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Demographic Transition and Economic Growth

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Acknowledgements

The "Population Debate" asks whether demographic growth hampers or fosters economic growth, and convincing arguments have been offered in support of both views. In my thesis I try to show that, in the dynamic framework of demographic transition, both cases are possible. Nevertheless, when working on my study, I learned to appreciate why that most ardent of optimists, Julian Simon, argued in 1981 that people are "The Ultimate Resource".

I am thankful to Professor Seppo Honkapohja, whose continuous support made it possible for me to undertake this study. His comprehensive approach to economic theory helped me to set my work in a wider context of current economic analysis. He encouraged me to face new, challenging questions concerning population economics but also provided his help in my down-to-earth problems. I wish to thank Professor Mikael Linden for his support to me, a theorist, to go through the demanding phases of empirical research. His principle was that empirical work is not so much appreciated on the basis of it being correctly or incorrectly, but rather, well or poorly done. I also gratefully remember my meeting with Professor Seppo Salo, who gave important mathematical advices to continue my work.

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My research was conducted mainly while a lecturer in the Department of Economics at the University of Helsinki. Due to the persistent work of the head of the department, Professor Vesa Kanniainen, the personnel of the department, myself included, enjoyed good research conditions even under ever increasing pressures upon the scarce time of academics. The daily concerns of my academic life were alleviated by the gentle care of administrator Ritva Teräväinen and secretaries Hillevi Forsberg, Leena Harinen, and Päivi Leskinen. Among my colleagues, I especially wish to thank Professor Tapio Palokangas for his helpful comments in theoretical and practical problems that I encountered in my work, and for his encouraging me to participate in broader academic conversations. I also greatly enjoyed the academic comradery of Professor Anne Mikkola and research assistants Mai Allo, Anu Alander, Anna-Leena Asikainen, and Helena Holopainen. Our stimulating and pleasurable discussions ranged from new economic theories to computational problems and from professional topics to everyday joys and sorrows.

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Helsinki, June 17, 2003

Ulla Lehmijoki

Contents

A Demographic Introduction

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Abstract

The thesis deals with the effects of demographic transition on economic growth. As an introduction, we provide a survey on the determinants of demographic transition itself. Both a theoretical and an empirical survey are given. In the empirical survey, we concentrate on the determinants of the fertility rate and its change. Standard panel techniques are used. The survey shows that, in terms of fertiliy behavior, the role of demographic variables themselves is important. Total fertility remains high as long as mortality does the same. But as mortality decreases, so does fertility, and the tempo of the decrease in fertility is fast if its level was high initially. Economic growth, the level of per capita income, and the share of the labor force in agriculture are other important determinants of fertility behavior. Therefore, fertility and population growth might be endogenous to economic variables. This result is utilized and discussed in the essays of the thesis.

Together with industrialization, a demographic modernization, called demographic transition, has taken place. In this demographic transition, there has been a shift in fertility and mortality rates to lower levels. However, because the shift in fertility far lagged that in mortality, population growth has accelerated temporarily (Coale 1987). In Europe, mortality started its decline at the end of the eighteenth century. Since then, life expectancy has increased from 30-35 years to 75-80 years (see Chesnais 1992 and Livi-Bacci 1997). The decline in fertility started around 1900, and the total fertility rate has decreased from five children per woman to less than two (see Coale and Treadway 1986). During this transition, the population in Europe has increased fourfold. The same history was repeated in countries where the roots of the population were European. On the contrary, in other countries, demographic transition began only during the twentieth century, and it is expected to continue through this century. The central demographic variables since 1950-55 are shown in Figure 1 for these two groups of countries, called *Early* and *Others*. The former contains countries in Europe and its offshoots and the latter countries in the rest of the world.

Figure 1 shows that a rapid decrease of (infant) mortality and fertility rates, as well as a rapid increase in life expectancy is ongoing in the *Other* countries, but the levels of these variables are still far from those in Europe. The population growth rate in the *Other* countries has declined only recently.

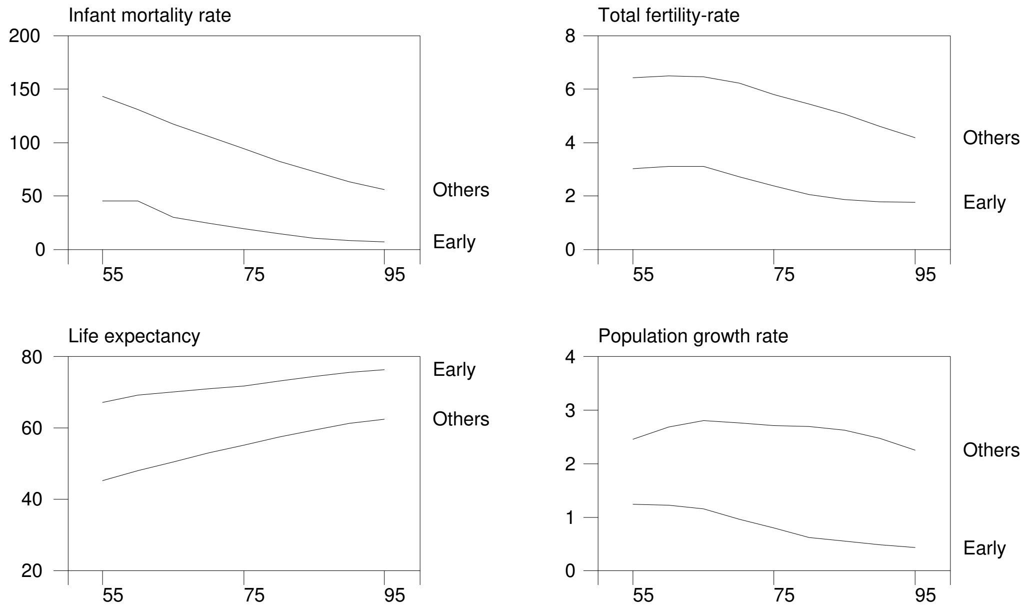

Figure 1: Development of demographic variables from 1950-55 to 1990-95 for 22 countries in Europe and its offshoots (*Early*) and 49 countries in the rest of the world (*Others*). For the data, see United Nations (1999), for the concept *Early* and *Others*, see United Nations (1991).

The first theory on demographic transition was presented by Notestein (1945), even if he had as his predecessors Thompson (1929) and Landry (1934). Notestein argued that the socio-economic change since the industrial revolution was accompanied by a concomitant change in human reproductive behavior. He also argued that the decline in mortality, caused by agricultural, technical, and medical innovations, was a necessary condition for a decrease in fertility to take place, even if fertility decrease realized only after a considerable time lag:

> Until very recently, it has been impossible from a technological point of view to achieve low death rates. The populations that could survive to modern times in spite of the inevitably high death rates were those which maintained correspondingly high birth rates. These populations had to have social systems conductive to high birth rates. The organization of their economic, community, and familial life; their normative orders of religious and moral sentiment; their educational system; and the means by which men and women gained status with each other and with their fellows were all oriented in mutually supporting fashion to provide ample reproduction. Such arrangements are woven tightly into the social fabric and are slow

to change. Notestein (1951).

Notestein emphasizes that, even if some of the decrease in fertility came through increased use of contraceptive methods, the appearance of new contraceptives was not the primary reason for declining fertility rates. The primary reason was the socio-economic change that, on the one hand, made survival possible even if fertility was low, and, on the other, introduced new living conditions and values favoring low fertility rates. Ever increasing urban populations became less dependent on the status of the family, which then lost many of its economic and educational functions. New labor market opportunities for women arose. This made it possibly for women to have roles and economic interests outside their homes.

After some years of a "honeymoon," however, Notestein's theory came under criticism (see Kirk 1996). The first reason was that it was very general in nature. The theory's generality made it difficult to test it empirically. New, more detailed questions dealing with the escalation of transition, and the role of political and ideological movements in determination of fertility, also arose. Economists were interested to examine, whether Notestein's theory was compatible with the microeconomic theory for rational, utility-maximizing households.

The second reason was that some new statistics called Notestein's theory into question. For example, the European Fertility Project organized by Ansley Coale in 1963, established that in France the fertility decline took place prior to that in mortality (see Coale and Treadway 1986).[1] In addition, the escalation of the fertility decline in Europe took place along its linguistic and cultural, rather than economic borders (see Watkins 1990).

Subsequent theories concentrated either on the exact role of economic factors, such as demand for and supply of children, and the costs of family planning, or on the role of the emergence of new philosophical and political ideas and their diffusion from country to country. The former can be labelled as "economic," the latter as "cultural" theories of demographic transition. Both economic and cultural theories lay emphasis on the deliberate choice of individual couples in assessing their fertility behavior. On the other hand, "biological" theories are concerned with such deterministic biological factors as nutrition and sexual drives, which give conditions to reproductive behavior of humans as well as animals. To summarize, instead of trying to tell a whole story, the new theories concentrate on some specific aspects of demographic transition.

The rest of the paper is organized as follows: in Section 1 we offer a short review concerning recent theories of demographic transition. The review provided here differs from that given by Szreter (1993), van de Kaa (1996), Kirk (1996), and Mason (1997) in that it is directed to the reader with interests in economics. In Section 2 we try to quantify determinants explained in these theories by regressing the total fertility rate as well as the its rate of change against the explanatory variables proposed in these theories. These regressions differ from those given by Barro and Lee (1994) and by Barro and Sala-i-Martin

[1] But see Chesnais 1992, chapter 11.

(1995) in that they are explicitly based on demographic theories. Section 3 discusses the contents of the three essays in the thesis.

1 Six Theories of Fertility Transition

All economic theories discussed here— the households demand theory, the excess supply theory, and the wealth flow theory — work in the framework of economic rationality. However, only the households demand theory and the excess supply theory utilize the building blocks of microeconomic theory of consumer choice. Both economic and cultural theories concentrate on fertility behavior and take mortality as given. On the other hand, the homeostatic theory deals with population growth, because it is the population, not simply the number of newcomers, which determines the resources available per person. The short-hand term for each theory is in italics. The term for Notestein's theory is *traditional*.

1.1 Economic Theories

The households *demand* approach for fertility was formulated by Gary Becker (1960) (see also Willis 1973). In this approach, the household maximizes its utility from the number (n), and the quality (q) of children, and from other goods (Z), as given by the utility function $U = U(n, q, Z)$. All children in the family are assumed to have the same quality. In the beginning of their reproductive career, households make a single decision concerning their demand for children of a given quality (qn). Households are facing the budget constraint

$$p_c qn + \pi_z Z = I,$$

in which p_c stands for the constant cost of unit quality, which includes the costs of foregone working time and material costs, π_z is the market price of other goods, and I is income. The standard utility maximizing yields

$$\begin{aligned} MU_n &= \lambda p_c q = \lambda \pi_n(q), \\ MU_q &= \lambda p_c n = \lambda \pi_q(n), \\ MU_z &= \lambda \pi_z. \end{aligned}$$

This leaves us with two equilibrium conditions, namely:

$$\begin{aligned} \frac{MU_n}{MU_z} &= \frac{\pi_n}{\pi_z}, \\ \frac{MU_z}{MU_q} &= \frac{\pi_z}{\pi_q}. \end{aligned}$$

Note that, in the optimum, the price of an extra child $\pi_n(q)$ depends positively on the quality of the children already chosen by the parents, because the new-comer should be provided with the same quality as her or his elder siblings. Correspondingly, the optimum price of the additional quality $\pi_q(n)$ depends on the number of children chosen. Therefore, it is more informative to write the budget constraint in terms of optimizing prices and full-time income as follows:

$$\begin{aligned} p_c qn + \pi_z Z &= I, \\ (p_c q)n + (p_c n)q + \pi_z Z &= I + p_c qn, \\ \pi_n(q)n + \pi_q(n)q + \pi_z Z &= I + p_c qn = R, \end{aligned}$$

in which R stands for full-time income. The household demand approach can explain the decrease in fertility in two ways. First, it is possible that there has been a shift in relative prices of child quality and quantity in favor of the former. Assume that $\pi_q(n)$ decreases due to, say, greater supply of public schooling. This bends the parental choice toward quality, and q increases, which might increase the price of an extra child $\pi_n(q)$, leading to a decrease in n. A lower number of children leads to a further decrease in the price of the additional quality $\pi_q(n)$ (remember that each child has same quality), and chances in child quality and quantity can be considerable for some initial decrease in $\pi_q(n)$ (Becker 1982).

The second reason for fertility changes comes from the increase in incomes along with economic growth. An income increase raises the demand for both child quantity and quality, assuming that both goods are normal. This is a pure income effect. If the income elasticity of child quality is larger than that of quantity, the demand increase in quality exceeds that in quantity. The increase in income also has a price-of-time-effect: because income increase is mainly due to an increase in wages, the price of time increases. Since child bearing and rearing is time intensive, the price-of-time effect decreases the demand for children in terms of other goods.[2] If there is a gender bias in family efforts, the dominance between these effects might depend on *relative wages* between women and men because the wage of both sexes has the same (pure) income effect, but only women's wages have the price-of-time effect in terms of children. Galor and Weil (1996) supply an ingenious explanation for the increase in women's relative wages coincidental to economic growth: The supply of labor has two components, physical labor, L^P, and mental labor, L^M. The sexes are provided with equal ability to provide mental labor, but the supply of physical labor of men far exceeds that of women. Therefore, women have a comparative advantage in supply for mental labor L^M. As the economy now develops, technology becomes more complementary to mental labor and the demand for mental labor relative to the demand for physical labor increases, leading to dominance of the (mother's) price-of-time effect and to decrease in fertility.

The effect of women's relative wages on fertility in Sweden from 1869 to 1910 was studied by Schultz (1985). The most ingenious feature in his study was that he was able to find a situation, in which the change in relative wages was exogenous to fertility decisions: From 1750-1880 the price of grain in Europe

[2] The increase in the price of time, has both (negative) income effect and (negative) substitution effects on child demand. Above, we simply discuss the latter.

increased steadily, and the trade liberalization from Sweden increased the grain export fourfold from 1851 to 1880. However, the European grain crisis, due to widening of grain import from the USA lead to a decrease in grain export but increase in butter export in Sweden. Because milk processing was women's work, the price of their time increased. However, agriculture relied on grain and animal products differently in different parts of the country, and in his regressions, Schultz is able to show that fertility was lower in those parts of countries, in which dependence of animal products was higher.

Lucas (2002) gives a series of dynamic models to show what is essential in demographic transition. Assume that all children are of constant quality and denote their number by C. The unit price of child raring π_n can be rewritten as π_c. Assume that the price of the commodity is normalized to unity, i.e., $\pi_z = 1$. Assume that the parents at time 0 have the utility function $U_0 = U(C_0, Z_0, U_1)$, in which U_1 is the lifetime utility of each child. Starting from the simplest possible economic environment of the hunter-gatherers, Lucas shows that if land is commonly owned, there is nothing that parents can do to improve the utility of their children. Therefore, they simply choose in a steady state

$$\frac{MU_c}{MU_z} = \pi_c. \tag{1}$$

Lucas (2002) then assumes that instead of common ownership, property rights are established, and land is initially equally divided among families, each having x units of it. The land per family then produces $f(x)$ units of good. The thing to do is to leave the family property to an optimal number of descendants. Therefore, in a steady state

$$\frac{MU_c}{MU_z} = \pi_c + \frac{f'(x)x}{\rho},$$

in which ρ is the discount factor so that $\frac{f'(x)x}{\rho}$ is the discounted rental income of land. Because each newcomer decreases the per capita rental income, their marginal utility must be larger than in the common ownership case and the population growth must be smaller. Further, if we add a reproducible factor, the accumulation of which takes place at the cost of current consumption, the steady state condition becomes

$$\frac{MU_c}{MU_z} = \pi_c + k + \frac{f'(x)x}{\rho},$$

in which k is the reproducible input per capita and the population growth must decrease in the steady state. Assume now that the possibility of human capital accumulation is introduced, and that per capita human capital H accumulates according to $\dot{H} = H \cdot \varphi(r)$, in which $\varphi(r)$ is the productivity of r, the share of the time devoted to educate one child. This formulation says that human capital accumulation is the more efficient, the higher is its level. Assume that human capital is the only input in production, i.e., $f(H) = H\left[1 - (r + \pi_c)\, C\right]$, in which π_c now is interpreted as the time cost of growing up one child so that the time available to work is $1 - (r + \pi_c)\, C$. Then the altruistic parents have to choose how much to work (and consume), and how much time to devote to each child's education, and how many children to have. Because the choice to educate a child changes also the future production, each child's life-time utility also change. Therefore, we have two steady state conditions:

$$\frac{MU_c}{MU_z} = r + \pi_c, \tag{2}$$

$$\frac{MU_z}{MU_{U_1}} = \frac{\mathcal{U}\varphi'}{C}. \tag{3}$$

Because we assumed neither land nor other input, equation (2) is to be compared to equation (1): the time cost of child quality reduces the fertility in the steady state. In equation (3), the marginal rate of substitution between current consumption and the child utility depends on the total utility per child ($\mathcal{U}/C$) reached in the optimum, multiplied by the productivity of each time unit devoted to child quality. The optimizing parent is willing to give away just this amount of current consumtion to increase the child quality. The households budget constraint is $Z_0 \leq H\, [1 - (r + \pi_c)\, C]$. Using the Cobb-Douglas preferences, which in a steady state can be written $U(C, Z, U) = C^{\eta} Z^{1-\beta} U^{\beta}$, and an exponential formula for human capital accumulation, given by $\varphi\,(r) = r^{\varepsilon}$, the marginal conditions can be solved to give

$$\begin{aligned} C &= \frac{1}{\pi_c}\frac{\eta - \beta\varepsilon}{1 + \eta - \beta}, \\ r &= \frac{\beta\varepsilon}{\eta - \beta\varepsilon}\pi_c. \end{aligned}$$

These equations show that an increase in human capital productivity ε of the parental time share r decreases the fertility in a steady state. The models introduced by Becker *et al.* (1990) and Galor and Weil (1999) also share this feature: an improvement in production technology decreases the population growth. According to Lucas (2002), a *transition* from a low-income steady state to a high-income steady state is possible only, if the economy has two types of technologies, a land dependent agricultural technology and a human capital dependent modern technology, and if one can postpone a mechanism, through which the former is gradually replaced by the latter.

In their simulation study dealing with US demographic transition from 1800 to 1940, Greenwood and Seshadri (2002) combine the elements of household demand theory with technical progress, which has been different in agricultural and manufacturing sectors. The three period overlapping-generations model let parents choose both the child quality and quantity. Because technical progress in agriculture is lower (total factor productivity estimated to increase 1.95 fold from 1800 to 1940) than in manufacturing (4.11 fold), its relative output price increases, demand decreases, and labor demand in that sector decreases. Because an uneducated labor force is only demanded in the agricultural sector, the relative wage for an uneducated labor force decreases, and more and more parents (deriving utility from their decedents wages) choose to have fewer but better educated children. The simulation results closely resemble the actual data and give support to the basic model.

Instead of concentrating on demand for children, Richard Easterlin (1978) presents a theory combining demand for and supply of children to the concept of *excess supply* of children. The excess supply of children is the sole motivation of fertility control, while the actual control used depends on the costs of this control. Easterlin's ideas can be represented graphically, if we assume that the quality of children is constant, so that the parental demand is in terms of child numbers only. Further, assume that we live in a "perfect contraceptive" society (see Bumpass and Westoff 1970), in which the information of contraceptives is perfect, and their use is painless and costless. In this case the utility function an budget constraint of a household are

$$\begin{aligned} U &= U(C,\ Z), \\ \pi_c C + \pi_z Z &= I, \end{aligned}$$

in which C is the number of children and Z is that of other goods and services (Easterlin 1978, 63). In Figure 2 the budget line is given by fe. The demand for children, C_d, refers to the tangency point of fe and the indifference curve at point A. Easterlin's synthesis, however, combines the supply of fertility with the demand side. The concept of natural fertility, C_n, refers to the number of children born to a couple in the absence of any deliberate birth control. Natural fertility is drawn as the vertical line in Figure 2, in which the considerable difference between C_n and C_d says that children are in large excess supply. In a perfect contraceptive society, the gap of excess supply is costlesly eliminated by use of contraceptives. In this case the actual number of children born were equal to the number demanded C_d.

In the real world, the use contraceptives is not painless or costless. The use of contraceptives may create disutility, such as inconvenience or moral resistance, and market costs. Therefore, both the utility function and the budget constraint are affected. The indifference curves of households grow steeper (the marginal rate of substitution between children and goods increases), and the budget line shifts down due to the fixed costs of contraception dg, and rotates counterclockwise due to the variable costs of contraception. The new budget line hg is tangent to the new indifference curve at B. The new equilibrium demand for children is C, which is also the number of children born to a couple. The number of avoided children is $C_n - C$. Note that some unwanted children $C - C_d$ still exist due to the costs of contraceptives.

Figure 2 only shows an instantaneous situation, describing a case, in which $C_n > C_d$. It is possible that the C_n-line is located left of C_d. A stylized history of a fertility revolution in the course of modernization can be told as follows (see Easterlin 1975, and Easterlin and Crimmins 1985): In premodern society, children usually were in excess demand. Due to bad nutrition of fertile women

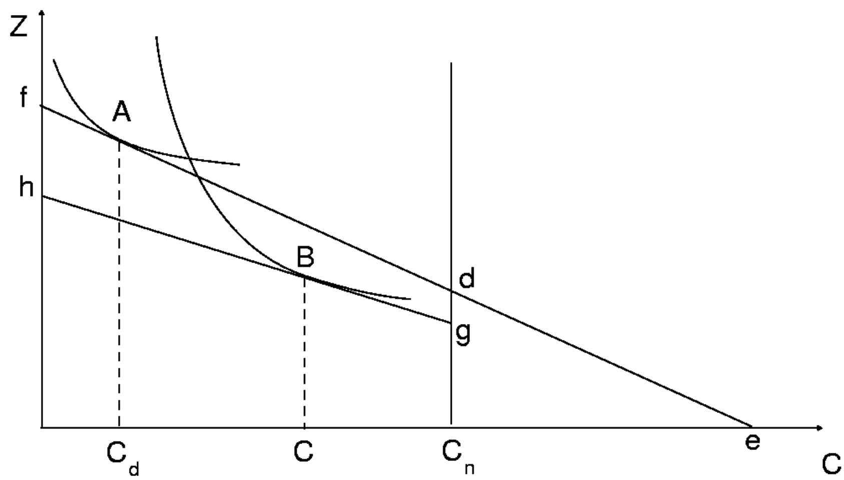

Figure 2: Natural fertility C_n, the number of children demanded in a "perfect contraceptive" society C_d, and the number of children demanded C in the precence of costs for contraception. Easterlin and Crimmins (1985).

and high infant mortality, C_n tended to be low, while, due to high preferences toward children, C_d tended to be high. The actual number of children C was limited by the supply factor C_n. This situation holds left of m in Figure 3. As society goes through modernization, the demand for children decreases, while the supply can increase. However, even in the case of excess supply, high control costs might prevent the use of contraceptives, which starts only at h. Because, in the course of modernization, the disutility and market costs of contraceptives decrease, the actual number of children C approaches that demanded, C_d. Fertility is perfectly demand determined after p.

John Caldwell's theory of *wealth flows* is based on the economic rationality of the reproductive behavior in a society. Because the framework of this rationality is established by social goals and conditions, these goals and conditions must be properly understood (Caldwell 1982). There are basically two modes of production accompanied by two modes of fertility behavior. The familial mode of production is typical in primitive and traditional societies. It is characterized by intergenerationally exploitative economic relations, which favor the older generation in terms of the younger (Caldwell 1982). The net flow of wealth — money, goods, services and guarantees — goes from children to parents, and therefore the maximum number of children maximizes these flows.

Based on his investigations in Nigeria's Yorubaland, Caldwell argues that fertility starts to fall as the net flow of wealth turns from parents to children. This takes place only if the labor market mode of production, and the nuclear

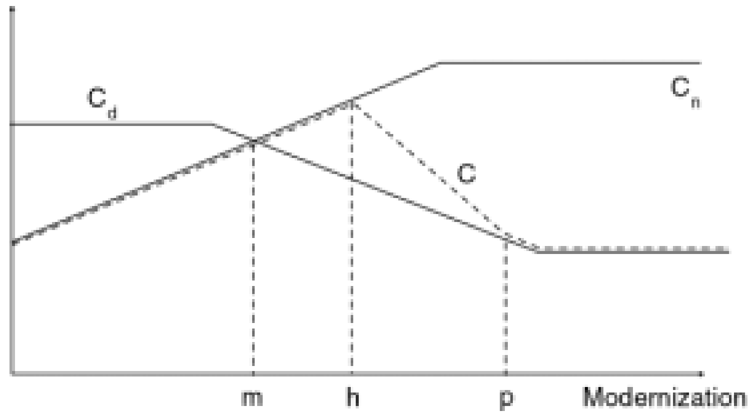

Figure 3: Demand for and supply of children in the course of modernization. Easterlin and Crimmins (1985).

family mode are adopted (Caldwell and Caldwell 1997). However, the familial system is supported by a morality that justifies the exploitation of the younger generation. Therefore, even if the direction of the wealth flow has changed, the decline of fertility is not immediate. The length of the fertility lag is shortened by two phenomena. The first is mass education, which tends to increase the status of the younger generation and the cost of children to their parents. The second is westernization — the import of western values and customs above those required by the sole modernization of the economic system — which also erodes the traditional values and morality.

Kaplan's (1994) study shows that the methods of sociologists' can be different from, but not less interesting than those, used by economists. Kaplan argues that the direction of wealth flows between the generations — the sole determinant of fertility according to Caldwell — can be best studied in small size in-kind economies for the reason that the wealth in this case can directly be identified by food. The arrangements were the following: Kaplan found three hunter-gatherers, the Aches in Paraquay (a tribe with 200 members), Piros in Peru (200 members), and Machiquengas (105 members) also in Peru. During the day, the hunting and gathering groups were accompanied by a research worker, who wrote down all the food received and transformed different types of foods to calor equivalents. The age of person supplying that "wealth" was recorded. The food consumption in each age was taken to be the average energy expenditure calculated as a function of age, sex, and weight in World Health Organization 1985. Every individual in each tribe was weighted. The results were similar in each tribe: children under 18 were net receivers of wealth, a result which seems to contradict Caldwells theories, which say that the high fertility in primitive economies (familial mode of production) were due to wealth flows from children to parents.

1.2 Cultural Theories

A modification of the fertility demand theory, promoted by Lesthaeghe and Surkyn (Lesthaeghe 1983, and Lesthaeghe and Syrkyn 1988) concentrates on the role of changing *tastes*. Tastes have changed along with great cultural and ideological movements. These movements have their roots in the philosophy of the Enlightenment, which emphasizes individual freedom and choice. Actually, the "...fertility decline is in essence part of broader emancipation process" (Lesthaeghe 1983).

The changes in taste take place in three different dimensions. First, political liberalization, accompanied with secularization, moved fertility out of the domain of the sacred to that of individual choice. Second, the newly promoted value of an individual shifted tastes from collective responsibilities toward the utility maximization of a single couple or family. This process was favored by the formation of the bourgeoise nuclear family, where child "quality" was cherished. Third, economic growth made it possible to satisfy new types of needs. Because needs are hierarchically ordered (according to Maslow 1954), new needs are different from old ones. Typically, higher needs are individually oriented, like the need of freedom and self-fulfillment (Lesthaeghe and Surkyn 1988). Strong economic growth also favors feelings of self-confidence and bright prospects, which makes (young) people less willing to adopt traditional modes of behavior. Taken together, the demand theory of fertility can explain the past and the present fertility decline only if it is augmented by the changes in tastes due to changes in the culture.

The *"ideational"* theory of fertility transition lays emphasis on the diffusion of ideas and information between areas and countries.[3] Even if the economic reasoning of child demand and supply might tell something of the very origins of fertility transition, economic theories do not provide a plausible explanation of fertility trends during the last century (Cleland and Wilson 1987). For example, the rapid speed of transition in Europe within a remarkable short period of time in spite of large economic differences across the continent, refers to the spread of knowledge of fertility control practices (Knodel and de Walle 1979). Susan Watkins (1990) counts three factors that promoted the diffusion of ideas and information in Europe, namely, the integration of national markets, the expansion of the role of states, and the nation-building process, which lead to a decrease in language barriers.

The diffusion of ideas and information can take place both through "contagion" from person to person, and through some external source, like mass media, or deliberate family planning programs (Rosero-Bixby and Casterline 1993). In their study of developing countries, Bongaarts and Watkins (1996) noted that even if fertility was negatively related to the level of socioeconomic

[3] For the term "ideational", see Mason (1997).

development in a country, which is what economic theories suggest, two problems between development and fertility decline appeared.[4] First, if the fertility decline started in a macro-region, neighboring countries followed much earlier than what was predicted by the development level itself. Next, if a country entered the transition at a relative high level of development, the pace of fertility decline was faster. The explanation proposed by Bongaarts and Watkins concerns social interactions, personal, local, and international, which tend to move the fertility transition from country to country. Their thesis is that, at a high level of development, the channels of social interaction are more developed. This leads to a more rapid transmission of new information. Barriers to interaction, such as ethnic diversity or language difficulties, tend to impede the diffusion of fertility transition, whereas participation in the global society and population movements tend to accelerate it.

Carlsson's (1966) study is worth mentioning here, because this early article preceded and apparently influenced the theoretical papers above. Carlsson argued, that birth control was not new to mankind. Instead of being explained by the innovation of new contraceptive methods, the fertility decline in Sweden from 1870 to 1960 can be explained by diffusion of new, more liberal values and attitudes. The diffusion has two dimensions. First, the diffusion should start earlier in metropolises than in rural areas; Carlsson obtained (descriptive statistics only), that if fertility in Stockholm even initially was lower than in other urban or rural areas, the tempo of its decline was still faster in the capital. Second, it is also possible that some regions are reached later. Carlsson indeed found that fertility in the northern Sweden was initially much higher than elsewhere. In addition, Carlsson discovered that in these areas the decline of fertility started later than in the other parts of the country.

1.3 Homeostatic Theory

The *homeostatic* or equilibrium theory of population growth is a biologically oriented theory. It deals with the conflict between the carrying capacity of the environment and the potential to rapid growth of mankind, fueled by the "passion between sexes" (Hirschman 1994).

Recent homeostatic theories derive from Robert Malthus's ideas of reproductive cycles which are due to the constant tendency of population "to increase beyond the nourishment prepared for it" (Malthus 1914, 5). According to Malthus, because the most fertile land were first utilized by man, the productivity of land decreases as new areas are cultivated. Instead, the reproductive capacity of people stays constant. Therefore, every discovery of a new piece of land or agricultural technique leads to an increase in wages and employment but, unfortunately, to a much larger increase in population. This in turn increases the demand for food (over the increase in supply), and the food price increases, eroding the real wages to the subsistence level again. In the cases in

[4] Socioeconomic development was measured by a human development index that combined life expectancy at birth, literacy rate, and real GDP per capita.

which the tendency for an exponential increase in population is not restrained by a preventive (negative) check of postponed marriages, it is mastered by a positive check of increased mortality.

Malthus, relying totally on the diminishing returns to land, was not able to see the revolution of technical progress which invalidated his gloomy predictions. Some economists even assume that population growth has stimulated this progress: Esther Boserup (1965) argues that population pressure makes the adoption of new technology compelling for survival. Romer (1990), Kremer (1993) and Jones (1997) propose that the number of new ideas is directly proportional to the number of people trying to invent them. Along the same lines, Fogel (1999) argues that better economic functioning of the current race is due to the abatement of chronic malnutrition, which was virtually universal three centuries ago.

There is also another stumbling block in the homeostatic (Malthusian based) theory. It seems to be unsuccessful in explaining the ongoing phase of demographic transition. Recently, the relationship between the demographic rates and the level of income seems to be in wrong direction: the poorest are the most fertile. Ronald Lee (1987) sees this as follows: "National production came to depend very little on land, mortality became largely independent of income, and fertility came to respond perversely to growing productivity of labor." Because of this deficiency in explaining the short run transitional details, the homeostatic theory might most successfully be applied in the long-term analysis of the demographic trends in the past and in the future.

2 Some Empirical Results

2.1 Model for Fertility Level

According to Mason (1997), the theories above should be seen as alternative or partial rather than as competitive explanations of the fertility transition. From an empirical point of view, this means that — instead of trying to fit a model with a single explanatory variable — we should try to fit a model with a combination of variables mentioned in these theories. We have chosen a set of ten variables that measures the central dimensions of the theories, and for which reliable data are available for a broad sample of countries. The level of per capita income (GDP) is chosen for three reasons. First, because the household demand theory relates the number of children demanded to the level of income. Second, the ideational theory assumes that the channels of social interactions are more developed in high income countries. Third, the traditional theory and the excess supply theory assume that the level of fertility depends on the level of socio-economic development or modernization in a country, and we propose that these factors are best measured by GDP per capita. The rate of economic growth ($GROWTH$) and an index of political freedom ($FREEDOM$) are included because tastes theory assumes that liberal, anti-natalistic values are typical during economic growth and with a politically free, uncontrolled atmosphere.

Variable	Related theory	Sign	Symbol
Dependent variable			
Log of total fertility			$logTFR$
Explanatory variables			
Log of p.c. income	Demand, Traditional, Ideational, Supply	+ / -	$logGDP$
Growth of p.c. income	Tastes	-	$GROWTH$
Log of number of radios	Ideational	-	$logRADIOS$
Without schooling, %	Ideational, Wealth Flow	+	$NOSCHOOL$
Export+import, % of GDP	Ideational	-	$TRADE$
Freedom	Tastes	-	$FREEDOM$
Agricultural labor force, %	Wealth Flow, Demand	+	$AGRILAB$
Female labor force, %	Traditional, Demand	-	$FEMLAB$
Log of population density	Homeostatic	+ / -	$logPOPDEN$
Log of lagged infant mortality	Traditional	+	$logMORTIN$

Table 1: The variables used and their relation to different theories.

The number of radios per thousand people ($RADIOS$) refers to availability of information and ideas (ideational theory). The percentage of people without any schooling ($NOSCHOOL$) refers to poor functioning of the diffusion mechanism (ideational theory), and to the low status of the younger generation (wealth flow theory). The openness of trade ($TRADE$) is included because information and ideas are transported together with goods. The percentage of the agricultural labor force ($AGRILAB$) refers to the generality of the familial mode of production, central in the wealth flow theory, but it is also present in the version of household demand theory provided by Lucas (2002). The percentage of female labor force ($FEMLAB$) refers to the status of females, mentioned in the traditional theory. This variable can also interpreted as an indicator of women's price-of-time and as an incentive for investments in human capital (household demand, especially Lucas 2002). Population density ($POPDEN$) is important in reference to the carrying capacity of the environment (homeostatic theory). We also include the infant mortality rate ($MORTIN$), because a decrease in infant mortality is assumed to be a necessary condition for a decrease in fertility (traditional theory).

Table 1 summarizes the variables and their functional formulas and gives the expected signs for each variable. Both signs are possible for $logGDP$ and $logPOPDEN$. The former refers to the fact that the household demand theory leaves the dominance between the pure income effect and the price effect of the mother's time open. The latter refers to the two competitive interpretations for the effect of the variable: the old Malthusian (1914) interpretation says that availability of land (and other natural resources) increases fertility, whereas the new interpretation given by Lee (1987), says that the reverse is true.

