

**COMPARING INCOME DISTRIBUTIONS BETWEEN  
ECONOMIES THAT REWARD INNOVATION AND THOSE  
THAT REWARD KNOWLEDGE**

**Rodney Beard & Kam-Ki Tang**  
School of Economics  
The University of Queensland  
Brisbane Qld 4072  
Australia  
fax: 61 7 3365 7299

August 2002

Discussion Paper No 314

ISSN 1446-5523

© Beard & Tang, 2002

This discussion paper should not be quoted or reproduced in whole or in part without the written consent of the author.

# COMPARING INCOME DISTRIBUTIONS BETWEEN ECONOMIES THAT REWARD INNOVATION AND THOSE THAT REWARD KNOWLEDGE

**Rodney Beard  
&  
Kam-Ki Tang**

## **Abstract:**

In this paper, we develop an optimal control model of labor allocation in two types of economy – one economy is for innovative workers and the other one for knowledge workers. In both economies, workers allocate time between learning and discovering new knowledge. Both markets consist of a continuum of heterogeneous agents that are distinguished by their learning ability. Workers are rewarded for the knowledge they possess in the knowledge economy, and only for the new knowledge they create in the innovative economy. We show that, at steady state, while human capital accumulation is higher in the knowledge economy, the rate of knowledge creation is not necessarily higher in the innovative economy. Secondly, we prove that when the cost of learning is sufficiently high, the distribution of net wage income in the knowledge economy dominates that in the innovative economy in the first degree.

## INTRODUCTION

Considering the following dilemma of career choice faced by a PhD graduate. The graduate is offered jobs in a research-oriented university and in a teaching-oriented university, respectively. In the research university, her salary is linked to her research output. She can allocate time between knowledge learning (e.g. reading the literature) and knowledge creation (e.g. writing research papers). Learning will broaden her knowledge base and thus enhance her research ability. But she will be rewarded by how much new knowledge she creates, not by how much knowledge she possesses.

On the other hand, in the teaching university, the graduate's salary depends on her teaching performance, which is positively related to the amount of knowledge she possesses. While creating new knowledge is not directly rewarded, it, nevertheless, will enrich her knowledge base. Therefore, she still may allocate a certain amount of time to knowledge creation.

Suppose the income distribution in each university depends on the distribution of its academics' ability. The question this paper attempts to answer is that, if the fresh graduate is not sure about her ability *compared to* her peer academics, in which university will she be better off as far as income is concerned? In other words, can we rank the income distributions of the teaching university and the research university and on what will the ranking depend?

The implication of the dilemma will become clearer when one considers the two universities described above as miniatures of two polar "knowledge-based economies." In one economy, workers are rewarded for their innovations only, whereas in the other, workers are rewarded according to their knowledge capacity. We term the former an "innovative economy" and the latter a "knowledge economy," and the workers in the two economies "innovative workers" and "knowledge workers," respectively. Thus, the graduate's dilemma can be generalized into a thesis about comparing the income distributions between the two types of knowledge-based

economy. The policy implication of the analysis is underpinned by the fact that the cost of learning plays a key role in determining the result.

Recently, the idea of knowledge-based economies has captured the attention of policy makers, business leaders and academics. The OECD defines knowledge-based economies as: "...those which are directly based on the production, distribution and use of knowledge and information" (OECD 1996). Nevertheless, such a definition is rather imprecise.<sup>1</sup> For instance, both teaching and research can be equally considered as knowledge-based jobs. Notwithstanding this, the graduate's dilemma indicates that the specific nature of a knowledge-based economy could have important implications for human capital accumulation, income distribution and, consequently, social welfare. What distinguishes this paper from others in the literature is precisely its differentiated treatment of different types of knowledge-based economy.

