed a power law viscosity with $\omega = 0.72$. The cross shock mean free path λ_{12} depends on the downstream number density n_2 and the average cross-section for high collision speed $g \approx u_1 - u_2$. It can be shown that it is

approximately the same as the nominal mean free path

$$\lambda_{\text{nom}} = 2\mu / (\rho \bar{c}) \quad (1)$$

based on the post shock conditions μ_2 , ρ_2 and $\bar{c}_2 = (8RT_2/\pi)^{1/2}$. Navier-Stokes calculations, following the method of Gilbarg and Paolucci [5], show that for different power law viscosity formulae with $0.5 \leq \omega \leq 1$, the shock thickness Δ is $3.61\lambda_{\text{nom},2} \pm 1.4\%$. A similar length scale, based on the viscosity characteristic of collisions occurring within the shock, is the sonic length scale L^* used by Robben and Talbot [3], defined by

$$L^* \equiv \mu^* / (\rho u), \quad (2)$$

where μ^* is the viscosity at the sonic temperature T^* . Boyd [10] used DSMC, with the Sutherland collision model of Kušćer [11], to calculate the structure of shocks in nitrogen and found that the ratio Δ/L^* approached a constant value at high M_1 .

Thus the DSMC, Mott-Smith and Navier-Stokes calculations all suggest that shock thickness for high M_1 approaches a constant multiple of a suitably defined length scale based on the energy of collisions occurring within the shock. In other words, the shock thickness is primarily a measure of the viscosity of the gas at the high temperatures within and downstream of the shock, rather than a measure of the exact form of the viscosity law $\mu = \mu(T)$. This general result has not been demonstrated with experimental data except for the low stagnation temperature experiments of Robben and Talbot [3], because the viscosity of gases has not been measured reliably for temperatures above $\sim 2,000$ K.

If the general result is true, it has two important consequences.

1. Firstly, for DSMC collision models that display a viscosity law that cannot match the viscosity of real gases at all temperatures, it emphasises the importance of selecting the parameters defining the collision model such that the viscosity of the model gas is matched to that of the real gas at a temperature characteristic of the flow under consideration.
2. Secondly, the measured shock thickness can be used to *deduce* the viscosity of the gas at the high temperatures within and downstream of the shock. This would provide a direct measure of the high temperature viscosity, and does not rely on knowing or assuming a particular form of the viscosity law.

It is the purpose of this paper to investigate, with DSMC simulations, the extent to which shock thickness is a measure of high temperature viscosity, independent of the particular details of the collision model used. A modification of DSMC, called v-DSMC [12], which can simulate arbitrary viscosity laws with reasonable accuracy, is also used. Finally, the experimental argon shock thickness results of Alsmeyer [4] are used to estimate limiting values of $\Delta/\lambda_{\text{nom},2}$ and Δ/L^* . The analysis is limited to those results where T_2 and T^* are $\lesssim 2,000$ K, for which the viscosity of argon is reliably known from experiments. These limiting values, together with Alsmeyer's results for shock thickness where T_2 and T^* are $\gtrsim 2,000$ K are then used to estimate the viscosity of argon for $T > 2,000$ K, for which there appears to be no other reliable experimental data.

DSMC MODELS AND VISCOSITY FORMULAE

The DSMC method, described in detail by Bird [13], can be used to simulate the internal structure of a normal shock. The variable hard sphere (VHS) molecular model is the most common model adopted in DSMC simulations. It has the same variation of cross-section with collision speed as an inverse power potential but the scattering is isotropic, as for hard spheres. The Chapman-Enskog viscosity approximation for both VHS and inverse power molecules is

$$\mu = \mu_{\text{ref}} (T/T_{\text{ref}})^{\omega},$$

where ω is a constant and μ_{ref} is the viscosity measured at the reference temperature T_{ref} . $\omega = 0.5$ and 1 correspond to hard sphere and 'Maxwell' VHS molecules respectively. The Sutherland viscosity formula gives accurate viscosity predictions over intermediate temperature ranges, and is given by

$$\mu = \mu_{\text{ref}} (T/T_{\text{ref}})^{3/2} (T_{\text{ref}} + T_s) / (T + T_s),$$

where T_s is the Sutherland temperature that is characteristic of each gas. A variable hard sphere molecular model that reproduces a Sutherland viscosity formula was introduced by Kušćer [11]. The total cross-section of this Sutherland

