

Sensitivity to measurement perturbation of single atom dynamics in cavity QED

X. M. Liu , M. Hug , and G. J. Milburn

*The Center for Laser Sciences, Department of Physics,
The University of Queensland, St. Lucia, Brisbane, Qld 4072, Australia*

(May 3, 2000)

Abstract

We consider continuous observation of the nonlinear dynamics of single atom trapped in an optical cavity by a standing wave with intensity modulation. The motion of the atom changes the phase of the field which is then monitored by homodyne detection of the output field. We show that the conditional Hilbert space dynamics of this system, subject to measurement induced perturbations, depends strongly on whether the corresponding classical dynamics is regular or chaotic. If the classical dynamics is chaotic the distribution of conditional Hilbert space vectors corresponding to different observation records tends to be orthogonal. This is a characteristic feature of hypersensitivity to perturbation for quantum chaotic systems.

arXiv:quant-ph/0001056 v3 3 May 2000

I. INTRODUCTION

The study of quantum nonlinear dynamics, especially systems which classically exhibit Hamiltonian chaos, has recently begun to focus on the response of such systems to external sources of noise and decoherence [1,2]. This direction was prompted by the observation that nonintegrable classical systems, when quantized will exhibit dynamics that departs from that expected classically on a very short time scale [3,4,5], so short that even macroscopic systems should show observable quantum features in their motion [6]. There have now been numerous experimental observations of the short time deviations between quantum and classical dynamics of nonlinear systems [7,8,9,10]. The nonlinear dynamics of cold atoms in optical dipole potentials has proved to be a particularly fertile field for quantum nonlinear dynamics. Recently the effects of decoherence in quantum chaotic dynamics was studied using cold atoms [11,12,13]. However in all experimental observations so far, the results were obtained from an ensemble of systems, not from repeated observations on a single quantum system. Recent progress in single atom dynamics in small optical cavities [14] indicate that it will soon be possible to study the quantum nonlinear dynamics of a single quantum system subject to repeated measurements [15] and it is towards describing such systems that this paper is directed.

It is in the context of such single system dynamics that the information approach of Schack and Caves [16] based on hypersensitivity to perturbation becomes significant. In that approach the response of classical and quantum nonlinear systems to external perturbations is considered. In particular they show that for a chaotic system it requires a huge amount algorithmic information to track the classical (phase space trajectory) or quantum (Hilbert space vector) of a single chaotic system when it is subjected to small external perturbations. It is better in such cases to average over the perturbation and pay a much smaller cost in von Neumann entropy. In the quantum case the signature of this hypersensitivity to perturbation has been shown to be the distribution of Hilbert space vectors resulting from dynamical sequences with different perturbation histories. While this approach seems to offer

considerable insight into quantum and classical chaos, it is far from clear what it means for an experiment where the dominant source of perturbation is likely to be the measurement back action associated with the attempt to continuously monitor the dynamics.

In reference [17] an attempt was made to study hypersensitivity to perturbation arising from quantum measurements made on a single quantum system: a nonlinear kicked top. In that study the measurements were not continuous in time but rather a sequence of discrete readouts applied at the same time as the kicks. The results confirmed in general terms the observation of Schack and Caves for measurement induced perturbation. Specifically it was shown that if a system was initially localized on a chaotic region of phase space, the Hilbert space vectors resulting from different measurement histories tended to become orthogonal, while for initial regular states the Hilbert space vectors for different histories tended to remain closer together. In this paper we extend that study to the case of a continuously monitored single quantum nonlinear system: a single atom trapped by an intracavity optical dipole field. The motion of the atom changes the phase of the cavity field which may be monitored using phase sensitive detection of the light leaving the cavity. Chaos is introduced by externally modulating the intensity of the light inside the cavity. We use the now established techniques of quantum trajectories [18,19] to study the distribution of Hilbert space vectors for different measurement histories. Previous studies that use quantum trajectories to describe the dynamics of open quantum nonlinear systems include the work of Brun et al. [20].