The main intellectual heritage that the paper is built upon is human capital theory. Human capital theory has had a major influence on the theory of personal income distribution. Seminal papers include Blinder and Yoram (1976), Becker (1975) and Ben-Porath (1967); for more recent publications, see Glomm and Ravikumar (1992), Mincer (1997) and Neal and Rosen (2000). However, the application of human capital theory to the study of income distribution has been criticized by Ramser (1987, p. 43) on three grounds: (1) the human capital concept is not operational; (2) the individual decision problem is usually formulated in a much too restrictive way; and (3) institutional aspects regarding the impact of investment in human capital on the distribution of income are largely ignored. In particular, Ramser (1987, p. 44) criticizes an overemphasis on the productivity function of human capital, to the neglect of the information function of human capital.

---

<sup>1</sup> Empirically, OECD (1999) and APEC (2000) attempt to use a wide-range of indicators to measure to what extent their member countries have advanced towards a knowledge economy.

The model presented in this paper addresses these criticisms to some extent. Regarding the first point, whilst the human capital concept itself is not operational, a human capital theory that leads to the analytic derivation of income distributions is, in principal operational. For instance, if individual income data are available, the derived distributions could be fitted to the data using maximum likelihood techniques.

Regarding the last two points, our model emphasizes the interaction between learning from an existing knowledge base and the discovery of new knowledge, as well as the payment function which is related to the institutional setting. We have expanded the range of individual choices usually considered, and have attempted to focus not just on the productivity function of human capital, but also on its role in generating new knowledge.

The model presented here extends Stiglitz's (1975) work to take into account dynamic accumulation of human capital, and discovery of new knowledge, and to compare the distribution of income in different institutional scenarios which we term "economies." It, therefore, also lies well within the tradition of so-called *public income distribution theories* (Sahota 1978).

Models in new growth theory do capture the discovery of new knowledge in the form of new products or research output that are able to be protected by patents and copyrights (Grossman and Helpman 1991; Aghion and Howitt 1998). However innovation and the creation of new knowledge often occur as a result of workers addressing new problems that emerge as part of their daily activities. For example, managerial staff need to provide new solutions to new problems that arise as company or market conditions change; these new ideas are usually unable to be copyrighted. Furthermore, much knowledge created by academics is also not protected by copyright or patent. In this respect, our model considers an aspect of innovation that is not considered traditionally in the new growth theory.

The remainder of the paper is organized as follows. Section 2 lays out the basic set up of the model. Sections 3 and 4 derive the analytical solutions for the innovative economy and the knowledge economy, respectively. Section 5 applies stochastic dominance theory to compare the income distributions of the two economies. Section 6 concludes.

## BASIC MODEL SETTING

In this section, we sketch out the basic setting that is common to the two types of economies.<sup>2</sup>

The next two sections will add features that distinguish the two economies.

In both economies, workers allocate time between knowledge learning and knowledge creating.

The dynamics of knowledge accumulated through learning is given by:

$$\dot{K}^L = T(\pi + d(H)) \quad (1)$$

where  $K^L$  is the stock of learned knowledge;  $T$  is the time devoted to learning;  $\pi$  is the efficiency of learning, which may be equated to the ability of individuals; and  $d(H)$  is the “efficiency dividend” of investment in human capital.

Efficiency dividend  $d(H)$  is a linear function of accumulated human capital  $H$ :

$$d = \phi H; \phi > 0 \quad (2)$$

As will be seen below, workers are heterogeneous in term of having different values of  $\pi$ . However, we assume that each worker knows her own value of  $\pi$ ; therefore  $\pi$  is treated as a predetermined and hence deterministic variable rather than a stochastic variable in the worker’s optimization problem. While  $\pi$  is given,  $d$  is an endogenous function of previously accumulated human assets (see below). A more knowledgeable person will be more efficient in learning (and creation). Furthermore, a more efficient learner will need a smaller amount of effort or time to learn a given amount of knowledge.

---

<sup>2</sup> Strictly speaking, what we model are merely two labor markets, as we do not explicitly model the production side. But “knowledge labor market” sounds a little awkward to us.