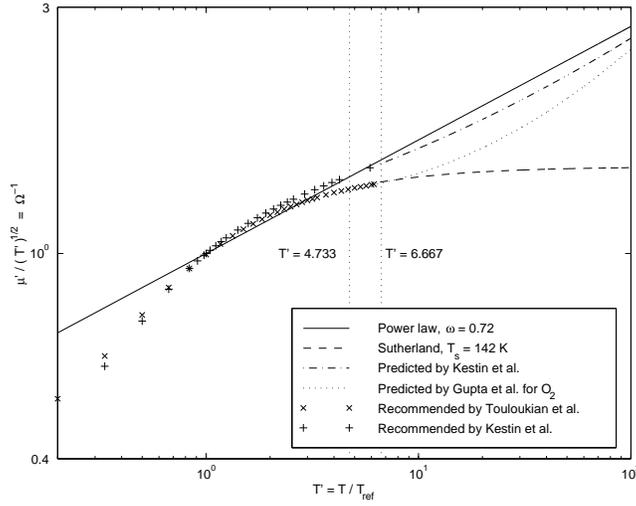


FIGURE 1. Reduced viscosity $\mu' / \sqrt{T'} = \Omega^{-1}$ versus reduced temperature $T' = T/T_{\text{ref}}$. From the values recommended by Kestin *et al.* [16], $\mu_{\text{ref}} = 2.283 \times 10^{-5}$ Pa.s at $T_{\text{ref}} = 300$ K. Both sets of recommended values [14, 16] are reported to be within 2% of the experimental data sets upon which they are based. Lines for $T' = 4.733$ and $T' = 6.667$, which correspond to $T = 10T_s$ and $T = 2,000$ K respectively, are shown.

variable hard sphere (SVHS) model is given by $\sigma = \sigma_s(1 + g_s^2/g^2)$, where σ_s is a reference cross-section, g is the collision speed and $g_s^2 = 12kT_s/m = 12RT_s$, where m is the molecular mass.

The viscosity of a hard sphere gas at some reference temperature T_{ref} is

$$\mu_{\text{ref}}(T_{\text{ref}}) = (5m/16) (\pi RT_{\text{ref}})^{\frac{1}{2}} \cdot (\pi d_{\text{ref}}^2)^{-1},$$

where d_{ref} is the hard sphere diameter evaluated at T_{ref} . It is convenient to express the viscosity of a gas in non-dimensional form as a reduced viscosity μ' , given by

$$\mu'(T') = \mu(T) / \mu_{\text{ref}}(T_{\text{ref}}) = \sqrt{T'} \cdot \Omega(T')^{-1}, \quad (3)$$

where $T' = T/T_{\text{ref}}$ is a reduced temperature and $\Omega(T')$ measures the departure of the viscosity law from the hard sphere law $\mu \propto T^{\frac{1}{2}}$. Eq. 3 can be rearranged to get $\mu' / \sqrt{T'} = \Omega^{-1}$, which has been used in Fig. 1 to plot viscosity data for Ar.