We use the nonlinear stochastic Schrödinger equation to parallel the discussion in reference [17] based on how information is extracted. The nonlinear Schrödinger equation describes a true measurement in which actual information about the quantum state of the monitored system is extracted from the external field. Our results confirm that a chaotic system, subject to different continuous observation histories, will produce a distribution of states that tend to be orthogonal. This means that a very tiny error in recording the measurement history will suggest a final state that is very likely orthogonal to the actual final state. In this way the intuitive idea that chaos constrains predictability is carried over to

continuously observed single nonlinear quantum systems.

II. THEORETICAL MODEL OF HOMODYNE MEASUREMENTS ON SINGLE ATOM DYNAMICS IN CAVITY QED

Recently, experiments in cavity QED have achieved the exceptional circumstance of strong coupling, for which single quanta can impact the atom-cavity system. The trapping of a single atom in a high-finesse cavity has been realized [14]. In these systems we must treat quantum mechanically both the optical and electronic degrees of freedom as well as the center-of-mass motion of the atom. In our model the atom is in the optical dipole potential of a cavity standing wave which is blue detuned from an atomic resonance so that there is a net conservative force acting on the atom in the direction of decreasing intensity. This interaction does not change the intensity of the optical field but it does change the phase by an amount that depends on the atomic position. As the atom moves in the cavity it changes the phase of the field and if this phase change can be monitored we can effectively monitor the atomic position. This can be accomplished by a homodyne measurement of the field leaving the optical cavity. Mabuchi et al [23] have already demonstrated this kind of measurement at the level of a single atom. A similar model for an atom trapped in a harmonic optical potential was recently discussed by Doherty et al [15]

The basic theoretical description can be given as master equation for a two-level atom coupling to a single electromagnetic mode via the Jaynes-Cummings interaction Hamiltonian, including the quantization of the atomic center-of-mass [21,22]. The Hamiltonian in Schrödinger picture can be written as

$$\hat{H} = \frac{\hat{p}^2}{2M} + \hbar\omega_A\sigma^+\sigma^- + \hbar\omega_c a^+a + \hbar E_0(ae^{-i\omega_L t} + a^+e^{i\omega_L t}) + \hbar g \sin(k_L\hat{x})(a\sigma^+ + a^+\sigma^-), \quad (1)$$

where p is the momentum of atom, M is its mass, ω_A , ω_c , and ω_L are the two-level resonance frequency, the cavity frequency, and the frequency of the driving laser field, respectively. The term E_0 is a constant proportional to the amplitude of the driving field, g

is the coupling constant of the interaction between driving field and atom, σ^+ and σ^- are the raising and lowering operators for the two-level atom, and a^+ and a are the creation and annihilation operators for the cavity field. We assume that the detuning Δ is positive and $\Delta = \omega_A - \omega_L \gg g, \Gamma$ and $\omega_c = \omega_L$, where Γ is the atomic dipole decay rate. In the interaction picture the Hamiltonian can be simplified as

$$\hat{H}' = \hat{H}_{eff} + \hbar E_0(a + a^+) \quad (2)$$

where

$$\hat{H}_{eff} = \frac{\hat{p}^2}{2M} - \frac{\hbar g^2}{2\Delta} a^+ a \cos(2k_L \hat{x}), \quad (3)$$

is the effective Hamiltonian [21]. Note that the effective interaction does not include the driving laser field.

We denote Λ the density operator for the joint state of the atom and the cavity. Then the master equation for Λ is [24,25],

$$\frac{d\Lambda}{dt} = \frac{1}{i\hbar} [\hat{H}_{eff}, \Lambda] - iE_0[a + a^+, \Lambda] + \frac{\kappa}{2}(2a\Lambda a^+ - a^+ a \Lambda - \Lambda a^+ a), \quad (4)$$

where κ is the cavity decay rate. Note that if the cavity is driven by a strong coherent field and if it is strongly damped at the rate κ , the field state will relax to approximately a coherent state with amplitude $\alpha = \frac{-2iE_0}{\kappa}$.