The dynamics of accumulation of knowledge through discovery (i.e. knowledge creation) is described by:

$$\dot{K}^D = (1-T)d(H) \quad (3)$$

where  $K^D$  is the stock of created knowledge. The total time available is normalized to one, so the amount of time spent on making discovery is equal to  $(1-T)$ .

The dynamics of human capital accumulation is governed by:

$$\dot{H} = \dot{K}^L + \dot{K}^D - \delta H = \pi T + d(H) - \delta H \quad (4)$$

Both learned and discovered knowledge contribute to the accumulation of human capital. On the other hand, human capital depreciates continuously because of both internal and external reasons. Individuals have a tendency to forget overtime, especially with increased age. In addition, previously learned knowledge will become obsolete as new knowledge emerges, or as a result of changes in the social environment. For instance, the knowledge of using typewriters was rendered redundant when computers became the standard office typesetting equipment.

Since  $d$  enters both (1) and (3), it means that a higher level of human capital will give rise to higher efficiency in both learning and creation. Furthermore, when human capital level is equal to zero, the worker will be unable to make any discovery. However, she will still be able to learn. In other words, creation must be based on existing knowledge.

We assume that: (a) the time workers spent on leisure is fixed, and (b) there is a perfect financial market. As a result, consumption-investment decision and time allocation decision (over learning and creation) can be separated, and we can model the worker's decision-making process as an income maximization problem instead of a utility maximization problem. A merit of this specification is that the analysis can be isolated from elements about the distribution of initial wealth.

The problem faced by a given innovative worker is to:

$$\text{Maximize } \int_0^{\infty} Y e^{-rt} dt \quad \text{subject to}$$

$$Y = W - pT \quad (5)$$

$$\dot{H} = (\phi - \delta)H + \pi T \quad (6)$$

where  $Y$  is the net wage income;  $W$  is wage income, those specification is stated below;  $p$  is the cost of learning, such as tuition fee. The time subscript is omitted for notational clearness.

Briefly, (5) states that net wage income is equal to wage income minus the cost of learning. Equation (6) describes the dynamics of human capital, and is obtained by substituting (2) into (4)

The stability of the system requires the following assumption.

**Assumption 1:**  $\delta > \phi$ .

## INNOVATIVE ECONOMY

In an innovative economy, an innovative worker is rewarded for the amount of new knowledge she produces per period:

$$W = f(\dot{K}^D) \quad (7)$$

where  $f(\cdot)$  is a wage function.

Equation (7) can be interpreted as that, in our graduate example, the wage earned by an academic in a research university depends on how many publications she produces every year.

To the extent that knowledge is not a (standardized) commodity, the innovative worker should face a downward sloping demand curve, i.e.  $f'(\cdot) > 0$ ,  $f''(\cdot) < 0$ . Here we specify the wage function as

$$f(\dot{K}^D) = w_D \ln(\dot{K}^D) \quad (8)$$

where  $w_D$  is a parameter.

## 1.1 Analytical Solution

Substituting (8) into (5), then we can formulate the current value Hamiltonian of the optimization problem as:

$$A = w_D \ln[(1-T)\phi H] - pT + \lambda[(\phi - \delta)H + \pi T]$$

where  $T$  is the control variable,  $H$  the state variable,  $\lambda$  the costate variable.