Touloukian *et al.* [14] have recommended Ar viscosity values that are within 2% of 30 sets of experimental data in the range $58 \text{ K} \leq T \leq 1,868 \text{ K}$. For $T \gtrsim 100 \text{ K}$, excellent agreement with their recommended values is achieved using a Sutherland viscosity formula with $T_s = 142 \text{ K}$ and $\mu_{\text{ref}} = 2.272 \times 10^{-5} \text{ Pa.s}$ at $T_{\text{ref}} = 300 \text{ K}$. Fig. 1 shows some of the recommended values for $T \leq 1,868 \text{ K}$, and the fitted Sutherland law for $T > 2,000 \text{ K}$. The CRC Handbook of Chemistry and Physics [15] recommends the use of Ar viscosity values given by Kestin *et al.* [16] and Younglove and Hanley [17]. Kestin *et al.* give equations for viscosity that agree within 2% of 12 sets of experimental data for Ar over the range $100 \text{ K} \lesssim T \lesssim 2,000 \text{ K}$. The recommended values up to the temperature of the experiments upon which they are based are shown in Fig. 1, as well as the fitted equation for $T > 2,000 \text{ K}$. At high temperatures, there is some discrepancy between the two sets of recommended values [14, 16]. The reason for the discrepancy is not clear but the two sets of recommended values are based on different experimental data sets. The recommended values of Kestin *et al.* agree closely with those of Younglove and Hanley, and are probably more accurate than the earlier recommendations of Touloukian *et al.* Fig. 1 also shows a power law viscosity $\mu = \mu_{\text{ref}}(T/T_{\text{ref}})^{0.72}$. It is clear that this viscosity law does not match the Ar viscosity data for $T < T_{\text{ref}}$, but is in reasonable agreement with the data over the range $1 \lesssim T' \lesssim 7$.

TABLE 1. Summary of symbols that appear in Figs. 2, 3 and 4.

Model	Symbol	Model	Symbol
DSMC, hard spheres	○⋯○	v-DSMC, hard spheres	○---○
DSMC, VHS, $\omega = 0.72$	□⋯□	v-DSMC, power law, $\omega = 0.72$	□---□
DSMC, VHS, $\omega = 0.81$	+⋯+	v-DSMC, power law, $\omega = 0.81$	+---+
DSMC, VHS, $\omega = 0.9$	◇⋯◇	v-DSMC, power law, $\omega = 0.9$	◇---◇
DSMC, VHS, $\omega = 1$	×⋯×	v-DSMC, power law, $\omega = 1$	×---×
DSMC, SVHS	△⋯△	v-DSMC, Sutherland	△---△
Mean experimental results of Alsmeyer [4]	—	v-DSMC, hybrid	▽---▽
DSMC results of Erwin <i>et al.</i> [19], VHS, $\omega = 0.72$	▷		

The v-DSMC method and a hybrid viscosity formula

A variant of the DSMC method, recently proposed by Macrossan [12], can simulate, with reasonable accuracy, any specified viscosity law $\mu = \mu(T)$. In v-DSMC, the collision rate in any computational cell is set to the theoretical collision rate required to achieve the specified viscosity, derived from the kinetic temperature in the cell. v-DSMC was used to test a hybrid viscosity law which followed the Sutherland formula up to a temperature of $10T_s$, and thereafter followed the recommended curve fits given by Gupta *et al.* [18] for high temperature viscosity, which are based on theoretical predictions derived from an exponential repulsive potential. The curve fit for O_2 can be converted into a form suitable for Ar by considering the different Sutherland constants σ_s and T_s for O_2 and Ar. For this hybrid viscosity law, with $T_{\text{ref}} = 300$ K,

$$\Omega = \begin{cases} (T' + T'_s) / (1 + T'_s) & \text{for } T' \leq 4.733, \\ a(293.6T')^{b \ln T' + c} & \text{for } T' > 4.733, \end{cases}$$

where $a = 0.088$, $b = -0.049$ and $c = 0.371$ are constants and $T'_s = T_s/T_{\text{ref}} = 142/300$. The resulting viscosity relation for $T' > 4.733$ is shown in Fig. 1.