We assume that $E_0/\kappa \ll 1$. Then we can transform the total state by

$$\tilde{\Lambda} = D^+(\alpha)\Lambda D(\alpha). \quad (5)$$

Therefore $a \rightarrow a + \alpha$ and $a^+ \rightarrow a^+ + \alpha^*$. We then expand $\tilde{\Lambda}$ [24]

$$\tilde{\Lambda} = \rho_0 \otimes |0\rangle_a \langle 0| + (\rho_1 \otimes |1\rangle_a \langle 0| + H.c.) + \rho_2 \otimes |1\rangle_a \langle 1| + (\rho'_2 \otimes |2\rangle_a \langle 0| + H.c.). \quad (6)$$

The reduced density operator is $\rho = Tr(\tilde{\Lambda}) = \rho_0 + \rho_2$ and the master equation after adiabatic elimination is

$$\frac{d\rho}{dt} = \frac{1}{i\hbar} [\hat{H}_0, \rho] - D[\hat{J}, [\hat{J}, \rho]]. \quad (7)$$

Here

$$D = \frac{2g^4 E_0^2}{\Delta^2 \kappa^3} \quad (8)$$

is the diffusion constant and

$$\hat{J} = -\cos(2k_L \hat{x}), \quad (9)$$

$$\hat{H}_0 = \frac{\hat{p}^2}{2M} + \hbar \chi \hat{J}, \quad (10)$$

where

$$\chi = \frac{2g^2 E_0^2}{\Delta \kappa^2}. \quad (11)$$

The conditional master equation for the optical field undergoing continuous Homodyne measurement is [24,25]

$$\left(\frac{d\rho_c}{dt}\right)_{field} = \frac{\kappa}{2}(2a\rho_c a^+ - a^+ a \rho_c - \rho_c a^+ a) + \sqrt{\kappa} \frac{dW(t)}{dt} (a\rho_c + \rho_c a^+ - \langle a + a^+ \rangle_c \rho_c), \quad (12)$$

where $dW(t)$ is the infinitesimal Wiener increment. In this equation ρ_c is the density matrix that is conditioned on a particular realization of the Homodyne current up to time t . The corresponding stochastic Schrödinger equation is

$$d|\psi_c(t)\rangle = dt[-i\hat{H}_{eff}/\hbar - \frac{1}{2}\kappa a^+ a + I(t)a]|\psi_c(t)\rangle, \quad (13)$$

where

$$I(t) = \kappa \langle a + a^+ \rangle + \sqrt{\kappa} \frac{dW(t)}{dt} \quad (14)$$

is the measured current. Using Eq. (7), we can derive the nonlinear stochastic Schrödinger equation by adiabatic elimination,

$$d|\psi_c(t)\rangle = dt[-i\hat{H}_0/\hbar - D\hat{J}^2 + I_A(t)\hat{J}]|\psi_c(t)\rangle, \quad (15)$$

where $I_A = 4D\langle \hat{J} \rangle + \sqrt{2D} \frac{dW(t)}{dt}$.

The normalized nonlinear stochastic Schrödinger equation is

$$d|\psi_c(t)\rangle = dt[-i\hat{H}_0/\hbar - D(\hat{J} - \langle\hat{J}\rangle_c)^2 + \sqrt{2D}(\hat{J} - \langle\hat{J}\rangle_c)\frac{dW(t)}{dt}]|\psi_c(t)\rangle. \quad (16)$$

Given the modulation frequency ω , we can define dimensionless parameters by $\tilde{t} = \omega t$, $\tilde{p} = (\frac{2k_L}{M\omega})p$, $\tilde{x} = 2k_L x$, $\tilde{H}_0 = \frac{4k_L^2}{M\omega^2}\hat{H}_0$, $\tilde{g} = g/\omega$, $\tilde{E} = E/\omega$, $\tilde{\Delta} = \Delta/\omega$, $\tilde{\kappa} = \kappa/\omega$, $\tilde{D} = D/\omega$, and $\tilde{\chi} = \chi/\omega$. This yields the commutator relation

$$[\tilde{x}, \tilde{p}] = i\tilde{\kappa}, \quad (17)$$

where $\tilde{\kappa} = \frac{4\hbar k_L^2}{M\omega}$ is the dimensionless Plank constant.