Besides the original constraints, the other first order conditions are

$$\lambda\pi = w_D/(1-T) + p \tag{9}$$

$$\dot{\lambda} = (r + \delta - \phi)\lambda - w_D/H \tag{10}$$

The steady state solutions for  $H$  and  $T$  are:

$$\begin{aligned} \tilde{H} &= \frac{1}{2} \left[ \left( \frac{w_D}{p} \right) \left( \frac{r+2\delta-2\phi}{r+\delta-\phi} \right) + 1 \right] \left( \frac{\pi}{\delta-\phi} \right) \\ &\quad \pm \frac{1}{2} \left\{ \left[ \left( \frac{w_D}{p} \right) \left( \frac{r+2\delta-2\phi}{r+\delta-\phi} \right) + 1 \right]^2 \left( \frac{\pi}{\delta-\phi} \right)^2 - \left( \frac{w_D}{p} \right) \left[ \frac{4\pi^2}{(\delta-\phi)(r+\delta-\phi)} \right] \right\}^{1/2} \end{aligned} \tag{11}$$

$$\begin{aligned} \tilde{T} &= \frac{1}{2} \left[ \left( \frac{w_D}{p} \right) \left( \frac{r+2\delta-2\phi}{r+\delta-\phi} \right) + 1 \right] \\ &\quad \pm \frac{1}{2} \left\{ \left[ \left( \frac{w_D}{p} \right) \left( \frac{r+2\delta-2\phi}{r+\delta-\phi} \right) + 1 \right]^2 - \left( \frac{w_D}{p} \right) \left[ \frac{4(\delta-\phi)}{(r+\delta-\phi)} \right] \right\}^{1/2} . \end{aligned} \tag{12}$$

Since the terms inside the squared-root bracket in both equations are positive,  $\tilde{H}$  and  $\tilde{T}$  have two positive real solutions:  $(H^*, T^*)$  and  $(H^{**}, T^{**})$ ;  $H^{**} > H^*$ .<sup>3</sup> However,  $T^{**} > 1$ , violating the constraint of non-negative time allocation in either learning or creation. Therefore, only  $(H^*, T^*)$  is the only feasible solution. The feasible equilibrium human capital and net wage income are:

$$H^* = \frac{1}{2} \left[ \left( \frac{w_D}{p} \right) \left( \frac{r+2\delta-2\phi}{r+\delta-\phi} \right) + 1 \right] \left( \frac{\pi}{\delta-\phi} \right) - \frac{1}{2} \left\{ \left[ \left( \frac{w_D}{p} \right) \left( \frac{r+2\delta-2\phi}{r+\delta-\phi} \right) + 1 \right]^2 \left( \frac{\pi}{\delta-\phi} \right)^2 - \left( \frac{w_D}{p} \right) \left[ \frac{4\pi^2}{(\delta-\phi)(r+\delta-\phi)} \right] \right\}^{1/2} \quad (13)$$

$$Y^* = w_D \ln \left\{ \phi H^* \left[ 1 - (\delta - \phi)(H^* / \pi) \right] \right\} - p(\delta - \phi)(H^* / \pi) \quad (14)$$

In order to examine the stability of the system, we substitute (9) into (6) and (10) to obtain two dynamic equations that characterize the system

$$\dot{H} = (\phi - \delta)H + \pi - w_D / X \quad (15)$$

$$\dot{X} = -w_D / H + (r + \delta - \phi)X + p(r + \delta - \phi) / \pi \quad (16)$$

where  $X = \lambda - p / \pi = w_D / [(1 - T)\pi]$ .

Since  $1 > T^* > 0$ ,  $\pi > 0$ , and  $w_D > 0$ , we have  $X^* > 0$ .

---

<sup>3</sup> In the case of free-education, i.e.  $p = 0$ , the two solutions collapse into one:  $H^* = \pi / (r + 2\delta - 2\phi)$  and  $T^* = (\delta - \phi) / (r + 2\delta - 2\phi)$ . Note that to obtain the solution, substitute  $p = 0$  into the first order conditions, instead of directly into (11) and (12).