Data for these variables can be found in periods of five years from 1965 to 1995 (seven periods) for 73 countries, 22 of which are in European origin (*Early*

Regression Model	1 OLS	2 Two-way FEM	3 Panel $MEANS$	4 Two-way FEM	5 Panel $MEANS$
$logGDP$	-3.001	26.427	-33.802	-9.183	4.364
	(1.00)	(2.97)	(1.28)	(3.28)	(0.61)
GR	-0.891	-3.103	-4.486		
	(2.92)	(4.34)	(0.544)		
$\log RADIOS$	-1.054				
	(1.75)				
$NOSCHOOL$	0.270				
	(3.67)				
$TRADE$	0.084				
	(3.04)				
$FREEDOM$	-1.227				
	(1.99)				
$AGRILAB$	0.531	1.217	-0.179	0.523	0.53
	(5.37)	(3.46)	(0.28)	(3.10)	(2.30)
$FEMLAB$	-0.625			-2.124	-0.149
	(5.76)			(7.06)	(0.49)
$logPOPDEN$	-4.458			64.636	-3.842
	(7.62)			(6.68)	(2.26)
$logMORTIN$	28.604	9.224	-12.542	10.694	37.914
	(8.95)	(2.19)	(0.77)	(2.59)	(4.97)
R^2	0.87	0.90	0.14	0.94	0.77
Sample	All	Early	Early	Others	Others
Countries	73	22	22	49	49

Table 2: The regression results for total fertility rate. In pooled OLS, heteroscedasticity corrected standard errors are used. All coefficients multiplied by 100. Absolute t-values given in parenthesis.

countries) and 49 are countries in other continents (*Other* countries). Most of the latter are developing countries. For a list of the countries, see Appendix A and for some demographic statistics, see Figure 1. The functional formulas of the variables follow the standard rule that a logarithm is used, if the variable is given in absolute numbers. Because the percentage of women in the labor force might be endogenous to the contemporary total fertility rate, it is proxied by the lagged value of the same variable. In the following discussion, we limit our methods to those given by standard panel technique alone.

We now regress the level of (the log of) total fertility rate (TFR) against the variables above. As a starting point, we fit the pooled OLS-model $y_{it} = \alpha + \beta' x_{it} + \epsilon_{it}$. The results are reported in regression 1 in Table 2.

The model explains 87 % of the variation in (the log of) total fertility rate.

Most of the variables are of the expected sign. Unfortunately, the sample is very heterogenous, a fact that is shown by the high value of Breush-Pagan test ($\chi^2 = 39.08$, $p = 0.0000$), and it is necessary to ask, whether it is justified to fit a single model for this large sample of countries. The answer is no. The demographic transition began in Europe and its offshoots more than fifty years earlier than in the rest of the world, and it is not — at least on a historical or a theoretical basis — reasonable to fit the same model to both group of countries. The empirics confirms this. The F-value for common slope coefficients for the large sample and sub-sample *Early* is 5.84 ($p = 0.000$).

Therefore, we continue with two sub-samples *Early* and *Others*.[5] A closer look on the sub-sample *Early* shows that it contains two problematic countries, Argentina and Uruguay. The demographic transition in these countries started early, but during the research period, their political and economic situation has been notably different from that in the rest of the group. Recently, they are outliers in the sub-sample *Early*, and we exclude these countries from this sub-sample.

Even in the sub-samples, some heteroskedasticity is left, and the pooled OLS estimators might be — even if consistent — inefficient. A random effect model (REM) $y_{it} = \beta' x_{it} + \mu_i + \epsilon_{it}$, in which μ_i is a country-specific error term, corrects the inefficiency. The Lagrange multiplier test value 94.53 ($p = 0.0000$) favors REM over OLS in the *Early* countries and the value 241.30 ($p = 0.0000$) in the *Other* countries. However, if the country specific error term is correlated with the explanatory variables both OLS and REM are inconsistent, but a fixed effect model (FEM) $y_{it} = \alpha_i + \beta' x_{it} + \epsilon_{it}$, in which α_i is a country-specific fixed effect, is still consistent. The Hausman test value is 84.54 ($p = 0.0000$) in the *Early* countries and 60.40 ($p = 0.0000$) in the *Other* countries. This favors FEM over REM in both sub-samples.

The next problem is with the dimension of time. Total fertility rates and explanatory variables have very different values in 1965 than in 1995, and the next question is, how much of the fertility changes can be explained by time itself. In both sub-samples, the time dummies are significant: $F = 9.27$ ($p = 0.0001$) for the *Early* countries and $F = 9.58$ ($p = 0.0000$) for the *Others*. Therefore, we ultimately fit for both sub-samples a two way fixed effect model

$$y_{it} = \alpha_i + \lambda_t + \beta' x_{it} + \epsilon_{it},$$

in which λ_t is the coefficient for each period dummy variable. The models for sub-samples contain many variables that are not significant at 5 % level. By moving step-wise backwards, we eliminate these variables. The results are given in regressions 2 and 4 in Table 2. Let us concentrate on these results.

First, note that the variable $NOSCHOOL$ is not present in either of the models. This confirms the results derived by Barro and Lee (1994), and Barro and Sala-i-Martin (1995): the role of education is far from self-evident in fer-

[5] Countries in the subsample *Early* have reached the total fertility of five children per woman before 1950/55. Most countries actually reached the value $TFR < 4$. The definition is originally given by United Nations (1999). For the list of countries, see Appendix A.

tility choice. Next, both models contain variables $logGDP$, $AGRILAB$, and $logMORTIN$. All these variables have changed significantly during the research period in both groups of countries. The share of agricultural labor force decreased from 17.8 to 5.1 in the *Early* countries, and from 61.1 to 41.2 in the *Other* countries. This induced a decrease of 1.2 and 1.1 children per woman in these countries. The infant mortality rate decreased from 45.5 to 9.0 in the *Early* and from 130.9 to 64.0 in the *Other* countries (see Figure 1). This decreased fertility by 20.4 % and 9.1 % respectively. Figure 4 illustrates the relationship between the (log of) lagged infant mortality and total fertility rates in both sub-samples.

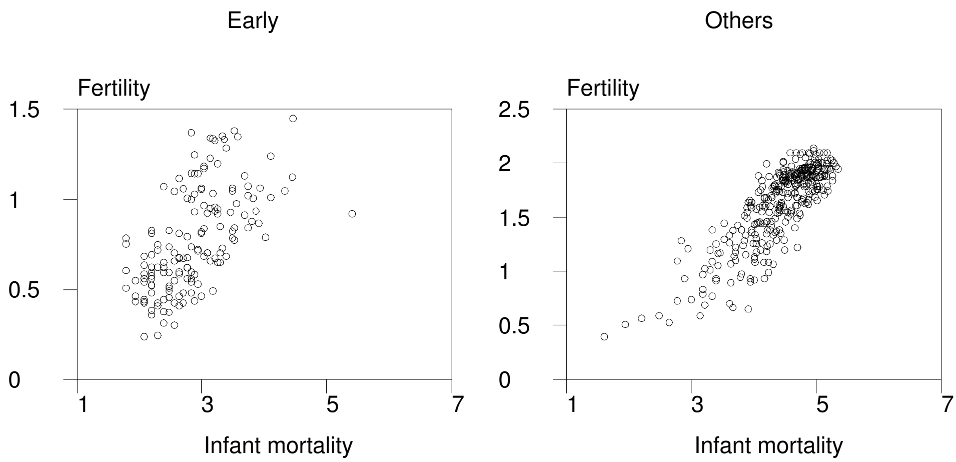

Figure 4: The relationship between the (log of) lagged infant mortality and total fertility rates in sub-samples *Early* and *Others*.

In both sub-samples, the coefficient for $logGDP$ is significant, but its sign is positive for the *Early* countries and negative for the *Others*. This means that in the *Early* countries, the income effect dominated the price-of-time effect, but in the *Other* countries the reverse was true. This is what one expects. In the *Other* countries, the level of income was low but the number of children was high initially. In this situation, an increase in income is likely to bring great gains in utility through additional consumption possibilities (income elasticity of goods is large), but only small gains through additional children (income elasticity of children is small). On the other hand, an increase in the price of (the mother's) time along with income increases, leads to decreases in fertility. This decrease can be large because, due to large number of children, the marginal utility of the last child is low. Thus, the net effect is expected to be in direction of fertility decrease. In the *Early* countries, income and consumption was high but fertility was low initially. Assuming that the marginal utility of both goods and children is decreasing, the marginal utility of one extra unit of goods is small, but that of an extra child is large. Therefore, the income elasticity of goods is small but that of children is large in the *Early* countries. This explains the dominance of the income effect over the (mother's) price-of-time effect, and fertility is expected to increase as income increases. Note, however, that these results are contrary to those derived by Barro and Sala-i-Martin (1995), who predict that income is positively related with fertility in poor countries, but negatively in rich ones.

In the *Early* countries, the coefficient of the economic growth rate is negative, as was expected. Note that economic growth, referring to strong feelings of self-confidence and bright prospects (see Lesthaeghe and Surkyn 1988), is not important in the *Other* countries, most of whom are less developed countries.

In the *Other* countries, the coefficient of $logPOPDEN$ is highly significant. The positive sign of the coefficient shows that fertility has been highest where population is most densely located. This refers to the modern transitional, rather than the Malthusian interpretation. If the land per head can be seen as the ultimate measure of prosperity — which is what Malthus did —, we can see that the poorest have really been the most fertile, and Malthusian logic does not hold (see Lee 1987). However, the possibility of reversed causality can not be completely excluded. The population density in a country is determined by its demographic history, including diseases, wars, migrational movements, and past fertility and mortality rates. However, high or low fertility rates have been long-lasting trends. Therefore, it is possible that high fertility rates experienced by some countries during the research period have lasted for such a long time that they have had a significant effect on the population density in these countries. In this case, the causal order is not necessary from density to fertility, but from fertility to density.

Because demographic transition got started in different time in different countries, and because transition so closely was related to change of "social fabric" as a whole (see Notestein 1951), one has a good reason to ask, whether the good explanatory power and the high significance of some variables in models 2 and 4 refer to correlation rather than to causality. To get *some* insight, we compare the results of regressions 2 and 4 to the results of panel means models 3 and 5, in which the average value of the regressand is regressed against average value of regressors. In regression 3 for the sub-sample *Early*, none of the regressors suggested by regression 2 is significant. The *average* level of, say, mortality, or the *average* rate of economic growth from 1960 to 1995 had no effect on the *average* rate of fertility in a country even if the *decrease* in mortality and the *increase* in economic growth *decreased* fertility during this same period. Therefore, not the difference in levels but the variance of regressor during the period was important, and we are inclined to conclude that causality, not plain correlation has been discovered.

The sub-sample *Others* is more problematic. The explanatory power of regression 5 is rather high and significant regressors are present. For example infant mortality seems to be a more powerful explanatory variable on the average than in the details of its change. Still it is too far-fetching to say that this refers to presence of correlation and to absence of causality. Even if infant mortality decreases in all developing countries, short-run fluctuations are still common, and it is possible that just these occasional fluctuations decrease coefficient and its t-value in regression 4. To increase the quality of the model, longer periods (now five years) would possibly eliminate this type of noise. Unfortunately, the availability of data made this impossible to us. In regression 4 also the high value $F = 11.34$ in a test against similar country averages in TFR (standard analysis of variance) refers to the possibility that these countries really are on different stages of their economic and demographic development, and that this stage, reflected as typical values of regressors, rather than individual regressors themselves, might provide a better explanation for the fertility in a country.[6]

2.2 Model for Fertility Change

Instead of being seen as explanatory theories for the level of the total fertility rate, the theories described above can be interpreted in terms of the change in the fertility rate. In this case, we are interested in the pace at which the demographic (fertility) transition proceeds, and which are the factors accelerating or retarding the demographic modernization process. Therefore, our dependent variable is the change in the total fertility rate ($DTFR_t = TFR_t - TFR_{t-1}$) during each period of five years between 1965 and 1995. There are six such periods.

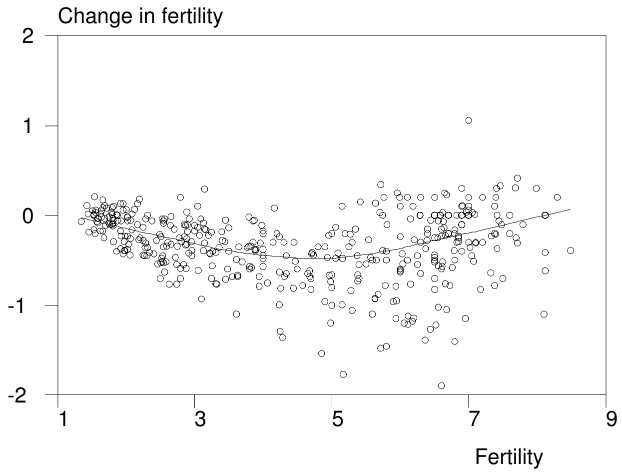

Figure 5: Total fertility rate and its change in 73 countries. Lowess-smoothing shown. Estimated minimum point is $TFR = 4.57$ births per woman. Other variables uncontrolled.

The explanatory variables are those used earlier (see Table 1), augmented by the level of total fertility rate itself. The traditional theory refers to the

[6] For the *Early* countries $F = 3.26$, but even this number rejets the null at 1 % level because of large number of observations.

demographic transition as a by-product of modernization, which takes place if the level of fertility is high. Thus the level of TFR can be seen as an important explanatory variable for the change in TFR. Figure 5 shows that TFR enters nonlinearly into the model. Therefore, a squared variable $TFR2$ is included. Figure 6 shows the conditional correlation between $DTFR$ and TFR before and after the squared term is included. The variable $GROWTH$ refers here to average growth during the period. The other variables, including $FEMLAB$, refer to the values in the beginning of the period. Both fertility and mortality are left unlogaritmized to keep the symmetry with the fertility change.

The countries in the two sub-samples, *Early* and *Others*, have advanced to very different levels in their demographic transition. Therefore, in regressing the level of fertility, two separate sub-samples were needed. However, in spite of the large differences in levels, the changes in demographic variables are not too dissimilar because the transition is still in progress in both groups. Figure 1 confirms this. Therefore, in regressing $DTFR$, the sample is homogenous enough for common slope coefficients.[7] Again, we start with OLS as given in regression 6, and then perform the standard tests to choose consistent estimators.[8] As a result, we run a fixed effect model

$$y_{it} = \alpha_i + \beta' x_{it} + \epsilon_{it},$$

for the large sample of 73 countries. The variables that are significant at 5 % level are reported in regression 7 in Table 3.

In regression 7, the explanatory role of the fertility rate is most prominent. Both the coefficients for TFR and $TFR2$ are highly significant. The estimated coefficients -33.123 for TFR and 4.529 for $TFR2$ together imply that fertility decreases most intensively at fertility level 3.66 children per woman but then abates as the number of children grows smaller. On the other hand, averaging over fixed effects, using the estimated coefficients in regression 7 and the average values for $\log RADIOS$, $NOSCHOOL$, $AGRILAB$, and $MORTIN$ in the sample, we can calculate the constant term 1.003. By using this constant term and the coefficients -33.123 for TFR and 4.529 for $TFR2$ we can see that the effect of fertility on its change is positive, if fertility was higher than 7.28 children initially.[9] Values above 7.28 were reached in nine countries, five of which come from Africa. In four of these, even the period average was above the critical value saying that the transition had not yet really started. Still there were differences between these countries. In Rwanda, fertility increased from 7.68 children in 1965 to 8.49 children in 1980. On the other hand, Kenya reached the maximum number of 8.12 children already in 1965. This maximum

[7] The F-test for common coefficients in the large sample versus the sub-sample Early gives $F = 1.52$, $p = 0.1129$.

[8] The Lagrange multiplier $LM = 20.74$ ($p = 0.0000$) favours REM over OLS, Hausman $\chi^2 = 64$, ($p = 0.0000$) favours FEM over REM, $F = 0.42$ ($p = 0.8377$) shows that period dummies are insignificant.

[9] The smaller root of the squared fertility equation is only 0.003 children per woman saying that, at this very low level of total fertility (not observed in the data), fertility were so low that its effect on fertility change would be positive again.

Regression	6	7	8
Model		One-Way	Panel
	OLS	FEM	$MEANS$
$logGDP$	-0.5614		
	(0.41)		
GR	-0.4289		
	(0.85)		
$\log RADIOS$	-2.9106	-2.495	-0.7824
	(3.01)	(2.16)	(0.28)
$NOSCHOOL$	0.2934	0.708	0.0512
	(2.19)	(2.14)	(0.24)
$TRADE$	0.1253		
	(2.64)		
$FREEDOM$	0.4955		
	(0.46)		
$AGRILAB$	-0.0053	1.118	-0.0080
	(0.31)	(2.50)	(0.04)
$FEMLAB$	0.4772		
	(2.67)		
$logPOPDEN$	-4.3734		
	(4.00)		
$MORTIN$	0.3580	0.545	0.2409
	(3.795)	(3.50)	(1.85)
TFR	-15.6716	-33.123	-6.4969
	(7.12)	(11.87)	(2.15)
$TFR2$	4.1079	4.529	4.4010
	(7.41)	(6.03)	(5.01)
R^2	0.35	0.59	0.35
Sample	All	All	All
Countries	73	73	73

Table 3: The regression results for fertility change. In pooled OLS, heteroscedasticity corrected standard errors are used. All coefficients multiplied by 100. Absolute t-values given in parenthesis.

stayed still until 1980 from which on a rapid decrease started and as low number as 5.40 was reached in 1995.

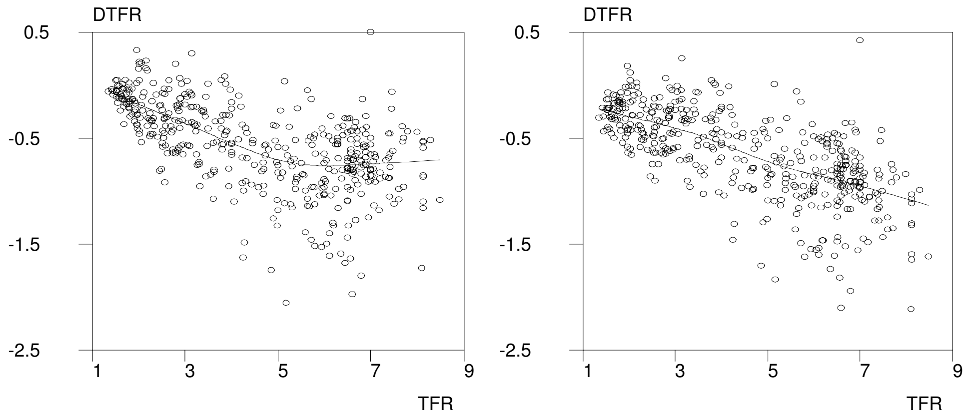

Figure 6: The change in fertility ($DTFR$) after controlling variables as in regression 6 in Table 3 without $TFR2$ on the left. $TFR2$ also controlled on the right. Component and residual plot and lowess smoothing shown.

Note, however, that regressing $DTFR_t = TFR_t - TFR_{t-1}$ against TFR_{t-1} (or its square) makes the regression equation one with lagged dependent variable as regressor. In this case the country-specific fixed effect (part of the error term) necessarily correlates with the lagged regressor and its coefficient tends to be biased. A solution would be to take first differences to eliminate the fixed effects. This leaves a nondegenerate correlation between the differenced regressor and the differenced error term $\Delta\epsilon_{it}$, and there still is a need to find an instrument for the regressor (see Bond 2002). Unfortunately, the instruments available (a two-period lagged regressor or some other variables in regression 1) tend to be weak, and large finite sample biases can be present (see Bond *et al.* 2001). In addition, our model, having originally a differenced regressand, would be difficult to interpret in this context.[10] Therefore, we continue with regression 7 using its results with the reservation needed in a case like this.

The important role of infant mortality is also confirmed: in countries where infant mortality was high, the transition advanced only slowly (positive or small negative values for $DTFR$). The average decrease from 1965 to 1990 in $MORTIN$ by 43.38 per mil points accelerated the tempo of transition by $0.00545 \cdot 43.38 = 0.24$ children per woman (see regression 7 in Table 3). The same accelerating effect (0.27 children per woman) was reached by the decrease of $AGRILAB$ (14.21 % points) and $NOSCOOL$ (15.86 % points) together.

[10] Both Bond (2002) and Bond *et al.* (2001) recommend the adoption of GMM estimation in the situation described above, but in this introduction, we limit ourselves to standard panel methods alone.

The effect of the increase in the number of radios was less considerable, 0.05 children per woman.

We run the panel means regression to see, that a good deal of explanatory power is lost by neglecting the time dimension of the data. Especially, the significancy of total fertility decreases, if handled as a mean during the period. This is what one expects. The average decrease of fertility from 1965 to 1990 was as large as from 5.35 to 3.72 children per woman. Therefore, pretty many countries moved from the stage, in which fertility, say, decreases fast to the stage, in which the decrease already levels off. In these countries, the period average fertility is not the adequate measure to understand the mechanism in question.

2.3 Concluding Comments

Mason (1997) has suggested that the theories represented in the field of demographic transition should be seen as complementary rather than as competitive. Instead of purporting to present a general theory, such as initially given by Notestein (1945), it is more accurate to provide partial explanations of the transition. From an empirical point of view, if demographic transition really has many partial explanations, then we should test them all together. To do this, we have chosen ten variables, suggested by the (complementary) theories to explain fertility and its change in 73 countries from 1965 to 1995.

The regression results in regressions 2, 4, and 7 show that the role of demographic variables is strong. All regressions 2, 4, and 7 have the variable $MORTIN$ (or its log) in common. This confirms the idea of Notestein (1945): a decrease in mortality has been a necessary condition for fertility decrease to take place. In addition, the role of the level of fertility is dominant in regressing $DTFR$. Total fertility stays high as long as mortality does so. But as mortality decreases, fertility follows, and the decrease in fertility is great if its original level was high. Outside the demographic variables, the variable $AGRILAB$ is present in each regression, and the level of GDP or its growth is always present in the regressions for TFR.

The analysis given here is too preliminary to discriminate between the traditional and alternative theories of demographic transition. For example, the choice of variables used in the model are highly limited by the availability of data, which also tended to make our sample biased toward the *Early* countries. What one can say is that neither the traditional theories (see Notestein 1945) nor economic theories (see Becker 1960, Lucas 2002, Easterlin 1978, and Caldwell 1982) are undermined by the data. The latter result is utilized in our thesis, in which the reasoning is reverted and the effects of demographic transition on economic growth is studied. To see the connection, some things should be pointed out.

First, it is the basic idea in the field of theoretical population economics that population growth is endogenous to economic variables, such as the level of per capita income or economic growth, and this is the starting point in our thesis as well. The empirical results of this introduction give support to this idea.

Note, however, that in the regressions above, we have chosen our regressand to be fertility (or its change) rather than of population growth rate, even if the latter enters as a variable in our theoretical and empirical essays in the thesis. The reason for this choice is that population growth is the difference between fertility and mortality. If population growth were used as the regressand, it is difficult to see, which of its components have really changed. Thus, even if the results of this introduction give some support to the endogeneity of population growth, the question is far from being settled.

Second, the implied endogeneity of population growth made us aware of the problems to be met in empirical studies using population growth as regressor for economic growth. Fortunately, the results of this introduction also provided some tools to evaluate them, and we were able to derive some empirical results in spite of the endogeneity.

3 Contents of the Thesis

All the theories described above provide interesting explanations for demographic transition, but only the theories of household demand are developed to well formulated mathematical models, in which adequate microfoundations are explicitly stated. In these models, the households choose, simultaneously with their consumption and savings, the child quality and quantity. Children, however, have the special feature that they are not available as goods of their own but only exist together with another good, namely sexual satisfaction. In modelling the demand for children, it is then essential to know whether these two goods can be freely combined, i.e., whether contraceptive techniques are at sufficient level to prevent unwanted births. Unless this is the case, interior equilibrium in terms of both goods is not warranted, and the actual number of children may be determined by supply rather than by demand.

In empirical data, it is hard to enter into final conclusions in terms of these two alternatives. The demographic history in Europe provides an example of the difficulties met. In the beginning of the nineteenth century, the Europeans faced a "supply shock" as mortality of adults but especially that of children decreased. This mortality decrease had its roots in technical progress, increases in income, and medical discoveries, such as invention of the small-box vaccination by Jenner. The decrease of mortality is clearly stated also in the Swedish data given in Figure 7.[11] On the other hand, the long-run data collected by Lucas (2002) show that, during the nineteenth century, the per capita GDP increased in the Scandinavian countries to 2.6 fold, referring to possible increases in demand for children. The problem is that it is hard to say to which extend the increase in supply was absorbed by the increase in demand because we have no direct data on preferences. On the other hand, we know a good deal about the history of birth control, telling that efficient techniques were discovered only

[11] The Finnish data is characterized by famine shocks and wars and is less suitable to serve as an example.

recently. Therefore, if the households in industrializing Europe would have preferred to have fewer children, they would have failed convenient, safe, and cheep methods to achieve this goal.

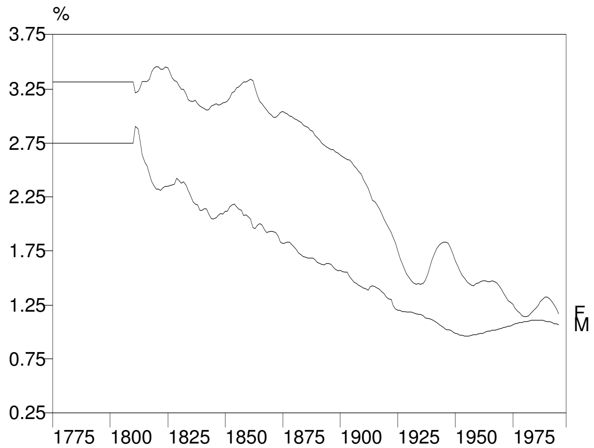

Figure 7: Mortality (M) and fertility (F) in Sweden, 10 years moving averages, period average shown for 1775-1800. Data source: Statistical centralbyron i Sweden, authors calculations.

To take the possible excess supply into account, one could explicitly model in terms of joint goods. Alternatively, one could follow the tracks of Lucas (2002) who argues that to model a "transition," it is necessary to work with two production technologies, the old and the new, to show how the erosion of the old technology led to increased incentives to invest in child quality. One could then complete the model with two reproductive technologies, the old and the new, to show to which extend this technology permits the realization of these goals. In this thesis, however, instead of trying to give a detailed description of demographic transition, we try to give a model, which is simple enough to be conveniently used in applications where demographic transition has a role. For example the following fields and questions seem to be of interest:

- Environmental economics: The environmental Kuznets curve shows that there is an inverted-U relationship between environmental degradation and income. In rich countries, many indicators of pollution decrease, whereas in developing countries, these indicator are increasing (Pasche 2002). Dasgupta (1995) points out that there may exist a causal link from population growth to environmental degradation, especially in poor countries. Because population growth in poor countries is high, their ability to save and invest is limited (Coale and Hoover 1958). This applies to environmental investments as well. On the other hand, the consumption of resources and pollution increase together with population growth. Later, these problems are alleviated as population growth decreases. Actually, one has a good reason to ask for the environmental implications of the family planning programs in poor countries. The model discussed in this thesis can be enlarged to deal with population-dependent environmental problem.

- International trade: Cleland and Wilson (1987) suggested that demographic transition has diffused from developed to developing countries and Rosero-Bixby and Casterline (1993) argued that this diffusion can take place through "contagion." Because contagion between countries has been facilitated by trade flows, international trade has possibly accelerated the onset of transition in developing countries. In some developing countries, the advantages of trade can have been masked by simultaneous disadvantages of increasing population growth typical in the beginning of transition. The "contagion" models are perhaps most successfully formulated in the North-South framework recently discussed by Baldwin *et al.* (2001). An open economy two-country (two-pole) version of the model, given in this thesis, could serve as a basis of this type of discussion.

- Convergence of international incomes: The convergence debate derives from two sources, namely from decreasing returns and from diffusion of technology. Countries with low initial capital per head should grow at a high rate and catch up to countries initially well provided with capital (Barro and Sala-i-Martin 1992), and they should also enjoy the advantage of backwardness because of the diffusion of technology (Barro and Sala-i-Martin 1997). Demographic transition has an implication in terms of both arguments. As initially poor countries started to get richer, population growth accelerated, and economic growth was lower than what one would expect. In empirical investigation, therefore, divergence, not convergence was present due to the demographic drag for growth in initially poor countries during the twentieth century. In the essay "Convergence, Income Inequality, and Demographic Clubs" we deal with this question.

To achieve the goal of simplicity, we abandon from complete microfoundations and adopt the framework provided by Easterlin and Grimmins (1985) (see Section 1.2, Figure 3) according to which demographic transition has contained stages of excess demand, excess supply, as well as stages of equilibrium. The term "modernization" used by Easterlin and Grimmins has its operational counterpart in the per capita income. The stylized history of demographic transition then becomes: At a low level of income, children were in excess demand and the actual number of children was limited by supply. This situation holds left of m in Figure 3. As income increases, there is a change in child demand. On the other hand, supply increases as mortality decreases and fertility increases.

Even in the case of excess supply, high costs and poor information of contraceptives prevent their use. However, developing markets and increased contacts between people decrease these barriers (Watkins 1990) and the use of contraceptives starts at h. The actual number of children approaches that demanded, but fertility is perfectly demand determined only after p. Population growth — the actual number of children — endogenously increases and decreases as a function of income

The stylized history is summarized in a bell-shaped population function. Through most of the transition, we assume that the number of children is not determined by demand but by supply. Therefore, at each level of income, the households take as given the income-typical number of children, as shown by this function. As long as we think of the beginning of the transition, this is meant to be a true description of the situation, but after point p, this solution should be seen as a simplification alone. Further, we assume that rational households choose consumption and investments as to maximize their utility. But because population growth is endogenous to income, by making their choice between consumption and savings, the households implicitly determine the long run behavior of population growth and, as a function of income, population growth first increases and then decreases (see C in Figure 3). The lack of complete microfoundations is necessarily a deficiency, but our model, instead of trying to give an as-accurate-as-possible description of demographic transition, tries to give as-simple-as-possible solution, which still maintains the most fundamental fact, namely that population growth endogenously increases and decreases, and this model is mainly build up to serve in macroeconomic applications, such as described above.

The thesis consists of three essays. The essay "Demographic Transition in the Ramsay Model" introduces the basic theoretical framework, the essay "Learning by Living: Early Development" gives a theoretical application. The essay "Convergence, Income Inequality, and Demographic Clubs" supplies some empirical applications of the basic model.

3.1 Demographic Transition in the Ramsey Model

The essay takes as its starting point the controversy between empirical results and growth models. Demographic data show that population growth rates vary because of demographic transition, but most economic growth models take them as a constant. Figure 8 gives an example.

We argue that population growth in demographic transition is a bell-shaped function of per capita income y. Further, because the growth models are usually formulated in terms of per capita capital, k, rather than in terms of per capita income (a monotonously increasing function of the former), we introduce a *population function*, $n = n[k(t)]$, with the properties

$$\begin{aligned} n'[k(t)] > 0 &\Leftrightarrow k(t) < \mu, \\ n'[k(t)] = 0 &\Leftrightarrow k(t) = \mu, \\ n'[k(t)] < 0 &\Leftrightarrow k(t) > \mu, \end{aligned} \tag{4}$$

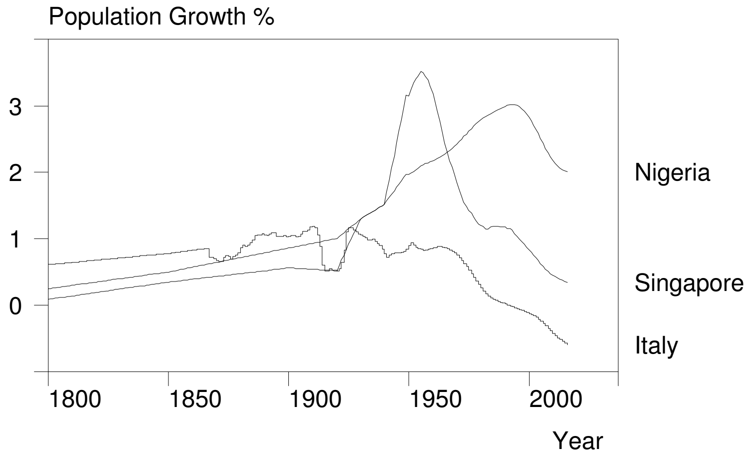

Figure 8: Population growth rates in Italy, Nigeria, and Singapore.

in which $n[k(t)]$ and $n'[k(t)]$ are the population growth rate and its change, and the per capita capital $k(t) = \mu$ is the level from which on the population growth rate starts its decrease.[12] We then discover the dynamic implications of demographic transition by introducing the population function $n[k(t)]$ into a standard version of the Ramsey model.

The first problem to be solved is that the discount rate of the model becomes variable. Instead of writing the objective functional as

$$U = \int_0^\infty u\left[c\left(t\right)\right] e^{-(\rho - n)t} dt,$$

it is to be written as

$$U = \int_0^\infty u\left[c\left(t\right)\right] \cdot \exp\left\{- \int_0^t \left\{\rho - n\left[k\left(\tau\right)\right]\right\} d\tau\right\} dt,$$

in which $u\left[c\left(t\right)\right]$ and ρ are the instantaneous utility derived from per capita consumption $c(t)$ and the time preference factor respectively. Uzawa (1968), in studying variable time preferences, has first shown, how to use the substitution rule of integration in this context. To put it more colorfully, the problem is to be transformed from “natural” time to “virtual” time, in which the discount rate is constant. In this time the model is regular, because both utility and production functions are concave. Earlier theoretical results are available, and the problem can be solved by using standard methods.

[12] The idea that population growth could be modelled as a function of k can be already found in Solow (1956), even if he was not yet able to give the interpretation of demographic transition to this function.

The analysis of the results shows that multiple equilibria (steady states) may or may not exists. Because the necessary conditions are non-linear, it is hard to limit the theoretical number of steady states, and we concentrate on the generic cases with one or three of them. A standard local stability analysis shows that the single steady state is saddle stable. In the case of three steady states, the first and third are saddles, but the second is an unstable focus or node. By using conventional reasoning we show that all other candidates for optimal paths are dominated by the stable branches of the saddle paths toward steady states.

Multiple equilibria in neoclassical growth models exist where

- competition fails and monopolistic competition is present, or where
- external effects or some other factors create increasing returns to the production function.

Many models share one (or both) of these features, but have their main focus in endogenous growth or in other interesting features of the model (see, for example, Azariadis and Drazen 1990, Romer 1990). However, in some models the features of these equilibria, such as the local and global stability, are studied for their own right. These models are closely related to the models presented in this thesis.

Matsuyama (1991) studies the career selection problem between agricultural and manufacturing industries with increasing returns prevailing in the latter. Matsuyama describes the global bifurcation technique to study the effect of the value of the parameters in the model. Benhabib and Gali (1995) provide a generalized framework that is useful in dealing with both externalities and market imperfections. Gali (1996) discusses a model with monopolistic competition and variable demand elasticities, and Yip and Zhang (1997) introduce a model with endogenous fertility and productive externalities. These models share the feature that the market solution is indeterminate. Some initial states may be such that several steady states can be reached, but no self-evident rule exists to say which of them is selected (is the realized equilibrium by the decentralized system).[13] One solution suggested is that, if each individual is optimistic, i.e., believes that the high income steady state is reached in the future, and behaves accordingly, then these beliefs will be self-fulfilling. The essay gives a solution to the equilibrium selection problem by seeking the optimal solution for the central planner's problem in a way first discussed by Skiba (1978) and later by Tahvonen and Salo (1996). We also discuss the problems arising in the competitive case.

The results show that the stable path toward the high income steady state can run from the origin or it can spirally emanate from the unstable steady state between the two saddles. The former path can be shown to be globally optimal. In the latter case, starting with high initial capital stock, it is optimal to proceed

[13] Indeterminacy also arises in cases in which the single steady state is a stable node. In this case several equilibrium paths, all satisfying the necessary conditions, lead toward this steady state.

towards high income steady state, but starting with low initial capital stock, the path leading to low income steady state is optimal. Therefore, the model has a poverty trap (the low income steady state). The trap can be escaped only if the productive capital available rises above the threshold value, from which on the path toward the high income steady state becomes optimal. To summarize, three globally different cases arise: the single steady state case, the case of three steady states with globally optimal path from the origin, and the case of three steady states, one of which is a poverty trap. The exact functional formulas specified (in particular, the parameters used) in the model then determine which of these cases is realized.