Omitting all the tildes, the equivalent equations are similar except \hbar is replaced by $\tilde{\kappa}$ and the dimensionless Hamiltonian is

$$\hat{H}_0 = \frac{\hat{p}^2}{2} - \xi \cos \hat{x}, \quad (18)$$

where $\xi = \frac{4k_L^2}{M\omega^2}\hbar\chi$. In order to study chaos in a quantum system, we consider a periodic modulation of the driving field $E_0(t) = E_0\sqrt{1 - 2\epsilon \cos t}$. The expressions of the stochastic equations (14) and (15) will not change except that \hbar is replaced by $\tilde{\kappa}$ and D replaced by $D(1 - 2\epsilon \cos t)$ and ξ in Eq. (17) is replaced by $\xi(1 - 2\epsilon \cos t)$.

III. SENSITIVITY TO DIFFUSION CONSTANT

We assume that initially the atomic center-of-mass wave function is in a Gaussian minimum uncertainty state with the position representation

$$\psi(x) = \left(\frac{1}{2\pi\sigma_x}\right)^{1/4} \exp\left[-\frac{(x - x_0)^2}{4\sigma_x} + \frac{ip_0x}{\tilde{\kappa}}\right]. \quad (19)$$

We take $x_0 = 0$, $p_0 = 1.0$ as for these values the state is localized on a second order period one resonance and is thus localized in a regular region of phase space (see Fig. 1). For $\sigma_x = 0.3906$, $\tilde{\kappa} = 0.25$, $\xi = 1.2$, Dyrting et al. [26] have shown that the system will coherently tunnel between the two corresponding second order period one resonances. We use a Split Operator Method [27] and FFT (Fast Fourier Transformation) [28] to obtain

the numerical solution of the stochastic Schrödinger equations. In this scheme the kinetic operator and potential operator are used separately to propagate the wave function:

$$\exp[-i\hat{\mathbf{H}}\delta t/\hbar] \sim \exp[-i(\hat{\mathbf{P}})^2\delta t/4\hbar] \exp[-i(\hat{\mathbf{V}})\delta t/2\hbar] \exp[-i(\hat{\mathbf{P}})^2\delta t/4\hbar]. \quad (20)$$

The computing errors are of $O(\delta t^3)$. Here \hat{V} is the effective potential which includes a stochastic term,

$$\hat{V} = -\xi(t) \cos \hat{x} - i\hbar[D(t)(\hat{J} - \langle \hat{J} \rangle_c)^2(1 + \frac{dW(t)^2}{dt}) - \sqrt{2D(t)}(\hat{J} - \langle \hat{J} \rangle_c)\frac{dW(t)}{dt}]. \quad (21)$$

where $\xi(t) = \xi(1 - 2\epsilon \cos t)$ and $D(t) = D(1 - 2\epsilon \cos t)$. We introduced the $\frac{(dW(t))^2}{dt}$ term to keep the expression consistent with the normalized nonlinear stochastic Schrödinger equation after an expansion of the exponential function.

In order to compare the quantum and semi-classical stochastic evolutions, we calculate the Wigner function [29], [31]

$$P(x, p) = \frac{1}{2\pi\hbar} \int dy \langle x - \frac{y}{2} | \rho | x + \frac{y}{2} \rangle \exp(ipy/\hbar). \quad (22)$$

This expression can be interpreted as the Weyl-Wigner correspondence [31] of the density operator. To give the dynamical equation for the Wigner function that is the quantum correspondence of a classical Liouville equation we use the Weyl-Wigner correspondence of an operator $\hat{F} = \hat{A}\hat{B}$ which is [30,31]

$$F(x, p) = A(x, p)XB(x, p), \quad (23)$$

where $X = \exp[\frac{\hbar}{2i}(\frac{\overleftarrow{\partial}}{\partial p} \frac{\overrightarrow{\partial}}{\partial x} - \frac{\overleftarrow{\partial}}{\partial x} \frac{\overrightarrow{\partial}}{\partial p})]$ and the arrows on the operators denote the term on which the operator is to be applied. Alternatively, we obtain

$$F(x, p) = A(x - \frac{\hbar}{2i} \frac{\partial}{\partial p}, p + \frac{\hbar}{2i} \frac{\partial}{\partial q})B(x, p). \quad (24)$$