Around the steady state, the system can be linearized into

$$\begin{bmatrix} \dot{H} \\ \dot{X} \end{bmatrix} = \begin{bmatrix} \phi - \delta & w_D / X^{*2} \\ w_D / H^{*2} & r + \delta - \phi \end{bmatrix} \begin{bmatrix} H - H^* \\ X - X^* \end{bmatrix}. \quad (17)$$

Let the solutions for the characteristic equation to be  $\mu_1$  and  $\mu_2$ , then

$$\mu_1, \mu_2 = \frac{r \pm \sqrt{r^2 + 4(\delta - \phi)(r + \delta - \phi)[w_D / (X^* H^*)]^2}}{2}. \quad (18)$$

Since  $X^*$  and  $H^*$  are real, both roots are real, and  $\mu_1 \mu_2 < 0$ . That is, the steady state is a saddle point.<sup>4</sup>

**Proposition 1:** In the innovative economy, the lower the cost of learning, the higher the efficiency of knowledge creation, and the higher an individuals ability, the higher the equilibrium level of human capital.

**Proof:** Using (13), it is straightforward to compute the following comparative statics results:  $\partial H^* / \partial p < 0$ ;  $\partial H^* / \partial \phi > 0$ ; and  $\partial H^* / \partial \pi > 0$ .

**Proposition 2:** In the innovative economy, the lower the cost of learning the higher, the efficiency of knowledge creation, and the higher an individual's ability, the higher the equilibrium rate of knowledge creation.

---

<sup>4</sup> The other steady state solution  $(H^{**}, T^{**})$ , while infeasible, is also a saddle point.

**Proof:** Using (2) and (3), the equilibrium rate of knowledge creation is given by:  $\dot{K}^D = [1 - (\delta - \phi)H^* / \pi] \phi H^*$ . Making use of Proposition 1 and noting that  $1 > 2(\delta - \phi)H^* / \pi$ , gives the following comparative statics results:  $\partial \dot{K}^D / \partial p < 0$ ;  $\partial \dot{K}^D / \partial \phi > 0$ ; and  $\partial \dot{K}^D / \partial \pi > 0$ .

If accumulation of human capital and knowledge creation by individuals results in externalities to the economy, such as better citizens or knowledge spillovers, our results suggest subsidizing education.

## KNOWLEDGE ECONOMY

In a knowledge economy, a worker is paid according to her productivity, which is positively related to her stock of human capital:

$$W = w_H \ln H \tag{19}$$

where  $w_H$  is a parameter. Here we, again, assume that the worker faces a downward sloping labor demand curve.

Substituting (19) into (5), we obtain another optimal control problem. The steady state solution for which is given by:

$$H^\# = \left( \frac{w_H}{p} \right) \left( \frac{\pi}{r + \delta - \phi} \right) \tag{20}$$

$$T^\# = \left( \frac{w_H}{p} \right) \left( \frac{\delta - \phi}{r + \delta - \phi} \right) \tag{21}$$

$$Y^\# = w_H \ln \left\{ \left( \frac{w_H}{p} \right) \left( \frac{\pi}{r + \delta - \phi} \right) \right\} - w_H \left( \frac{\delta - \phi}{r + \delta - \phi} \right). \tag{22}$$

**Proposition 3:** In the knowledge economy the lower the cost of learning the higher the efficiency of knowledge creation, and the higher an individuals ability the higher the equilibrium level of human capital.

**Proof:** Using (20), it can be easily computed that:  $\partial H^\# / \partial p < 0$ ,  $\partial H^\# / \partial \phi > 0$ , and  $\partial H^\# / \partial \pi > 0$ .

**Proposition 4:** In the knowledge economy the lower the cost of learning the higher the efficiency of knowledge creation, and the higher an individuals learning ability the higher the equilibrium rate of knowledge creation.

**Proof:** The proof is essentially that same as that of Proposition 2. Making use of Proposition 3, we can compute that:  $\partial \dot{K}^D / \partial p < 0$ ;  $\partial \dot{K}^D / \partial \phi > 0$ ; and  $\partial \dot{K}^D / \partial \pi > 0$ .