The empirical examples (see Figure 8) show that economies have experienced different transitions in terms of timing and shape. In the model, these differences are reflected as differences in productive capital during the transition, as differences in sensitivity of population growth to income (or capital), and as differences in peak population growth rate. Three types of transitions, *weak*, *moderate*, and *strong*, refer to three types of global dynamics described above. To analyze these types, a calibrated version of the model is given. The production function and the utility function, as well as their parameters are the standard ones, and this essay concentrates on the role of demography. The parametrized version of the population function is

$$n(k) = \eta \cdot \exp\left\{-\frac{1}{2}\left(\frac{k-\mu}{\sigma}\right)^2\right\},$$

in which η is the peak population growth rate and μ is the value of productive capital from which on population growth starts to decrease. The parameter σ gives the dispersion of the transitional period. The lower is this dispersion, the higher is the sensitivity of population growth to income (capital). This functional formula satisfies the important limit condition $\lim_{k\to 0}\{n(k)\} < \infty$ keeping the slope of the effective depreciation line below the production function in the origin, and thus warranting the existence of interior steady states in the model. The parametric calculations show that high values of μ and η and low value of σ refer to strong type of transition, i.e. to the existence of a poverty trap.

The parameter η has its straightforward counterpart in empirical data and can be read in Figure 8. Instead, the parameters μ and σ are given in terms of productive capital, not directly in terms of time, and the calculation of the value of these parameters, for example, in Italy, Singapore, and Nigeria needs some further efforts. But based on what is known of high peak population growth rates in developing countries, it can be argued that developing countries are (or have been) in danger to be captured by the poverty trap, and it is of some value to evaluate this possibility.

Obviously, Singapore has already escaped, and we think that Nigeria will also do so. Actually, we would like to suggest that all developing countries will succeed in avoiding poverty trap. But then, if poverty trap is only a theoretical possibility, why should we care of it? Or, to put it differently, why should we care of the number of steady states? The transitional dynamics of the model show that the convergence result of the standard Ramsey model (growth rate decreases monotonously) breaks down. In the augmented model, as transition proceeds and population growth accelerates, it is optimal to increase the rate of capital accumulation to get over the transition peak as fast as possible. This increase is done at the cost of consumption which is stagnated for a long period of time.

3.2 Learning by Living: Early Development

Goodfriend and McDermot (1995) show that during the epoch that can be called "early development," some unique events took place, and models working in historical contexts are needed. In purely demographic context, "early development" is defined to be the stage of demographic transition during which population growth accelerates because mortality decrease preceded the decrease in fertility. This period ran approximately from 1800 to 1900 in Europe and its offshoots. As mortality decreases, the expected length of life increases. From 1800 to 1900 life expectancy in Europe increased from some 30 years to some 50 years and practical possibilities for learning through formal schooling, and the expected returns for this schooling increased. We argue that learning through family teaching also increased because the overlap of generations increased.

In the model of the essay, learning takes place by living (see Arrow 1962 for the concept of learning by doing). Human capital accumulation is a costless side effect of mortality decrease. This approach emphasizes possibilities and demand for learning but leaves aside the costly supply side of formal schooling. The essay derives from the notations and ideas in the field of "new growth theory," but has the distinguishing feature that rewardable resources to accumulate human capital are not used.

We motivate the accumulation of human capital as a function of mortality decrease or life expectancy increase "semi-endogenously" (see Kremer 1993 and Jones 1995), and we give the accumulation equation in terms of partial derivatives, and in parametric functional forms. Using this technique, the reversed causality, i.e. from human capital to mortality is modelled. Some historical connections are given. Further, causal links from human capital to fertility and from income to mortality and fertility are derived. Finally, because population growth during early development mainly took place through mortality decreases, we create a causal link from population growth, n, to human capital accumulation. Another causal link then goes from human capital and per capita income, $\frac{Y}{L} = y$ to population growth. Let H be the stock of human capital and $h(n)$ its rate of change. Then

$$\begin{aligned} \frac{\dot{H}}{H} &= h(n), \\ \frac{\dot{L}}{L} &= n = n(\frac{Y}{L}, H) = n(y, H). \end{aligned}$$

Equations for population growth and human capital accumulation are introduced into a standard Ramsey-type model of consumer optimization, where production function is

$$Y = F(K, HL).$$

The solution of the model is technically very similar to that explained in essay "Demographic Transition in the Ramsey Model." The model might have multiple equilibria, and we ask, under which conditions (at which values of the parameters) the poverty trap does exist.

In a multiequilibrium model, the global dynamics can be such that the high growth steady state may or may not be available from all initial states. The comparative dynamics of the model show that the role of the income share of capital is decisive. If this share is high, the high growth steady state can be reached even from capital stocks which are initially low. But if it is low, accumulation of both types of capital is low, and the economy stagnates in a low income poverty trap. The result can be interpreted in terms of importance of income increase in demographic transition: capital's income share is essentially the incentive to invest and increase income in the future. If these incentives are high, a process which leads to decrease in mortality, increase in life-expectancy and population growth and accumulation of human capital can take place.

3.3 Convergence, Income Inequality, and Demographic Clubs

International income distribution has recently been studied in two quite different frameworks. The modern approach derives from the theory of economic growth and from the econometric theory of time series unit root tests, whereas the traditional framework derives from measures that originally were devoted to study all kinds of inequality. Another difference is that the traditional approach is static in the meaning that it gives a single measure of inequality for each year, whereas the modern is dynamic, capturing the information of several years into a single testable parameter. This article supplies both types of analyses.

The transitional dynamics of the augmented Ramsey model are summarized in the following time paths of the population growth rate, consumption, per capita capital, and the growth rate of per capita income.

Figure 9 shows that the economic growth rate decreases (due to decreasing marginal productivity of capital) toward the beginning of transition, but as transition really starts, capital accumulation and economic growth rate increase at the cost of consumption, and economic growth reaches its maximum during the peak of transition. The result can be understood in the light of the optimizing behavior in the model: it is best to "get over the worst" as soon as possible.

Ramsey model, based on neoclassical production function with decreasing marginal productivity of capital, implies that economic growth rate decreases as countries get richer in capital. This is the convergence result. Figure 9 shows that no such unilateral convergence of incomes is implied in the augmented model. Even so, the underlying reason for convergence — the decreasing pro-

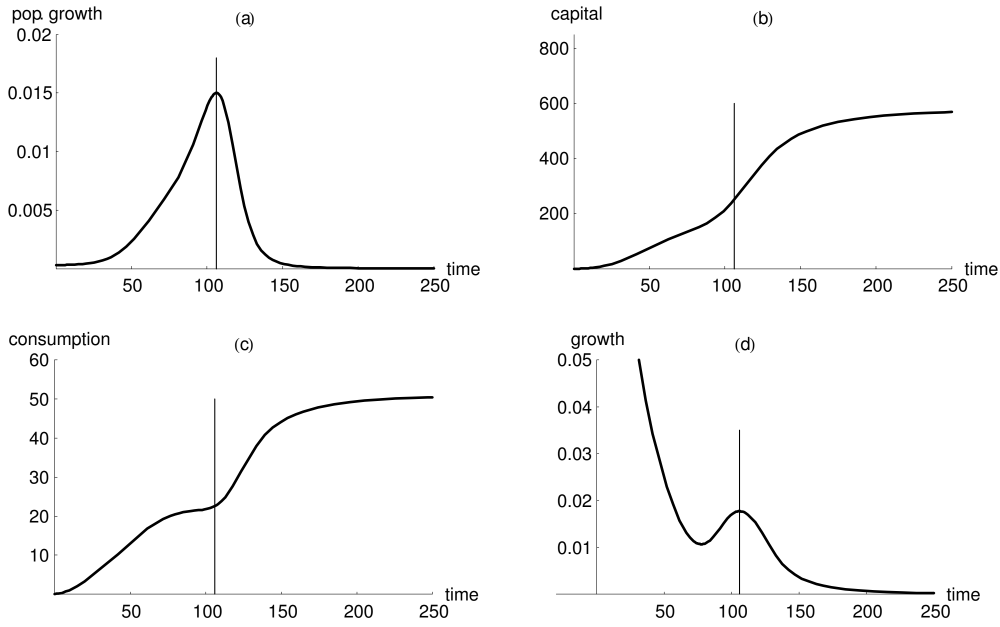

Figure 9: The transitional dynamics in the augmented Ramsey model.

ductivity of capital — is still present in the model and is only overshadowed by demographic transition. To uncover the convergence, demographic transition must be adequately controlled for. Sala-i-Martin (1996) argues that two strategies exist to perform this control. One can either explicitly introduce the shadowing factors as regressors into the model or one can partition the sample into sub-samples — convergence clubs — which are homogenous in terms of the shadowing variables so that they can then safely be omitted. In our case, there is a serious limitations are against the former strategy because due to the long time period (from 1960 to 1995) used, the availability of data for other variables in developing countries is poor.[14] Each new variable included leads to a more severe sample selection bias which in this study would be most undesirable.[15]

Therefore, in this essay we follow the latter strategy and try to create convergence clubs by partitioning the sample into sub-samples. However, neither Figure 9 nor the augmented Ramsey model give any theoretical implication, how to find the number and borderlines of such sub-samples. Therefore, we try to identify the demographic clubs directly from the data by using regression

[14] Instead, United Nations (1999) provides demographic variables for a broad sample of countries.

[15] The sample selection bias was certainly present when we derived the results for Tables 2 and 3. As compared to the sample of 110 countries in the essay "Convergence, Income Inequality, and Demographic Clubs", the introduction of the ten variables in these regressions led to a loss 37 developing countries. In this study, however, this loss was unavoidable,

tree analysis earlier applied by Durlauf and Johnson (1995). The idea of the regression tree analysis is the following: First we argue that economic growth ($GROWTH$) is, among other variables (X), determined by population growth (N) and its change (DN) so that the following equation holds:

$$GROWTH = F\left(DN, N, X\right) + \varepsilon.$$

The regressors of interest are DN and N, both of which vary within a given range giving us a two dimensional space. We now want to partition the $DN - N -$space into (approximate) level sets of the regressor $GROWTH$, i.e., to those values of DN and N producing as constant as possible values for $GROWTH$. The algorithm calculates each possible split of the space and chooses the first split such that the sum of deviancies of $GROWTH$ in the two areas created is as small as possible, i.e., the areas are as-good-as-possible level sets. The two areas are further divided by successive splits to reach the desired result. Once the number and borderlines of the regressands are found, we can divide the sample into sub-samples respectively.

Demography, however, is not the only factor that can mask the convergence of incomes, even if decreasing capital productivity holds in the model. In general, convergence is exhibited only if all other variables are adequately controlled, but we argue that demographic clubs are relatively homogenous in terms of the other variables as well, such as labor supply, education, or savings. Therefore, the only source for variation of $GROWTH$ within a convergence club is the underlying tendency for convergence. Evans and Karras (1996) have suggested a version of the augmented Dickey-Fuller unit root test to state the convergence of incomes. In Approach I this test is run in each demographic club separately with the result that three of them exhibit conditional convergence.

In Approach II we concentrate on the relationship between demographic transition and inequality of incomes at the level of the entire sample. Kuznets (1967) delivered the inverted-U hypothesis of intercountry income inequality and population growth. In its simple form the hypothesis assumes that constant resources are owned by each country. Then, because the population growth rate first decreases in rich countries, per capita income in these countries increases, whereas high population growth rate in poor countries decreases the per capita income in these countries, and income inequality between these two groups increases. Later, as population growth rates in poor countries start to decrease, income inequality decreases. We calculated the Gini coefficients for the sample of 110 countries for every fifth year between 1960 and 1995. The time profile of these coefficients is similar to the time profile of population growth rates in initially poor countries, a result that gives support to the inverted-U hypothesis.

In the future, demographic transition proceeds and countries move through successive demographic stages. The United Nations (2000) supplies projections for future population growth rates in three variants, Low, Medium, and High. We assume that the stage-specific growth rates hold in the future. Using the UN-projections for future demography, we are able to allocate each country to one of the stages and to calculate the country-specific growth rates and incomes, as well as the dispersion of these incomes in the future. Furthermore, the three variants provide three projections for future income dispersion. One can apply the inverted-U hypothesis by arguing that a fast decrease in population growth (the Low Variant) leads to lower income inequality in the future. We calculated the Gini coefficients for each variant from 1995 until 2030. The result is unexpected. The income is less unequally distributed in the High Variant than in the Low Variant. The reason is the deterioration of the worker / dependent ratio in two richest country in the sample, if the High Variant is realized.

A Appendix: Data and Variables

A.1 List of Data Sources

- Barro, Robert and Jong-Wha Lee (2000): "International Data on Educational Attainment: Updates and Implications," manuscript, Harvard University. (BL)
- Easterly, William (1999): "Life During Growth," *Journal of Economic Growth* 4(3), 239-276.
- Freedom House (2000): *Freedom in the World.* New York. (FR)
- Sachs, Jeffrey D. and Andrew M. Warner (1997): "Fundamental Sources of Long-Run Growth," *American Economic Review, Papers and Proceedings* 87, 2 (May), 184-188. (SJ)
- Summers, Robert and Alan Heston (1991): "The Penn World Table (Mark 5.): An Expanded Set of International Comparisons, 1950-1988," *Quarterly Journal of Economics,* May. Updated diskette version 5.6a (SH)
- United Nations (1999): *World Population Prospects.* The 1998 Revision, Vol I: Comprehensive Tables. New York. (UN)
- World Bank (2000): *World Development Indicators.* CD-rom version 4.2. Washington. (WDI)

A.2 Variables

- $MORTIN$: Infant mortality rate. The number of deaths of infants younger than one year. Per 1000 births in a given year. UN-estimate for years t-(t-5). (UN).
- $FEMLAB$: Female labor force, % of total. (WDI)
- $AGRILAB$: Agricultural labor force, % of total. (WDI)
- TFR: Total fertility rate. The number of live birth per woman. Total fertility rate represents the number of children that would be born to a woman if she were to live to the end of her childbearing years and bear

children in accordance with prevailing age-specific fertility rates. UN-estimate for years t-(t-5). (UN).

- *GDP*: Gross domestic product per person. In 1985 international dollars. (Chain index). (SH) and (SJ)
- *RADIOS*: Number of radio receivers per 1000 people. (WDI)
- *TRADE*: The sum of exports and imports of goods and services, % of gross domestic product. (WDI)
- *FREEDOM*: An index of political rights and civil liberties. High values of index refer to low level of liberties. (FR)
- *GR*=Average (5 years) annual growth in gross domestic product per person. (SH) and (SJ)
- *POPDEN*: Population density. Midyear population per land area in square kilometers. (WDI)
- *NOSCHOOL*: Percentage of "no schooling" in the total population. (BL)

A Note: The following replacements in the data are made: The missing data (the whole variable) for *RADIOS* for 1965, *FREEDOM* for 1965 and *AGRILAB* for 1995 is replaced by linear estimates. Missing individual observations for *NOSCHOOL* in 1965 (4 replacements) and 1970 (2 replacements) are replaced by their linear estimates. The variables $TFR, MORTIN, POPDEN$ and *GDP* for Rwanda in 1995 are replaced by linear estimates.

A.3 Samples

Early (Summers-Heston country codes in parenthesis):

Barbados (52), Canada (54), U.S.A. (72), Israel (94), Austria (116), Belgium (117), Denmark (121), Finland (122), France (123), Greece (126), Iceland (128), Ireland (129), Italy (130), Malta (132), Netherlands (133), Norway (134), Portugal (136), Spain (138), Switzerland (140), United Kingdom (142), Australia (145), New Zealand (147).

Others :

Algeria (1), Benin (3), Botswana (4), Congo (12), Egypt (14), Ghana (18), Kenya (22), Lesotho (23), Malawi (26), Mauritius (29), Niger (33), Rwanda (36), Senegal (37), South Africa (41), Swaziland (43), Togo (45), Uganda (47), Zaire (48), Zambia (49), Costa Rica (55), Dominican Rep. (57), El Salvador (58), Guatemala (60), Haiti (61), Honduras (62), Jamaica (63), Mexico (64), Nicaragua (65), Trinidad & Tobago (71), Bolivia (74), Brazil (75), Chile (76), Colombia (77), Equador (78), Guyana (79), Paraquay (80), Suriname (81), Venezuela (84), Bangladesh (86), China (88), Indonesia (91), Japan (95), Korea, Rep. (97), Malaysia (100), Philippines (106), Sri Lanka (110), Syria (111), Thailand (113), Fiji (146).

Sample All contains countries in the sub-samples *Early* and *Others* and Argentina (73), and Uruguay (83).

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Demographic Transition in the Ramsey Model

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Abstract

The paper introduces a Ramsey-type of model, augmented by a population function that summarizes the main features of demographic transition. The non-linear discounting factor problem is solved in virtual time. The central planner's solution can have several steady states and history-dependent optimal paths. The augmented model suggests that, if the population growth rate is very sensitive to income and reaches high values during the transition, and if capital stocks are large during the transition, the economy can be caught in a poverty trap. A calibrated version illustrates comparative and transitional dynamics in the model.

1 Introduction

The continuous time consumer optimization model, originally delivered by Ramsey (1928), currently serves as one of the basic approaches in many macroeconomic problems. This model, however, has the deficiency that population growth is assumed to be a given constant. The empirical argument against the constant population growth assumption is in demographic transition, which has produced a shift in fertility and mortality rates to a lower level and, because the decrease in fertility has much lagged that in mortality, has also produced a temporary increase in population growth.

Even if the principle of such a transition seems to be much the same everywhere, there exists great variety in the details of timing, duration, and population growth rates during the transition. In Europe, the onset of transition took place at the end of the eighteenth century. After having lasted some 200 years, it is now almost completed (Coale and Treadway 1986). In less developed countries, the transition emerged only after World War I and is expected to continue through this century. The average peak of the population growth rate, 2.5 %, was reached in less developed countries in the mid-1960s. Many countries have experienced rates close to 4 %, while population growth rate rarely exceeded 2 % in Europe (Livi-Bacci 1997). Figure 1 shows the demographic growth rates in three countries. The European example is Italy, Singapore and Nigeria give examples from other continents. The goal of this paper is to introduce demographic transition into the Ramsey model in a simple way, such that the model still could serve as the basic approach in continuous time but be in line with the demographic numbers described above.

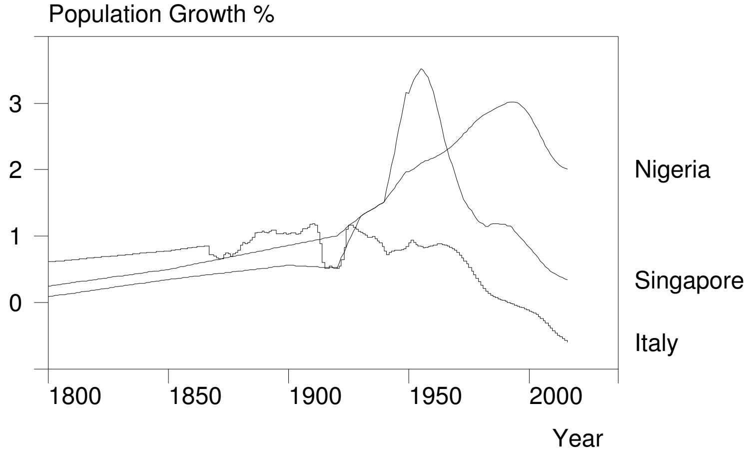

Figure 1: Population growth rates in Italy, Nigeria, and Singapore. For data, see Durand (1977), US Census Bureau (2002) , Kuznets (1980), and Mitchell (1975), authors calculations.

The microfoundations of demographic transition are mainly based on the household theory of child demand delivered by Becker (1960). Later modifications derive demographic transition as an increase in incentives to invest in child quality (Becker *et al.* 1990, Lucas 2002), or as an increase in the price of mother's time (Galor and Weil 1996). The number of children, however, can be freely chosen only if contraceptive techniques are at sufficient level to prevent unwanted births. Because modern contraceptives became available only lately, it is possible that the maximizing households have been pushed out of their optimum during the period discussed. The excess supply theory, originally delivered by Easterlin (1975) and completed by Easterlin and Grimmins (1985) takes this possibility into account. According to Easterlin and Grimmins, the stylized history of demographic transition can be told as follows: At a low level of income, children usually were in excess demand. Due to bad nutrition of fertile women and high infant mortality, the supply of children (C_s) tended to be low, while, due to high preferences toward children, child demand (C_d) tended to be high. The actual number of children (n) was limited by the supply. This situation holds left of m in Figure 2. As income increases, there is a change in child demand. Both increases and decrease are possible, but we assume that the demand for children decreases. On the other hand, as income increases, there is an increase in supply because mortality decreases and fertility increases. Even if children now are in excess supply, high costs and poor availability prevent the use of contraceptives. However, as income increases, developing markets lead to increased contacts between people and information and availability of contraceptives increases (Watkins 1990) and their use starts at h. The actual number of children approaches that demanded, but fertility is perfectly demand determined only after p. Population growth — the actual number of children — endogenously increases and decreases as a function of income.

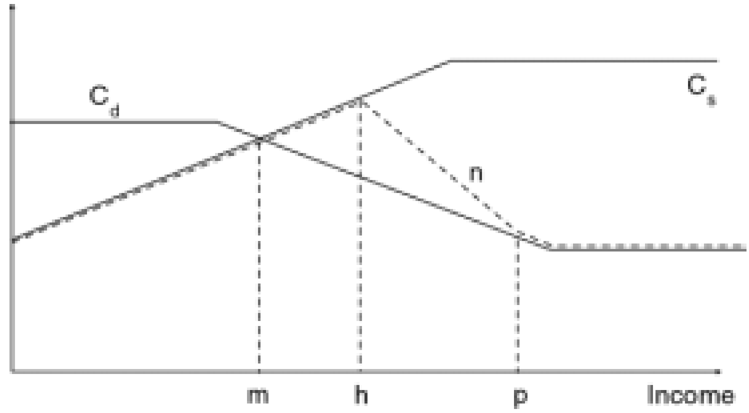

Figure 2: The stylized history of demographic transition (Easterlin and Crimmins 1985).

At each level of income, the households consider the number of children to be outside their control but behaving in some predictable way. As long as we think of the beginning of the transition, this is to be taken as a true description of the situation, but after point p, this solution should be seen as a simplification alone. Further, we assume that rational households choose consumption and investments as to maximize their utility. Because population growth is endogenous to income, by making their choice between consumption and savings, the households implicitly determine the long run behavior of population growth.

The main result of the augmented model is that the changing population growth rate can produce multiple steady states and history-dependent equilibrium paths. Some recent modifications of neo-classical growth models concentrate on the roles of productive externalities and market imperfections to introduce multiple steady states, poverty traps, and history-dependent equilibrium paths (see Azariadis and Drazen (1990), Matsuyama (1991), Benhabib and Farmer (1994), and Benhabib and Gali (1995), Gali (1996). Less attention is paid to the role of demography to produce closely related results. In this paper we will show that the relaxation of the constant population growth assumption alone can lead to a multiequilibrium model, the dynamic properties of which rely on the demographic features specified. No other special features are included. The returns to inputs are decreasing everywhere and externalities are not present in the model. Thus the central planner's solution, first given, is identical to the competitive solution also discussed in the paper.

Demographic features — the sensitivity of the population growth rate to income increases, the height of population growth rates during the transition, as well as in timing of the transition — have not only quantitative, but important qualitative implications in the model. The number of steady states and dynamic properties of the equilibrium paths change from case to case. Three types of demographic transitions are discovered. In a *weak* case, in which the transition takes place at low levels of income, with a weak reaction of the population growth rate to income increases, and with a low peak population growth rate, only a single steady state exists. But in a *strong* case, with high per capita income during the transition, and with a population growth rate reaching extreme numbers and reacting strongly to increases in incomes, the economy is led toward a poverty trap that can be escaped only if income is somehow raised above its threshold value. In the *moderate* case, finally, multiple steady states appear, but the equilibrium path toward the high-income steady state is still globally optimal.

The outline of the paper is as follows: In Section 2 we introduce the Ramsey model, which will be augmented by the population function that gives the population growth rate as a function of per capita income. The Ramsey model assumes that the utility of a representative family increases if the number of family members enjoying a given level of consumption increases. For this reason, the discounted stream of utility from consumption has as an element in its discounting factor the population growth rate. Because the population growth rate in our model is not constant, the model tends to be rather complicated. However, following the tradition begun by Uzawa (1968), it is possible to move from natural time to "virtual" time so that the discounting factor is constant, and the problem can be solved by using standard methods. In Section 3 we study the dynamic properties of the augmented model. In Section 4 we introduce a calibrated version to deal with the comparative dynamics of the model and Section 5 discusses the transitional dynamics. The decentralized version of the model is discussed in Section 6 and Section 7 closes the paper.

2 The Ramsey Model Augmented

2.1 Economy and Population

In the textbook Ramsey model,[1] the economy consists of an infinitely lived family or many identical families, the preferences of which are given by

$$U = \int_0^\infty u\left[c\left(t\right)\right] e^{-(\rho-n)t} dt,$$

in which $c(t)$ is consumption per head. The per capita utility function $u[c(t)]$ has $u'[c(t)] > 0, u''[c(t)] < 0$ and satisfies the Inada conditions $\lim\limits_{c(t)\to 0} u'[c(t)] = \infty$

[1] For a good representation, see Barro and Sala-i-Martin (1995), Chapter 2.

and $\lim_{c(t)\to\infty} u'[c(t)] = 0$. The time-preference factor ρ and the population growth rate n are assumed to be given positive constants. The multiplication of $u[c(t)]$ by the family size $L(t) = e^{nt}$ ($L(0) = 1$) represents the adding up of utils for all family members alive at time t.

The economy has the productive capital $K(t)$ and a constant-returns-to-scale production function $F[K(t), L(t)]$. In per capita terms, the productive per capita capital $k(t) = \frac{K(t)}{L(t)}$ can produce the per capita output $y(t) = f[k(t)]$. The per capita production function $f[k(t)]$ is assumed to be strictly concave and to satisfy the Inada conditions $\lim_{k(t)\to 0} f'[k(t)] = \infty$ and $\lim_{k(t)\to\infty} f'[k(t)] = 0$. The per capita output is either consumed or invested. Assuming a closed economy, the per capita productive capital accumulates according to

$$\dot{k}(t) = f[k(t)] - c(t) - (\delta + n) k(t),$$

in which δ is the depreciation rate of capital. It is assumed that initially there exists some capital, so that $k(0) > 0$.

We modify this basic model by assuming that — instead of being constant — the population growth rate n changes as a function of per capita income. At low levels of per capita income y, the demand for children — the population growth rate — increases as income increases. At higher level of income, the reverse is true. Because the per capita income y is a monotonously increasing function of per capita capital k, it is convenient, for modelling reasons, to write n as a function of k. *The population function* $n = n[k(t)]$ described above then assumes

$$\begin{aligned} n'[k(t)] > 0 &\Leftrightarrow k(t) < \mu, \\ n'[k(t)] = 0 &\Leftrightarrow k(t) = \mu, \\ n'[k(t)] < 0 &\Leftrightarrow k(t) > \mu. \end{aligned} \tag{1}$$

The per capita capital $k(t) = \mu$ is the level of per capita capital (income) that is large enough to make demographic behavior turn toward its modern phase in which additional income decreases the number of children per family. In addition, we assume $\lim_{k\to 0} \{n[k(t)]\} < \infty$, $\lim_{k\to\infty} \{n'[k(t)]\} = 0$. In addition $n[k(t)]$ and $n'[k(t)]$ are continuous in k. Defined in this way, the population function $n = n[k(t)]$ is in line with the microeconomic results explained above, and with existing empirical evidence. Figure 3 illustrates. Note, however, that the details of the transition, that is, the sensitivity of n to k, and the level of capital stock at which the transition takes place, as well as the maximal population growth during the transition, have been markedly different in different economies.[2]

[2] In Chapter 4 we introduce a parametric version of the population function given by

$$n[k(t)] = \eta \cdot \exp\left\{ -\frac{1}{2} \left[\frac{k(t) - \mu}{\sigma} \right]^2 \right\}.$$

Because countries have experienced different transitions, the parameters of the above function are country-specific.

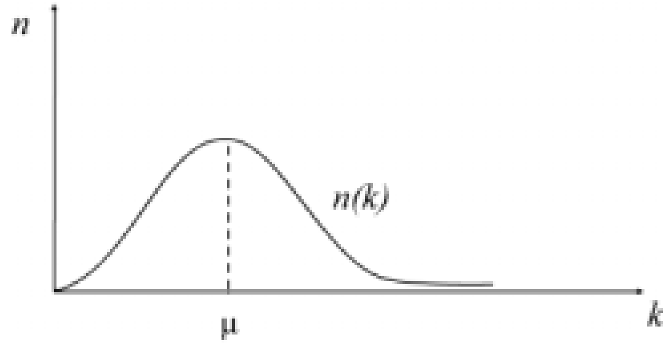

Figure 3: The population function.

Because $L(t) = L(0)\, e^{\int_0^t n[k(\tau)]d\tau}$, for $L(0) = 1$ the expressions for U and $\dot{k}$ can be replaced by

$$U = \int_0^\infty u[c(t)] \cdot \exp\left\{-\int_0^t \{\rho - n[k(\tau)]\}\, d\tau\right\} dt, \tag{2}$$

$$\dot{k}(t) = f[k(t)] - c(t) - (\delta + n[k(t)])\, k(t)\,. \tag{3}$$

To make the objective function bounded, we assume $\rho > n(k)$ for all k. This means that family members are selfish enough to prefer their own instantaneous utility to that possibly derived later by their descendants.

The logic of the model is the following: the planner chooses between consumption and capital accumulation facing the fact that this accumulation leads to some predictable changes in population growth rate summarized in $n[k(t)]$. Therefore, by making this choice, the planner implicitly also chooses the population growth rate that maximized the intertemporal utility as expressed in (2).

2.2 The Planner's Problem

Assume we have a central planner who, at time $t = 0$, wants to maximize the family's discounted utility stream (2), subject to the capital accumulation constraint (3), and the initial condition $k(0) > 0$. Equations (2) - (3) define an infinite horizon discount problem in which the discount rate is variable. The solution to the problem is easier if we follow the tradition of Uzawa (1968) and move from unit steps in natural time t to those in a virtual time Δ by defining

$$\Delta(t) = \int_0^t \{\rho - n[k(\tau)]\} \, d\tau,$$

so that $\frac{d\Delta(t)}{dt} = \rho - n[k(t)]$ and $dt = \frac{d\Delta(t)}{\rho - n[k(t)]}$. The problem can be written in terms of $\Delta(t)$:

$$U = \int_0^\infty \frac{u[c(t)]}{\rho - n[k(t)]} e^{-\Delta(t)} d\Delta(t), \tag{4}$$

$$\frac{dk(t)}{dt} \cdot \frac{dt}{d\Delta(t)} = \frac{dk(t)}{d\Delta(t)} = \frac{f[k(t)] - c(t) - (\delta + n[k(t)])\,k(t)}{\rho - n[k(t)]}. \tag{5}$$

Note that in this virtual time, we have a constant discounting factor problem, which can be solved by using standard methods. The current value Hamiltonian $H[k(t), c(t), \lambda(t)]$ and the necessary conditions for this problem are (see Benveniste and Scheinkman 1982):

$$H[k(t), c(t), \lambda(t)] = \frac{u[c(t)]}{\rho - n[k(t)]} + \lambda[\Delta(t)] \left\{ \frac{f[k(t)] - c(t) - (\delta + n[k(t)])\,k(t)}{\rho - n[k(t)]} \right\},$$

$$\begin{aligned} \frac{\partial H[k(t), c(t), \lambda(t)]}{\partial c(t)} &= 0; \\ \frac{d\lambda[\Delta(t)]}{d\Delta(t)} &= -\frac{\partial H[k(t), c(t), \lambda(t)]}{\partial k(t)} + \lambda[\Delta(t)]; \\ \lim_{\Delta(t)\to\infty} \left\{ \lambda[\Delta(t)] \cdot e^{-\Delta(t)} \cdot k(t) \right\} &= 0, \end{aligned}$$

together with (5). The conditions $\frac{\partial H[k(t),c(t),\lambda(t)]}{\partial c(t)} = 0$ and $\frac{d\lambda[\Delta(t)]}{d\Delta(t)} = -\frac{\partial H[k(t),c(t),\lambda(t)]}{\partial k(t)} + \lambda[\Delta(t)]$ give[3]

$$u'(c) = \lambda, \tag{6}$$

$$\begin{aligned} \frac{d\lambda}{d\Delta} &= -\left\{ \frac{u(c)n'(k)}{[\rho - n(k)]^2} + \lambda \left[\frac{[\rho - n(k)][f'(k) - (\delta + n(k)) - n'(k)k] + n'(k)[f(k) - c - [\delta + n(k)]k]}{[\rho - n(k)]^2} \right] \right\} + \lambda \\ &= -\lambda \left(\frac{f'(k) - (\delta + \rho) - n'(k)k}{\rho - n(k)} \right) - \frac{n'(k)}{\rho - n(k)} H(k, c, \lambda). \end{aligned} \tag{7}$$

We now transform the necessary conditions to natural time again. In natural time, (5) implies (3). The time derivative of λ becomes

$$\frac{d\lambda}{d\Delta} \cdot \frac{d\Delta}{dt} = \dot{\lambda} = -[f'(k) - (\delta + \rho) - n'(k)k]\lambda - n'(k)H(k, c, \lambda). \tag{8}$$

[3] Instead of the notations $c(t)$, $k(t)$, and $\lambda(t)$, identical but longer notations $c[\Delta(t)]$, $k[\Delta(t)]$, and $\lambda[\Delta(t)]$ can be used. Because the model contains no time lags, we now abandon reference to time (virtual or natural).

Equation (8) and constraint (3) form a system of two ordinary differential equations in which $\dot{\lambda}$ and $\dot{k}$ are dependent on λ and k and $u(c)$. Equation (6) relates $u(c)$ to λ implying $u''(c)\dot{c} = \dot{\lambda}$. Therefore, we can eliminate λ, and (8) simplifies to

$$u''(c)\dot{c} = -u'(c)\left[f'(k) - (\delta+\rho) - n'(k)k\right] - n'(k)H(k,c),$$

in which $H(k,c) = \frac{u(c)}{\rho-n(k)} + u'(c)\left\{\frac{f(k)-c-[\delta+n(k)]k}{\rho-n(k)}\right\}$ refers to optimized Hamiltonian. Solving this for (net) marginal product of capital gives the Euler equation:

$$f'(k) - \delta = -\frac{u''(c)\,c}{u'(c)} \cdot \frac{\dot{c}}{c} + \rho + n'(k)k - \frac{n'(k)}{u'(c)}H(k,c).$$

The Euler equation says that it is optimal to invest one unit if the (net) marginal product it produces covers the loss of utility due to a unit decrease in consumption. This loss of utility, the right-hand side of the Euler equation, consists of the terms $n'(k)k$ and $\frac{n'(k)}{u'(c)}H(k,c)$, in addition to the ordinary terms of elasticity of intertemporal substitution and time preference. The term $n'(k)k$ says that because investment changes per capita capital, population growth-rate changes as well. A changed number of new people must be provided with new capital. Note that if $n'(k) < 0$, this factor alleviates the productivity requirement for the last unit of capital. The last term on the right refers to the changed discounting factor or, as an equivalent, the family size in the future. Because the optimized Hamiltonian refers to the total utility of one unit of good, a change in population growth changes the future size of the family and the total flow of utils in the future. The marginal product of new capital must cover this effect.