SIMULATION METHOD, RESULTS AND DISCUSSION

All simulations were performed using the normal shock simulation code DSMC1S, supplied by Bird [13], with modifications to incorporate the SVHS model and v-DSMC routines, and were executed on the University of Queensland SGI Origin 3000 computer. Each simulation used 300 cells, with six subcells per cell. Final sample sizes were typically $\sim 10^7$ particles per cell. DSMC simulations were performed using the VHS model with $\omega = 0.5, 0.72, 0.81, 0.9$ and 1, and the SVHS model. v-DSMC simulations were performed using the power law viscosity formula with the same ω values, and the Sutherland and hybrid viscosity formulae. For each model, simulations were performed at integral Mach numbers in the range $2 \leq M_1 \leq 10$. The upstream temperature T_1 was 300 K in all simulations.

Shock thickness results

The absolute shock thickness Δ is defined as the distance spanned by the maximum gradient of the density profile between the upstream and downstream densities. Typically, Δ is normalised with respect to some kinetic length scale. The reciprocal normalised shock thickness is usually presented, and is given here by $\delta_1 \equiv \lambda_{\text{nom},1}/\Delta$, where $\lambda_{\text{nom},1}$ is the nominal upstream mean free path from Eq. 1. Subscripts 1 and 2 refer to upstream and downstream conditions respectively. The simulation results, plotted as δ_1 versus M_1 , are presented in Fig. 2. For clarity, the symbols used in Figs. 2, 3 and 4 are presented in Table 1. The mean experimental results of Alsmeyer [4], and the simulation results of Erwin *et al.* [19] for VHS molecules with $\omega = 0.72$, are also shown in Fig. 2. The conventional DSMC simulation results with $\omega = 0.72$ are in reasonable agreement with the results of Alsmeyer for $M_1 \gtrsim 6$. At low M_1 , the DSMC results predict a higher value of δ_1 than given by experiment. This result is consistent with the results of Erwin *et al.*, and is probably due to the fact that viscosity for the VHS model with $\omega = 0.72$ for $1 \lesssim T' \lesssim 4$ is too low, as illustrated in Fig. 1. Using the more realistic Maitland-Smith potential [20], Erwin *et al.* found values of δ_1 closer to the experimental values at low M_1 . For low collision energies, the attractive intermolecular forces, which are ignored

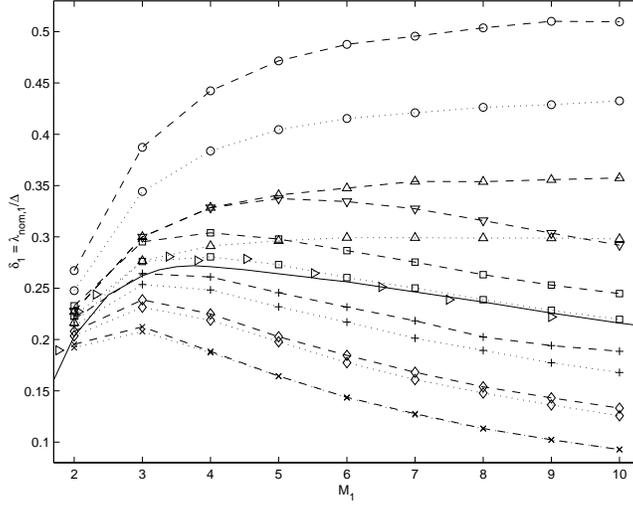


FIGURE 2. Normalised reciprocal shock thickness $\delta_1 \equiv \lambda_{\text{nom},1}/\Delta$ versus Mach number M_1 . Symbols are as given in Table 1.

in the VHS model, can be expected to have a significant influence, and explains the better agreement at low M_1 for the Maitland-Smith potential.

For $\omega = 1$, the ν -DSMC results are very close to those for DSMC. This result was expected as conventional DSMC and ν -DSMC are identical techniques when considering $\omega = 1$. For the other ω values and the Sutherland results, it is clear that both methods give variations of δ_1 with M_1 that are of the same form. Given this result, it may be assumed that ν -DSMC using the hybrid viscosity law will give a variation of shock thickness with M_1 that is of the same form as that which could be obtained using DSMC if a molecular model existed that could reproduce the hybrid law.