When we apply this formula to the products appearing in the Master equation we can readily obtain the phase space equation for the Wigner function we are looking for

$$\frac{\partial P}{\partial t} = [\frac{\partial H_0(t)}{\partial q} \frac{\partial P}{\partial p} - \frac{\partial H_0(t)}{\partial p} \frac{\partial P}{\partial q}] + D(t)\hbar^2 \sin^2 x \frac{\partial^2 P}{\partial p^2}, \quad (25)$$

where $H_0(t)$ is the classical Hamiltonian including modulation $H_0(t) = \frac{p^2}{2} - \xi(t) \cos x$. We give the corresponding classical stochastic F-P equation from quantum nonlinear stochastic Schrödinger equation [32]

$$\frac{dx}{dt} = p, \quad (26)$$

$$\frac{dp}{dt} = -\xi(t) \sin x + \sqrt{2D(t)} \hbar \sin x \frac{dW(t)}{dt}. \quad (27)$$

To describe the classical distribution we use the classical Q function [33]. The initial state is a bivariate Gaussian centered on (x_0, p_0) with position variance δ_x and momentum variance δ_p ,

$$Q_0(x, p) = \frac{1}{2\pi\sqrt{\delta_x\delta_p}} \exp\left[-\frac{(p-p_0)^2}{2\delta_p}\right] \exp\left[-\frac{(x-x_0)^2}{2\delta_x}\right], \quad (28)$$

where the classical variances δ_x and δ_p are related with quantum parameters $\delta_x = \frac{\hbar^2}{2\xi} + \frac{\hbar^2}{4\sigma_x}$, $\delta_p = \frac{\hbar\sqrt{\xi}}{2} + \sigma_p$. The evolution of Q function is $Q(x, p, t) = Q_0[\bar{x}(x, p, -t), \bar{p}(x, p, -t)]$, where $\bar{x}(x, p, -t), \bar{p}(x, p, -t)$ is the trajectory generated by Hamilton's equations.

To compare the quantum dynamics with the classical conditional dynamics, for quantum system we study the ensemble with the same initial condition but with random trajectories. It shows that when D is very small, for Homodyne measurement, the evolution of average momentum $\langle p \rangle$ and average variance of momentum $(\langle p^2 \rangle - \langle p \rangle^2)$ for the ensemble show coherent tunneling. Therefore the perturbation is not serious for small D when the initial state is in the regular region of the classical phase space.

For the classical dynamics for small D the results are close to the no diffusion case [26]. Obviously we therefore expect that we obtain different results between classical and quantum conditional dynamics (Fig. 2).

However, if the diffusion constant is large enough, we obtain almost the same result as in the classical case (Fig. 3). For a single stochastic measurement, the terms in the normalized nonlinear Schrödinger stochastic equation due to the measurement depend on the quantity

$(\hat{J} - \langle \hat{J} \rangle_c)$. We expect that for some range of values of D , the stochastic measurement terms would drive the system towards an oscillating trajectory for which

$$\langle \hat{J}^2 \rangle_c \simeq \langle \hat{J} \rangle_c^2. \quad (29)$$

Therefore for the ensemble which includes many random trajectories, the results will approach that of the classical conditional dynamics.

IV. SENSITIVITY TO CHAOTIC AND REGULAR INITIAL STATES

We again assume that the wave function is initially in a minimum uncertainty state in the position representation. We choose two initial locations in classical phase space. In the first case, $x_0 = 0$, $p_0 = 1.0$ it is in the regular region of classical phase space. In the second case $x_0 = -2.5$, $p_0 = 1.0$, it is in the chaotic region (see Fig. (1)).