The Euler equation can be solved as a differential equation of consumption:

$$\frac{\dot{c}}{c} = -\frac{u'(c)}{u''(c)\,c}\left[f'(k) - (\delta+\rho) - n'(k)\,k + \frac{n'(k)}{u'(c)}H(k,c)\right].$$

The equation is easier to handle if we adopt a constant-elasticity-of-intertemporal-substitution utility function $u(c) = \frac{c^{1-\theta}}{1-\theta}, \theta > 0, \theta \neq 1$, which has the convenient property that $\frac{u'(c)}{u''(c)c}$, the reciprocal of the elasticity of substitution is constant, in this case $-\frac{1}{\theta}$. We assume $\theta > 1$.[4] We can write the optimized Hamiltonian and a system of two ODEs as follows:

$$H(k,c) = \frac{c^{1-\theta}}{(1-\theta)\left[\rho - n(k)\right]} + c^{-\theta}\left[\frac{f(k) - c - [\delta + n(k)]k}{\rho - n(k)}\right],$$

$$\dot{c} = \frac{c}{\theta}\left[f'(k) - (\delta+\rho) - n'(k)k + \frac{n'(k)}{c^{-\theta}}H(k,c)\right], \tag{9}$$

[4] High values of θ refer to the unwillingness of households to accept any deviations from the uniform pattern of consumption over time. In the isocline $\dot{c} = 0$, the expression $\frac{\theta-1}{\theta}$ appears as a multiplier (see equation (12)), so that the figure of $\dot{c} = 0$ turns "upside-down" for $\theta < 1$. However, the conclusions of the augmented model are unchanged even in this case. The alternative $\theta > 1$ is chosen because Hall (1988) suggests that high values for θ are empirically most plausible.

$$\dot{k} = f(k) - c - [\delta + n(k)]k. \tag{10}$$

3 Dynamic Properties of the Augmented Model

We can now discover the dynamics of the augmented Ramsey model. In Section 3.1 we introduce the isoclines of the model. We assume that the demographic transition takes place at low levels of capital stock. Technically, this makes the limit behavior of the isoclines unique. The interpretation of the assumption is clear: decrease in the mortality rate and the accompanied increase in the population growth rate begin as soon as the level of income (and capital) increases above its subsistence level. Therefore, it is impossible for the demographic transition in any economy to be related to high levels of income and capital.

It turns out that the number of interior steady states is either three or one. In Section 3.2 the local stability of these steady states is discussed. We will show that if the number of the steady states is three, the two extremes of them are saddles, and they have an unstable node or focus between them. However, two qualitatively different cases arise. In the first case the saddle paths start in the steady state between them. In the second case the saddles start in the origin. The global properties of the former are analyzed in Section 3.3 and of the latter in 3.4.

3.1 Phase Diagram

To analyze the dynamics of the system (3) through (9), we plot the isoclines on the $k-c-$ space. The first isocline is given by

$$\dot{k} = 0 \Rightarrow c = f(k) - [\delta + n(k)]\, k. \tag{11}$$

The isocline starts at the origin and intersects the k-axis at $\tilde{k}$ where $f(\tilde{k})/\tilde{k} = \delta + n(\tilde{k})$. Even if the production function $f(k)$ is strictly concave, the isocline $\dot{k} = 0$ has a non-concave area due to changing $n(k)$.[5] The slope of the isocline is given by $f' - [\delta + n(k)]\ - n'(k)k$.

The second isocline is given by

$$\begin{aligned}
\dot{c} &= 0 \Rightarrow c = \frac{\theta-1}{\theta}\{[f'(k) - (\delta+\rho) - n'(k)k]\,(\frac{\rho - n(k)}{n'(k)}) + [f(k) - [\delta + n(k)]\,k]\} \\
&= \frac{\theta-1}{\theta}\{[f'(k) - (\delta+\rho)]\,(\frac{\rho - n(k)}{n'(k)}) + [f(k) - (\delta+\rho)\,k]\}. \qquad (12)
\end{aligned}$$

For $\theta > 1$, the isocline approaches $+\infty$ as $k \to 0$. For $k > 0$, note that the shape of the isocline is determined by the behavior of three expressions. The

[5] It is in principle possible that the isocline cuts the k-axis for $k < \tilde{k}$ due to a very strong demographic transition. This, however, would imply that population grows at a high rate even if consumption is zero — a situation impossible in the real life.

expression $f'(k)-(\delta+\rho)$ approaches -$(\delta+\rho)$, intersecting the $k-$axis from above at $\hat{k}$ where $f'(\hat{k})=(\delta+\rho)$. Under the assumption $\rho>n(k)$ for all k, the expression $\frac{\rho-n(k)}{n'(k)}$ approaches $+\infty$ as $k\to\mu$ from the left, and $-\infty$ as $k\to\mu$ from the right, having a point of discontinuity at $k=\mu$. Therefore, the value of the product $[f'(k)-(\delta+\rho)]\,(\frac{\rho-n(k)}{n'(k)})$ depends on the relation between μ and $\hat{k}$. The empirics imply that demographic transition always takes place at low levels of capital stock. Therefore, we assume that $\mu<\hat{k}$. In this case $f'(k=\mu)-(\delta+\rho)>0$ and

$$\begin{aligned}
\lim_{k\uparrow\mu}[f'(k)-(\delta+\rho)]\,(\frac{\rho-n(k)}{n'(k)}) &= +\infty,\\
\lim_{k\downarrow\mu}[f'(k)-(\delta+\rho)]\,(\frac{\rho-n(k)}{n'(k)}) &= -\infty,\\
\lim_{k\to\infty}[f'(k)-(\delta+\rho)]\,(\frac{\rho-n(k)}{n'(k)}) &= +\infty.
\end{aligned}$$

Therefore, the formula $[f'(k)-(\delta+\rho)]\,(\frac{\rho-n(k)}{n'(k)})$ produces a U-shaped graph for $k<\mu$, but swings from $-\infty$ to $+\infty$ when $k>\mu$. Finally, the expression $f(k)-(\delta+\rho)\,k$ in (12) is positive for $k<\breve{k}$, in which $\breve{k}$ is given by $f(\breve{k})/\breve{k}=(\delta+\rho)$. It has no effect on the limit behavior of the isocline $\dot{c}=0$, but can affect its shape in the vicinity of the k-axis. By definition $\hat{k}<\breve{k}<\tilde{k}$.

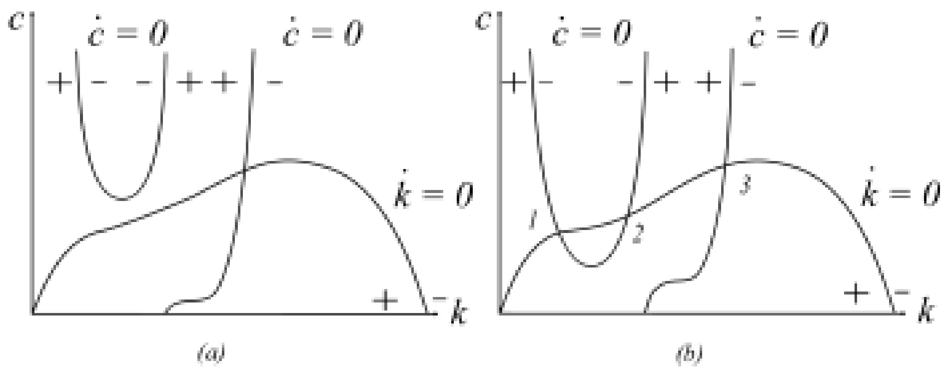

Figure 4: The phase diagrams for the augmented model.

The phase diagram depicted in Figure 4 shows that two cases arise. The U-part of the isocline $\dot{c}=0$ can lie high enough to avoid the intersection with the isocline $\dot{k}=0$ and the number of interior steady states is one.[6] Alternatively,

[6]The non-generic case in which the $\dot{c}=0$ line is tangent to $\dot{k}=0$ line is not analyzed. Because of non-concavities and discontinuity of the isoclines, additional intersections can not be excluded a priori.

the U-part of the isocline $\dot{c} = 0$ can lie so low that it intersects $\dot{k} = 0-$line and the number of interior steady states is three. The former is given in case a and the latter in case b in Figure 4.

3.2 Stability of the Steady States

In case b in Figure 4, three interior steady states exist. To state their dynamic properties, we write $\dot{k} = \varphi(k, c)$ and $\dot{c} = \phi(k, c)$. The slope of the isocline $\dot{k} = 0$ is $\frac{dc}{dk} = -\frac{\partial\varphi/\partial k}{\partial\varphi/\partial c}$ and that of $\dot{c} = 0$ is $\frac{dc}{dk} = -\frac{\partial\phi/\partial k}{\partial\phi/\partial c}$. The Jacobian of the system is given by

$$J = \begin{bmatrix} \partial\varphi/\partial k & \partial\varphi/\partial c \\ \partial\phi/\partial k & \partial\phi/\partial c \end{bmatrix}.$$

The stability of the system of equations $\dot{k} = \varphi(k, c)$ and $\dot{c} = \phi(k, c)$ linearized around the steady state depends on the values of the characteristic roots given by

$$\nu_1, \nu_2 = \frac{TR \pm \sqrt{TR^2 - 4DET}}{2},$$

in which DET is the determinant of the Jacobian around the steady state, and TR is its trace respectively. Then $\nu_1 \cdot \nu_2 = DET$ and $\nu_1 + \nu_2 = TR$ (Chiang 1984). In each of the steady states, $\dot{k} = \dot{c} = 0$ and equations (11) and (12) give

$$f'(k) - (\delta + \rho) - n'(k)\,k = \frac{n'(k)}{(\theta - 1)[\rho - n(k)]}\left\{f(k) - [\delta + n(k)]k\right\}. \qquad (13)$$

In the steady states, the elements of the Jacobian are

$$\begin{aligned}
\partial\varphi/\partial k &= f' - (\delta + n) - n'k, \\
\partial\varphi/\partial c &= -1, \\
\partial\phi/\partial k &= \frac{c}{\theta}\left\{ \begin{array}{c} f'' - (n''k + n') + \frac{n''(\rho-n)+(n'')^2}{(\rho-n)^2}\left[\frac{\theta c}{1-\theta} + f - (\delta+n)k\right] \\ + \frac{n'}{\rho-n}\left[f' - (\delta+n) - n'k\right] \end{array} \right\}, \\
\partial\phi/\partial c &= \frac{1}{\theta}\left\{f' - (\delta+\rho) - n'k + \frac{n'}{\rho - n}\left[f - (\delta+n)k\right]\right\} + \frac{2n'c}{(1-\theta)(\rho-n)} \\
&= \frac{-n'}{(\theta-1)(\rho-n)}\left[f - (\delta+n)k\right],
\end{aligned}$$

in which the last equation is derived by using the definitions of the steady state (13) and the isocline $\dot{k} = 0$ given by (11). Because $f(k) - [\delta + n(k)]k$ is positive (for $k < \tilde{k}$), the sign of $\partial\phi/\partial c$ is that of $-n'(k)$. Unfortunately, the expression for $\partial\phi/\partial k$ contains the second derivative of $n(k)$, the sign of which is unknown unless further assumptions are made. To still find the sign of the determinant of the Jacobian, we write

$$\begin{aligned} DET &= (\partial\varphi/\partial k)\cdot(\partial\phi/\partial c) - (\partial\phi/\partial k)\cdot(\partial\varphi/\partial c) \\ &= \left[\left(-\frac{\partial\varphi/\partial k}{\partial\varphi/\partial c}\right) - \left(-\frac{\partial\phi/\partial k}{\partial\phi/\partial c}\right)\right](-\partial\varphi/\partial c)\cdot(\partial\phi/\partial c), \end{aligned}$$

in which the expression in the square brackets is the difference in the slopes of the isoclines $\dot{k} = 0$ and $\dot{c} = 0$, and $\partial\varphi/\partial c$ and $\partial\phi/\partial c$ are as given above. In steady state 1 (see Figure 4), the isocline $\dot{c} = 0$ hits the isocline $\dot{k} = 0$ from above which makes the expression in the brackets positive. Because $n'(k) > 0$, we have $DET < 0$, and the steady state is a saddle point (ν_1 and ν_2 of different sign, see Chiang 1984). In steady state 3, the isocline $\dot{c} = 0$ intersects the isocline $\dot{k} = 0$ from below, which makes the brackets negative. Because $n'(k) < 0$, the sign of the determinant is negative, and this steady state is again a saddle.

In steady state 2, the isocline $\dot{c} = 0$ intersects the isocline $\dot{k} = 0$ from below, but because $n'(k) > 0$, we have $DET > 0$ (ν_1 and ν_2 of same sign). The trace of the Jacobian is given by $TR = \partial\varphi/\partial k + \partial\phi/\partial c$. Because in any steady state (13) holds, and because $\rho > n(k)$ by assumption, we can write

$$\begin{aligned} TR &= f' - (\delta + n) - n'k - \frac{n'}{(\theta-1)(\rho-n)}[f - (\delta+n)k] \\ &> f' - (\delta + \rho) - n'k - \frac{n'}{(\theta-1)(\rho-n)}[f - (\delta+n)k] \\ &= \frac{n'}{(\theta-1)(\rho-n)}[f - (\delta+n)k] - \frac{n'}{(\theta-1)(\rho-n)}[f - (\delta+n)k] = 0. \end{aligned}$$

The result $TR > 0$ implies that the real part of the eigenvalue of the linearized system is positive, which makes the second steady state unstable($\nu_1 + \nu_2 > 0$). Because we do not know the sign of $(TR)^2 - 4DET$, we must conclude that the second steady state is an unstable node or focus (Chiang 1984). Appendix A shows that there exists no limit cycle around this steady state. In case b, the single steady state is a saddle.

The dynamics outside the steady states is the following: because $\partial\varphi/\partial c = -1$, the capital stock increases (decreases) below (above) the isocline $\dot{k} = 0$. The behavior of consumption is given by $\partial\phi/\partial c = \frac{-n'(k)}{(\theta-1)[\rho-n(k)]}\{f(k) - [\delta + n(k)]k\}$. Therefore, consumption decreases (increases) above (below) the isocline $\dot{c} = 0$ for positive $n'(k)$, but increases (decreases) above (below) it for negative $n'(k)$ (see Figure 4). This implies that the stable branches of the saddle paths approach steady states 1 and 3 (the single steady state in case a) from the southwest and northeast, while the unstable branches run to the northwest and south-east. However, in case b, with three steady states, two possibilities arise:[7] the south-western stable branch toward steady state 3 can untwist from the unstable steady state, as depicted in Figure 5, or it can emanate from the origin, as depicted in Figure 6.[8] We now turn to the dynamics of the former possibility.

3.3 Optimal Path: The Spiral Case

All candidates for optimal paths in the $k-c-$ space satisfy the necessary conditions (3) and (9). Figure 5 shows two types of paths that satisfy the necessary conditions above: the stable branches of the saddle paths A and B, which spiral out of steady state 2, and paths I and II which do not lead to any steady state. In Appendix B we show that the stable branches A and B dominate all other optimality candidates like I or II. Therefore, from initial incomes $k(0)$ between 0 and k_l, it is optimal to choose $c(0)$ on the stable branch A, which leads to steady state 1 (see Figure 5). If the capital stock is larger than k_h initially, it is optimal to choose $c(0)$ on the stable branch B leading to steady state 3.

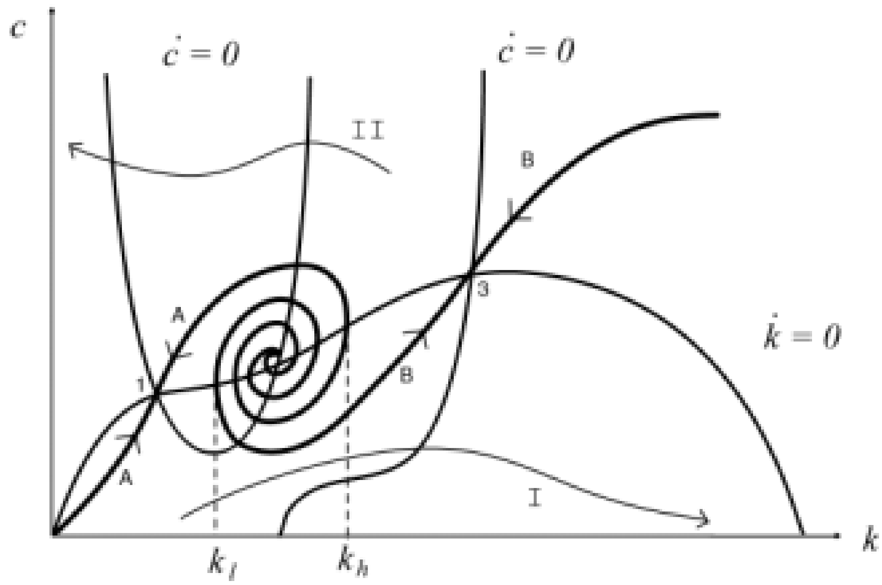

Figure 5: The steady states and the saddle paths in the spiral case. The capital stock k_l (k_h) is the lowest (highest) capital stock from which steady state 3 (1) can be reached.

[7] We checked by using the parametric formulas given in Chapter 4 that these cases really exist.

[8] We assume, for simplicity, that steady state 2 is a focus ($(TR)^2 < 4DET$). The case of node ($(TR)^2 \geq 4DET$) is not very dissimilar (Chiang 1984).

Let k_l (k_h) be the lowest (highest) capital stock from which a path to steady state 3 (1) can be reached (see Figure 5). The problem to choose between two alternative saddle paths is for initial capital $k_l < k(0) < k_h$. The steps for discriminating between two saddle paths are provided by Skiba (1978). We utilize the alternative proof given by Tahvonen and Salo (1996).

First, for an infinite time problem, along any trajectory leading to a steady state, the value of the objective function is equal to the optimized Hamiltonian evaluated at time zero and divided by the discount rate. This result translates to our problem with the discount rate equal to unity (for a proof, see Appendix C). Therefore, it holds that

$$\int_0^\infty u\left[c\left(t\right)\right] \cdot \exp\left\{-\int_0^t \left\{\rho - n\left[k\left(\tau\right)\right]\right\} d\tau\right\} dt = H\left[k\left(0\right), c\left(0\right)\right],$$

and two alternative saddle paths can be compared by comparing their Hamiltonians at their initial points. To see how the value of the Hamiltonian changes, if we choose path A instead of B (for a given initial capital), we adopt again the general form of utility function to have

$$\begin{aligned} H(k,c) &= \{u(c) + u'(c)[f(k) - c - [\delta + n(k)]k]\} \frac{1}{\rho - n(k)} \\ &= \{u(c) + u'(c)\dot{k}\} \frac{1}{\rho - n(k)}. \end{aligned}$$

It is now possible to state the following result:

$$\frac{\partial H(k,c)}{\partial c} = \left[u'(c) + u''(c)\dot{k} - u'(c)\right] \frac{1}{\rho - n(k)} = \frac{u''(c)}{\rho - n(k)}\dot{k}. \tag{14}$$

Note that equation (14) can be used to compare two paths if they lie on the same side of the $\dot{k} = 0-$line. Assume that $k(0) = k_l$ (see Figure 5). Denote the initial consumption chosen on path A and B by c_l^A and c_l^B, respectively. Then $H(k_l, c_l^A)$ is the value of the program if path A is chosen, and $H(k_l, c_l^B)$ is the value of the program if path B is chosen. Note that $c_l^A > c_l^B$. Because (k_l, c_l^B) necessarily lies on the isocline $\dot{k} = 0$ but (k_l, c_l^A) above it, equation (14) gives $H(k_l, c_l^A) > H(k_l, c_l^B)$. Therefore, for $k(0) = k_l$, the value of the program is larger on path A than on path B. By the same argument, we can show that for $k(0) = k_h$ the value of the program is larger on path B.

Each optimal path satisfies

$$\frac{dc}{dk} = \frac{\dot{c}}{\dot{k}} = \frac{\frac{u'}{u''}\left\{f'(k) - (\delta + \rho) - n'(k)k + \frac{n'(k)}{\rho - n(k)}H(k,c)\right\}}{f - c - [\delta + n(k)]k}.$$

Therefore, along any optimal path, $c = c(k)$. The derivative of H in terms of k along an optimal path then becomes

$$
\begin{aligned}
\frac{dH[k,c(k)]}{dk} &= \frac{\partial H[k,c(k)]}{\partial k} + \frac{\partial H[k,c(k)]}{\partial c} \cdot \frac{\dot{c}}{\dot{k}} \\
& \left\{ u + u' \; \dot{k} \right\} \frac{n'(k)}{[\rho - n(k)]^2} + \\
& \frac{u'}{\rho - n(k)} \left\{ f'(k) - [\delta + n(k)] - n'(k)k \right\} \\
& -\frac{u''\dot{k}}{\rho - n(k)} \cdot \frac{\frac{u'}{u''}\left[f'(k) - (\delta+\rho) - n'(k)k + \frac{n'(k)}{u'(c)} H(k,c) \right]}{\dot{k}} \\
&= u' > 0.
\end{aligned} \tag{15}
$$

Equation (15) can be used to compare two paths as k changes. Because $u''(c) < 0$, the increase of $H[k, c(k)]$ as a function of k is faster, the lower the value of $c(k)$. Assume for a while that for some $k(0) \in (k_l, k_h)$, path A is optimal. Path A can be reached by choosing one of the several initial consumptions (see Figure 5). Assume that the lowest possible initial consumption is chosen. It is then first necessary to increase capital stock by $k_h - k\,(0)$ units, each increasing the value of the program by $u'(c)$ (see 15), where each c lies on path A. Next, the movement along A means that capital stock is to be decreased by $k_h - k\,(0)$ units, each decreasing the value of program by $u'(c)$. But because each c is now larger, each $u'(c)$ is smaller, and the value of the program increases when moving from lowest to highest initial consumption. Therefore, for those initial capital stocks for which path A is optimal, it is always best to choose the highest possible consumption initially. By the same argument, if B is optimal, the lowest possible consumption should be chosen. By this argument, steady state 2 is dominated both by path A and B.

Because for all $k \in [k_l, k_h]$ the best value of $c(k)$ is lower on B than on A (see Figure 5), $H[k, c(k)]$ increases faster along B than along A as k increases. Therefore, because $H(k_l, c_l^A) > H(k_l, c_l^B)$, but $H(k_h, c_h^A) < H(k_h, c_h^B)$, and because $H[k, c(k)]$ is continuous in k, there must exist a unique $k_m \in (k_l, k_h)$ such that $H(k_m, c_m^A) = H(k_m, c_m^B)$. For $k\,(0) = k_m$, the planner is indifferent between paths A and B. We conclude that for all initial capital stocks $k\,(0) < k_m$, it is optimal to choose saddle path A toward steady state 1, but for all $k\,(0) > k_m$, path B toward steady state 3 is optimal.

3.4 Optimal Paths: Saddles from the Origin

Assume that the south-western stable branch of saddle path B toward the steady state 3 has its starting point in the origin, as depicted in Figure 6.

For $k(0) \leq k_1^*$, the path toward steady state 1, given by A, lies above B, which leads to steady state 3. In addition, they both lie below the isocline $\dot{k} = 0$. Therefore, equation (14) implies that $H(k, c^A) < H(k, c^B)$. For $k_1^* < k(0) < k_h$, B further lies below A. Therefore, equation (15) implies that the value of the program increases faster along B as $k(0)$ increases. For $k(0) \geq k_h$, B is the only stable branch available. Thus, we reach the conclusion that stable branch B leading toward steady state 3, is a globally optimal solution.

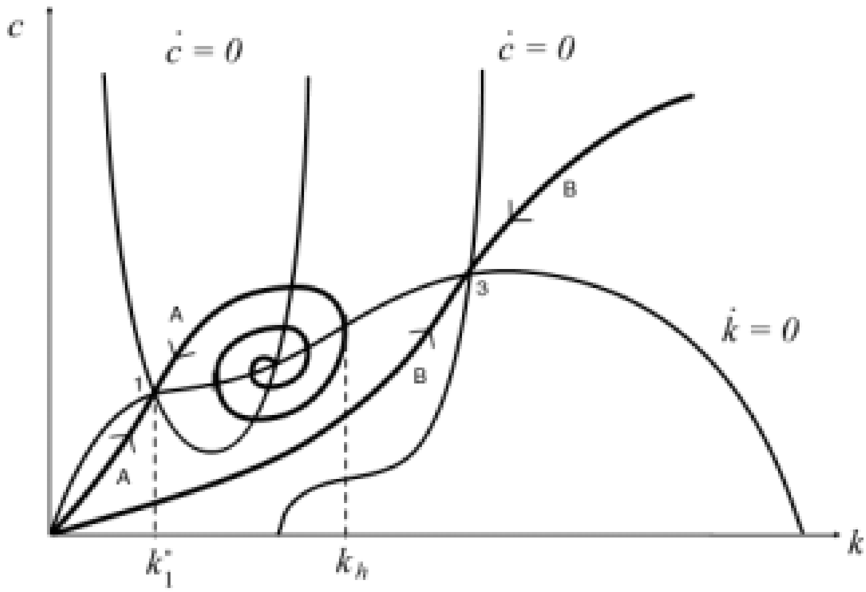

Figure 6: The steady states and the saddle paths starting from the origin.

To summarize, in case b in which three interior steady state exist, two qualitatively different dynamics arise. In the case in which the south-western saddle path toward steady state 3 untwists from the focus, the initial capital determines the final outcome. In the case in which the saddle path starts from the origin, the central planner always chooses the saddle path leading to the high income steady state 3.

In case a there exists a single steady state, which is a saddle point. This case is like the standard Ramsey model with constant population growth, in that there exists a unique saddle path from the origin toward this single steady state.

4 Comparative Dynamics

We will now show that cases a and b in Figure 4 refer to differences in the features of demographic transitions experienced by economies. For the given time preference factor ρ, the specific features of demographic transitions determine the discount rate of the model. Matsuyama (1991) has analyzed the effect of a constant discount rate on the global dynamics of the model by using the global bifurcation technique. Instead of providing a generic theorem in the case of a variable discount rate, we perform parametric calculations to give some illustrative results.

Let a parametrized population function be given by

$$n(k) = \eta \cdot \exp\left\{-\frac{1}{2}\left(\frac{k-\mu}{\sigma}\right)^2\right\},$$

in which η is the peak population growth rate and μ is the turning point of population growth. The parameter σ gives the dispersion of the transitional period. For any given η and μ, a small (large) value for σ refers to strong (weak) reaction of the population growth rate to increases in capital and income. The simple functional form used here assumes that population growth is normally distributed, an assumption that is adopted in the absence of any closer information.

From the parameters above, data on the peak population growth rate η is most directly available. To see the effect of η on the number of steady states, we derive from equation (12)

$$\frac{\partial c}{\partial \eta}\mid_{\dot{c}=0} = -\frac{\theta-1}{\theta}\left[f'(k)-(\delta+\rho)\right]\frac{\rho}{\eta \cdot n'(k)}.$$

The expression in the parenthesis is positive for $k < \mu < \hat{k}$, so that $\frac{\partial c}{\partial \eta}\mid_{\dot{c}=0}$ is negative for $n'(k) > 0$ and positive for $n'(k) < 0$. For $k > \hat{k}$ the contrary is true.[9] The most important partial effect of the peak population growth rate η is thus that high peak population growth rates tend to push the $U-$part of the isocline $\dot{c} = 0$ down. This makes the dynamics of the model deviate from the standard model and increases the possibility of the spiral case in which a poverty trap — a demographic trap — appears.

To illustrate the effects of η on the number of steady states and the nature of the optimal paths, we perform calculations using a calibrated version of the augmented model. The functional forms and parameters used are reported in Table 1. The parameters of production and utility functions are close to those applied by Barro and Sala-i-Martin (1995). The limits for the value for

[9] The value of η has an effect on the isocline $\dot{k} = 0$, too. We can derive from (11)

$$\frac{\partial c}{\partial \eta}\mid_{\dot{k}=0} = -\exp\left\{-\frac{1}{2}\left(\frac{k-\mu}{\sigma}\right)^2\right\}k < 0.$$

The higher the value of η, the larger the deviation from the standard concave isocline $\dot{k} = 0$.

$\alpha = 0.7$	The share of broad capital
$\rho = 0.045$	Time preference factor
$\theta = 3$	The negative of the elasticity of marginal utility
$\delta = 0.05$	The rate of depreciation
$10 < \sigma < 120$	The dispersion (duration) of demographic transition
$120 < \mu < 778.5 = \hat{k}$	The capital stock at which population growth is peaked
$0.01 < \eta < 0.045$	Peak population growth rate
$y = k^{\alpha}$	Cobb-Douglas production function
$u(c) = \frac{c^{1-\theta}}{1-\theta}$	CIES utility function
$n(k) = \eta e^{-\frac{1}{2}(\frac{k-\mu}{\sigma})^2}$	Population function

Table 1: The functional forms and the values of the parametres used.

η are directly empirically justifiable. To find limits for σ, note that $L(t) = L(0) \cdot \exp\left\{\int_0^t \eta e^{-\frac{1}{2}(\frac{k(\tau)-\mu}{\sigma})^2} d\tau\right\}$. The expression $\exp\left\{\int_0^t \eta e^{-\frac{1}{2}(\frac{k(\tau)-\mu}{\sigma})^2} d\tau\right\}$ is the population multiplier, which tells how many fold the population grows during the transition. The empirical multipliers estimated are between 2.5 and 20 (see Livi-Bacci 1997). This gives us the approximate limits $10 < \sigma < 120$. The upper limit $\mu < 778.5 = \hat{k}$ is given by the values of parameters α, δ, and ρ, and the lower limit $120 < \mu$ is chosen to accommodate the values of μ with those of σ.

Panel a in Figure 7 illustrates the effect of η on the shape and position of the isoclines such that η increases in the direction of the arrow. Assume that η is initially low and the $U-$part of the isocline $\dot{c} = 0$ lies high enough for the model to have just a single steady state. As η now slightly increases, there is a change in the coordinates of *this* steady state. This type of effect is called a *small effect* by Honkapohja and Turunen-Red (2002). As η increases, the $\dot{c} = 0-$line moves down and becomes tangent to the $\dot{k} = 0-$line and the number of steady states increases to two (not shown). At this point, any slightest increase in η induces a local bifurcation as the number of steady states increases to three. The effects of parameter shifts that lead to changes in the number of steady states are called *large effects* by Honkapohja and Turunen-Red (2002). As η further increases, the coordinates of steady states change but their number stays as three. That is, small but no large effects take place. Even so, the structure of the vector field satisfying (9)-(10) changes continuously and — in the area of interest, left of μ — starts to run more and more from the northwest as the $U-$part of the isocline $\dot{c} = 0$ moves down. As an element of this change, the stable branch B toward steady state 3 gets more curved and runs closer to steady state 1. For some critical value of η, B runs out of steady state 1, and for η larger still, B spirals out of steady state 2. Therefore, in addition to small and large effects, the parameter η also has an effect that we call a *global effect.* Panels $b - d$ in Figure 7 illustrate the changes in the slope of the saddle paths induced by successive increases in η. Panel b refers to the lowest value of η and to the highest $U-$part of the isocline $\dot{c} = 0$ in panel a. Panels c and d refer to the

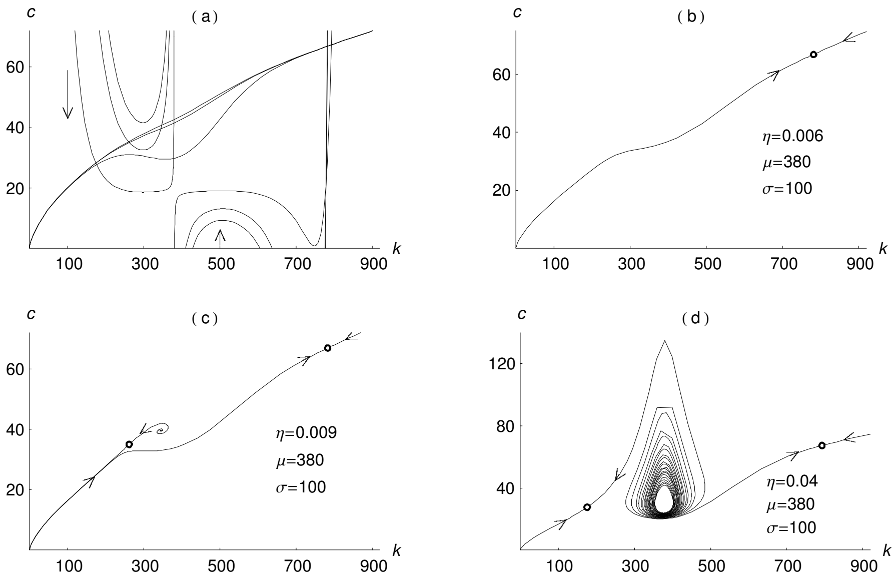

Figure 7: The isoclines (panel a) and the stable saddle paths (panels $b - d$) for parameters $\eta = (0.006,\ 0.009,\ \text{and}\ 0.04)$, $\mu = 380$, and $\sigma = 100$. The value for η increases in the direction of the arrow in panel a. Mathematica 4.02. program to calculate and draw the figure is available from the author.

other two isoclines in panel a respectively.

The role of μ is to shift the $U-$part of the isocline $\dot{c} = 0$ horizontally to the right as μ increases. Large capital stocks during the transition tend to increase the possibility of a poverty trap. The explanation is found in the decreasing marginal product of capital: if the capital stock is large during the transition, the low marginal product of a unit increase in capital is easily "eaten up" by an induced increase in population growth rate, and the deepening of capital is not possible. For any given η and μ, small values of σ make the slope $n(k)$ steeper. This means that the population growth rate strongly reacts to an increase in per capita capital and income. For $n'(k) > 0$, a unit increase in capital is then accompanied by a large increase in the population growth rate, and this tends to lead the economy toward a poverty trap.

Figure 8 shows the combined effect of parameters η, μ, and σ on the dynamics of the augmented model. The two surfaces divide the space into three areas. In area I, the values of μ and η are low but that of σ is high. This is the *weak* case of demographic transition. In this case, the transition takes place at low stocks of productive capital, the population growth is not very sensitive to income (capital), and holds low value even in its peak. Only a single steady state exists, and it is optimal to proceed toward it from all initial capital stocks. In area III, the transition takes place with relatively large capital stocks, peak

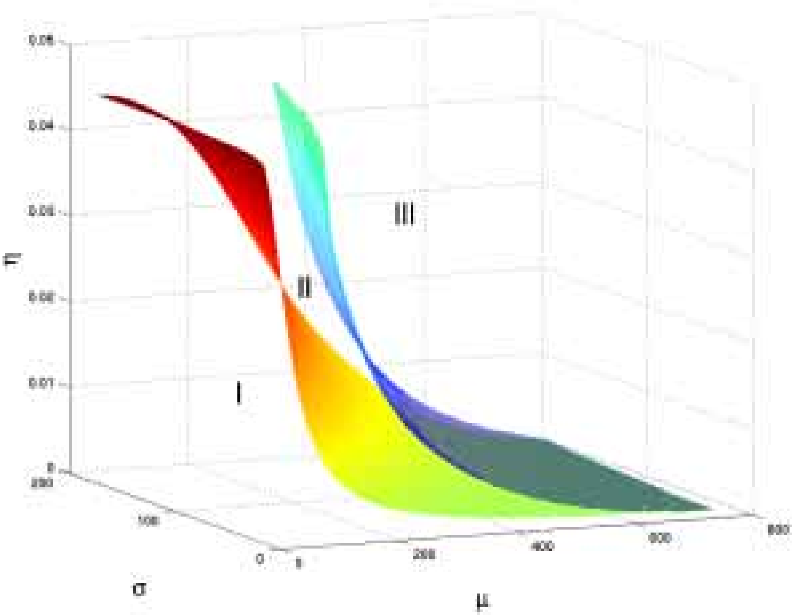

Figure 8: Effect of the parameters in the calibrated model. Area I: single steady state. Area II: three steady states, the south-western saddle path B starts from the origin. Area III: three steady states, the south-western saddle path B emanates spirally from steady state 2. The figure was calculated and drawn by Yrjö Leino from CSC.

population growth is high, and its reaction to income and increase in capital is intensive. In this *strong* case of demographic transition, a poverty trap arises, and the economy can escape it only if its productive capital is raised above the threshold value by some exogenous factor like foreign aid. Figure 9 gives a cross section of Figure 8 for $\sigma = 100$.

