Application Of Linear Fracture Mechanics To Timber Engineering

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ABSTRACT: This paper describes the application of linear elastic fracture mechanics to the design of structural timber members. These include non-zero angle notches, glued lap joints and the butt joints of laminated timber members. The results of this research were used in the development of design rules given in AS 1720, the Australian Timber Engineering Design Code.

1 INTRODUCTION

The theory of fracture mechanics is often applied to the study of material properties and very small or "micro" cracks. Frequently the cracks are invisible or may even be intrinsic to the material microstructure as in the case of cracks related to metal grain size or concrete aggregate. However, in the drafting of timber engineering design codes, it has been found that linear elastic fracture mechanics can form a very useful basis for predicting the load carrying capacity of full size structural elements. The following, will revisit some examples that were used in drafting the Australian Standard AS 1720: SAA Timber Engineering Code (Standards Association of Australia, 1975), a particularly early example of the application of fracture mechanics to an engineering code of practice.

2 EIGEN EQUATIONS

The stresses $\{\sigma_r, \sigma_\theta, \sigma_{r\theta}\}$ in the vicinity of a notch root such as that shown in Figure 1 can be written in terms of the polar coordinates (r, θ) in the following form (Leicester 1971)

$$\{\sigma_r, \sigma_\theta, \sigma_{r\theta}\} = \frac{K_A}{(2\pi r)^s}\{f_1(\theta), f_2(\theta), f_3(\theta)\} \quad (1)$$

and

$$\{\sigma_r, \sigma_\theta, \sigma_{r\theta}\} = \frac{K_B}{(2\pi r)^q}\{f_4(\theta), f_5(\theta), f_6(\theta)\} \quad (2)$$

where $f_1(\theta), f_2(\theta)$, …. are functions of θ only, s and q are constants that will be termed "size factors" and K_A and K_B are stress intensity factors defined by

$$\sigma_\theta\big|_{\theta=\pi} = K_A/(2\pi r)^s \quad (3)$$

$$\sigma_{r\theta}\big|_{\theta=\pi} = K_B/(2\pi r)^q \quad (4)$$

where $q \le s$.

The notation shown in Figure 1 will be used throughout this paper. It is assumed that the grain of the wood lies along the line $\theta = 0°$. and that the notch has free edges along the lines $\theta = 0$ and $\theta = 2\pi - \beta$, where β denotes the notch angle. Equations for other orientations of the notch have also been derived (Leicester, 1969).

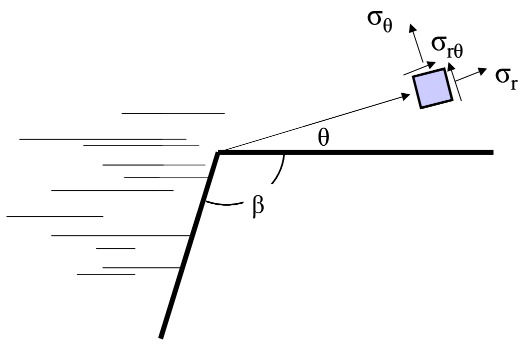


Figure 1. Notation used at a notch root with an included angle, β

Both equations (3) and (4) describe stress singularities at $r = 0$. It was found that the singularity described by equation (3) exists for all notch angles $0 \leq \beta < \pi$ and that the singularity of equation (4) exists for a notch angle $0 \leq \beta < \alpha$, where α is roughly about 0.5π.

3 FAILURE CRITERIA

For the condition $q < s$, the stress singularity described by equation (3) will always give rise to larger stresses near the notch root than those of equation (4) and hence the fracture criterion can be written

$$K_A = K_{AC} \tag{5}$$

where K_{AC} is termed the critical stress intensity factor.

The condition $q = s$ corresponds to the case where the notch degenerates to a sharp crack and the two stress fields are equally important.

For this case the notation K_I and K_{II} will by used for K_A and K_B and the failure criterion for the cases of pure tension and pure shear respectively will be

$$K_I = K_{IC} \tag{6}$$

$$K_{II} = K_{II\,C} \tag{7}$$

where K_{IC} and K_{IIC} denote critical stress intensity factors.