**Proposition 5:** If  $w_H = w_D$ , the equilibrium human capital level of a worker will be higher in the knowledge economy than in the innovative economy, but the result for the equilibrium rate of knowledge creation is ambiguous.

**Proof:** Using (13) and (20), it is easy to show that  $H^\# - H^* > 0$ . The difference in the rates of knowledge creation is equal to:  $[1 - (\delta - \phi)H^\# / \pi] \phi H^\# - [1 - (\delta - \phi)H^* / \pi] \phi H^*$ . Factorizing, it gives:  $\phi(H^\# - H^*) [1 - (\delta - \phi)(H^\# + H^*) / \pi]$ . The sign of  $[1 - (\delta - \phi)(H^\# + H^*) / \pi]$  is ambiguous. For instance, if  $p$  is very large relative to  $w_H$  and  $w_D$ , the term can be positive; that is, a worker may have a higher rate of knowledge creation in the knowledge economy than in the innovative economy.

## STOCHASTIC DOMINANCE ANALYSIS

In order to address the question raised in the introduction as to which institution the fresh PhD graduate should choose it is necessary to compare the net income distributions of the knowledge and innovative economies. To do this we employ stochastic dominance analysis. Stochastic

dominance analysis allows one to compare distributions and rank them without specific reference to utility functions.

**Definition 1** (First Order Stochastic Dominance):

If for all  $c$ ,  $F(c) \leq G(c)$  and the inequality is strict over some interval, the distribution  $F$  exhibits first-order stochastic dominance over  $G$ . This indicates that the distribution  $F$  will lie everywhere to the right of the distribution  $G$ . A consequence of first-order stochastic dominance is that any risk-averse individual with positive marginal utility of income, will prefer  $F$  to  $G$  (Hirshleifer and Riley 1992, p. 106).

## 1.2 Distribution of Net Wage Income

In order to derive income distributions, we require the following assumption.

**Assumption 2:**  $\pi$  is uniformly distributed with unit interval support.

Lemmas 1 and 2 derive these distributions of net wage income for the two types of worker based on Assumption 2 and the results in the previous section. Proposition 6 relates to stochastic dominance of net wage income.

**Lemma 1:** The distribution of net wage income for the innovative workers is given by

$$P_D(Y \leq y) = \frac{1}{[1 - (\delta - \phi)Z] \phi Z} \exp \left\{ \frac{Y}{w_D} + \frac{(\delta - \phi)pZ}{w_D} \right\} \quad (23)$$

where  $P_D(Y \leq y)$  is the probability that the net wage income  $Y$  is less than or equal to  $y$ ;

$$Z = \frac{1}{2} \left[ \left( \frac{w_D}{p} \right) \left( \frac{r + 2\delta - 2\phi}{r + \delta - \phi} \right) + 1 \right] \left( \frac{1}{\delta - \phi} \right) - \frac{1}{2} \left\{ \left[ \left( \frac{w_D}{p} \right) \left( \frac{r + 2\delta - 2\phi}{r + \delta - \phi} \right) + 1 \right]^2 \left( \frac{1}{\delta - \phi} \right)^2 - \left( \frac{w_D}{p} \right) \left[ \frac{4}{(\delta - \phi)(r + \delta - \phi)} \right] \right\}^{1/2} .$$

**Proof:** From (13) one obtains:  $Z = H^* / \pi$ .

Substituting this into (14) yields:  $Y = w_D \ln \{ \phi Z \pi [1 - (\delta - \phi)Z] \} - p(\delta - \phi)Z$ .

Inverting (14) gives:  $\pi = \frac{1}{[1 - (\delta - \phi)Z] \phi Z} \exp \left\{ \frac{Y}{w_D} + \frac{(\delta - \phi)pZ}{w_D} \right\}$ .