Shock thickness results normalised with respect to $\lambda_{\text{nom},2}$ and L^*

The shock thickness results can be normalised using the downstream nominal mean free path $\lambda_{\text{nom},2}$. A reciprocal normalised shock thickness δ_2 may be defined where

$$\delta_2 \equiv \lambda_{\text{nom},2}/\Delta = \delta_1 (\mu_2/\mu_1) (\rho_1/\rho_2) (T_1/T_2)^{\frac{1}{2}}. \quad (4)$$

Fig. 3 gives a plot of δ_2 versus M_1 for all of the conventional DSMC results and the ν -DSMC results for the hybrid formula. For all models examined, except the hard sphere and Sutherland models, δ_2 is approximately constant for $M_1 \gtrsim 3$. There is a systematic increase in δ_2 with increasing ω , which has also been observed in the Navier-Stokes solutions obtained using the method of Gilbarg and Paolucci [5].

The sonic length scale L^* of Robben and Talbot [3], from Eq. 2, may also be used to normalise Δ . The sonic temperature T^* , at which the sonic viscosity μ^* is evaluated, is given by $T^* = 2T_0/(\gamma + 1)$, where T_0 is the stagnation temperature and γ is the ratio of specific heats. For a monatomic gas, it can be shown that $L^* \approx 0.618\lambda_{\text{nom}}^*$, where λ_{nom}^* is the nominal mean free path evaluated at the sonic conditions. Therefore, L^* is simply a measure of the mean free path within the shock. Using $u_1 = M_1(\gamma RT_1)^{\frac{1}{2}}$, a reciprocal shock thickness normalised with respect to L^* , denoted δ^* , may be defined by

$$\delta^* \equiv L^*/\Delta = \delta_1 (\mu^*/\mu_1) [2/(\gamma\pi)]^{\frac{1}{2}}/M_1. \quad (5)$$

Robben and Talbot presented shock thickness data in terms of δ^* . The current shock thickness data, presented as δ^* versus M_1 is given in Fig. 4. These results suggest that δ^* for a given molecular model also converges to a constant value. Again, there is a systematic increase in δ^* with ω . Boyd [10] performed DSMC simulations of shock structure in nitrogen using the VHS model with $\omega = 0.72$ and the SVHS model, and obtained curves for δ^* versus M_1 that were of a form similar to those shown in Fig. 4.

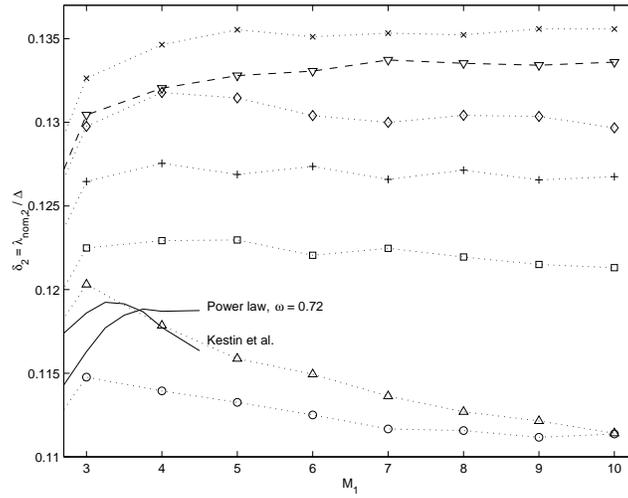


FIGURE 3. Normalised reciprocal shock thickness $\delta_2 \equiv \lambda_{\text{nom},2}/\Delta$ versus Mach number M_1 . Symbols are as given in Table 1.