4 THE SIZE FACTOR

An important aspect of the eigen equations (1) and (2) is that they imply a significant size factor in strength predictions. Using dimensional analysis, it can be shown that the value of K_A can be written in the form

$$K_A = AL^s\,\sigma_{nom} \tag{8}$$

where L is a characteristic dimension of the member, σ_{nom} is the nominal applied stress and A is a constant that is proportional to the applied load.

Using the failure criterion of equation (5), the nominal stress at fracture for two members of different sizes (but having the same geometrical shapes) are related by

$$(\sigma_{ult,1}) / (\sigma_{ult,2}) = (L_2/L_1)^s \tag{9}$$

where the subscripts 1 and 2 refer to the two geometrically similar members, $\sigma_{ult,1}$ and $\sigma_{ult,2}$ denote the values of σ_{nom} at failure and L_1 and L_2 are the characteristic dimensions. The concept of a size factor is illustrated schematically in Figure 2. Here it is seen that the linear elastic fracture equations apply only if the characteristic dimensions of a member are larger than a critical value denoted by L_{crit}.

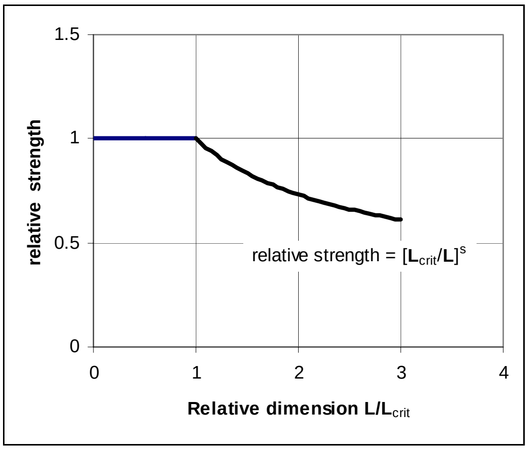


Figure 2. Illustration of the concept of the size factor.

Some typical examples of nominal stress values used by engineers are illustrated in Figure 3. These examples correspond to design cases that were covered by the Australian Timber Engineering Design Code AS 1720 (Standards Association of Australia 1975). For the members loaded in tension as shown in Figures 3a and 3b, the nominal tension and shear stresses, denoted by $\sigma_{t,nom}$ and $\sigma_{s,nom}$ respectively, are taken to be

$$\sigma_{t,nom} = T/2bd \tag{10}$$

$$\sigma_{s,nom} = V/2bd \tag{11}$$

where b denotes the thickness of the element.

For the beam shown in Figure 3c, the nominal stress in bending tension and beam shear, denoted by $\sigma_{b,nom}$ and $\sigma_{v,nom}$ respectively, are taken to be

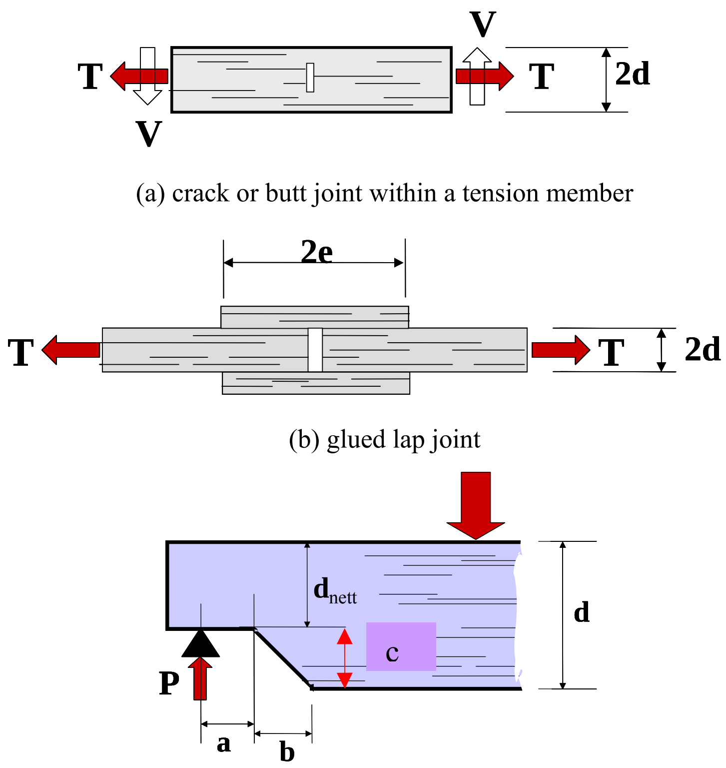
$$\sigma_{b,nom} = Pa/(bd_{nett}^2/6) \tag{12}$$

$$\sigma_{v,nom} = 1.5P/(bd_{nett}) \tag{13}$$

where again b denotes the thickness of the element.