We can also depict the population functions in weak and strong cases by choosing the parameters appropriately, as shown in Figure 10. The similarity of Figures 10 and 1 is only too apparent, but complete comparison is not possible because the former is given in terms of time, whereas the latter refers to capital stock. Nevertheless, one is quite confident that demographic transition in Italy has been weak. The relevant question is, whether demographic transition in Singapore or Nigeria has been strong. In the case of Singapore, the answer is apparently “no.” Singapore is already well on its way toward prosperity, and, in spite of its very high peak population growth rate, it has obviously avoided the trap. Furthermore, it is likely that Nigeria will also do so. Even if Singapore and Nigeria (and most of the developing countries) have experienced high peak population growth rates, the combination of the demographic parameters in these countries has been such that these countries have avoided (or shall avoid)

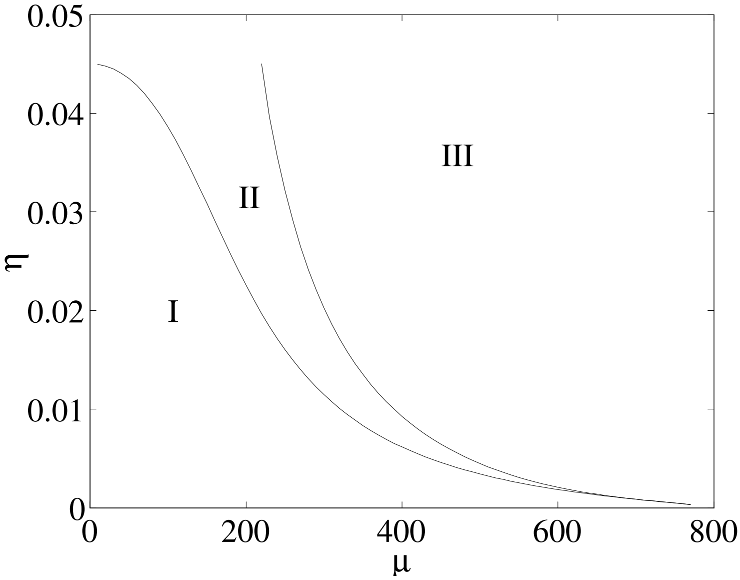

Figure 9: A cross section of Figure 8 for $\sigma = 100$. The figure was calculated and drawn by Yrjö Leino from CSC.

the poverty trap. Data, reporting that population growth everywhere currently is decreasing (United Nations 2000), supports the view that demographic trap is a possibility, introduced by the model but not often materialized in the real world. Therefore, it is necessary to ask, which really are the empirical implications of the augmented Ramsey model? To give an answer, we turn to the behavior of the endogenous variables of the model.

5 Transitional Dynamics

The transitional dynamics of the model is summarized in Figure 11, which gives the time paths for population growth, per capita capital and consumption, and the growth rate of per capita output γ_y for the moderate case. The phase diagram of this case is given in Figure 7 *c*. The growth rate of capital stock is given by

$$\gamma_k = \frac{\dot{k}}{k} = \frac{f}{k} - \frac{c}{k} - (\delta + n).$$

Figure 7 *b* shows that, along the stable arm, capital stock k increases monotonously as the economy proceeds toward the steady state. The Cobb-Douglas production function $y = k^\alpha$ implies $\gamma_y = \frac{\dot{y}}{y} = \alpha\gamma_k$. Because $f(k)/k$ decreases as the economy gets richer in capital, γ_y tends to decrease monotonously in cases

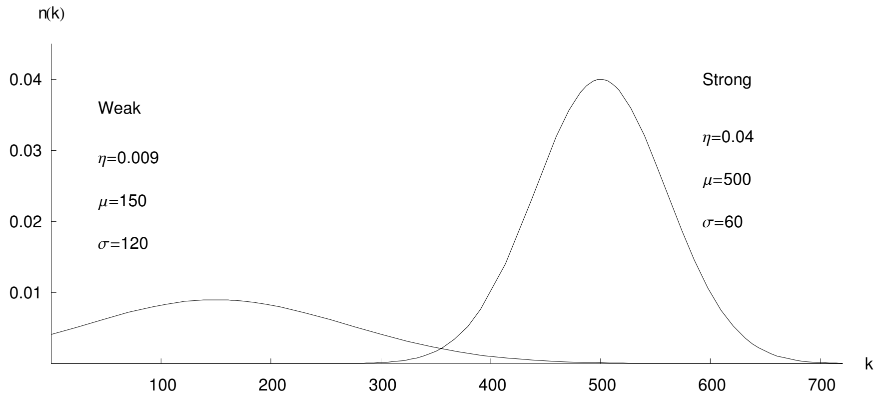

Figure 10: Two sets of demographic parameters and two population functions.

in which population growth rate n is a given constant.[10] This is the convergence result currently much discussed in growth empirics (for a summary, see Temple 1999). In the augmented Ramsey model, however, this monotony breaks down because $n(k)$ both decreases and increases as a function of k, and γ_y greatly varies along the optimal path. Figure 11 shows that, as transition proceeds and population growth accelerates, it is optimal to increase the rate of capital accumulation to get over the transition peak as quickly as possible. The increase γ_y is done at the cost of consumption, which stagnates for a long period of time.

The time paths in Figure 11 show that the convergence result does not hold in the augmented model. On the contrary, the economic rate of growth varies and the economy goes through successive growth stages in the course of its demographic transition. The cross-sectional implication of this result seems to be that countries experience both periods of convergence and divergence because of the uneven pace of transition between individual countries and, especially, between developed and developing countries.

6 Decentralized Model

Instead of being formulated as a central planner's optimization problem, the model above can be formulated as a decentralized model in a competitive economy. Let us concentrate on a family facing wages $w(t)$ and interests $r(t)$ and owning the stock of assets $A(t)$. In this instance, let the notations $L(t)$ and $n(t)$ refer to the number of family members and to the growth rate of this number respectively. The family allocates its incomes $w(t)L(t) + r(t)A(t)$ between consumption $C(t)$ and investments so that its wealth accumulates according to $\dot{A}(t) = w(t)L(t) + r(t)A(t) - C(t)$. The growth rate of the family size depends

[10] Barro and Sala-i-Martin (1995) give a proof for a model with constant n in their Appedix 2C.

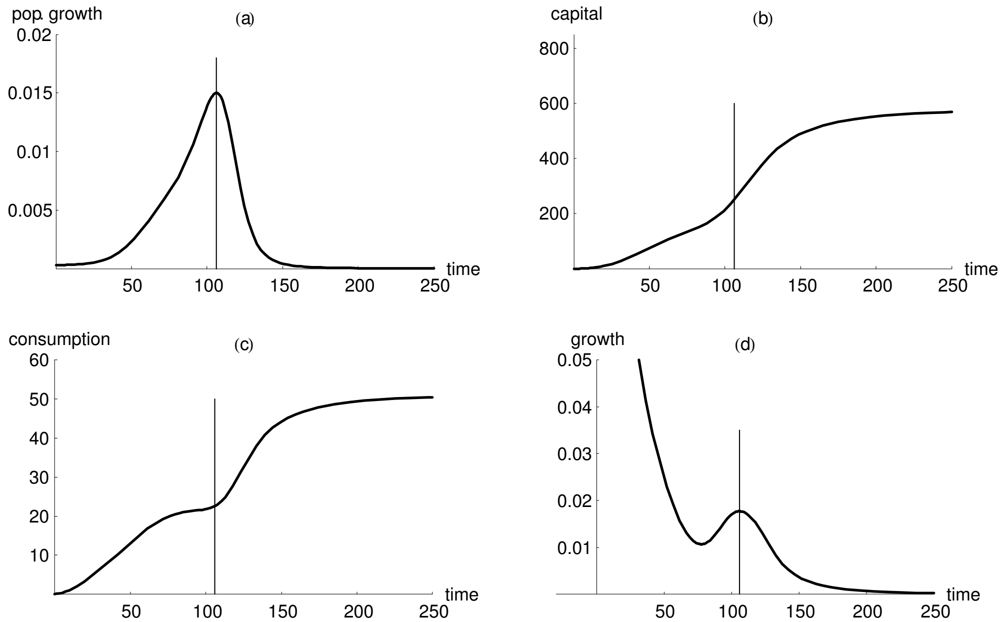

Figure 11: The time paths of the endogenous variables in the moderate case. Parameter values are $\eta = 0.009,\ \mu = 380, \sigma = 100$.

on the family's wealth per head $a(t) = A(t)/L(t)$, i.e., $n(t) = n[a(t)]$ so that $L(t) = e^{\int_0^t n[a(\tau)]d\tau}$ for $L(0) = 1$. The growth of the per capita wealth then becomes

$$\dot{a}(t) = w(t) - c(t) - (r(t) + n[a(t)])\ a(t), \tag{16}$$

in which $c(t)$ is the per capita consumption. We assume that $n[a(t)]$ satisfies $n'[a(t)] \gtreqless 0$ if $a(t) \lesseqgtr \psi$, i.e., the family's growth function is a bell-shaped function peaking at $a(t) = \psi$. The family head derives utility from two things, namely the size of his family, and the utility of each member individually. Therefore, the family head faces the objective functional and the no-chain-letter constraint

$$U = \int_0^\infty u[c(t)] \cdot \exp\left\{-\int_0^t \{\rho - n[a(\tau)]\} d\tau\right\} dt,$$

$$\lim_{t\to\infty} \left\{a(t) \cdot \exp\left\{-\int_0^t \{r(\tau) - n[a(\tau)]\} d\tau\right\}\right\} \geq 0, \tag{17}$$

the latter saying that ultimately, the wealth of the family must be non-negative.[11] The family head, provided with perfect foresight, knows the shape of $n\left[a\left(t\right)\right]$ but takes it as given. Writing $\Delta\left(t\right)=\int_0^t\left\{\rho-n\left[a\left(\tau\right)\right]\right\}d\tau$ and following the methods used in Section 2.2, the current value Hamiltonian $H=H\left[a\left(t\right),c\left(t\right),\lambda\left(t\right)\right]$ and the necessary conditions become

$$H=\frac{u\left[c\left(t\right)\right]}{\rho-n\left[a\left(t\right)\right]}+\lambda\left[\Delta\left(t\right)\right]\cdot\frac{w(t)-c\left(t\right)-\left(r(t)+n\left[a\left(t\right)\right]\right)a\left(t\right)}{\rho-n\left[a\left(t\right)\right]}, \tag{18}$$

$$\begin{aligned}
\frac{\partial H}{\partial c\left(t\right)} &= 0,\\
\frac{d\lambda\left[\Delta\left(t\right)\right]}{d\Delta(t)} &= -\frac{\partial H}{\partial a\left(t\right)}+\lambda[\Delta\left(t\right)],\\
\lim_{\Delta(t)\to\infty}\left\{\lambda\left[\Delta\left(t\right)\right]\cdot e^{-\Delta(t)}\cdot a\left(t\right)\right\} &= 0.
\end{aligned}$$

The equation of $\frac{d\lambda}{d\Delta}$ becomes

$$\begin{aligned}
\frac{d\lambda}{d\Delta} &= -\lambda\frac{r-n(a)-n'(a)\ a}{\rho-n(a)}-\frac{n'(a)}{\rho-n(a)}H(a,c,\lambda)+\lambda &\quad (19)\\
&= -\lambda\frac{r-\rho-n'(a)\ a}{\rho-n(a)}-\frac{n'(a)}{\rho-n(a)}H(a,c,\lambda). &\quad (20)
\end{aligned}$$

Further, because $\dot{\lambda}=\frac{d\lambda}{d\Delta}[\rho-n(a)]$, and because the condition $\frac{\partial H}{\partial c}=0$ implies $u'=\lambda$, we have $u''\dot{c}=\dot{\lambda}$. Using CIES utility function $-\frac{u'}{u''c}=\frac{1}{\theta}$, we derive the differential equation for family's consumption:

$$\frac{\dot{c}}{c}=\frac{1}{\theta}\left(r-\rho-n'(a)a+\frac{n'(a)}{c^{-\theta}}H\left(a,c\right)\right), \tag{21}$$

in which $H\left(a,c\right)$ is the optimized Hamiltonian. To aggregate over families, assume that the economy consists of N identical families (N constant). Then the population of the economy becomes $N\cdot L$. Therefore, at each point of time, the population of the economy grows at the rate $\frac{\dot{L}}{L}=n(a)$, which is identical to the growth rate of the size of an individual family. Note also that the wealth per head in a family is equal to the wealth per capita in the economy so that ψ (the value of wealth at which family's growth peaks) can be replaced by μ (the value of wealth at which population growth peaks). Therefore, instead of working with the $N-$multiplicity of an individual family, it is possible to work with the framework of a representative family by arguing that the family size L refers to the size of the entire population which grows at the rate $n(a)$.

Goods are supplied by competitive firms. A representative firms takes wages and interests as given and faces the linearly homogenous production technology

[11] Analogously to the planner's problen, the decentralized economy always proceeds toward an interior steady state $a^*>0$ so that the no-chain-letter constraint is always valid.

$$Y = F(K, L),$$

in which Y, K, and L refer to output, capital and labor input (population). The per capita production function is $y = \frac{Y}{L} = F(\frac{K}{L}, 1) = f(k)$. In each period, the competitive firm hires labor and capital according to

$$F_K = f'(k) = R = r + \delta, \tag{22}$$

$$F_L = f(k) - f'(k)k = w, \tag{23}$$

in which R is the rental rate of capital consisting of the interests and depreciations. Due to homogeneity, profits are zero for each choice of K and L and the scale of the output of the firm is indeterminate.

In a closed economy, the representative family owns all the productive capital. On the other hand, no other type of assets are available. Therefore, $a(t) = k(t)$ for each t. Substituting (22) and (23) into (16) and (21) gives

$$\dot{c} = \frac{c}{\theta}\left[f'(k) - (\delta + \rho) - n'(k)k + \frac{n'(k)}{c^{-\theta}}H(k, c)\right],$$

$$\dot{k} = f(k) - c - [\delta + n(k)]k.$$

These differential equations are identical to their planner's counterparts (9) and (10). Therefore, the competitive economy has the phase diagram and global dynamics completely identical to those of the central planner. Instead, what fails is the equilibrium selection system applied by the planner in cases in which two alternative saddle paths and two steady states are available. To see this, think for example the situation depicted in Figure 5 and let $k(0) \in (k_l, k_h)$. The central planner was able to choose between paths A and B by comparing the value of the program as expressed by the value of the optimized Hamiltonian on paths A and B at time zero. Analogously, the representative consumer can compare the value of his program facing the different competitive interests and wages given by (22)-(23) as implied by the alternative paths of $k(t)$ toward steady state 1 or steady state 3. But on the contrary to the central planner, the choice of the representative household is not sufficient to guarantee that the best path is realized in the economy. Even if the modelling technique of the representative household is a convenient simplification, we still have to think that the household behaves competitively and takes the realized prices as *given*, not as *chosen* by its own activities. Therefore, not knowing which prizes are realized, the situation stays as indeterminate to the household.

Several ways out of the dilemma are suggested. Matsuyama (1991) posits that if the consumer adopts optimistic expectations and believes that the best path is realized, and then behaves accordingly, his expectations become self-fulfilling. The same solution is suggested by Benhabib and Farmer (1994), Benhabib and Gali (1995), and Gali (1996). Further, if we give up the idea of identical consumers, the expectations of agents should be coordinated to find the solution, as is suggested by Matsuyama (1991). On the contrary to the models delivered by the authors above, our model contains no externalities, increasing returns, or market imperfections. Therefore, one idea were to suggest that, even if difficult to state explicitly, the competitive solution is the same as the central planner's solution. Apparently, none of these suggestions is completely satisfactory. Solutions, in which the expectational process is explicitly stated and some steady states defined to be stable (or unstable) under adaptive expectations are provided by Evans *et al.* (1998), Evans and Honkapohja (2001), and Honkapohja and Turunen-Red (2002). The final solution of the point is outside the scope of this paper. Note however, that the most serious outcome of the possible indeterminacy, namely the difficulty to derive robust empirical implications from the model (see Levine and Renelt 1992), is to some extend avoided here. As discussed above, the empirical proof against poverty trap is convincingly given by decrease in population growth rate, currently stated everywhere. Therefore, in spite of the problem of theoretical indeterminacy, empirical investigators can still relatively safely be based on our model.

7 Conclusions

Jeffrey Williamson (1998) has asked "[W]hy has it taken economists so long to learn that demography influences growth?" The explanation proposed in this paper is that neoclassical growth models unrealistically assume a constant population growth rate. The dominance of the constant population growth assumption is all the more striking given that the ongoing demographic transition is a well-known and a well-documented fact.

Once the standard Ramsey model is augmented by a *population function* that summarizes the basic features of the demographic transition, a rich variety of dynamics appears. Economies, like those in Europe, have experienced a weak form of demographic transition. The dynamics of these economies was not qualitatively affected by the transition. If the transition is strong, the economy is led into a poverty trap for demographic reasons alone. Nevertheless, the moderate case of demographic transition may be most relevant even in developing countries. Many of them have been badly hit by demographic transition after World War I, but we may expect that, other things equal, these countries also experience some demography-led booms in the future, and it is possible that periods of convergence between developed and developing countries may appear.

A Appendix

Following Skiba (1978), the policy function $\frac{dc}{dk} = \frac{\dot{c}}{\dot{k}}$ can be rewritten as

$$\{f(k) - c(k) - [\delta + n(k)]k\}\, dc - \frac{c(k)}{\theta} df + \frac{c(k)}{\theta[\rho - n(k)]}[\frac{\theta c(k)}{1-\theta} + f(k) - (\delta + \rho)\, k]dn$$
$$= \quad \frac{-(\delta + \rho)}{\theta} c(k) dk.$$

Taking line integrals of both sides of the equation along the possible limit cycle makes the right side to be $\frac{-(\delta+\rho)}{\theta} \oint c(k)dk$, which is the $\frac{-(\delta+\rho)}{\theta}-$ multiple of the area of the possible limit cycle.[12] Because the left side contains only continuous functions, this area is equal to zero, that is, there exists no limit cycle around steady state 2.

B Appendix

We demonstrate that the stable branches of the saddle paths, leading the economy ultimately to steady states 1 or 3, dominate all other candidates for optimal paths. First, note that a saddle path leading to a steady state meets the transversality condition $\lim_{\Delta \to \infty} \left\{ \lambda\left[\Delta\left(t\right)\right] \cdot e^{-\Delta} \cdot k\left(t\right) \right\} = 0$. This is because in any interior steady state, we have $k > 0$, $c > 0$, and $u'\left[c\left(\Delta\right)\right] = \lambda(\Delta)$ finite and continuous in Δ. Consider candidate paths like path I, which lie below stable branches A and B (see Figure 5). Along these paths, the capital stock $k(t)$ approaches a positive value, consumption approaches zero, and $\lambda[\Delta(t)] = u'\left[c\left(\Delta\right)\right]$ approaches $-\infty$ at an ever increasing rate. Because $e^{-\Delta(t)}$ approaches zero at rate unity, these paths ultimately violate the transversality condition. Consider then candidate paths like path II, which start above stable branches A or B. Note that, ultimately, such a path approaches the $c-$axis. To see that the path actually hits the $c-$axis in finite time, take the time derivative of (3), which is $d\dot{k}/dt = [f' - (\delta + n) - n'k]\, \dot{k} - \dot{c}$. Close to the $c-$axis, we have $\dot{c} > 0$ and $\lim_{k \to 0} f'(k) = \infty$, and $\dot{k} < 0$ above the isocline $\dot{k} = 0$. Therefore, we have $d\dot{k}/dt < 0$ close to the $c-$axis, and $k(t) = k(0) + \int_0^t \dot{k}(\tau)d\tau$ will be zero in finite time. At this point, consumption jumps from a positive value to zero and stays there forever. But because the disutility of the last unit of consumption lost is ∞, this path is necessarily dominated by any path leading to positive consumption forever.

C Appendix

For an infinite time problem, along any trajectory leading to a steady state, the value of the objective function is equal to the current value Hamiltonian

[12] Limit cycle is a closed trajectory, such that there exists some trajectories approaching (or leaving) it from one (or both) sides. For formal proof, see Arrowsmith and Place (1996).

evaluated at time zero and divided by the discount rate (see Skiba 1978). This result translates to our problem with the discount rate equal to unity. The current value Hamiltonian of the problem is $H = H[k(t), c(t), \lambda(t)] = \frac{u[c(t)]}{\rho - n[k(t)]} + \lambda[\Delta(t)]\left\{\frac{\dot{k}}{\rho - n[k(t)]}\right\}$. It is defined

$$\Delta(t) = \int_0^t \{\rho - n[k(\tau)]\}\, d\tau,$$

so that $\frac{d\Delta(t)}{dt} = \rho - n[k(t)]$ and $dt = \frac{d\Delta(t)}{\rho - n[k(t)]}$. Then, along any path satisfying the necessary conditions $\frac{\partial H}{\partial c} = 0$, and $\frac{d\lambda[\Delta(t)]}{d\Delta(t)} = -\frac{\partial H}{\partial k} + \lambda[\Delta(t)]$, as well as $\dot{\lambda} = \frac{d\lambda[\Delta(t)]}{d\Delta(t)}\frac{d\Delta(t)}{dt} = (\rho - n)\left(-\frac{\partial H}{\partial k} + \lambda[\Delta(t)]\right)$, it holds

$$\begin{aligned}
-\frac{d\left[e^{-\Delta}H\right]}{dt} &= -e^{-\Delta}\left[\frac{dH}{dt} - (\rho - n)H\right] \\
&= -e^{-\Delta}\left\{\frac{\partial H}{\partial c}\dot{c} + \frac{\partial H}{\partial k}\dot{k} + \frac{\partial H}{\partial \lambda}\dot{\lambda} - (\rho - n)H\right\} \\
&= -e^{-\Delta}\left\{\frac{\partial H}{\partial k}\dot{k} + \frac{\partial H}{\partial \lambda}\left\{(\rho - n)\left(-\frac{\partial H}{\partial k} + \lambda[\Delta(t)]\right)\right\} - (\rho - n)H\right\} \\
&= -e^{-\Delta}\left\{(\rho - n)\frac{\partial H}{\partial \lambda}\lambda - (\rho - n)H\right\} \\
&= e^{-\Delta}u[c(t)] \\
&= e^{-\int_0^t\{\rho - n[k(\tau)]\}d\tau}u[c(t)].
\end{aligned}$$

Taking the integral gives

$$\begin{aligned}
&\int_0^\infty e^{-\Delta}u[c(t)]dt \\
=\ & -\int_0^\infty \left\{e^{-\Delta}\left[\frac{dH}{dt} - (\rho - n)H\right]\right\}dt \\
=\ & H[k(0), c(0), \lambda(0)] - \lim_{t\to\infty} e^{-\int_0^t\{\rho - n[k(\tau)]\}d\tau} H[k(t), c(t), \lambda(t)].
\end{aligned}$$

Along any optimal path leading to a steady state, $k \to k^*$, $c \to c^*$, $\lambda \to u'(c^*)$ and $H[k(t), c(t), \lambda(t)]$ tends to a constant and we have

$$\lim_{t\to\infty} e^{-\int_0^t\{\rho - n[k(\tau)]\}d\tau} H[k(t), c(t), \lambda(t)] = 0,$$

because $\rho > n[k(t)]$ for all k. The value of the program is thus given by $\int_0^\infty u[c(t)]e^{-\int_0^t\{\rho - n[k(\tau)]\}d\tau}dt = H[k(0), c(0), \lambda(0)]$. Because on a saddle path $\lambda(0) = u'[c(0)]$, we have $H[k(0), c(0), \lambda(0)] = H[k(0), c(0)]$, i.e., the value of the program is the value of the optimized Hamiltonian at time zero. For an analogous proof, see Tahvonen and Salo (1996).

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Learning by Living: Early Development

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Abstract

Mortality decrease and the accompanied increase in life expectancy increased demand for formal schooling in early development. Possibilities for home teaching increased because generations overlapped ever more. Accumulation of human capital endogenously occurs as a side effect of a mortality decrease. An infinite horizon continuous time optimization model with human capital accumulation described may have multiple equilibria. Poverty trap can be avoided if private returns to capital are high enough to lead to a high rate of accumulation of both types of capital. Because population growth is due to a decrease in mortality, high population growth refers to high human capital accumulation and high growth of income in the long run.

1 Introduction

The industrial revolution in Europe was accompanied by a quiet revolution of a mortality decrease and a lengthening of life. The increase in life expectancy from some thirty years in 1800 to some fifty years in 1900 changed man's image of self, time, and the universe and rational attitudes toward life gained ground from religious and deterministic ones (Chesnais 1992). As an outcome, a new epoch in accumulation of human capital arose. Learning through formal schooling became possible and rewarding, and informal learning increased because the overlap of generations increased.

Goodfriend and McDermott (1995) have shown that the increase in population density, which led to efficient markets and specialization, only took place in an epoch that they called "early development." In the spirit of Goodfriend and McDermott, we argue that mortality decrease and an increase in life expectancy had a special role only during this epoch. First years of life lengthening made learning possible and rewarding. In later stages of development, the importance of life lengthening decreased, and other sources of human capital accumulation, such as intentional R&D, became important.

In this paper we introduce a model in which learning takes place by living (see Arrow 1962); human capital accumulation is a costless side-effect of the mortality decrease. This approach emphasizes the increasing possibilities and demands for learning but leaves aside the costly supply side of formal schooling. Therefore, financial or time resources to accumulate human capital are not necessary, imperfect competition is not needed (see Romer 1990), and the model is in one sector only (see Uzawa 1965 and Lucas 1988).

As mortality initially decreases and life expectancy increases, human capital accumulation starts. This leads to increases in income, which, together with the accumulated human capital, keeps mortality falling and human capital accumulating. The endogenous process of a mortality decrease and human capital accumulation is introduced into a infinite horizon continuous time consumer optimization model of economic growth. However, models of economic growth deal with the growth of per capita income, and population growth, rather than mortality changes, are needed in these models. The link between a mortality decrease and population growth is that a decrease in mortality, rather than an increase in fertility, was the reason for population growth in early development.

The model discusses the role of mortality and fertility separately and suggests some benchmark names and dates in the history of medicine. These dates show that contraceptives were discovered quite lately, and it is possible that optimizing households were unable to reach the number of children that would have maximized their utility. Therefore, we assume that at each level of income, households take the child number as given, but by choosing their consumption and savings, the households choose the level of income and the population growth rate in the future.

The solution of the model shows that multiple steady states might appear. One of them is characterized by low income, low long-run growth, and low accumulation of human capital, and another is characterized by high income, high consumption, and high rate of accumulation of human capital. As compared to other multiple equilibria models in the field of population economics, such as those given by Becker *et al.* (1990) or Galor and Weil (1999), our model has the distinguishing feature that the poverty trap is associated with a low, not with high, population growth rate. The reason is in our emphasis on mortality. High population growth, low mortality, long life, the rapid rate of human capital accumulation, and high economic growth all exist simultaneously in this model.

Closely related are the papers of Strulik (1997) and Blackburn and Cipriani (1998 and 2002). Strulik posits a positive correlation between population growth and economic growth through the population pressure hypothesis, according to which new technology is adopted to ensure survival. Sturik also provides parametric functional relationships between income and mortality, and between income and fertility. Blackburn and Cipriani (1998) give an overlapping-generations model in which child mortality is reduced by altruistic parents who devote resources to take care of the health of their children. In Blackburn and Cipriani (1998), as in our paper, longevity (probability to survive in Blackburn and Cipriani) increases as human capital accumulation takes place. Our model in continuous time macroeconomic framework is, despite of quite similar starting points, technically quite different from that presented by Blackburn and Cipriani.

The outline of the paper is the following: Section 2 discusses the causality from a mortality decrease to an accumulation of human capital. Then the reverse causality, from human capital to decrease in mortality and fertility is discussed. Mortality and fertility is also connected to income increases. The modelling is "semi-endogenous," and technical simplicity permits us to state some historical connections. We discuss Europe and its offshoots. Parametric functional formulas are also supplied. In Section 3 we combine the model of human capital accumulation with a macroeconomic infinite horizon optimization model and deal with local and global properties of the solution of this model. The most important result is that the model may have multiple steady states. Section 5 provides a calibrated version of the model. It is shown that the income share of capital is important. If this share is below some critical value, the model is led to poverty trap, whereas, for only slightly higher values of this share, the high income steady state is available. Section 6 discusses this result.

2 Human Capital, Mortality, and Life Expectancy

The concept of "early development" emphasizes the unique nature of certain events and refers mostly to early years of industrialization and economic growth (Goodfriend and McDermott 1995). For most purposes this level of accuracy in the definition is suitable. In a purely demographic framework, however, a more detailed definition is available. Let Sweden serve as an example. In the beginning of nineteenths, mortality started its straightforward decrease and life-expectancy increased (Figure 1). On the other hand, because a permanent decrease in fertility started only after a lag of almost half a century, population growth started to accelerate.

In Europe, mortality decreased in two waves from 1800 onwards. The first wave reached France, Czechoslovakia and Scandinavian countries (Finland excluded), and the second wave, which started around 1870, decreased mortality in the rest of western and northern Europe (Chesnais 1992). Fertility started its continuous decrease around 1900 (see Coale and Treadway 1986).[1] If we define the period of "early development" to be the stage of demographic transition in which fertility decreases lagged behind mortality decreases and population growth accelerated, then the time span of the "early development" in Europe was thus some one hundred years. During this period, population growth more than doubled from 0.4 % to 1.1 %, and the population in Europe increased threefold (Chesnais 1992). In 1800, the average life expectancy at birth was 30-35 years, but increased to some 50 years in 1900 and to some 60 years in 1930 (Livi-Bacci 1997).[2]

[1] Fertility permanently fell below 90% of its traditional level (see Coale and Treadway 1986).

[2] The somewhat loose concept of mortality actually consists of many age-specific sub-concepts, such as infant mortality (under 1 year), child mortality (under 5 years) *etc.* These age-specific measures can be summarized in two ways. Firstly, the concept "crude death rate" calculates all deaths per population. This measure depends on the age structure of the population. In the concept of life expectancy (at birth), the age-specific mortality rates are

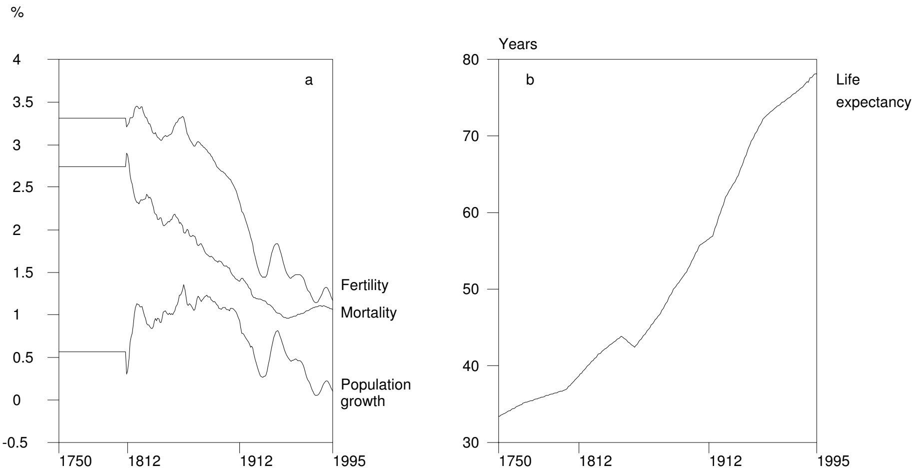

Figure 1: Panel a: Fertility (number of births per 100 inhabitants), mortality, and population growth in Sweden. The period average is shown for 1759-1810. Panel b: Life expectancy at birth. Data source: Statistical centralbyron i Sweden (2002), authors calculations.

2.1 From Mortality to Human Capital

Papers that discuss technical progress or new ideas that are produced through intentional research and development efforts, have been written by Romer (1990), Kremer (1993) and Jones (1995), to mention but some. In these papers, the non-rivalrous stock of knowledge is produced by a number of research workers, each discovering ideas at some rate. We follow certain ideas and notations in these papers with the difference that the accumulation of human capital comes as a side product of an increase in life expectancy, not as a result of intentional efforts. Another difference is that the stock of human capital accumulated is embodied in human beings, and is therefore rivalrous.

Let $e = e^a - \bar{e}$, where e^a is the actual life expectancy in years, and $\bar{e}$ is the life expectancy prevailing in a primitive subsistence economy. Thus, e gives the extra years above $\bar{e}$. Then, let

$$\dot{H} = \bar{\delta} e, \tag{1}$$

in which $\bar{\delta}$ gives the accumulation of human capital, H, per year above $\bar{e}$. This specification assumes $\dot{H} = 0$ for $e^a = \bar{e}$.

Equation (1) gives a linear relationship between years of living and $\dot{H}$. This need not be the case. It might be argued that the first ten years above $\bar{e}$

summarized to show what will be the expected length of life, if, in each age, the probability to die is as expressed in the age-specific measures of mortality. Both the concepts "mortality" and "life expectancy" refer to same phenomenon and are used here interchangeably.

(life expectancy increases, say, from 30 to 40 years) are more human capital productive than the latter ones (life expectancy increases, say, from 50 to 60 years). This is possibly true, if we think of the accumulation of human capital through formal education. The demand for schooling increases more in the former case. But human capital accumulation is essentially a social process (Lucas 1988), and once generations are overlapping enough, there is no need for each generation to start from the beginning; something can be inherited from previous generations. In general, we write

$$\dot{H} = \bar{\delta} e^{\gamma}, \tag{2}$$

and assume $\gamma < 1$ to keep the accumulation function of human capital concave. Low values for γ refer to high productivity of first years (the formal-schooling argument), while high values of γ refer to relative importance of the generations-overlap argument.

Next, we argue that human capital grows faster if the level already accumulated is larger. This is because of better formal education and better possibilities to learn valuable things from previous generations. Instead of being constant, the per capita accumulation $\bar{\delta}$ can be written as δH^{ϕ}, where ϕ is a positive constant. Human capital accumulation becomes

$$\dot{H} = \delta e^{\gamma} H^{\phi}. \tag{3}$$

In what follows, we use the simplifying assumption $\phi = 1$ saying that the rate of accumulation of human capital is still independent on its level.

As a final step, we move from life expectancy to population growth. For a given rate of fertility, life expectancy translates to population growth at the macro level. In a primitive subsistence economy, population growth rate is close to zero (Kremer 1993), and only years above $\bar{e}$ are relevant. Assume that an approximately constant multiplier $\omega \in (0, 1)$ transforms the years of life (above $\bar{e}$) to percentages of population growth. Then $\frac{\dot{L}}{L} = n = \omega e$ is the growth rate of population, L. Substituting $e = n/\omega$ into (3) and writing $\xi = \delta/(\omega^{\gamma})$ we finally have

$$\frac{\dot{H}}{H} = \xi n^{\gamma}. \tag{4}$$

In theoretical developments discussed below, the notation $\frac{\dot{H}}{H} = h(n),\ h'(n) > 0,\ h''(n) < 0$ is also used.

2.2 From Human Capital and Income to Mortality and Fertility

In Section 2.1 we argued that there exists a causality from mortality to accumulation of human capital, H. In this Section we concentrate on reversed causality, from human capital to mortality. We also briefly discuss the causality from human capital to fertility. In addition, the role of income is studied.