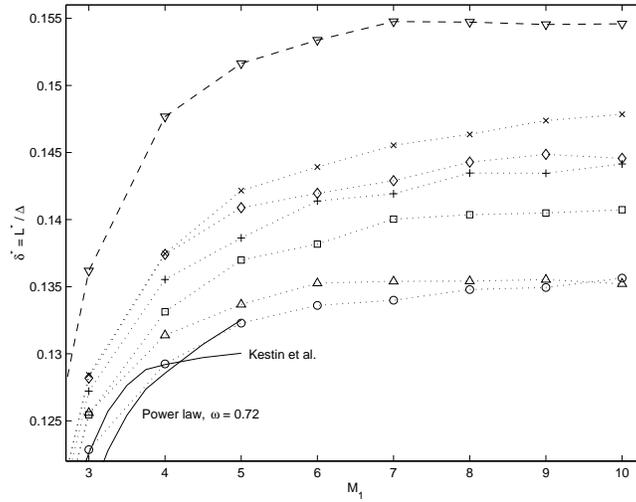


FIGURE 4. Normalised reciprocal shock thickness $\delta^* \equiv L^*/\Delta$ versus Mach number M_1 . Symbols are as given in Table 1.

It is important to note that the ν -DSMC results for the hybrid method have been included in this analysis simply to demonstrate that δ_2 and δ^* both approach constant values at high M_1 , irrespective of the viscosity relation employed. It should also be noted that the ν -DSMC results using the power law and Sutherland formulae gave curves of δ_2 and δ^* versus M_1 of the same form as those obtained using the corresponding conventional DSMC models.

To check the observation that δ_2 and δ^* approach constant values at high M_1 , several simulations were performed at $M_1 \gg 10$. These results are listed in Table 2, and demonstrate that beyond $M_1 \sim 10$, δ^* is approximately constant for each model.

TABLE 2. Summary of results for simulations performed at high M_1 .

Model	M_1	δ_2	δ^*	Model	M_1	δ_2	δ^*
DSMC, hard spheres	10	0.111	0.136	DSMC, SVHS	10	0.111	0.135
	20	0.111	0.136		20	0.111	0.136
	50	0.111	0.137				
DSMC, VHS, $\omega = 0.72$	10	0.121	0.141	v-DSMC, hybrid	10	0.134	0.155
	20	0.122	0.143		20	0.135	0.153

USING SHOCK THICKNESS TO ESTIMATE HIGH TEMPERATURE VISCOSITY

The preceding discussion has demonstrated that for any molecular model, $\delta_2 = \lambda_{\text{nom},2}/\Delta$ tends towards a constant value, which may be denoted A_2 , at high M_1 . With an experimentally determined value of δ_1 , this suggests that A_2 can be used to obtain an estimate of the viscosity at the downstream temperature, because $\lambda_{\text{nom}} = \lambda_{\text{nom}}(\mu, \rho, T)$. Using Eqs. 1 and 4, the expression

$$\mu_2 \approx A_2 \rho_2 (2RT_2/\pi)^{\frac{1}{2}} \Delta = \mu_1 (A_2/\delta_1) (\rho_2/\rho_1) (T_2/T_1)^{\frac{1}{2}} \quad (6)$$

is obtained. Alternatively, if δ^* approaches a constant value A^* at high M_1 , $A^* \approx L^*/\Delta$. Therefore, using Eqs. 2 and 5,

$$\mu^* \approx A^* (\rho u) \Delta = \mu_1 M_1 (A^*/\delta_1) (\gamma\pi/2)^{\frac{1}{2}}. \quad (7)$$

If this hypothesis is correct, it is reasonable to assume that experimentally determined shock thicknesses can give an estimate of the viscosity at T_2 and T^* , which often far exceed the limit of existing experimental data at $T \sim 2,000$ K.

Experimental results are required to determine A_2 and A^* for a real gas. The mean experimental results of Alsmeyer [4] are plotted in the form of δ_2 and δ^* versus M_1 in Figs. 3 and 4 respectively. The values of δ_2 and δ^* depend on the viscosity formula used, and a power law formula with $\omega = 0.72$ and the equations of Kestin *et al.* [16] have been used. It has been assumed that Alsmeyer performed all experiments at $T_1 = 300$ K. For δ_2 , the analysis has been limited to $M_1 = 4.5$ where $T_2 \sim 2,000$ K. Similarly, the analysis for δ^* has been limited to $M_1 = 5$. The results presented in Fig. 3 indicate that $A_2 \approx 0.119$ is appropriate, as this is the maximum of the curve derived using the predictions of Kestin *et al.* If the hypothesis that δ_2 approaches a constant value for high M_1 is correct, the decrease in δ_2 beyond $M_1 \approx 3.2$ as shown in Fig. 3 must be a result of either incorrect viscosity predictions by Kestin *et al.*, or error in the experimental results of Alsmeyer.