By Assumption 2,  $\pi$  is uniformly distributed with unit interval support. As  $\pi$  varies across workers in the economy, so will be  $Y$ . So now we can treat  $Y$  as a random variable in the economy, and that  $Y \in [0, \infty)$ . Let us define  $y$  as the realization of  $Y$ . Using Assumption 2, the probability distribution of  $Y$  is given by:

$$P_D(Y \leq y) = \pi = \frac{1}{[1 - (\delta - \phi)Z] \phi Z} \exp \left\{ \frac{Y}{w_D} + \frac{(\delta - \phi)pZ}{w_D} \right\}.$$

**Lemma 2:** The distribution of net wage income for the knowledge workers is given by:

$$P_H(Y \leq y) = \frac{(r + \delta - \phi)p}{w_H} \exp \left\{ \frac{Y}{w_H} + \frac{\delta - \phi}{r + \delta - \phi} \right\}. \quad (24)$$

**Proof:** Inverting (22) gives  $\pi = \frac{(r + \delta - \phi)p}{w_H} \exp \left\{ \frac{Y}{w_H} + \frac{\delta - \phi}{r + \delta - \phi} \right\}$ ; and then apply the argument in the proof of Lemma 1.

**Lemma 3:**  $\partial V / \partial p < 0$ , where  $V = [1 - (\delta - \phi)Z] \phi Z$ .

**Proof:**  $\partial V / \partial p = (\partial Z / \partial p)[1 - 2(\delta - \phi)Z] \phi = (\partial H^* / \partial p)[1 - 2(\delta - \phi)Z] \phi / \pi$ . From Proposition 1 we have  $\partial H^* / \partial p < 0$ . Furthermore, it is straightforward to show that  $[1 - 2(\delta - \phi)Z] > 0$ . Therefore,  $\partial V / \partial p < 0$ .

**Proposition 6:** The distribution of net wage incomes of knowledge workers stochastically dominates that of innovative workers in the first degree if  $p$  is sufficiently large.

**Proof:** The distribution of net wage income of knowledge workers stochastically dominates that of innovative workers in the first degree if  $P_H(Y \leq y) < P_D(Y \leq y)$ ,  $\forall Y$ . This implies that the following inequality condition needs to hold:

$$\frac{(r + \delta - \phi)p}{w_H} \exp\left\{\frac{Y}{w_H} + \frac{\delta - \phi}{r + \delta - \phi}\right\} < \frac{1}{[1 - (\delta - \phi)Z]\phi Z} \exp\left\{\frac{Y}{w_D} + \frac{(\delta - \phi)pZ}{w_D}\right\}, \forall Y \quad (25)$$

The term on the left hand side increases linearly with  $p$ . Using Lemma 3, we also obtain that the term on the right hand side also increases with  $p$ . However, as the term on the right hand side increases by more than exponentially, the inequality will hold for a sufficiently large  $p$ .

The result implies that, as far as income distribution is concerned, a higher learning cost will favor participants in the knowledge economy rather than those in the innovative economy. In our early example of academic jobs, if the graduate is risk averse and the cost of learning is sufficiently high, then she would prefer to accept the offer from the teaching university rather than an offer from a research university.

A policy implication of our result is that, if knowledge creation is deemed to be critical to the long-term growth, policy makers may want to encourage workers to enter an innovative labor market instead of a knowledge labor market. However, if the cost of education is not low enough, then a condition for the income distribution of innovative workers to dominate that of knowledge workers is that  $w_D$  is large relative to  $w_H$  (as well as relative to  $p$  because of Proposition 5). This can be seen easily from the fact that in (25), if  $w_H$  decreases, while keeping other parameters including  $w_D$  constant, eventually the inequality will be reversed.

## CONCLUSION

In this paper, we examine income distribution issues regarding innovative workers versus knowledge ones, and the impact of the cost of learning on income distribution.

In the model, we have analyzed time allocation in two types of labor markets. In the innovative economy, workers are rewarded for the new knowledge they discover, while in the knowledge economy, workers are rewarded for the knowledge they have learnt, regardless whether it is newly generated or inherited.