Name	**Years**	**Discovery**
Jenner	1749-1823	Small-box vaccination (1796)
Morton	1819-1868	Ether (1846)
Semmelweiss	1818-1865	Asepsis (1847)
Pasteur	1822-1895	The germ theory of diseases (1860)
Lister	1827-1912	Antisepsis (1867)
Fleming	1881-1955	Penicillin (1928)
Goodyear	1800-1860	Vulcanized rubber (1839)
Pincus	1903-1967	The pill (1951)

Table 1: Benchmark inventions in medicine.

Figure 1 shows that it is necessary in calculating population growth, to discuss both fertility and mortality, because population growth is their difference. Figure 1 also shows that population growth, our focus in this Sections as a difference of two non-linear functions, may take a rather complex functional form. Therefore, we start by discussing the causal link from human capital to mortality and fertility in Section 2.2.1, and from income to mortality and fertility in 2.2.2. Note that the discussion in Sections 2.2.1 and 2.2.2 is a partial analysis only. The functional formulas are discussed in Section 2.2.3.

2.2.1 Role of Human Capital

The *mortality* decreases due to human capital accumulation for example because greater care is devoted to health of children and adults (see Blackburn and Cipriani 1998). In this paper, we concentrate on the type of human capital which manifested itself as inventions, and we argue that just these inventions were typical to early development. During that epoch, inventions decreased mortality both directly, due to achievements in medicine, and indirectly, due to achievements in production of food and other goods. To avoid double counting, only direct effects are discussed here, whereas indirect effects are captured as increases in income.

Early inventions in medicine were important causes of mortality decrease. This can be exemplified by the discovery of asepsis. Ignaz Semmelweiss, when working in two obstetrical wards in Vienna in 1847, noticed that the death rate for puerperal fever between the wards was large and in the wrong direction. In ward II, conducted by midwives, it rarely exceeded three per cent, while in ward I, where students of medicine and their teachers were working, the death rate sometimes reached twenty or thirty per cent. Semmelweiss realized that the explanation was in the daily routine of the teachers and students, which first led them to the autopsy room, and then to ward I, were the examination of mothers was carried out. Once Semmelweiss required his staff to follow tight aseptic methods, the death rate in ward I decreased below that in ward II.

Table 1 gives some benchmark names and years in the history of early medicine. The first wave of mortality decrease took place in Europe soon after Jenner's findings, and the second wave started, once Pasteur had published his final conclusions between microbes and diseases, and earlier methods, such as those suggested by Semmelweis, ultimately adopted in large scales. Mortality, m, can be written

$$m = m(H); \;\; m'(H) < 0; \;\; m''(H) > 0,$$

in which the positive second derivative refers to "fishing out;" we can not expect as great discoveries to be done again.

When discussing the causality from human capital to *fertility*, it is essential to explain the lag between declines in mortality and fertility. Without this lag, the population growth rate would never have increased in early Europe (see Figure 1). There exists two types of explanations for fertility lag, social and technical. The former says that social and moral rules were developed to promote high fertility which, in the conditions of high mortality, was necessary for survival (Notestein 1945). This "social factory" (see Notestein 1945) was then slow to change, even if its original function disappeared. The second explanation, the technical one, relies on poor availability and high costs of effective contraceptives (Easterlin 1978).

Both approaches lead us to conclude that human capital has decreased fertility and, importantly, shortened the fertility lag: Human capital accumulation apparently tends to increase rational thinking and destroy meaningless habits. Table 1 gives some benchmark years of contraceptive techniques. Modern contraceptives increased effectiveness and convenience of birth control, but they were by no means the first ones. High fertility among Canadian Hutterites (about 11 children per woman) shows that the European average (about 5 children per woman in 1800, Livi-Bacci 1997) was a result of regulation of some kind. As human capital increased, new methods were discovered, and traditional methods were used in more rational ways. Therefore, we write the fertility function

$$f = f(H); \;\; f'(H) < 0.$$

The second derivative is left unspecified here. In summary, the population growth rate, being the difference between mortality and fertility, can be given as a function of human capital as

$$n = n(H) = f(H) - m(H). \tag{5}$$

The sign of the first derivative $n'(H) = f'(H) - m'(H)$ depends on the absolute sizes of $f'(H)$ and $m'(H)$. We assume that $n'(H)$ is negative. This is based on the reasoning that even if decrease in mortality was large, and took place in spectacular waves, it was more than off-set by gradual decreases in fertility along with increases in human capital.[3]

[3] The analysis here is partial; that mortality (in the data) decreased more than fertility is not contradictory.

2.2.2 Role of Income

The income per capita in 1820 in Europe and some of its offshoots was 1.055 (1985 US dollars) but increased to 3.068 in 1913 (Maddison 1994).[4] The decrease of *mortality* as a function of per capita income $y = \frac{Y}{L}$ hardly needs comments. We write

$$m = m(y); \quad m'(y) < 0.$$

The models of *fertility* as a function of income are the core field of the microeconomics of family, first analyzed by Becker (1960). These models derive from time budget constraint, which implies that

- as income increases, the demand for children increases if they are normal goods (pure income effect);
- because income increases mainly comes through increases in wages, the price of time increases, and the relative demand for time intensive goods, such as children, tends to decrease (price-of-time effect).

The net effect of income is an empirical question. Its is usually argued that, at low levels of income, the pure income effect dominates whereas the contrary is true, as income further increases.

The Beckerian model actually deals with the demand for surviving descendants, not fertility alone. If it is rewritten explicitly in terms of mortality and fertility components of population growth rate, the dominance of the substitution effect says that fertility decreases slowlier than mortality. Instead of doing this, we directly summarize the income discussion by writing the population growth as a function of per capita income, y, as follows:

$$\begin{aligned} n &= n(y), \\ n'(y) &> 0 \text{ for } y < \bar{y}; \end{aligned}$$

in which $\bar{y}$ is the level of per capita income below which per capita income stays during early development. The partial effects of human capital and income on population growth can now be combined to give:

$$\begin{aligned} n &= n(y, H), \\ \frac{\partial n}{\partial H} &< 0; \ \forall y, \\ \frac{\partial n}{\partial y} &> 0 \text{ for } y < \bar{y}; \ \forall H. \end{aligned} \tag{6}$$

[4] 11 countries in Europe, Australia, Canada, United States.

2.2.3 Functional Formula for Population Growth

Several authors have proposed useful functional formulas for population growth (see Kremer 1993 and Jones 1995). We take as our starting point the additive formulation provided by Lee (1988):

$$n(y,H) = \kappa \log y - \varkappa \log H. \tag{7}$$

Lee's equation can be rewritten as

$$n(y,H) = \log(\frac{y^{\kappa}}{H^{\varkappa}}),$$

In the case in which $\kappa = \varkappa = 1$, equation (7) becomes $n(y,H) = \log \tilde{y}$, in which $\tilde{y} = \frac{Y}{HL} = \frac{y}{H}$ is the output per efficiency units of labor. This is a convenient expression because models of economic growth are formulated in terms of efficiency units. To develop Lee's equation (7) further, note that it implies $\lim\limits_{\tilde{y}->0} \frac{\partial n}{\partial \tilde{y}} = \infty$. In economic growth models, it is necessary that the slope of the effective depreciation line (the derivative of population growth plus the depreciation of capital) is less than infinity close to the origin; this is needed to keep the depreciations below the production function to guarantee the existence of an interior steady state in the model. Therefore, the limit condition $\lim\limits_{\tilde{y}->0} \frac{\partial n}{\partial \tilde{y}} = 0$ is needed here. On the other hand, to keep population growth bounded, the limit condition $\lim\limits_{\tilde{y}->\infty} \frac{\partial n}{\partial \tilde{y}} = 0$ is also necessary.[5] Therefore, a better functional formula is needed. The problem is that no algebraic function or simple transcendental function can satisfy *both* limit conditions. We suggest that the algebraic counterpart of the logistic equation, given by

$$n(y,H) = n(\tilde{y}) = \frac{\eta}{1 + (\mu\tilde{y})^{-\beta}}$$

is useful. In this formulation, $\eta > 0$ refers to maximal population growth rate reached, $\beta \in (0,1)$ gives the curvature of the function, and $\mu > 0$ gives its slope. A high β makes the function very $S-$shaped, and a high μ refers to a steep slope of the function. Note, that these two are different properties.[6]

In what follows, the function above is combined with a neoclassical model of growth. In this model, argumentation takes place in terms of the productive capital per efficiency units of labor, $\tilde{k} = \frac{K}{HL}$, rather than in terms of its concave function $\tilde{y} = f(\tilde{k})$. Therefore, as a final simplification, we replace $\tilde{y}$ by $\tilde{k}$ to derive:

[5] This limit condition is satisfied by Lee.

[6] The derivative becomes

$$\frac{dn}{d\tilde{y}} = \frac{\beta\mu\eta(\mu\tilde{y})^{-\beta-1}}{\left[1 + (\mu\tilde{y})^{-\beta}\right]^2} = \frac{\beta\mu\eta}{(\mu\tilde{y})^{\beta+1} + 2\mu\tilde{y} + 1/(\mu\tilde{y})^{\beta-1}}.$$

Then, $\lim\limits_{\tilde{y}->0} \frac{\partial n}{\partial \tilde{y}} = \lim\limits_{\tilde{y}->\infty} \frac{\partial n}{\partial \tilde{y}} = 0$.

$$n(\tilde{k}) = \frac{\eta}{1+(\mu\tilde{k})^{-\beta}}. \tag{8}$$

The partial derivative formulation of (8) is $n = n(\tilde{k}), n'(\tilde{k}) > 0$ (plus limit conditions). Figure 2 illustrates the role of the parameters in function (8). Curves A and B share the same curvature parameter $\beta = 3$, but the slope parameter, μ, is smaller in A. On the other hand, curves B and D have the same slope parameter, $\mu = 0.2$, but the curvature parameter, β, is larger in D.

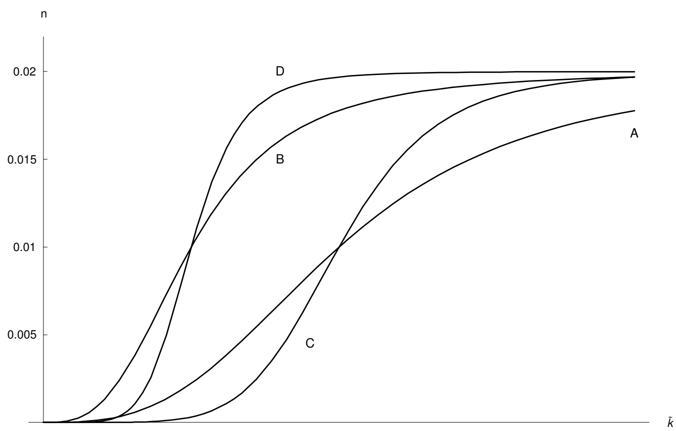

Figure 2: Population growth rate as a function of capital per efficient labor. In each curve, the maximum population growth rate is $\eta = 0.02$. Other parameters are in curve A: $\beta = 3,\ \mu = 0.1,$ in curve B: $\beta = 3,\ \mu = 0.2,$ in curve C: $\beta = 6,\ \mu = 0.1,$ in curve D: $\beta = 6,\ \mu = 0.2.$

To summarize in terms of derivatives, we argued that human capital accumulates because mortality decreases. Then, because a mortality decrease implies population growth, we wrote $\frac{\dot{H}}{H} = h(n),\ h'(n) > 0$. The reverse causality from human capital to population growth is due to the fact that fertility is, more than mortality, sensitive to human capital: $n = n(H), n'(H) < 0$. In addition, in early development, the pure income effect dominates: $n = n(y), n'(y) > 0$. Finally, for modelling reasons, income is replaced by capital to derive $n = n(\tilde{k}), n'(\tilde{k}) > 0$. Because $\tilde{k} = \frac{K}{HL}$, we have $\frac{\partial n}{\partial H} < 0$ and $\frac{\partial n}{\partial k} > 0$ as was, remembering that capital proxies income, argumented above.

3 The Planner's Problem

Assume we have a benevolent planner who maximizes the utility of a representative household in a closed economy. Under perfect foresight, the solution of this problem is identical to that of the competitive model. The instantaneous utility is a function of per capita consumption $u[c(t)] = u[C(t)/L(t)]$ and the number of family members $L(t)$. The future utils are discounted by the time preference factor ρ. Therefore, the planner's infinite horizon objective functional becomes

$$U = \int_0^\infty L(t)u[c(t)]e^{-\rho t}dt. \tag{9}$$

Romer (1990) has pointed out that the aggregate production function with a non-rivalrous stock of knowledge can not be linearly homogenous. To duplicate the output, there is no need to duplicate the stock of knowledge, only other inputs. In our case, however, human capital is of embodied type, and Romer's argument is not valid. Therefore, we assume that output Y produced by a linearly homogenous production function $Y(t) = F(K(t), H(t)L(t))$ satisfying the Inada conditions. The capital accumulation equation becomes

$$\dot{K} = F(K(t), H(t)L(t)) - C(t) - \delta K(t), \tag{10}$$

in which δ is the constant rate of depreciation. Because the production function is homogenous, we can write $\tilde{y}(t) = Y(t)/H(t)L(t) = f[\tilde{k}(t)]$, where $\tilde{k}(t) = K(t)/H(t)L(t)$. The per efficient capita production function $f[\tilde{k}(t)]$ is strictly concave, i.e., diminishing returns hold for all $\tilde{k}(t)$ and $\lim\limits_{\tilde{k}(t)-0} f[\tilde{k}(t)] = \infty$ and $\lim\limits_{\tilde{k}(t)-\infty} f[\tilde{k}(t)] = 0$.

The accumulation of efficient capital-labor ratio $\tilde{k}(t)$ becomes $\dot{\tilde{k}}(t) = f[\tilde{k}(t)] - \tilde{c}(t) - (\frac{\dot{H}(t)}{H(t)} + \frac{\dot{L}(t)}{L(t)} + \delta)\tilde{k}(t)$.

In Sections 2, we derived $\frac{\dot{L}(t)}{L(t)} = n[\tilde{k}(t)]$ and $\frac{\dot{H}(t)}{H(t)} = h[n(t)] = h\left[n\left(\tilde{k}(t)\right)\right]$. After substituting these expressions into (9) and (10), the objective functional, and the accumulation of efficient capital-labor ratio $\tilde{k}(t)$ became:

$$U = \int_0^\infty u[c(t)] \cdot \exp\left[-\int_0^t \left\{\rho - n[\tilde{k}(\tau)]\right\} d\tau\right] dt. \tag{11}$$

$$\dot{\tilde{k}}(t) = f[\tilde{k}(t)] - \tilde{c}(t) - [h\left[n\left(\tilde{k}(t)\right)\right] + n[\tilde{k}(t)] + \delta]\tilde{k}(t). \tag{12}$$

Equations (11) and (12) now give an infinite horizon control problem with a single control variable $c(t)$ and a single state variable $\tilde{k}(t)$. It is assumed that the economy starts with a positive capital stock $\tilde{k}(0) > 0$. By making the choice between consumption and saving, the planner determines the capital accumulation $\dot{\tilde{k}}(t)$. This determines (through production and income) the population growth rate $n(t)$ which, in turn, determines the human capital accumulation.

The discount factor in (11) is not constant because the population growth rate is variable. Therefore, we first transform the problem to "virtual time" (Uzawa 1968) by writing

$$\Delta(t) = \int_0^t \left\{\rho - n\left[\tilde{k}(\tau)\right]\right\} d\tau,$$

so that $\frac{d\Delta(t)}{dt} = \rho - n\left[\tilde{k}(t)\right]$ and $dt = \frac{d\Delta(t)}{\rho - n\left[\tilde{k}(t)\right]}$. The problem now becomes a standard infinite horizon problem in terms of $\Delta(t)$. Because the problem in this paper is mathematically similar to that solved in Lehmijoki (2003), we only sketch the solution here. The objective functional, the equation of motion, the current value Hamiltonian $\mathcal{J} = \mathcal{J}[k(t), c(t), \lambda(t)]$, and the necessary conditions are (see Benveniste and Scheinkman 1982):

$$U = \int_0^\infty \frac{u[c(t)]}{\rho - n\left[\tilde{k}(t)\right]} e^{-\Delta(t)} d\Delta(t), \tag{13}$$

$$\begin{aligned}
\frac{d\tilde{k}(t)}{dt} \cdot \frac{dt}{d\Delta(t)} &= \frac{d\tilde{k}(t)}{d\Delta(t)} \\
&= \frac{f\left[\tilde{k}(t)\right] - \tilde{c}(t) - \left\{h\left[n\left(\tilde{k}(t)\right)\right] + n[\tilde{k}(t)] + \delta\right\}\tilde{k}(t)}{\rho - n\left[\tilde{k}(t)\right]},
\end{aligned}$$

$$\mathcal{J} = \frac{u[c(t)]}{\rho - n\left[\tilde{k}(t)\right]} + \lambda[\Delta(t)] \frac{f\left[\tilde{k}(t)\right] - \tilde{c}(t) - (h\left[n\left(\tilde{k}(t)\right)\right] + n[\tilde{k}(t)] + \delta)\tilde{k}(t)}{\rho - n\left[\tilde{k}(t)\right]},$$

$$\frac{\partial \mathcal{J}}{\partial c(t)} = 0, \tag{14}$$

$$\frac{d\lambda[\Delta(t)]}{d\Delta} = -\frac{\partial \mathcal{J}}{\partial \tilde{k}(t)} + \lambda[\Delta(t)], \tag{15}$$

$$\lim_{\Delta(t)\to\infty} \left\{\lambda[\Delta(t)] \cdot e^{-\Delta(t)} \cdot \tilde{k}(t)\right\} = 0. \tag{16}$$

These necessary conditions can be solved in virtual time and the results can the be transformed to natural time again (see Lehmijoki 2003).[7] The Euler equation of the problem becomes:

$$f' - \delta = -\frac{u''c}{u'} \cdot \frac{\dot{c}}{c} + \rho + (h'n' + n')\tilde{k} - \frac{n'}{u'H}\mathcal{J}(\tilde{k}, c). \tag{17}$$

For graphical purposes, we adopt here the standard constant-elasticity-of-intertemporal-substitution form of utility function given by $u(c) = \frac{c^{1-\theta}}{1-\theta}, \theta > 0, \theta \neq 1$. The final dynamic equations and the optimized Hamiltonian $\mathcal{J}(k, c)$ then are:

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{1}{\theta}\left\{f' - (\rho + \delta + \theta h) - (h'n' + n')\tilde{k} + \frac{n'}{u'H}\mathcal{J}(k, c)\right\}, \tag{18}$$

[7] We abandon time indices and functional references to make the lenghtly expressions more readable.

$$\dot{\tilde{k}} = f - \tilde{c} - (h + n + \delta)\tilde{k}, \tag{19}$$

$$\frac{n'\mathcal{J}(k,c)}{u'H} = \frac{n'}{\rho - n}\left\{\frac{\theta\tilde{c}}{1-\theta} + f - (h+n+\delta)\tilde{k}\right\}. \tag{20}$$

3.1 Local Analysis: Isoclines and Steady States

The isoclines of the model in the $\tilde{k} - \tilde{c}-$space become

$$\dot{\tilde{k}} = 0 \Leftrightarrow \tilde{c} = f - (h+n+\delta)\tilde{k}, \tag{21}$$

$$\begin{aligned}
\frac{\dot{\tilde{c}}}{\tilde{c}} &= 0 \Leftrightarrow \tilde{c} = \frac{(\theta-1)(\rho-n)}{\theta n'} \cdot \\
&\quad \left\{f' - (\rho + \delta + \theta h) - (h'n' + n')\tilde{k} + \frac{n'}{\rho - n}\left[f - (h+n+\delta)\tilde{k}\right]\right\} \\
&= \frac{\theta - 1}{\theta}\left\{[f' - (\rho + \delta + \theta h)]\frac{\rho - n}{n'} + [f - (h + \rho + \delta + h'(\rho - n))\tilde{k}]\right\}.
\end{aligned} \tag{22}$$

To see the shape of the isocline $\dot{\tilde{k}} = 0$, note that the effective depreciation factor $(h+n+\delta)\tilde{k}$ is non-linear because both h and n are functions of $\tilde{k}$. Therefore, the isocline $\dot{\tilde{k}} = 0$ is not concave. However, the variation in both h and n along $\tilde{k}$ is small in absolute size (see Figure 2 for n). Thus, $\dot{\tilde{k}} = 0-$line resembles its counterpart in the standard Ramsey-model. The $\dot{\tilde{k}} = 0-$line hits the $k-$axis at $\tilde{\tilde{k}}$. Figure 3 gives the shape of the $\dot{\tilde{k}} = 0-$line.

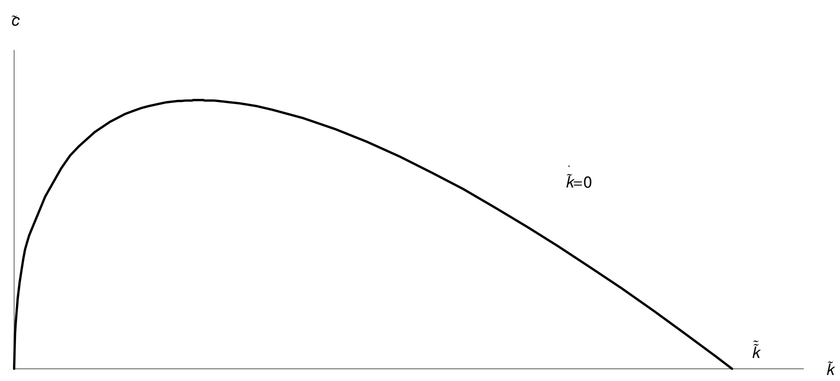

Figure 3: The isocline for capital accumulation.

The isocline $\dot{\tilde{c}} = 0$ consists of four parts, which will be analyzed in turn. To make it easy, we address letters $A-D$ for each. Because $\lim_{\tilde{k}\to 0} n'(\tilde{k}) = \lim_{\tilde{k}\to\infty} n'(\tilde{k}) = 0$, part $A = \frac{\rho-n}{n'} > 0$ is $U-$shaped and takes small numbers in the bottom of it. Part $B = f' - (\rho + \delta + \theta h)$ approaches $+\infty$ as $\tilde{k} - > 0$ and then decreases. The decrease is almost monotonous, i.e., even if θh is non-linear in $\tilde{k}$, its absolute size is always small. B hits the $k-$axis at $\overset{=}{\tilde{k}}$, and approaches $-\infty < -\rho + \delta + \theta h < 0$ as $\tilde{k} - > \infty$. The multiple $A \cdot B$ then swings from $+\infty$ to $-\infty$ and changes sign at $\overset{=}{\tilde{k}}$. Part $C = f - (h + \rho + \delta + h'(\rho - n))\tilde{k}$ is much like the $\dot{\tilde{k}} = 0-$line; the difference between the latter and the former is that between $n\tilde{k}$ and $(\rho + h'(\rho - n))\tilde{k}$. Part $D = \frac{\theta-1}{\theta} < 1$ is a positive constant.

To summarize, because both A and B take large values for small $\tilde{k}$, and because A is large for large $\tilde{k}$, the $\dot{\tilde{c}} = 0-$line is dominated by $A \cdot B$ for small and large $\tilde{k}$. For intermediate values of $\tilde{k}$, both A and B are small, and part C dominates. Therefore, the $\dot{\tilde{c}} = 0-$line swings from $+\infty$ to $-\infty$, but has a curved shape for intermediate values of $\tilde{k}$.

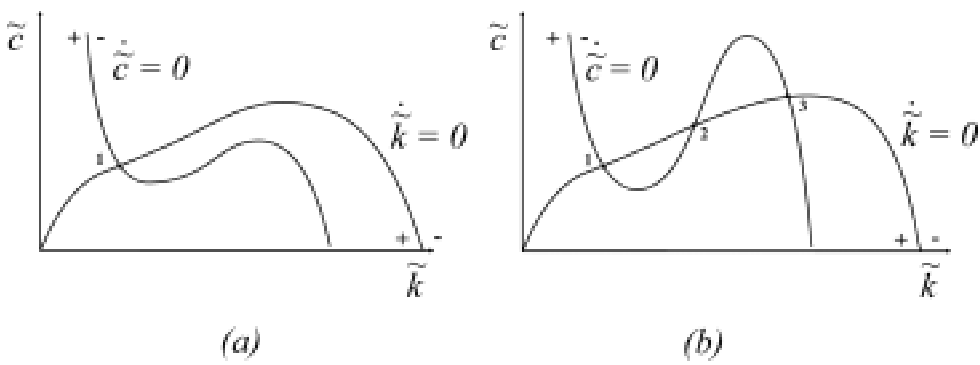

Figure 4: The alternatives for the phase diagram; a single steady state (panel a), and three steady states (panel b).

Appendix A gives a proof that the model has at least one steady state such that $\tilde{k}^* > 0$. However, due to the possibility that the shape of the $\dot{\tilde{c}} = 0-$line can be very curved, the number of steady states can also be three.[8] Figure 4 illustrates these cases.

A local stability analysis shows that the single steady state (Figure 4, a) is saddle stable. In the case of three steady states (Figure 4, b), the first and third steady states are saddles, and the second is an unstable focus or node. In each

[8] In principle, even a larger number of steady states is possible due to non-linearities in n and h. It is also possible that the isoclines are tangents, in which case the number of steady states is even. In this paper, we concentrate on the cases of one or three steady states.

case, the local analysis shows that, close to the saddle stable steady state, the stable paths toward saddle points run from south-west or from north-east. For a formal discussion, see Appendix A.

4 Global Solution

The global optimality in a system of a pair of non-concave differential equations was first analyzed by Skiba (1976). An alternative proof for his results was given by Tahvonen and Salo (1996). Lehmijoki (2003) applied the latter to show that in a case, in which the discount rate of the model is not constant the results are also valid. We discuss the proof here only briefly.

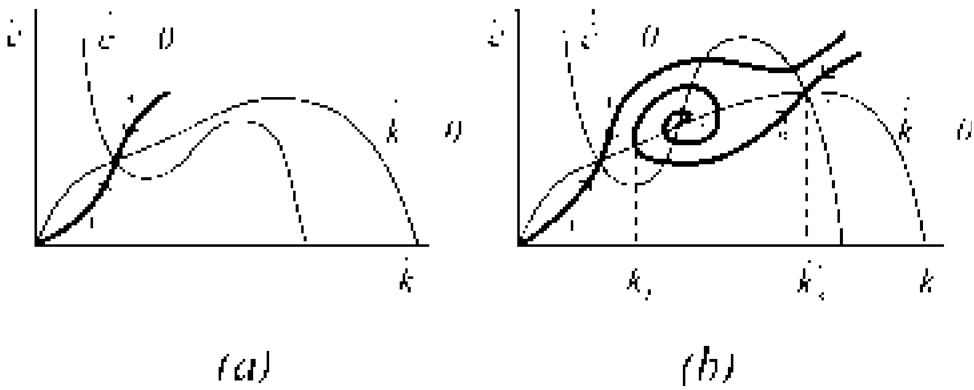

Figure 5: Possible shapes of the saddle paths: the single steady state case (panel a) and the three steady states case, in which path A is running non-spiraling from the north-east (panel b). The lowest possible initial capital $\tilde{k}_l$ to reach steady state 3 and the steady state capital $\tilde{k}_3^*$ shown in panel b.

In a case of a single steady state (see Figure 5, a), it is optimal for all initial capital stocks $\tilde{k}(0) = \tilde{k}_0$ to choose $\tilde{c}(0)$ such that it lies on a saddle path leading to the steady state (see Lehmijoki 2003). If the number of the steady states is three, and if two saddle paths are available for some initial capital stock, then the planner has to discriminate between these two paths to maximize the value of the program. Skiba (1978) has shown that the value of the program is equal to the value of the optimized Hamiltonian divided by the discount rate and evaluated at time zero. In Appendix B we show that this translates to our problem with the reformulation that in our problem the optimized Hamiltonian $\mathcal{J}(\tilde{k}, c)$ itself is already divided by the discount rate. Appendix B also states the following results:

$$\frac{\partial \mathcal{J}(\tilde{k}, c)}{\partial \tilde{c}} = \frac{u''(c)H^2}{\rho - n}\dot{\tilde{k}}, \tag{23}$$

$$\frac{d\mathcal{J}[\tilde{k}, c(\tilde{k})]}{d\tilde{k}} = \frac{u'H}{\rho - n}[\rho - n + (\theta - 1)h] > 0. \tag{24}$$

These results are analogous to their counterparts in Lehmijoki (2003):

In the case of three steady states, three sub-cases, depicted in Figures 5, b and in Figure 6 are possible.[9] Of these cases, only that depicted in Figure 5, b was not present in Lehmijoki (2003). Assume that this case is realized. Let $\tilde{k}(0) = \tilde{k}_l$, in which $\tilde{k}_l$ is the lowest initial capital from which steady state 3 can be reached. Then $\tilde{c}_0^B$ lies on the $\dot{\tilde{k}} = 0-$line, but $\tilde{c}_0^A$ lies above it. Equation (23) then implies $\mathcal{J}(\tilde{k}_0, c_0^A) > \mathcal{J}(\tilde{k}_0, c_0^B)$. For all $\tilde{k}(0) \in (\tilde{k}_l, \tilde{k}_3^*)$, path A lies above path B, and equation (24) implies that the value of the program increases faster along B than along A. However, for $\tilde{k}(0) = \tilde{k}_3^*$ it still holds $\mathcal{J}(\tilde{k}_0, c_0^A) > \mathcal{J}(\tilde{k}_0, c_0^B)$ (see 23*).* Because $\mathcal{J}[\tilde{k}, c(\tilde{k})]$ is continuous in $\tilde{k}$, $\mathcal{J}(\tilde{k}_0, c_0^A) > \mathcal{J}(\tilde{k}_0, c_0^B)$ then holds for all $\tilde{k}(0) \in (\tilde{k}_l, \tilde{k}_3^*)$. Therefore, for all initial states, it is optimal for the planner to choose the policy function A.

Cases given in Figure 6 a and b had their counterparts in Lehmijoki (2003) and we only state that in Figure 6 a there exists a threshold initial capital stock $\tilde{k}(0) = \tilde{k}_m \in (\tilde{k}_l, \tilde{k}_h)$ after (below of) which path B (path A) is optimal. In case given in Figure 6 b, path B is always optimal.

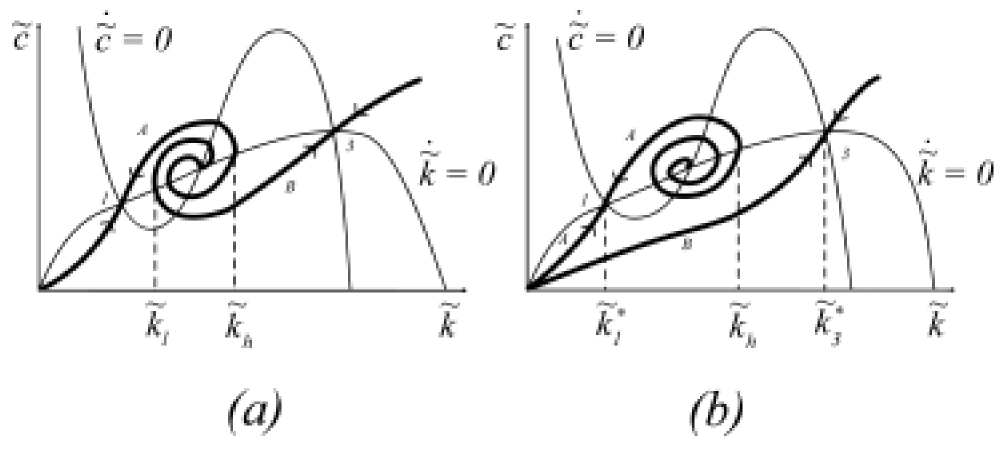

Figure 6: Possible shapes of the saddle paths: in panel a both paths spiral out of the focus; in panel b path B runs from the origin. In panel a, capital stocks $\tilde{k}_l$ and $\tilde{k}_h$ are shown, whereas $\tilde{k}_1^*$, $\tilde{k}_h$, and $\tilde{k}_3^*$ are shown in panel b.

[9] We used the parametric example discussed in Section 5 to state that these cases really exist. For parametric figures, see Figure 7.

To summarize, if the case is as depicted in Figure 5, b, the economy is always led to steady state 1, in which the efficient per capita capital is small, population grows at a low rate, and the rate of human capital accumulation is only low. Because, in any steady state, all per capita terms, such as per capita income and per capita consumption, grow at the rate of human capital accumulation, this case can, for a good reason, be denoted as "absolute poverty." Note that the single steady state case is of this same type because isoclines intersect only once and at a low level of $\tilde{k}$. In Figure 6 a, steady state 3 is reached only if the initial capital stock exceeds its threshold value $\tilde{k}_m$. Therefore, this situation denoted as "poverty trap" as usually; the destination of poverty or that of prosperity is determined by the initial state. Finally, in the case given in Figure 6 b, a path to steady state 3 is open from the origin and the economy proceeds toward it from all initial states. In steady state 3, capital stock is high, population grows at a high rate and human capital accumulates fast, and all per capita terms grow at a high rate. Therefore, the situation depicted in Figure 6 b can be denoted as "prosperity available." Note also that high (low) population growth, high (low) technical progress, and rapid (slow) increase in per capita income all exist simultaneously as is implied by the data (Maddison 1994, Kremer 1993).

The analysis above deals with the central planner's solution. Alternatively, the model can be formulated to a decentralized competitive model with a representative consumer and firm. The human capital accumulation $\dot{H}$ then refers to that accumulation within a family and its stock to the stock within a family respectively. Because human capital accumulation is a costless by-product, no markets for it are needed. Other features of the model are analogous to those discussed in Lehmijoki (2003); among these features again is the problem of indeterminacy of the decentralized model in a case of several possible steady states, but also relatively firm empirical data, which shows that the Western countries discussed in the model, have proceeded toward the high growth steady state, so that the indeterminacy is not a notably serious problem from the empirical point of view.

5 Comparative Dynamics

Mostly, comparative dynamics discuss the sensitivity of endogenous variables to exogenous parameters in the long-run, i.e., in a steady state. In a multiequilibrium framework, however, comparative dynamics have two new perspectives.[10] First, a shift in an exogenous parameter can lead to a change in the number of steady states (see Honkapohja and Turunen-Red 2002) as shown in Figure 4. Second, a shift in an exogenous parameter induces a change in the global structure of the vector field spanned by the solution curves. This change is then reflected as a change in the slope of policy functions A and B as depicted in Figures 5 b and 6 a and b.

In the model, the parameters η, μ, β, ξ, and ϕ appear in the formulas for population growth and human capital accumulation. The parameters δ, θ, and

[10] For a closer discussion, see Lehmijoki 2003.

Value	Parameter
$\eta = 0.02$	Maximum rate of population growth
$\mu = 0.2$	Steepness of population growth
$\beta = 6$	Curvature of population growth
$\gamma = 0.8$	Curvature of human capital accumulation
$\xi = 0.46$	Scales max. rate of hum. cap. to 0.02
$\delta = 0.03$	Rate of depreciation
$\rho = 0.03$	Rate of time preference
$\alpha \in [0.3 - 0.7]$	Capital's share of income
$\theta = 3$	Reciprocal of the elast. intertemp. subst.

Table 2: The values for the parameters.

ρ refer to the rate of depreciation, to the utility function $u(c)$, and to the rate of time preference. A shift in each of them induces some type of changes in the model. However, in the introduction, we argued that the decrease in mortality and the accompanied accumulation of human capital had their starting point in the initial increase in income due to capital accumulation. Therefore, we want to concentrate on the determinants of this starting point, and we argue that the profitability of capital accumulation was an important determinant of the development under the conditions described by the model.