Figure 4 shows the measured effect of size on the bending strength of notched messmate (*Eucalyptus obliqua*) timber beams. Figure 5 shows the predicted size parameters s and q for solid timber (Leicester 1969).The size factor derived from the slopes of the data in Figure 4 are plotted in Figure 5, and are seen to follow the predicted graph of the parameter s.

Since the strength of small clear pieces of messmate is about 180 MPa, the critical size for the notched beams corresponds roughly to d = 2.5 mm, i.e. linear fracture mechanics may be applied to beams with $d > 2.5$ mm. It is of interest to note that for plain concrete, the corresponding critical size would be about $d > 90$ mm (Walsh, 1972), so that for failure in tension perpendicular to the grain, wood may be considered to be considerably more brittle than plain concrete.



(a) crack or butt joint within a tension member

(b) glued lap joint

(c) notched beam

Figure 3. Examples of notches considered in AS 1720.

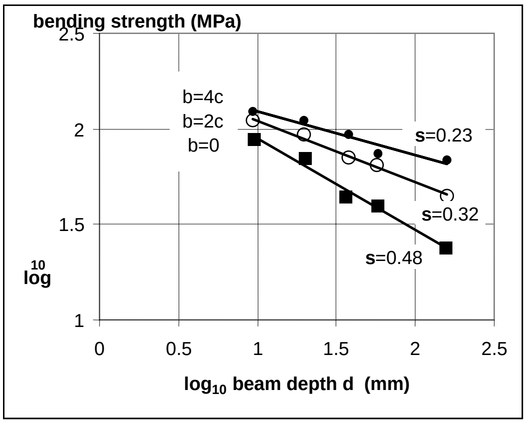


Figure 4. Measured bending strength of messmate (*E. obliqua*) for beams noted as shown in Figure 3c.

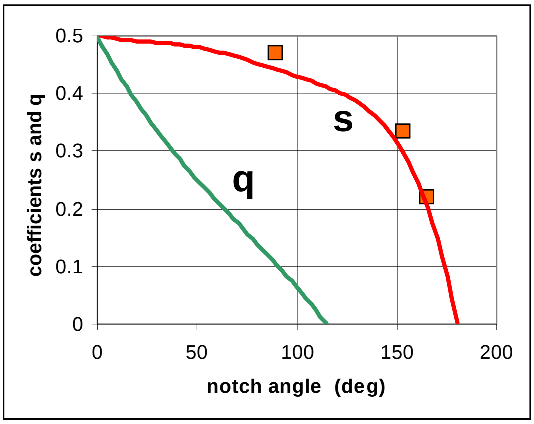
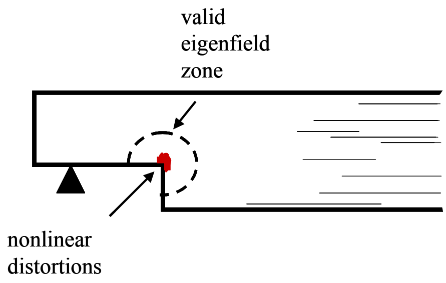


Figure 5. The predicted size coefficients for notches in timber beams; the plotted data points are taken from the slopes of the graphs shown in Figure 4.

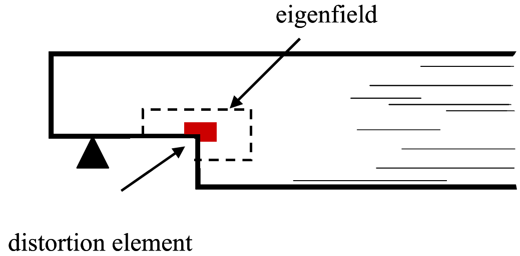
5 COMPUTATIONAL PROCEDURES

Computational procedures to evaluate stress intensity factors include the use of eigen function sets (Williams, 1952), and integral methods that make use of sets of point load solutions (Brahtz, 1933). However, in terms of current technology, the simplest procedure is to make use of conventional finite element packages (Leicester and Walsh, 1982).