Because the measurement will perturb the quantum state, we hope to be able to compare the effect of measurement noise of different trajectories. Here the angle $\theta_{ij}(t)$ is defined between two normalized state vectors $|\psi_i(t)\rangle$ and $|\psi_j(t)\rangle$ as [16,17]

$$\theta_{ij}(t) = \cos^{-1} |\langle \psi_i(t) | \psi_j(t) \rangle|. \quad (30)$$

In the position representation, this is

$$|\langle \psi_i(t) | \psi_j(t) \rangle| = \left| \int_{-\infty}^{\infty} \psi_i(x, t)^* \psi_j(x, t) dx \right|. \quad (31)$$

As a measure of the distribution of the state vectors in Hilbert space we can calculate the average angle between all pairs of vectors. We define

$$\theta_{ave}(t) = \frac{2}{(N^2 - N)} \sum_{i \neq j} \theta_{ij}(t), \quad (32)$$

where N is the number of trajectories.

In Fig. 4, we plot $\theta_{ave}(t)$ for the two initial states mentioned above, evolved up to 200 cycles. We used up to 40000 steps and $N = 1000$ trajectories for our calculation. As can be

seen, the average angle between vectors starting in the chaotic phase space region is larger than that of the regular initial state.

If the system started in the regular region, small errors in recording the results of the measurement will not be a very serious problem because the conditional states form trajectories which will remain close in Hilbert space. Therefore, if we consider the distribution of Hilbert angles at a fixed strobe number (Fig. 5), the distribution is centered at a small angle for an initial regular state. On the other hand, for an initial chaotic state, the peak location approaches $\pi/2$ (Fig. 6). It means that for an initial chaotic state most vectors are far apart from each other. Therefore if the initial state was in the chaotic region of phase space, it will be much more difficult to infer the system state reliably from the measurement results. This result is consistent with the results of the quantum kicked top [17].

In summary, we have demonstrated that for continuous Homodyne measurement of signals from the quantum system of single atom dynamics in cavity QED, the measured results are influenced by the diffusion constant and whether the initial states are in regular or chaotic phase space regions. We have shown that if the diffusion constant is large enough, the average measurement results are similar to classical conditional dynamics and for small diffusion constant the initial chaotic state will be more sensitive to errors in recording the measurement results than the initial regular state.

V. ACKNOWLEDGMENT

One of authors(XML) would like to thank Dr. J. F. Corney and Dr. M. Gagen for useful discussions on conditional master equations and Homodyne measurements.

REFERENCES

- [1] W.H.Zurek and J.P.Paz, Phys. Rev. Lett. **72**, 2508 (1994).
- [2] R.Schack, G.M.D'ariano and C.M.Caves, Phys. Rev E **50**, 972 (1994).
- [3] S.Fishman, D.R.Gremple and R.E. Prange, Phys. Rev. Lett. **49**, 509 (1982).
- [4] D.L.Shepelyansky, Phys. Rev. Lett. **56**, 677 (1986).
- [5] B.V.Chirikov, F.M.Israilev, D.L.Shepelyansky, Physica D **33**, 77 (1989).
- [6] W.H.Zurek, Physica Scripta **T76**, 186, (1998).
- [7] E.J.Galvez, B.E.Sauer, L.Moorman, P.M.Koch and D. Richards, Phys. Rev. Lett. **61**, 2011 (1988).
- [8] J.E.Bayfield, G.Casati, I.Guarneri and D.W.Sokol, Phys. Rev. Lett. **63**, 364 (1989).
- [9] M.Arndt, A. Buchleitner, R.N. Mantegna and H.Walther, Phys. Rev. Lett. **67**, 2435 (1991).
- [10] F.L. Moore, J.C. Robinson, C.F. Bharucha, Bala Sundaram, and M.G. Raizen, Physical Rev. Lett. **75**, 4598, (1995).
- [11] B. G. Klappauf, W. H. Oskay, D. A. Steck, and M. G. Raizen, Physical Rev. Lett. **81**, 1203, (1995).
- [12] B. G. Klappauf, W. H. Oskay, D. A. Steck, and M. G. Raizen, Physical Rev. Lett. **81**, 4044, (1995).
- [13] H. Ammann, R.Gray, I Shvarchuck, and N. Christensen. Physical Rev. Lett. **80**, 4111 (1998).
- [14] J.Ye, D.W.Vernooy, and H.J.Kimble, Phys. Rev. Lett. **83**, 4987(1999).
- [15] A.C.Doherty, S.M.Tan, A.S.Parkins and D.F.Walls, Phys. Rev A **60**, 2380 (1999).