We prove that in the two economies, both human capital accumulation and the rate of knowledge creation increase with lower cost of learning, higher efficiency in creation, and higher efficiency in learning. The finding is largely consistent with the literature of human capital theory.

However, we also show that while human capital accumulation is higher in the knowledge economy, the rate of knowledge creation is not necessarily higher in the innovative economy. For instance, if the cost of learning is sufficiently high, workers in the knowledge economy could have a higher rate of knowledge creation than those in the innovative economy.

Lastly, we have applied stochastic dominance analysis to the steady state income distributions derived from the optimal control problems of workers in each type of economies. This has allowed us to rank the income distributions of the two types of workers. We have established that if learning is costly, then innovative workers will be worse off than their knowledge counterparts in terms of first-degree stochastic dominance.

The research can be extended in a number of directions. Firstly, in our model workers are not allowed to choose which type of labor market they supply their labor to. A possible extension is to endogenize the choice of job, allowing workers to self-sort into either type of market according to their learning ability, e.g. Sattinger (1975). This approach will lead to the pooling and separating equilibrium in labor market screening models. Secondly, this paper assumes a wage function for each type of economy. A more comprehensive analysis should incorporate the production side and, hence, provide a foundation for the wage equations. Such a general equilibrium approach will be warranted if one wants to compare the long- term growth rates of the two types of knowledge-based economies.

Finally, in the above analysis we treat  $\pi$  as a random variable, but no other parameters, such as  $\delta$  and  $\phi$ . One may suggest that the depreciation rate of knowledge and the efficiency in knowledge creation are also likely to vary across workers. Nonetheless, as  $Y$  is not linear in both

parameters, a closed form solution for the income distribution in terms of either  $\delta$  or  $\phi$  cannot be obtained. Pursuing this issue in the current setting therefore requires numerical simulation.

## REFERENCES

- Aghion, P. and P. Howitt (1998). *Endogenous Growth Theory*. London, MIT Press.
- APEC (2000). *Towards Knowledge-based Economies in APEC*. Singapore, APEC.
- Becker, G. S. (1975). *Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education*. New York, NBER.
- Ben-Porath, Y. (1967). "The Production of Human Capital and the Life Cycle of Earnings." *Journal of Political Economy* 75(4): 352-65.
- Blinder, A. and W. Yoram (1976). "Human Capital and Labor Supply: A Synthesis." *Journal of Political Economy* 84(3): 449-72.
- Glomm, G. and B. Ravikumar (1992). "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality." *Journal of Political Economy* 100(4): 818-34.
- Grossman, G. and E. Helpman (1991). *Innovation and Growth in the Global Economy*. Mass., MIT Press.
- Hirshleifer, J. and J. C. Riley (1992). *The Analytics of Uncertainty and Information*. Cambridge, Cambridge University Press.
- Mincer, J. (1997). "The Production of Human Capital and the Life Cycle of Earnings: Variations on a Theme." *Journal of Labor Economics* 15(1): S26-47.
- Neal, D. and S. Rosen (2000). "The Theory of Earnings Distributions" in *Handbook of Income Distribution*. A. Atkinson and F. Bourguignon. Amsterdam, North Holland: 379-428.
- OECD (1996). *The Knowledge-based Economy*. Paris, OECD.
- OECD (1999). *Science, Technology and Industry Scoreboard*. Paris, OECD.
- Ramser, H. J. (1987). *Verteilungstheorie*. Berlin, Springer-Verlag.
- Sahota, G. S. (1978). "Theories of Personal Income Distribution: A Survey." *Journal of Economic Literature* 16(1): 1-55.
- Sattinger, M. (1975). "Comparative Advantage and the Distributions of Earnings and Abilities." *Econometrica* 43(3): 455-68.
- Stiglitz, J. E. (1975). "The Theory of Screening, Education and the Distribution of Income." *American Economic Review* 65(3): 283-300.