It is more difficult to select an appropriate value for δ^* . At $M_1 = 5$, $\delta^* = 0.130$ based on the viscosity predicted by Kestin *et al.* The results for other viscosity formulae indicate that δ^* increases by $\sim 3\%$ between $M_1 = 5$ and 10, so $A^* \approx 0.134$ has been selected.

These values of A_2 and A^* were used in Eqs. 6 and 7 to estimate viscosities for Ar, using the δ_1 values obtained from the mean results of Alsmeyer. The results are presented in Fig. 5, and indicate that the high temperature viscosity values estimated from the shock thickness results are approximately the mean of the predictions of the power law formula with $\omega = 0.72$, and those of Kestin *et al.* This explains why DSMC simulations using the VHS model with $\omega = 0.72$ closely reproduce the experimental results of Alsmeyer at high M_1 . For Ar, this model is appropriate for DSMC simulation of flows that are dominated by high collision energies. It is difficult to determine error bounds for the predicted viscosities, because A_2 and A^* were extrapolated from results obtained at relatively low M_1 .

The experimental results of Robben and Talbot [3] indicate that $A^* \approx 0.15$ at $M_1 = 17.4$ for Ar. This is in contrast to the result $A^* \approx 0.134$ estimated here. Robben and Talbot performed shock thickness measurements in a free expansion, and the freestream temperatures were therefore very low. It is possible that the discrepancy between the value of A^* determined here and that of Robben and Talbot may be due to some temperature dependence of A^* .

CONCLUSIONS

For various viscosity formulae, it has been demonstrated using DSMC simulations that shock thickness approaches a constant multiple of the downstream mean free path for $M_1 \gtrsim 3$. The shock thickness also approaches a constant multiple of the sonic length scale defined by Robben and Talbot [3] for $M_1 \gtrsim 10$. Using the experimental argon shock

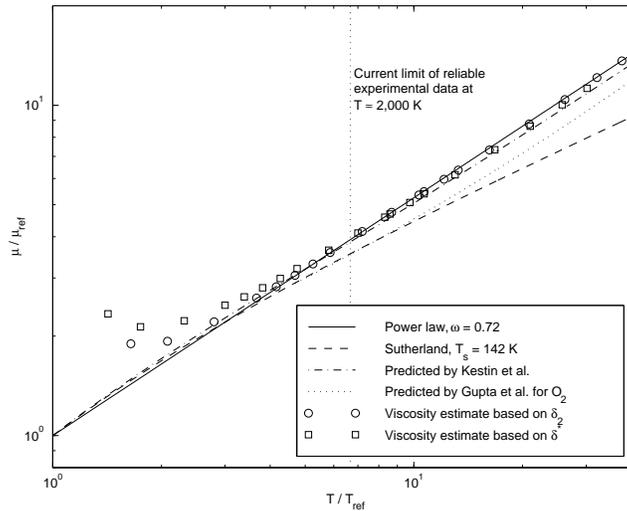


FIGURE 5. Estimates of high temperature viscosity using $A_2 = 0.119$ and $A^* = 0.134$, compared with various viscosity formulae.

thickness results of Alsmeyer [4], approximate values of the constants have been determined, and then used to estimate the viscosity of argon at temperatures that exceed the current limit of reliable experimental data at $T \sim 2,000$ K. It was found that the viscosities predicted at high temperatures were close to both those predicted using the power law viscosity formula with $\omega = 0.72$, and the predictions of Kestin *et al.* [16]. This result explains why the VHS model with $\omega = 0.72$ successfully predicts the structure of strong shocks in argon.

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