- [16] Rüdiger Schack, and Carlton M. Caves, Phys. Rev. A **53**, 3257(1996).
- [17] J.K. Berslin, and G. J. Milburn, Phys. Rev. A **59**, 1781(1999).
- [18] H.J.Carmichael, *An Open Systems Approach to Quantum Optics*, Lecture Notes in Physics: New Series m, Monographs, **m18**, (Springer, Berlin, 1993).
- [19] K. Molmer, Y.Castin, Quantum Semiclass. Opt. **8**, 49 (1996).
- [20] T.A.Brun, Gisin N, P.F.O'Mahony , M. Rigo, Phys. Lett A **229**, 267(1997).
- [21] A.M.Herkommer, H.J.Carmichael and W.P.Schleich, Quantum Semiclass. Opt. **8**, 189(1996).
- [22] Ralf Quadt, Matthew Collett, and Dan F. Walls, Phys. Rev. Lett. **74**, 351(1999).
- [23] H. Mabuchi, J.Ye, H.J.Kimble, Applied Physics B-Lasers and Optics **68**,1095,(1999).
- [24] H. M. Wiseman, and G. J. Milburn, Phys. Rev. A **47**, 642(1993).
- [25] J. F. Corney, and G. J. Milburn, Phys. Rev. A **58**, 2399(1998).
- [26] S. Dyrting, and G. J. Milburn, Phys. Rev. A **51**, 3136(1995).
- [27] Ronnie Kosloff, J. Phys. Chem. **92**, 2087(1988).
- [28] William H Press et al (1992), *Numerical Recipes in C* , Cambridge, University Press.
- [29] E. Wigner, Phys. Rev. **40**, 749(1932).
- [30] H. J. Groenewold, Physica **12**, 405(1946).
- [31] L. Wang and R. F. O'Connell, Found. Phys. **18**, 1023(1988).
- [32] D. F. Walls, G. J. Milburn(1994), *Quantum Optics*, Springer-Verlag Berlin Heidelberg.
- [33] Wenyu Chen, S. Dyrting, and G. J. Milburn, Aust. J. Phys. **49**, 777(1996).

FIGURES

FIG. 1. *Stroboscopic portrait of the system with Hamiltonian $H_0 = \frac{p^2}{2} - \xi(1 - 2\epsilon \cos t) \cos x$, where $\epsilon = 0.2$, $\xi = 1.2$.*

FIG. 2. *Evolution of average momentum $\langle p \rangle$ and average variance of momentum $\langle p^2 \rangle - \langle p \rangle^2$ for classical and quantum conditional dynamics when $D = 0.001$. 1000 random trajectories are taken. Solid line, classical conditional dynamics, dashed line, quantum conditional dynamics.*

FIG. 3. *Evolution of average momentum $\langle p \rangle$ and average variance of momentum $\langle p^2 \rangle - \langle p \rangle^2$ for classical and quantum conditional dynamics and $D = 0.1$. 1000 random trajectories are taken. Solid line, classical conditional dynamics, dashed line, quantum conditional dynamics.*

FIG. 4. *Evolution of average angles in Hilbert space for $D = 0.001$. Solid line, in chaotic region initially $x_0 = -2.5$, $p_0 = 1.0$. Dashed line, in regular region initially $x_0 = 0.0$, $p_0 = 1.0$.*

FIG. 5. *Distribution of angles in Hilbert space at strobe number 200 for initial regular state and $D = 0.001$.*

FIG. 6. *Distribution of angles in Hilbert space at strobe number 200 for initial chaotic state and $D = 0.001$.*