To perform an exercise of comparative dynamics, we adopt the Cobb-Douglas production function $\tilde{y} = \tilde{k}^{\alpha}$. We concentrate on the role of the parameter α, the income share of capital incomes, and let it vary from 0.3 to 0.7. The values of parameters δ, θ, and ρ are close to those used, for example, by Barro and Sala-i-Martin (1995). Of the demographic parameters, $\eta = 0.02$ (maximum rate of population growth) is directly empirically justifiable, whereas $\mu = 0.3$, $\beta = 6$, and $\gamma = 0.8$ merely give a modest shape to the functions. The parameter $\xi = 0.46$ (scales the maximal rate of human capital accumulation to 0.02) again is relatively firmly rooted. As a whole, however, the model is rich in parameters, each of which can, at best, be only a rough approximation of reality. Therefore, the principle, not the numerical values, is of importance here. Because functions (18) and (19) are continuous and smooth in the $\tilde{k} - \tilde{c}-$space, the isoclines and the flow of the vector field changes smoothly (Guckenheimer and Holmes 1983). Figure 7 shows the parametric counterparts of Figures 5 and 6.

Using other parameters as given in Table 2 and letting α to go from 0.3 to 0.7, we calculate and draw a series of phase diagrams to find the critical values for each class. Table 3 reports the limits of α for each class. It also reports the limits of the population growth rate and the rate of human capital accumulation, which are reached in the relevant steady state. Note that in the case of a poverty trap, both steady state are possible. Therefore, the first row refers to the values in steady state 1 and the second row to the values in steady state 3. These steady states are reached from $k(0) < k_m$ and $k(0) > k_m$ respectively.

Table 3 and Figure 7 show that as α increases, the realized case goes from "absolute poverty" to "prosperity available." This result can be understood in

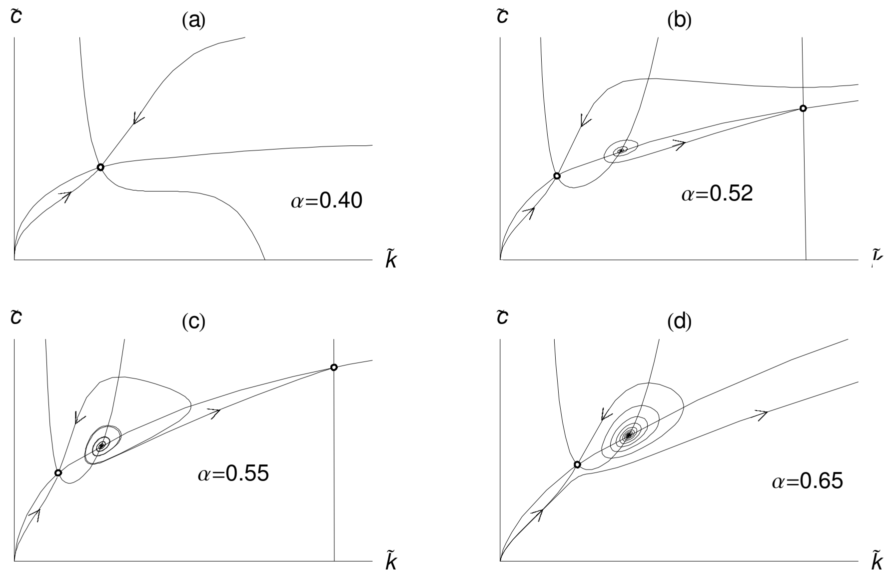

Figure 7: Parametric figures. Only the relevant parts of the figures shown. In panel d steady state k_3^* falls outside the figure range.

α	$n(k^*)(\%)$	$h(n^*)(\%)$	Nature
$0.300 - 0.528$	$0.142 \ - 0.400$	$0.241 \ - 0.560$	absolute poverty
$0.529 - 0.631$	$0.405 \ - 0.548$	$0.557 \ - 0.710$	poverty trap
	$1.908 - 2.000$	$1.926 - 2.000$	
$0.632 - 0.700$	2.000	2.000	prosperity available

Table 3: Limit for the value of capital share, population growth, and human capital accumulation in each class.

the light of the Euler equation (17). An increase in α increases the marginal productivity of capital for each $\tilde{k}$, and an investment, which previously was unprofitable, now becomes worth doing. Therefore, instead of decumulating, the planner tends to accumulate, mortality starts to decrease and population starts to grow. Life gets longer and human capital starts to accumulate, further decreasing mortality and increasing income. A virtuous cycle starts and the economy is led toward prosperity.

The importance of the limits between these classes is different in different situations. For example all European countries had to start their process toward modern incomes from very low levels of capital stocks and steady state 3 could be reached only if the value of α was high enough to lead to the realization of the case "prosperity available." The situation was different in the new continents, America, Australia, which were provided with affluent natural resources. As the immigrants arrived, they "jumped" to a high level of capital stock. In this situation, even if α would have been within the range of the "poverty trap," there would have been an escape, if the initial capital they faced exceeded the threshold value k_m.

6 Discussion

It is conventional to think that countries in which population growth is high are poor. This paper discusses an alternative possibility by presenting a model in which population growth accelerates because mortality decreases: a long life is available to learn and human capital accumulates at a high rate. The model has multiple steady states, and the high income steady state is characterized by high population growth rate, low mortality, high rate of human capital accumulation and high rate of increase of per capita consumption and income. The reasoning and the results seem to fit the experience in most of Europe and its offshoots during their early development. Continuous economic growth was accompanied by continuous acceleration of population growth and continuous increase in life expectancy.

Matsuyma (1991) has suggested that industrialization is a process that, for several reasons, can lead initially similar countries to diverging results; some countries proceed toward permanent poverty and some toward ever increasing prosperity. The model in this paper has this feature as well. In our model, the income share of productive capital, α, is decisive. If α is high enough, the economy accumulates at high rate and proceeds toward high income equilibrium, but for α only slightly lower, accumulation is low, and the economy is stagnated to low income poverty trap.

New growth theories say that the role of physical capital was not as important as was thought in earlier studies. Instead, they emphasize the role of ideas, technical progress, or accumulation of the stock of knowledge as the most powerful — and endogenous — engine of growth. As Romer (1993) puts it:

> We could produce statistical evidence suggesting that all growth came from capital accumulation with no room for anything called technological change. But we would not believe it.

Before coming to the conclusion that we are now suggesting that the physical capital was decisive after all, note that the threshold is in terms of α, the private return for capital, rather than in terms of capital itself. Since North and Thomas (1973), the new growth theorists have suggested that private returns to innovation activities have been crucial in growth (see Romer 1993 and Crafts 1995). In our model, intentional R&D is not present. Instead, human capital accumulation takes place as a side product of decreases in mortality. Because human capital accumulation, through mortality decreases, is tied to capital accumulation in this simple model, then, the more rewarding is this accumulation, the larger will also be the accumulation of human capital. In this way, we sug-

Name	Years	School
Pestalozzi	1746-1827	Children's school 1775
Froebel	1782-1852	Kindergarten 1840
Dewey	1859-1952	Laboratory school 1895

Table 4: Benchmark names in schooling.

gest, also the result of this model would ultimately refer to the importance of non-physical factors in production.

The model describes the role of mortality decrease in the early accumulation of human capital, and some evidence to support the importance of this role would be needed. Conventionally, schooling has been taken as the measure of human capital, and one possibility would be to compare the schooling and mortality statistics. Mortality statistics in Europe go far back into the past but only some data on early formal schooling is available and its quality has been questioned (Crafts 1995). Instead of relying on statistics, we provide some benchmark names and years in the field of schooling in Table 4. The model presented here lays emphasis on the increase in demand for learning. The authors mentioned in Table 4 were certainly affected by the idealistic movements of their time, but we can think that their work was at least partly motivated by increased demand for learning.[11] That they, and many others, became known just in the years of early development, seems to give some evidence for increase of such a demand.

A Appendix

A.1 Existence of the Solution

The $\dot{\tilde{c}} = 0-$line has the limits $-\infty$ as $\tilde{k} \to 0$, and $+\infty$ as $\tilde{k} \to \infty$. The $\dot{\tilde{k}} = 0-$line takes at least some positive values. The only thing to show, is that these lines intersect for some $\tilde{k}^* > 0$. For concave production functions $f\left(\tilde{k}\right)$, the marginal product $f'\left(\tilde{k}\right)$ is smaller than the average product $f\left(\tilde{k}\right)/\tilde{k}$, and the latter decreases for all $\tilde{k}$. The $\dot{\tilde{k}} = 0-$line hits the $\tilde{k}-$axis where $f\left(\widetilde{\tilde{k}}\right) / \widetilde{\tilde{k}} = h[n(\widetilde{\tilde{k}})] + \delta + n(\widetilde{\tilde{k}})$. The $C-$part of the $\dot{\tilde{c}} = 0-$line becomes zero, where $f\left(\bar{\tilde{k}}\right)/\bar{\tilde{k}} = h[n(\bar{\tilde{k}})] + \delta + \rho + h'[n(\bar{\tilde{k}})](\rho - n(\bar{\tilde{k}}))$. Because $\rho > n$ for all $\tilde{k}$ (assumption), and $h'[n(\bar{\tilde{k}})](\rho - n(\bar{\tilde{k}})) > 0$, we have $f\left(\bar{\tilde{k}}\right)/\bar{\tilde{k}} > f\left(\widetilde{\tilde{k}}\right) / \widetilde{\tilde{k}}$ implying $\bar{\tilde{k}} < \widetilde{\tilde{k}}$. The $B-$part becomes zero at $f'(\bar{\bar{\tilde{k}}}) = \left(\rho + \delta + \theta h[n(\bar{\bar{\tilde{k}}})]\right) >$

[11] The maxim "learning by doing" was first stated by John Dewey.

$h[n(\overset{\sim}{\tilde{k}})] + \delta + n(\overset{\sim}{\tilde{k}})$ implying again $\overset{=}{\tilde{k}} < \overset{\sim}{\tilde{k}}$. Thus, the $\dot{\tilde{c}} = 0-$line intersects the $\tilde{k}-$axis for some $\tilde{k} < \overset{\sim}{\tilde{k}}$, and the model has at least one steady state such that $\tilde{k}^* > 0$.

A.2 Stability of the Steady States

The isoclines of the model in the $\tilde{k} - \tilde{c}-$space are

$$\dot{\tilde{k}} = 0 \Leftrightarrow \tilde{c} = f - (h + n + \delta)\tilde{k}, \tag{25}$$

$$\begin{aligned} \frac{\dot{\tilde{c}}}{\tilde{c}} &= 0 \Leftrightarrow \tilde{c} = \frac{(\theta - 1)(\rho - n)}{\theta n'} \cdot \\ & \left\{ f' - (\rho + \delta + \theta h) - (h'n' + n')\tilde{k} + \frac{n'}{\rho - n}\left[f - (h + n + \delta)\tilde{k} \right] \right\} \\ &= \frac{\theta - 1}{\theta}\left\{ [f' - (\rho + \delta + \theta h)]\frac{\rho - n}{n'} + [f - (h + \rho + \delta + h'(\rho - n))\tilde{k})] \right\}. \end{aligned} \tag{26}$$

In a steady state, the isoclines intersect, giving the condition

$$f' - (\rho + \theta h + \delta) - (h'n' + n')\tilde{k} = \frac{n'}{(\theta - 1)(\rho - n)}\left[f - (h + n + \delta)\tilde{k} \right]. \tag{27}$$

To state the stability of the steady states, write $\dot{\tilde{k}} = \varphi(\tilde{k}, \tilde{c})$ and $\dot{\tilde{c}} = \phi(\tilde{k}, \tilde{c})$. The slope of the isocline $\dot{\tilde{k}} = 0$ is $\frac{d\tilde{c}}{d\tilde{k}} = -\frac{\partial\varphi/\partial\tilde{k}}{\partial\varphi/\partial\tilde{c}}$ and that of $\dot{\tilde{c}} = 0$ is $\frac{d\tilde{c}}{d\tilde{k}} = -\frac{\partial\phi/\partial\tilde{k}}{\partial\phi/\partial\tilde{c}}$. The Jacobian of the system is given by

$$J = \begin{bmatrix} \partial\varphi/\partial k & \partial\varphi/\partial c \\ \partial\phi/\partial k & \partial\phi/\partial c \end{bmatrix},$$

$$\begin{aligned} \partial\varphi/\partial\tilde{k} &= f' - (h + n + \delta) - (h'n' + n')\tilde{k}, \\ \partial\varphi/\partial\tilde{c} &= -1, \\ \partial\phi/\partial\tilde{k} &= \frac{\tilde{c}}{\theta}\left\{ \begin{array}{c} f'' - \theta h'n' - (h'n' + n') - (h''n' + n''h' + n'')\tilde{k} \\ +\frac{n''}{u'H}\mathcal{J}(k, c) + \frac{n'}{u'H}\frac{\partial\mathcal{J}(k,c)}{\partial\tilde{k}} \end{array} \right\}, \\ \partial\phi/\partial\tilde{c} &= \frac{1}{\theta}\left\{ f' - (\rho + \delta + \theta h) - (h'n' + n')\tilde{k} + \frac{n'}{u'H}\mathcal{J}(k, c) \right\} + \frac{n'\tilde{c}}{(1 - \theta)(\rho - n)} \\ &= \frac{-n'}{(\theta - 1)(\rho - n)}\tilde{c} = \frac{-n'}{(\theta - 1)(\rho - n)}[f - (h + n + \delta)\tilde{k}], \end{aligned}$$

in which the last equation is derived by using the definitions of the isocline $\dot{\tilde{c}} = 0$ given by (26). Because $f - [h + n + \delta]\tilde{k}$ is positive (for $\tilde{k} < \tilde{\tilde{k}}$), the sign of $\partial\phi/\partial\tilde{c}$ is that of $-n'$, which is always negative. Unfortunately, the expression for $\partial\phi/\partial\tilde{k}$ contains the second derivatives of n and h, the sign of which are unknown unless further assumptions are made. To still find the sign of the determinant of the Jacobian, we write

$$\begin{aligned} DET &= (\partial\varphi/\partial\tilde{k}) \cdot (\partial\phi/\partial\tilde{c}) - (\partial\phi/\partial\tilde{k}) \cdot (\partial\varphi/\partial\tilde{c}) \\ &= \left[\left(-\frac{\partial\varphi/\partial\tilde{k}}{\partial\varphi/\partial\tilde{c}}\right) - \left(-\frac{\partial\phi/\partial\tilde{k}}{\partial\phi/\partial\tilde{c}}\right)\right](-\partial\varphi/\partial\tilde{c}) \cdot (\partial\phi/\partial\tilde{c}), \end{aligned}$$

in which the expression in the brackets is the difference in the slopes of the isoclines. Because the expression $(-\partial\varphi/\partial\tilde{c}) \cdot (\partial\phi/\partial\tilde{c})$ is always negative, the determinant has the sign opposite to that of the brackets. In the case of a single steady state (Figure 4, panel a), the isocline $\dot{\tilde{k}} = 0$ hits the isocline $\dot{\tilde{c}} = 0$ from above so that the slope $\dot{\tilde{k}} = 0-$line is larger than that of $\dot{\tilde{c}} = 0-$line of which makes the brackets positive and $DET < 0$; the steady state is a saddle point. In the case of three steady states (Figure 4, panel b), in steady states I and III, the isocline $\dot{\tilde{k}} = 0$ hits the isocline $\dot{\tilde{c}} = 0$ from above, and steady states I and III are both saddles. In steady state II, $\dot{\tilde{k}} = 0$ intersects the isocline $\dot{\tilde{c}} = 0$ from below making the brackets positive. In this case, $DET > 0$. The trace of the system is given by

$$\begin{aligned} TR &= \left[f' - (h + n + \delta) - (h'n' + n')\tilde{k}\right] + \left[\frac{-n'}{(\theta - 1)(\rho - n)}[f - (h + n + \delta)\tilde{k}]\right] \\ &> \left[f' - (\theta h + \rho + \delta) - (h'n' + n')\tilde{k}\right] + \left[\frac{-n'}{(\theta - 1)(\rho - n)}[f - (h + n + \delta)\tilde{k}]\right] \\ &= 0, \end{aligned}$$

in which the inequality sign comes because $\theta > 1$ and $\rho > n$, and the last equality is based on (27). The result $TR > 0$ implies that the steady state II is unstable. We are unable to calculate $(TR)^2 - 4DET$, and thus unable to differentiate between the two types of unstable steady states, node and focus. The deficiency has a minor role in the analysis, and we assume, for simplicity, that steady state II is a focus.

Because $\partial\varphi/\partial\tilde{c} = -1$, the capital stock per effective worker increases (decreases) below (above) the isocline $\dot{\tilde{k}} = 0$. Because $\partial\phi/\partial\tilde{c} = \frac{-n'}{(\theta-1)(\rho-n)}\tilde{c} < 0$, the consumption per effective labor force increases (decreases) below(above) the $\dot{\tilde{c}} = 0-$isocline. Therefore, in the case of a single steady state, the stable branch of the saddle path runs from south-west to north-east, while the unstable branch runs from south-east to north-west. In the case of three steady states, the same conclusion holds for both saddles I and III in the vicinity of the steady states.

B Appendix : Some Elements of the Planner's Solution

First, note that the utility function $u(c) = \frac{c^{(1-\theta)}}{1-\theta}$ and the necessary condition $\frac{\partial \mathcal{J}(\tilde{k},c,\lambda)}{\partial c} = 0$ are given in terms of per capita consumption $c = \frac{C}{L}$ whereas the phase diagram and the pair of differential equations (18)-(19) are given in terms of consumption per efficient capita $\tilde{c} = \frac{C}{HL} = \frac{c}{H}$. This somewhat complicates the analogous analysis given in Lehmijoki (2003). The Hamiltonian is $\mathcal{J}(\tilde{k}, c, \lambda) = \frac{u(c)}{\rho-n} + \lambda \frac{\dot{\tilde{k}}}{\rho-n}$. The necessary conditions (14)-(15) and equation of motion (12) imply $\frac{\partial \mathcal{J}(\tilde{k},c,\lambda)}{\partial c} = 0$, $\dot{\lambda} = \left[-\frac{\partial \mathcal{J}(\tilde{k},c,\lambda)}{\partial \tilde{k}} + \lambda\right](\rho - n)$, and $\dot{\tilde{k}} = \frac{\partial \mathcal{J}(\tilde{k},c,\lambda)}{\partial \lambda}(\rho - n)$. Then

$$
\begin{aligned}
\frac{d\mathcal{J}(\tilde{k}, c, \lambda)}{dt} &= \frac{\partial \mathcal{J}(\cdot)}{\partial \tilde{k}}\dot{\tilde{k}} + \frac{\partial \mathcal{J}(\cdot)}{\partial c}\dot{c} + \frac{\partial \mathcal{J}(\cdot)}{\partial \lambda}\dot{\lambda} \\
&= \frac{\partial \mathcal{J}(\cdot)}{\partial \tilde{k}}\frac{\partial \mathcal{J}(\cdot)}{\partial \lambda}(\rho - n) + \frac{\partial \mathcal{J}(\cdot)}{\partial \lambda}\left[-\frac{\partial \mathcal{J}(\cdot)}{\partial \tilde{k}} + \lambda\right](\rho - n) \\
&= \frac{\partial \mathcal{J}(\cdot)}{\partial \lambda}\lambda(\rho - n) = \lambda\dot{\tilde{k}}.
\end{aligned}
$$

Then

$$
\begin{aligned}
-d\left[e^{-\int_0^t \{\rho-n[\tilde{k}(\tau)]\}d\tau}\mathcal{J}(\tilde{k}, c, \lambda)\right] / dt &= -e^{-\int_0^t\{\cdot\}\, d\tau}\left[\frac{d\mathcal{J}(\cdot)}{dt} - (\rho - n)\mathcal{J}(\cdot)\right] \\
&= -e^{-\int_0^t\{\cdot\}\, d\tau}[\lambda\dot{\tilde{k}} - (\rho - n)\mathcal{J}(\cdot)] \\
&= e^{-\int_0^t \{\rho-n[\tilde{k}(\tau)]\}d\tau}u(c).
\end{aligned}
$$

Taking integral gives the value of the program

$$
\begin{aligned}
&\int_0^\infty u(c)e^{-\int_0^t \{\rho-n[\tilde{k}(\tau)]\}d\tau}dt \\
&= -\int_0^\infty \left\{d\left[e^{-\int_0^t \{\rho-n[\tilde{k}(\tau)]\}d\tau}\mathcal{J}(\tilde{k}, c, \lambda)\right] / dt\right\}dt \\
&= \mathcal{J}[\tilde{k}(0), c(0), \lambda(0)] - \lim_{t\to\infty} e^{-\int_0^t \{\rho-n[\tilde{k}(\tau)]\}d\tau}\mathcal{J}\left[\tilde{k}(t), c(t), \lambda(t)\right],
\end{aligned}
$$

Along any stable saddle path, $\mathcal{J}\left[\tilde{k}(t), c(t), \lambda(t)\right] = \frac{u(c)}{\rho-n} + \lambda\frac{\dot{\tilde{k}}}{\rho-n}$ approaches $\frac{u(c)}{\rho-n}$. In a steady state, $\frac{\dot{c}}{c}$ is a constant given by $h^* = h\left[n\left(k^*\right)\right]$ and c goes to $+\infty$. However, because $\lim\limits_{c\to\infty} u'(c) = 0$, $\frac{u(c)}{\rho-n}$ is finite for all t. Because $\rho - n\left[\tilde{k}\left(t\right)\right] > 0$ for all t, we have $\lim\limits_{t\to\infty} e^{-\int_0^t\{\rho-n[\tilde{k}(\tau)]\}d\tau}\mathcal{J}\left[\tilde{k}(t), c(t), \lambda(t)\right] = 0$. Therefore, the value of the program is equal to$\mathcal{J}[\tilde{k}(0), c(0), \lambda(0)]$, where $c(0) = H(0)\cdot\tilde{c}(0)$ in which $\tilde{c}(0)$ is chosen such that it lies on a stable saddle path. Because any stable saddle path satisfies $\lambda(t) = u'(t)H(t)$ we further have $\int_0^\infty u(c)e^{-\int_0^t\{\rho-n[\tilde{k}(\tau)]\}d\tau}dt = \mathcal{J}[\tilde{k}(0), c(0)]$, i.e., along a stable saddle path, the value of the program is the value of the optimized Hamiltonian at time zero. For a similar proof in a case of a constant discount rate, see Tahvonen and Salo (1996).

For a given $\tilde{k}(0) = \frac{K(0)}{H(0)\cdot L(0)}$, a choice of $\tilde{c}(0)$ on different saddle paths leads to different values of $c(0)$ and $\mathcal{J}[\tilde{k}(0), c(0)]$. To calculate the effect of such a choice on the value of the program, write $\mathcal{J}(\tilde{k}, c) = \frac{1}{\rho-n}\left[u(\tilde{c}H) + u'(\tilde{c}H)H\dot{\tilde{k}}\right]$ where $\dot{\tilde{k}} = f(\tilde{k}) - \tilde{c} - (h+n+\delta)\tilde{k}$. Then

$$\frac{\partial\mathcal{J}(\tilde{k}, c)}{\partial\tilde{c}} = \frac{1}{\rho-n}\left[u'H - u'H + u''H^2\dot{\tilde{k}}\right] = \frac{u''(c)H^2}{\rho-n}\dot{\tilde{k}}.$$

In the expression above, $\frac{u''(c)H^2}{\rho-n} < 0$ so that $\frac{\partial\mathcal{J}(\tilde{k},c)}{\partial\tilde{c}}$ has the sign of $-\ \dot{\tilde{k}}$. The differential equation for consumption given in (18) can be rewritten as $\dot{\tilde{c}} = \frac{-u'}{u''H}\left[f' - (\rho+\delta) - (h\acute{n}+n')\tilde{k} + \frac{n'}{u'H}\mathcal{J}(\tilde{k}, c)\right] - h\tilde{c}$. The slope of a saddle path in the $\tilde{k} - \tilde{c}-$space is given by

$$\frac{d\tilde{c}}{d\tilde{k}} = \dot{\tilde{c}}/\dot{\tilde{k}} = \frac{\frac{-u'}{u''H}\left[f' - (\rho+\delta) - (h\acute{n}+n')\tilde{k} + \frac{n'}{u'H}\mathcal{J}(\tilde{k}, c)\right] - h\tilde{c}}{f - \tilde{c} - (h+n+\delta)\tilde{k}}.$$

Write $\tilde{c} = \tilde{c}(\tilde{k})$ to refer to those values of $\tilde{c}$, which lie on a saddle path. Then $c(\tilde{k}) = \tilde{c}(\tilde{k})H\left(\tilde{k}\right)$ respectively. Then, along any saddle path (see also Tahvonen and Salo 1996):

$$
\begin{aligned}
\frac{d\mathcal{J}[\tilde{k}, c(\tilde{k})]}{d\tilde{k}} &= \frac{\partial \mathcal{J}(\tilde{k}, c)}{\partial \tilde{k}} + \frac{\partial \mathcal{J}(\tilde{k}, c)}{\partial \tilde{c}} \cdot \dot{\tilde{c}}/\dot{\tilde{k}} \\
&= \frac{n'}{\rho - n}\mathcal{J}(\tilde{k}, c) + \frac{u'H}{\rho - n}\left[f' - (h + n + \delta) - (hn' + n')\tilde{k}\right] \\
&\quad + \frac{u''H^2}{\rho - n}\dot{\tilde{k}}\frac{\frac{-u'}{u''H}\left[f' - (\rho + \delta) - (hn' + n')\tilde{k} + \frac{n'}{u'H}\mathcal{J}(\tilde{k}, c) - \frac{u''H}{-u'}h\tilde{c}\right]}{\dot{\tilde{k}}} \\
&= \frac{u'H}{\rho - n}\left[\rho - n + (\theta - 1)h\right],
\end{aligned}
$$

in which the last expression is due to the fact that $\frac{u''H}{-u'}h\tilde{c} = \frac{u''H\tilde{c}}{-u'}h = \theta h$.

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Convergence, Income Inequality, and Demographic Clubs

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Abstract

The paper provides two studies in the field of international income inequality, a dynamic modern and a static traditional. Both studies are based on four demographic clubs that are identified by the regression tree technique from the data. The dynamic modern study shows that in three of the four clubs the time series of incomes tends to converge. The traditional study gives some support to the inverted-U hypothesis: the decrease of income inequality in the sample and the decrease in population growth rate in developing countries have the same time profile.

1 Introduction

International income distribution has recently been studied in two quite different frameworks. The modern approach derives from the theory of economic growth and from the econometric theory of time series unit root tests, whereas the traditional framework derives from measures that originally were devoted to study all kinds of inequality. Another difference is that the traditional approach is static in the meaning that it gives a single measure of inequality for each year, whereas the modern is dynamic, capturing the information of several years into a single testable parameter. This article supplies both types of analyses.

The transitional dynamics of the augmented Ramsey model are summarized in the time paths of the population growth rate, the per capita capital and consumption, and the growth rate of per capita income in Figure 1. Panel d shows that, due to decreasing marginal productivity of capital, economic growth rate first decreases, but as population growth accelerates, capital accumulation and economic growth increase at the cost of consumption, and economic growth reaches its maximum during the peak of transition. This result can be understood in the light of the optimization behavior of the households: it is best to "get over the worst" as soon as possible.

The standard Ramsey model implies that economic growth rate decreases as countries get richer in capital. Therefore, in a sample of countries, the initially poor countries should catch-up with the initially rich countries. This is the convergence result. Figure 1 shows that no such unilateral convergence of incomes is implied by the augmented model. On the contrary, the transitional dynamics of this model suggests that both divergence and convergence should take place in a sample, in which each country is in different stage of its demographic transition. Even so, the underlying reason for convergence — the decreasing productivity of capital — is still present in the augmented model and is only overshadowed by demographic transition. To uncover the convergence, demographic transition must be adequately controlled for. Sala-i-Martin (1996) argues that two strategies exist to perform this control. One can either explicitly introduce the shadowing factors as regressors into the model or one can partition the sample into sub-samples — convergence clubs — which are homogenous in terms of the shadowing variables so that they can then be safely omitted.

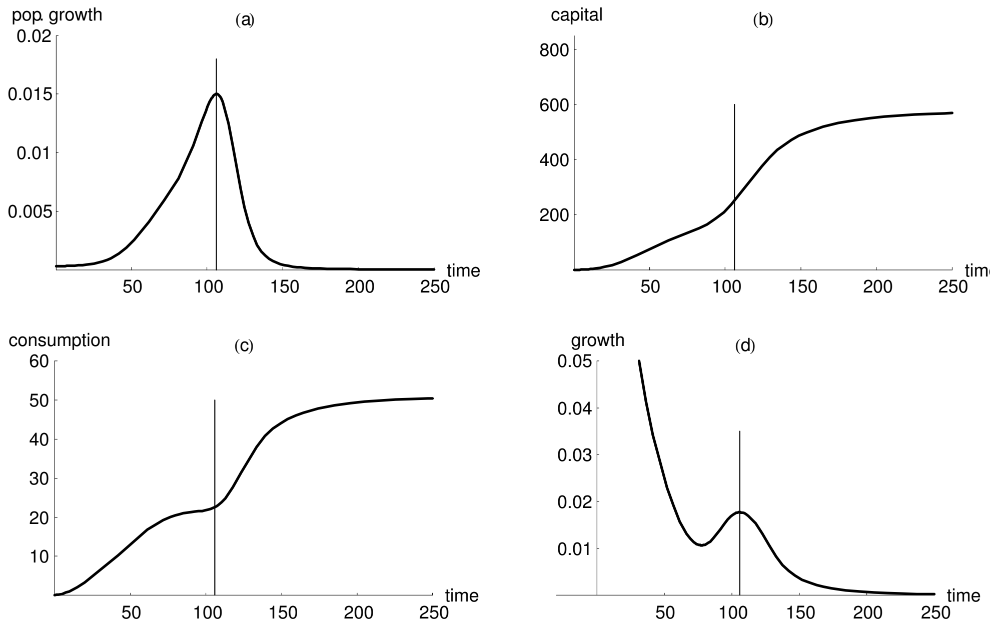

Figure 1: The transitional dynamics in the augmented Ramsey model. Lehmijoki 2003.

In this essay we follow the latter strategy and create demographic convergence clubs by partitioning the sample into homogenous sub-samples in terms of their demography. However, neither Figure 1 nor the augmented Ramsey model itself give any theoretical implication, how to find the number or borderlines of such clubs. Therefore, we identify them directly from the data by using regression tree analysis as applied by Durlauf and Johnson (1995). Regression tree analysis is a data sorting method that partitions the range of regressors into (approximate) level sets of the regressand, i.e., it tries to find those values of the explanatory variables which are as homogenous as possible in terms of the dependent variable, thus producing a club-typical value to the latter. The demographic regressors used are the level and the change of population growth rate as implied in Figure 1. The regressand is the growth rate of the per capita income. Once the number and limits of the clubs are identified in terms of the regressors, the sample is partitioned accordingly. The regression tree analysis shows that the number of demographic convergence clubs is four.

However, it is not enough to control for the demographic factors alone. To uncover the convergence, we have to show that the clubs are homogenous in terms of other potential determinants of growth as well. In earlier studies, the sample has been partitioned into convergence clubs on the basis of common history (Baumol 1986), geographical location (Maddison 1994), or mutual trade (Ben-David and Loewy1998). In these studies, the criterion of partitioning then serves as a justification for the homogeneity of the other relevant variables within the club. For example, Ben-David and Loewy argue that trade tends to equalize economies through factor prize equalization and through knowledge spillovers facilitated by trade flows. We suggest that demography also has such an equalizing power. The supply and the age structure of the labor force, the structure of demand for goods, the level of education, and the need for public services are closely related to the demographic situation in a country. Further, Coale and Hoover (1958) suggest that savings are determined by the worker / dependent ratio, and one might even argue that adoption of new technology is the faster the larger is the share of the young in the economy. Therefore, countries sharing common demography also share common values of many other variables.

Because demographic clubs then are homogenous in terms of demographic variables and also in terms of other variables, there is no factor that could mask the underlying tendency for convergence *within* a club. To state this convergence, the time series unit root convergence test in the version provided by Evans and Karras (1996) is run in each club separately in Approach I. The tests show that three of the four demographic clubs exhibited conditional convergence of incomes.

The inverted-U hypothesis represented by Kuznets (1967) says that, in the process of economic development, income inequality first increases and then decreases. Poor people tend initially to be more fertile but have limited access to resources, such as land and capital (Birdsall 1988) or education (Dahan and Tsiddon 1998, Kremer and Chen 1999). Therefore, their population share initially swells, but the income share stays constant. Later, as fertility among the poor decreases, their population share decreases and income becomes less unequally distributed. If the decrease in the population growth rate first starts in rich countries, and then expands to poor ones, we might expect that, as this decrease among poor countries starts, income inequality starts to decrease. In Approach II, Gini coefficients for 1965-1990 are calculated and their values are compared to demographic data. The timing of the onset of a permanent decrease in population growth rate in developing countries is identical to the timing of a decrease of income inequality.

In the nearest future, decreases of population growth rates have escalated everywhere. The question is, how fast will be its tempo? We propose that low population growth in the future will tend to decrease the inequality of incomes. The United Nations (2000) provides projections for future growth rates in three variants, Low, Medium, and High. Based on these projections and on the four demographic clubs derived, we calculate future incomes until 2030. Gini coefficients are derived for each variant to test the future version of the inverted-U hypothesis. The result is unexpected: The Low Variant leads to highest inequality of incomes in the future. Closer examination shows that this might be due to an increase of inequality within rich countries. Under the Low Variant, all rich countries stay in favorable demographic clubs, whereas, under the High Variant, the two richest of them (USA and Luxembourg) move to the a demographic club in which the age structure is unfavorable to growth. This result reminds us that the deterioration of the worker/dependent ratio in the future can lead to lower economic growth rates in currently rich countries.

The outline of the paper is as follows: Approach I in Section 2 derives the demographic clubs, and a time series convergence test for each club is performed. Forecasts for future incomes and Gini coefficients for each variant are derived, and inverted-U hypothesis is discussed and tested in Approach II, in Section 3. Section 4 concludes the paper.

2 Approach I: Demographic Convergence Clubs

2.1 Regression Tree Analysis

In his paper, Sala-i-Martin (1996) argues that, even if convergence is not usually present in a sample of heterogenous countries, it can be detected in a subsample or clubs of homogenous countries. Our paper argues that demography could serve as a criterion for such homogeneity, but the problem is, how to find the appropriate demographic clubs. Unfortunately, no theory dealing with the number or the borderlines of such clubs is available. A practical way out of the problem is to identify the clubs directly from the data. Durlauf and Johnson (1995) have used the regression tree method to find clubs with a single steady state, but the method is useful in identification of convergence clubs as well. The technique is also discussed in Breiman *et al.* (1984). The practical guidance for tree analysis in S-Plus language is provided by Venables and Ripley (1994).

The regression tree analysis partitions the space of regressors to (approximate) level sets of regressand. The regressors used are N (average annual population growth rate during the research period, percentages) and DN (change in N during the period, percentage points), whereas the regressand is $GROWTH$ (average annual growth rate of per capita GDP, percentages[1]) as implied by Figure 1, *d*. The time span of Approach I is from 1960 to 1995. The demographic data comes from The United Nations (1999). This data is in periods

[1] International 1985 dollars, chain index.