The finite element procedure is based on imitating the features of a real structural member. For a real structural member, as illustrated in Figure 6a, the nonlinear distortions at fracture are limited to a small zone around the notch root, well within the theoretical eigen field if $L > L_{crit}$. In a similar way, the theoretical analysis of the structural member may be undertaken by using a small distortion element at the notch root. This element is first calibrated by noting the distortions when it is placed at a notch root within a unit eigen field. Then the distortions of the element when placed at the notch root of a structurally loaded member may be used to calculate the stress intensity factor at the notch root.



(a) Conditions in a structural member just prior to fracture



(b) Eigen field and distortion element used for computing stress intensity factors

Figure 6. Comparison between the characteristics of real structural members and the computational procedure used for evaluating stress intensity factors.

6 COMPUTED STRESS INTENSITY FACTORS

Using the computational procedure outlined in the previous section, stress intensity factors for a variety of configurations involving sharp cracks and right angle re-enterant notches have been computed by Walsh (Walsh 1972, 1974, Walsh *et al*. 1973).

For example, for notched beams with a notch depth d_{nett}/d in the range of 0.3–0.7, the stress intensity factor is given by

$$K_A = d^{\,0.45}[0.05\ \sigma_{b,nom} + 0.25\ \sigma_{v,nom}] \qquad (14)$$

and for the glued lap-joint,

$$K_A = \sigma_{t,nom}\ d^{\,0.45}[0.06 + 0.3\ (d/e)] \qquad (15)$$

7 FRACTURE STRENGTH OF TIMBER

Measured values of the fracture strength of timber cracks and notches have been given in various reports (Leicester and Bunker 1969, Schienwind and Centeno 1971, Walsh *et al*. 1973, Leicester 1974, Wu 1977, Leicester and Poynter 1980, Barrett 1981).

For the case of sawn cracks orientated perpendicular to the grain of the wood as shown in Figure 3a, the values of K_{IC} and K_{IIC}, *stated in N-mm*$^{-1.5}$ *units,* are approximately given by

$$K_{IC} = 0.15\ \rho$$

$$K_{IIC} = 0.03\ \rho$$

where ρ denotes the density of the timber at a moisture content of 12%, stated in kg/m^3 units.

For cracks formed by the butt joints of laminated timber, the values are about 20% higher.

For combined modes of stress the interaction equation is

$$(K_I/K_{IC}) + (K_{II}/K_{IIC})^2 = 1 \qquad (16)$$

If there are N butt joints located within a tension member, then failure will occur at the weakest joint and so the stress at failure $\sigma_{t,nom}$ needs to be reduced by a factor of about $N^{0.1}$ to account for the variability between the strength of the various joints.

For a right angle notch formed by a saw cut, the critical stress intensity factor is approximately equal to

$$K_{AC} = 0.015\ \rho \qquad (17)$$

where K_{AC} is the critical stress intensity factor stated in *N-mm*$^{-1.55}$ *units, and* ρ denotes the density of the timber at a moisture content of 12%, stated in kg/m^3 units

Again, the value of K_{AC} is about 20% higher if the notch is formed by a glued joint as occurs in the case of the lap-joint shown in Figure 3b.

8 APPLICATIONS

The research described herein was used to develop design rules for the Australian Standard AS 1720 (Standards Association of Australia 1975, Leicester 1974, 1979). These apply to the design of structural members such as those shown in Figure 3.

Another application would be to investigate the effect of moisture induced stresses on the fracture strength of notched beams (Jensen and Hoffmeyer 1996) and also the cracking of exposed timber, an important aspect of durability assessments.

An interesting application of fracture mechanics occurred in the construction of a large span laminated timber structure for the 2000 Sydney Olympics. It was noted that within the completed structure there would be something like 5,000 finger joints located within the tension members. If a finger joint has been poorly fabricated, it behaves as a butt joint and for the lamination thickness used, 50% of the strength of the tension member is lost. Since the glulam fabricators were unable to demonstrate an ability to produce 5000 finger joints with a negligible chance of a single poor joint, and the structural design engineers believed that the failure of a single member would be catastrophic, the only solution available was to require the proof testing of every fabricated finger joint, an extremely costly requirement.

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