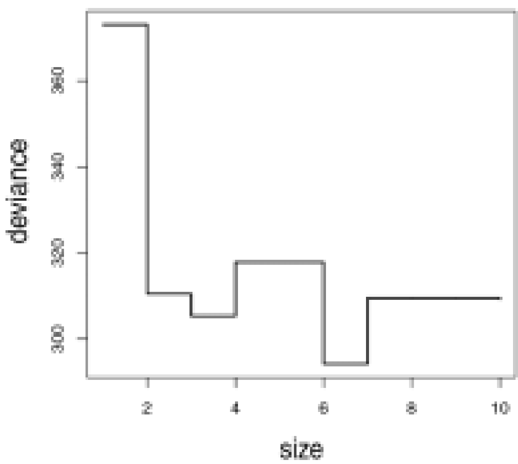

Figure 2: Average deviations from cross-validation procedure. Average deviation minimized for six terminal nodes.

of five years. Economic data comes from Summers and Heston, version 156b (1991). The data is at annual level and is mainly available from 1960 to 1990.[2] This data has recently been updated to 1995 by Easterly (1999). We exclude five countries because of strong migration arguing that migration has led to exceptional demographic situations. The members of OPEC are also excluded from the sample. We exclude Rwanda as well, because the mass murders there considerably decreased the accuracy of the demographic data. In this way we can collect a sample of 110 countries for which a complete set of data is available.

The following algorithm is used: The whole sample is the root node of the algorithm. Let $D = \sum_{i=1}^{N}(y_i - \mu)^2$ be the deviance of the regressand y in the root node (μ stands for sample mean). The root node can be split into two nodes. Only binary splits are allowed. Let D_u and D_v be the deviancies in these two nodes respectively. The deviance of the tree is defined as the sum of the deviancies of the nodes; after the first split this is $D_u + D_v$. The value of the split is $D - D_u - D_v$, i.e., the decrease in deviance of the tree. The algorithm calculates the value of the split in terms of all regressors and their values, and chooses the split regressor and its split value, such that the value of the split is as large as possible. Only one step lookahead is used, i.e., only the value of the next split is calculated. The tree is grown until the number of members in each terminal node is small enough, or until they are so homogenous that further splitting can have low values only.

By definition, each successive split decreases the deviance of the tree. The

[2] Version 6.1 became available in October 2002.

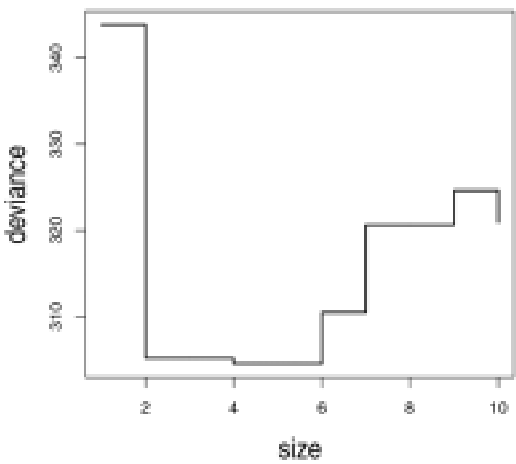

Figure 3: Average deviations from cross-validation procedure. Average deviation minimized for four or five terminal nodes.

	Splitting Variable and Value	Members	Deviance	$GROWTH$	
Root		110	338.00	1.7050	
	$DN < -0.105$	68	133.80	2.4100	
	$DN < -1.7$	5	28.09	4.4620	**A**
	$DN > -1.7$	63	82.98	2.2470	
	$N < 2.0275$	32	35.41	2.7810	**B**
	$N > 2.0275$	31	29.05	1.6960	**C**
	$DN > -0.105$	42	115.50	0.5623	**D**

Table 1: The regression tree.

larger the tree, the more constant is the value of the regressand within each node. There are two reasons, however, why the maximal tree is not necessarily the best tree. The first reason is that the maximal tree, being hard to interpret and comprehend, can be less useful for empirical purposes. The second reason is theoretical. The sample used to grow the tree might be "noisy," i.e., it contains cases that are not typical or important in the population from which the sample was derived. In this case, the algorithm overfitts the tree, and the decrease of the deviance of the tree might not take place if the same splits are applied to the population as a whole.

Because the sample at hand is typically the only data available, possible overfitting is controlled by a cross-validation as follows: The sample is randomly divided into K (say 4-10) parts. The tree is grown by the data in $K-1$ parts, and the deviance of the tree is calculated by fitting the tree to the last part. This is repeated K times, and the deviancies are averaged. As long as this average deviance decreases with successive splits, these splits are considered as typical to the population, but as the average deviance starts to increase, the splits are considered as sample-specific only. The best number of the splits is then the number that minimizes the average deviation, derived in the cross-validation procedure.

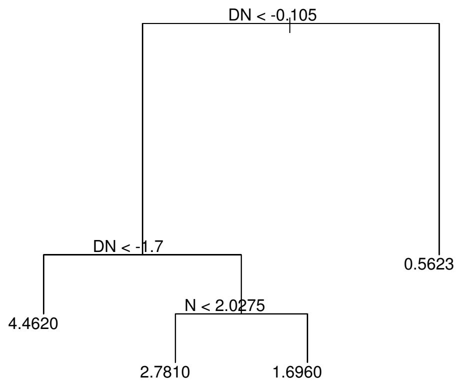

Figure 4: The regression tree with four terminal nodes.

We apply the regression tree analysis to find the number of the clubs (terminal nodes) and their actual borders in the $DN-N-$space. The procedure grows up the maximal tree of ten terminal nodes. To prune the maximal tree, the cross-validation procedure with $K = 5$ is run.[3] As one might expect, the sample being only of 110 cases, the cross-validation procedure is rather unstable. Each successive cross-validation gives a slightly different result. In each case, the first split ($DN \lesseqgtr -0.105$) greatly decreases the average deviance of the tree, but the results in terms of subsequent splits are less clear. Figures 2 and **??** give typical cross-validation results. In Figure 2, the average deviation is minimized when the number of the terminal nodes is six. In Figure **??**, four terminal nodes minimize the average deviation. Because smaller trees are easier to interpret, we choose to prune the tree until four nodes. The structure of the final tree derived in this way is reported in Table 1. The second row of the table gives the data for the root node (the sample). Subsequent rows report the splits such that each split can be read as a sub-set of its mother-node. Each row reports the splitting variable, the splitting value, the number of members, the deviance in the node, and average value of $GROWTH$ within the node. Symbols $A-D$ are attached

[3]The conclusions above are are unchanged for $K \in (4, 10)$.

to each terminal node. Figure 4 gives some of this information in graphical form. The node or club A contains the well known fast-growers of East-Asia (Hong Kong, Korea Rep, and Singapore) and Mauritius and Trinidad&Tobago. The node B mainly contains western developed countries, but Barbados, Uruguay, China, and Sri Lanka are also in this club. Nodes C and D contain developing countries alone. For a complete list, see Appendix A. Finally, Figure 5 gives the partition of the $DN-N-$space into clubs $A-D$.

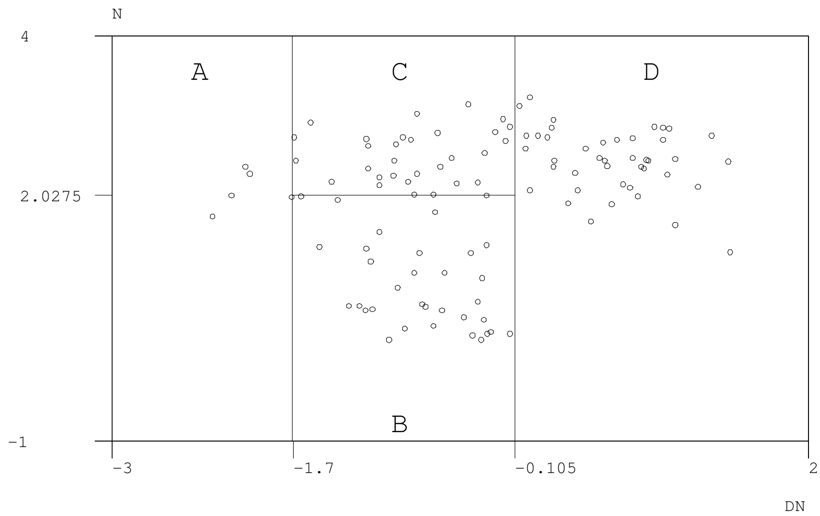

Figure 5: Four demographic clubs.

Low average value and large negative change in the population growth rate are associated with high economic growth rate, which varies from 0.5620 to 4.4620 %. The F-value 22.0583 in the standard analysis of variance gives the probability 0.0000 for the growth rates in clubs to be identical

2.2 Convergence in Demographic Clubs

2.2.1 Earlier Studies

Most recent studies in the field of convergence analysis have their roots in the work of Baumol (1986), who calculated the growth rates of the per working hour GDPs for 16 now rich countries, and obtained that, from 1870 to 1979, this growth rate was higher in countries that initially had lower per working hour GDP. Baumol's idea of convergence survived, even if his results were shown by DeLong (1988) to be distorted by a sample selection bias. New results were provided by Barro (1991), Barro and Sala-i-Martin (1992), and Mankiw *et al.*

(1992), to mention some. These studies were motivated by empirical interest in the field. Summarized by Lucas (1988):

> The consequence for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else.

But they also derived from theoretical sources. Since the works of Romer (1986) and Azariadis and Drazen (1990), models with endogenous technical progress or with discontinuities in the production function gained ground, and advocates of more traditional models seek support for their theoretical finding from empirical results. In terms of a simple Solow-model, the argument is the following: Let $y = f(k)$ be the per capita production function with $k = \frac{K}{AL}$, where K stands for productive capital (possibly including human capital), L and A stand for labor and level of technology respectively. Let f be continuous and strictly concave in k. The fundamental equation for capital accumulation is

$$\dot{k} = sf(k) - (n + \delta + \phi)k,$$

in which s is constant saving rate, and n, δ, and ϕ are population growth rate and the rate of depreciation and the rate of technical progress respectively. Then $\gamma_k = \frac{\dot{k}}{k} = sf(k)/k - (n+\delta+\phi)$, and $\frac{\partial \gamma_k}{\partial k} = s[(f'(k) - f(k)/k)/k]$. The expression in the brackets is negative if the marginal product of capital, $f'(k)$, falls short of the average product of capital, $f(k)/k$, which is the case if the production function is strictly concave. Because the growth rate of per capita income γ_y is a positively monotonous function of γ_k, its behavior follows the same pattern: A country's growth rate tends to decrease as it gets richer in capital. On the contrary, if the production function contains discontinuities or if the marginal product of capital is constant or increasing, $\frac{\partial \gamma_k}{\partial k}$ would not be negative for all k.

In cross sectional interpretations, if $\frac{\partial \gamma_k}{\partial k} < 0$ holds, initially poor countries tend to catch up with the initially rich ones. In such cases, regressing the growth rates against the (logs of) initial incomes in a sample of countries, the regression coefficient is expected to be negative. This is the concept of absolute $\beta-$convergence. For the purposes of empirical studies, controlling variables like population growth, savings, exports and imports, government spending, and inflation would be necessary to correct the omitted variable bias in the coefficient of initial income. If, after this control, the estimated coefficient for initial income is still negative, conditional $\beta-$convergence is said to prevail. This result has been confirmed in the empirical studies mentioned above. The theoretical conclusion then is that the production function is strictly concave, and that technical progress is exogenous (adopted from more advanced countries) rather than endogenous in nature.

The $\beta-$convergence results have been criticized in three ways. First, Friedman (1992) and Quah (1993) falsified the basic logic behind the above reasoning. Friedman argued that a country may be poor (rich) for temporary, not permanent reasons. Therefore, its increase (decrease) in rank is only natural. Quah's argument is the following: assume that income distribution is unimodal. Then, if a country is located in the lowest part of this distribution (is initially poor), its probability to move ahead in the distribution is, by definition, larger than its probability to fall back. For a country located in the upper end of the distribution (initially rich), the probability to move back is larger than the probability to move ahead. This property of any unimodal income distribution says nothing about the properties of the production functions of the countries.

The next type of criticism derives from econometric sources and suggests some refinements for the estimation procedure. Islam (1995) suggests that instead of simple cross-sectional estimates, panel methods would be more appropriate. In addition, he argues that country-specific aspects in production function were ignored by Barro (1991), Barro and Sala-i-Martin (1992), and Mankiw *et al.* (1992). Because these country-specific features, such as natural resources and level of technology, tend to related to initial income, the estimates for the latter will be biased due to these omitted variables. Therefore, in estimating the coefficient of initial income, country-specific differences in the growth rates should be allowed to concentrate on the convergence effect alone. According to Islam, it is not clear, however, whether the country-specific effect should be treated as fixed or random. Because of suspected correlation of country-specific effects, random effect (panel) estimates would be biased again. On the contrary, if country specific effects are assumed to be fixed, unbiased estimates for initial incomes would follow. Islam estimates both fixed effect models and random effect models, and derives quite similar results in both cases. Caselli *et al.* (1996) suggest that the country-specific growth rates should be eliminated by taking differences. They also propose, that the possibly endogeneity of the variables should be eliminated by using lagged values as instruments (see also Bond 2002). Lee *et al.* (1997) also evoked the question whether, in addition to allowing the country-specific fixed effects, the country-specific coefficients for the initial income also should be allowed. This of course, "...renders the notion of (conditional or unconditional) beta convergence meaningless in an economic sense: knowledge of the speed with which countries' outputs converge to their own equilibria provides no insights on the evolution over time of the cross-country variance of outputs..." Lee *et al.* (1998).

Third type of criticism is related to the above cited comment of Lee *et al.* (1998): even if $\beta-$convergence was estimated in most of the studies, the lack of any convergence, i.e., the lack of the intercountry income distribution to tail off, was apparent in the real world. In the real world, the poor were getting poorer and the rich were getting richer. Sala-i-Martin (1996) shows that absolute $\beta-$convergence does not imply diminution of the standard deviation of the income ($\sigma-$convergence) because of the possibility of leapfrogging. On the other hand, conditional $\beta-$convergence has no bearing on this deviation whatsoever. Therefore, methods to evaluate whether countries really approached each other, were sought in the field of time series analysis.

Bernard and Durlauf (1996) define the convergence between countries i and j in two alternative ways. Let y_i and y_j stand for the logs of per capita incomes in countries i and j, and let $\Im_t$ be the information available at time t. The two definitions are:

Definition 1: Convergence as catching up. Countries i and j converge between dates t and $t+T$ if the (log of) per capita output disparity at t is expected to decrease in value. If $y_{i,t} > y_{j,t}$, this can be written

$$E(y_{i,t+T} - y_{j,t+T} \mid \Im_t) < y_{i,t} - y_{j,t}. \tag{1}$$

Definition 2: Convergence as equality of long term forecasts at a fixed time. Countries i and j converge if the long term forecasts of (log of) per capita output for both countries are equal at a fixed time t:

$$\lim_{T\to\infty} E(y_{i,t+T} - y_{j,t+T} \mid \Im_t) = 0. \tag{2}$$

Note that (2) implies(1) but not vice versa; the latter requires, that the incomes in countries i and j become identical in fixed time, whereas the former only requires that they approach each other. Definition 1 is used in Approach I. For a sample of countries, definition 1 should be slightly modified such that it gives the convergence between country i and the sample average. We say that a country i converges to the sample average if, for $y_{i,t} > \bar{y}_t$ it can be written: $E(y_{i,t+T} - \bar{y}_{t+T} \mid \Im_t) < y_{i,t} - \bar{y}_t$, where $\bar{y}_t$ is the average of logs of income in the sample.

The tools of unit root tests in a time series can be used to test this type of convergence as follows: Let $x_{i,t} = y_{i,t} - \bar{y}_t$. If $x_{i,t}$ is constant over t, country i grows at a rate common to the sample average. This means that if country i was initially poor (rich) in terms of the sample average, its per capita GDP falls in absolute terms further behind (gets further ahead of) the average per capita GDP in the sample. On the other hand, if $x_{i,t}$ decreases over time, the growth rate of a poor (rich) country exceeds (is smaller than) the growth rate in the sample, and its per capita GDP converges to the sample average.[4] Formally, we write

$$x_{i,t} = \delta_i + \eta_i x_{i,t-1} + \varepsilon_{i,t}, \tag{3}$$

in which $\varepsilon_{i,t}$ is identically and independently distributed. In (3), $\eta_i = 1$ means that the growth rate differential between country i and the sample average is constant and $\eta_i < 1$ means that the growth rate differential decreases. To test the null hypothesis $H_0 : \eta_i = 1$ against $H_1 : \eta_i < 1$ we rewrite (3) as

$$\begin{aligned} \Delta x_{i,t} &= \delta_i + (\eta_i - 1)x_{i,t-1} + \varepsilon_{i,t} \\ &= \delta_i + \rho_i x_{i,t-1} + \varepsilon_{i,t}. \end{aligned} \tag{4}$$

The null now becomes $H_0 : \rho_i = 0$ and $H_1 : \rho_i < 0$. This is the standard Dickey-Fuller unit root test. If the error terms turn out to be autocorrelated, the estimate for ρ_i its $t-$value are biased. This can be corrected by using the augmented form of the above test:

[4] Ultimately, exeeds (falls behind) the sample average.

$$\Delta x_{i,t} = \delta_i + \rho_i x_{i,t-1} + \sum_{j=1}^{p} \varphi_{i,j} \Delta x_{i,t-j} + \varepsilon_{i,t}. \tag{5}$$

The problem is that we want to concentrate on the convergence, not of a single country, but of the sample as a whole. One alternative is to require that all countries in a sample converge individually (see Bernard and Durlauf 1996). This is obviously a too strong requirement. Levin and Lin (1993) suggest that the countries of the sample could be pooled to estimate ρ for the whole sample whereas the intercept term δ_i could be country-specific.[5] Then, the hypotheses would become $H_0 : \rho_i = 0$ for all i and $H_1 : \rho_i = \rho < 0$ for all i. This procedure was applied in the convergence framework by Ben-David (1993). Evans and Karras (1996) suggest that, instead of direct pooling of the countries, each country's time series $x_{i,t}$ should be first normalized by using the standard error for the estimate $\hat{\rho}_i$ in the country-specific unit root test. The possible autocorrelations of the error terms should also be handled at the country level. For a recent discussion of other alternatives, see Banerjee (1999), Karlsson and Löthgren (2000), and Levin *et al.* (2002).

Data on different countries is heterogenous and growth rates fluctuate different ways, and normalization of these fluctuations would lead to better comparability of the countries. Therefore, in this paper, we adopt the procedure suggested by Evans and Karras (1996) which runs as follows:

- Calculate the standard error $\hat{\sigma}_i$ of $\hat{\rho}_i$ for all i by applying ordinary least squares to (5). Calculate the normalized series $\hat{z}_{i,t} = x_{i,t}/\ \hat{\sigma}_i$ for all i.
- Calculate the pooled value $\hat{\rho}$ and its $t-$value $\tau\left(\hat{\rho}\right)$ by ordinary least squares from

$$\Delta \hat{z}_{i,t} = \hat{\delta}_i + \rho \hat{z}_{i,t-1} + \sum_{j=1}^{p} \varphi_{i,j} \Delta \hat{z}_{i,t-j} + \hat{\varepsilon}_{i,t}, \tag{6}$$

where $\hat{\delta}_i = \delta_i/\hat{\sigma}_i$ and $\hat{\varepsilon}_{i,t} = \varepsilon_{i,t}/\hat{\sigma}_i$.

- If $\tau\left(\hat{\rho}\right)$ exceeds the appropriately chosen limits, reject H_0 in favor of H_1.
- If H_0 is rejected, calculate $\Phi(\hat{\delta}) = \frac{1}{N-1}\sum_{i=1}^{N}\left[\tau\left(\hat{\delta}_i\right)\right]^2$, where $\tau\left(\hat{\delta}_i\right)$ is the $t-$ratio of the estimator $\hat{\delta}_i$ obtained in (5) and N is the number of countries in the sample.
- If $\Phi(\hat{\delta})$ exceeds the appropriately chosen limit, conclude that convergence is conditional. Otherwise, convergence is absolute. In the latter case, the differences in per capita incomes tend to be completely eliminated

[5] The equation to be estimated can contain a country-specific and common time trend (see Levin and Lin 1993).

Club	n	$\hat{\boldsymbol{\rho}}$	$\tau(\hat{\boldsymbol{\rho}})$	$\Phi(\hat{\delta})$
A	5	-0.0082 (0.0146)	-0.563	-
B	32	-0.0241 (0.0054)	-4.509	3.230
C	31	-0.0480 (0.0090)	-5.335	2.684
D	42	-0.0617 (0.0074)	-8.287	3.457

Table 2: The results of the Karras-Evans test for convergence toward the club average. Standard errors in the parenthesis.

in the long run. In the former, only the differences in growth rates are eliminated. This means that, on the average, countries, which initially were rich (poor), stay as such forever.

Evans and Carras (1996) show that, under the null, $\tau(\hat{\rho})$ converges in distribution to standard normal as T and N approach infinity and N/T approaches zero. As T approaches infinity, $\Phi(\hat{\delta})$ converges in distribution to $F[N-1,(N-1)(T-p-2)]$. In small samples, however, the asymptotic distributions do not accurately approximate the distributions above. Therefore, the small sample probability limits for $\tau(\hat{\rho})$ and $\Phi(\hat{\delta})$ must be approximated by Monte Carlo simulations (see Evans and Karras 1996).

2.2.2 Empirical Results

We apply the Evans-Karras procedure separately to each of our four clubs. The sample average $\bar{y}_t$ then refers to the average log of income in a given club at time t, and $x_{i,t} = y_{i,t} - \bar{y}_t$ refers to the difference of the log income of country i from the club average. The null is:

Proposition 1 *Countries within clubs A-D do not converge.*

The results of the Karras-Evans tests of are given in Table 2. Table 2 shows that clubs B, C, and D show relatively large $t-$values for the coefficient $\hat{\rho}$. At this level of analysis, we have not performed the Monte Carlo simulations needed to calculate the significancy of these values. Instead, we rely on the studies of Evans and Karras (1996) to get a rough idea. Evans and Karras estimated the convergence of 54 countries from 1960 to 1990 and derived the coefficient $\hat{\rho} = -0.0430$ with $t-$value -7.74, which they then simulated to have the $p-$value 0.0291. Evans and Karras estimated $\Phi(\hat{\delta}) = 3.46$, the $p-$value of which was simulated to be 0.0872. Based on these results, we make the conclusion that the clubs B, C, and D might have converged from 1960 to 1995. The convergence seems to have been only conditional.

Countries in club A do not converge. Figure 6 shows the time series for the logs of per capita incomes of countries in club A. Among these countries,

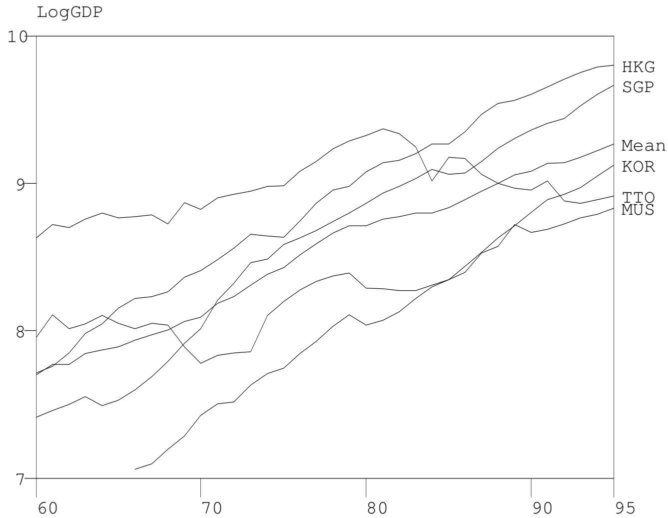

Figure 6: The logarithms of per capita GDPs in club A.

Korea (KOR), Hong Kong (HKG), and Singapore (SGP), the well known fast-growers, have steadily grown at a quite common rate. Yet, Trinidad & Tobago (TTO) experienced a serious and long-lasting depression due to falling oil prices in the 1980s.[6]

2.3 Caveats

To summarize, the Evans-Karras version of the time series unit root test for convergence of incomes within a club shows convergence in three demographic clubs of four. At least three types of caveats are present in the analysis.

First, note that the testing procedure only gives statistics within demographic clubs. To know what really was the role of demography in the analysis we should know, whether we have derived the same convergence result (three of four clubs converged) if the clubs were created randomly. If we forget the problem of unequal group sizes (from 5 to 42 members in our study), and simply randomly derive four groups of same size out of the data of 110 countries (to approximate, take 27 members to each group, note that $110/4 = 27.5$), we would enter with some $209847 \cdot 10^{20}$ possible ways of collecting the four random clubs. Once we had ran $4 \cdot 209847 \cdot 10^{20}$ units root tests, we were prepared to

[6] Trinidad & Tobago is not a member of OPEC and is not, for this reason, excluded from the sample, even much if its economy is based on the production of oil.

say, if the possibility to obtain convergence were smaller or larger than three of four. Actually, this type of analysis is performed by Ben-David (1996) in his study of trade based convergence clubs, but we leave this somewhat laborious exercise to future studies.

Second, because countries in club A did not converge, we are unable to identify, whether this result was due to lack of homogeneity of the club or whether the large decrease in population growth typical in this club was the reason for the non-convergence result. To find the answer, we rerun the analysis without Trinidad&Tobago with the result $\hat{\boldsymbol{\rho}} = -0.0668$, $\tau(\hat{\boldsymbol{\rho}}) = -3.988$, in which the estimated $t-$value is somewhat lower than in other clubs (see Table 2). This implies that the non-convergence result in club A, even if mainly due to the presence of exceptional country Trinidad&Tobago, might also be due to exceptionally rapid decrease in population growth rate (and very favorable worker/dependent ratio), which itself covers and dominates the possible convergence effect in this club. In a small club like A, all results are, however, much dependent on country-specific histories, and any far-fetching results should nor be derived.

Third, it is possible that the sample selection bias pointed out by DeLong (1988) is present in this study. In longitudinal studies the sample may be biased, if some *ex post* variables or factors are used in collecting it, because these variables might be endogenous to those variables that are to be explained by the study. We based the regression tree analysis to variables N (average population growth during the period) and DN (change in population growth during the period), both of which might be endogenous to economic growth, which is the variable to be studied in the paper. It would have been possible to avoid this caveat by dividing the time span (from 1960 to 1995) into two parts and by using the first part to sample the clubs, and the second part to run the unit root tests. However, the limited number of years made the goals of bias avoiding and accuracy of test results competitive. Fortunately, from our earlier investigations (see Lehmijoki 2003) we are able to see that demographic variables rely merely on the levels of income rather than on its growth, and we argue that even if some reservations are needed in terms of the results, this exercise can however give some information of convergence in demographic clubs.

3 Approach II: Demography and Income Inequality from 1960 to 2030

The dynamics of income inequality in the process of economic development was first studied by Kuznets (1967). His inverted-U hypothesis says that, in this process, income inequality first increases and then decreases again. High fertility in early development increases the number of people relative to land and capital. The share of wages decreases. Because land and capital is owned by a small minority of people, income tends to be more unequally distributed (Birdsall 1988). Later, as fertility decreases, the share of wages increases, and income inequality becomes less severe. Recently, Dahan and Tsiddon (1998) and Kremer and Chen (1999) have argued that the basic logic behind this process is not the initial ownership of land and capital, but the initial ownership of education: In the first stage, the fertility of educated people decreases, the number of the uneducated swells, and the wages for unskilled labor decrease leading to an increase in income inequality. Later, the fertility of uneducated decreases too, and the process is reverted.

We generalize these ideas to include income inequality between countries. Population growth rate first decreased in the rich countries which owned most of the (natural and other) resources and educated people in the world. High population growth rate in initially poor countries and low population growth rate in initially rich ones tended to increase the income inequality in the world. Therefore, as population growth rates in poor countries start to fall, world income should become less unequally distributed. Cross-country regressions supportive to this idea are provided by Sheehey (1996) and Williamson (1998).

In the nearest future (from 1995 to 2030), population growth rate is already decreasing everywhere. The question is, how fast will be the tempo of this decrease. The original hypothesis should be slightly modified to say that lower population growth is associated with lower income inequality. Therefore, we give two slightly different hypotheses, one for the past, and one for the future:

Proposition 2 *(Period 1960-1995). Income inequality increases as long as population growth rates in developing countries increase, but decreases as population growth in developing countries starts to decrease.*

Proposition 3 *(Period 1995-2030). Lower population growth rates lead to lower inequality of incomes.*

The income inequality in the sample of 110 countries from 1960 to 1995 can be calculated directly by using the data described on page 4. The United Nations (2000) provides projections for future population growth rates. Using these projections and the four demographic clubs with known economic growth rates during 1960-95, we estimate the growth rates from 1995 to 2030 for each country in the sample. Because the United Nations provides three variants — Low, Medium, and High — we are able to derive three estimates for future incomes and three measures of income inequality for each variant. We start by calculating future incomes.

3.1 Future Incomes

In the future, population growth rates decrease almost everywhere. This means that countries in the sample move from one demographic club to another. We calculate these movements for each country on the basis of what is known about future population growth rates. By assuming that economic growth rates in each club remains from 1995 to 2030 as they where from 1960 to 1995, we derive the future incomes for each country.

This might be speculative. It implicitly assumes that everything in economic growth depends on demography. If this were the case, knowing the future demographic situation of a country would be all that we would need to make valid projections for future incomes. Fortunately, we can alternatively assume that there exists no systematic reason for the club-specific growth rates from 1995 to 2030 to be different from the same growth rates from 1960 to 1995. In this case, our forecasts might be uncertain, but not biased.

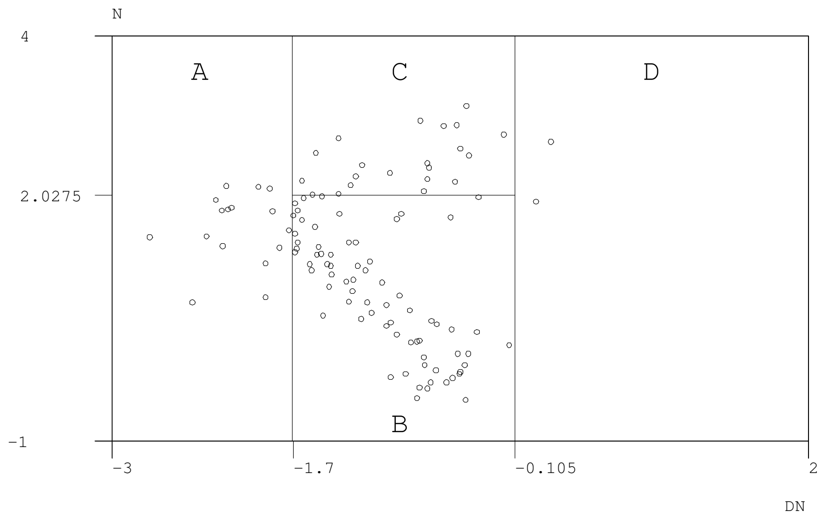

Figure 7: The Low Variant projection for the demographic situation in the future.

The three UN-projections for future population growth rates are given in three variants, which are

- Low Variant;
- Medium Variant;
- High Variant;

The details of the definitions are given in Appendix B. We concentrate on differences between High and Low Variants, but report the results for the Medium Variant for the sake of completeness.

Future incomes are derived as follows: First, for a given variant, calculate the change in population growth rate from 1995 to 2030 and average population growth rate from 1995 to 2030 for each country. Then locate each country in one of the four clubs derived above. Figures 7 and 8 depict future demographic

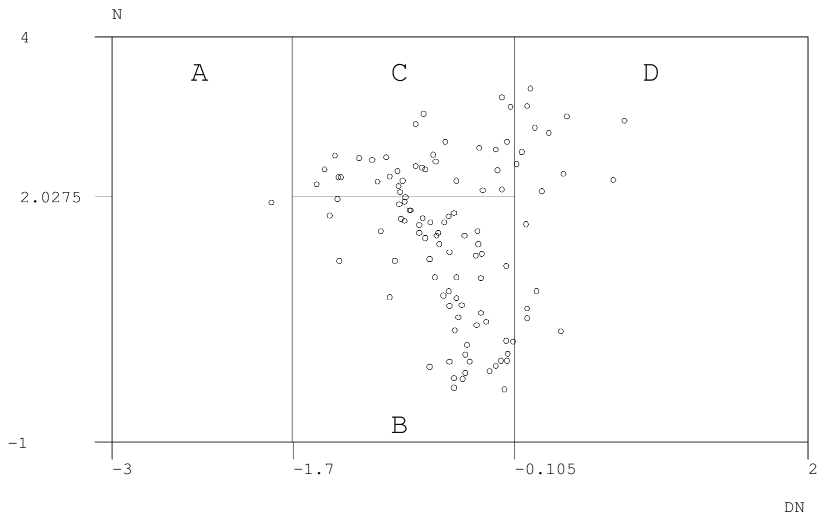

Figure 8: The High variant projection for the demographic situation in the future.

situations under Low and High Variants. Once each country is located to its appropriate club, its growth rate from 1995 to 2030 is assumed to be the average growth rate calculated for that club from 1960 to 1995. Then, calculate, for each variant

$$GDPxx = GDP95 * Exp(g_j * (xx - 1995)),$$

in which GDP is the per capita income, xx is the calendar year from 2000 to 2030, and g_j is the club-specific growth rate. Repeat this for each variant to have three different income projections for each five years between 2000 and 2030 and for each country. Table 3 summarizes the demographic and the economic development in the past and in the future.

Both total population and average GDP per capita increased steadily from 1960 to 1995. Obviously, our planet supplied an increasing well-being to an increasing number of people. According to our forecasts, this trend will continue from 1995 to 2030, but differences between the variants are considerable. In 2030, total population in the sample of 110 countries is 16 % larger and average GDP per capita is 19 % smaller under the High Variant than under the Low Variant. Therefore, both human and economic aspects favor low population growth rate in the future. In addition, assuming that deterioration of environment and sufficiency of natural resources are related to population growth, low population growth might have positive effects not completely discovered in this paper. Figure 9 shows the entire time path for population and per capita

Year	1960	1965	1970	1975	1980	1985	1990	1995
Population	2 353	2 605	2 900	3 208	3 501	3 819	4 161	4 488
GDP	2 266	2 692	3 234	3 669	4 193	4 354	4 798	5 150
Year	**2000**	**2005**	**2010**	**2015**	**2020**	**2025**	**2030**	
Low								
Population	4 802	5 079	5 327	5 551	5 752	5 929	6 068	
GDP	6 259	7 216	8 323	9 605	11 090	12 814	14 814	
Medium								
Population	4 802	5 116	5 426	5 731	6 024	6 304	6 565	
GDP	6 218	7 123	8 164	9 364	10 747	12 342	14 184	
High								
Population	4 802	5 151	5 516	5 895	6 280	6 668	7 059	
GDP	6 064	6 782	7 602	8 538	9 606	10 827	12 223	

Table 3: Total population (in millions) and average per capita GDP (in international 1985 dollars) in the sample.

incomes.

3.2 Inequality of Incomes

United Nations population projections are supplied in periods of five years. Therefore, the time series test for income convergence in Exercise I cannot be repeated here. Instead, to study the dispersion of future incomes, we are bound to use more traditional and direct measures of inequality, such as provided by Lorenz (1905), Gini (1936), and Theil (1967). These measures concentrate on estimation of an index for the inequality of the income division in a given year.

The desirable properties for this kind of index are discussed by Sen (1973), Atkinson (1970), Allison (1978), and Dasgupta *et al.* (1973). First, an index of inequality has to meet the Pigou-Dalton condition, which states that any transfer from a rich person to a poor person should decrease the value of the index (see Pigou 1912 and Dalton 1920). Second, the index should be scale invariant, meaning that if everyone's income is multiplied by a constant, the value of the index is unchanged. In nominal terms this requirement is indisputable: whether the incomes are measured in yen or dollars does not have any effect on inequality. Real proportionate increases are more problematic. If everyone's income increases by, say, 10 %, the ratios of incomes are left unchanged but the rich gain in absolute terms. Sen (1973), however, argues that this would not be necessary. Third, the index should be invariant to the size of the population, and fourth, it should be most sensitive to transfers in the lower end of the income division. The latter requirement says that the social welfare function, implied by the index, should be strictly concave (see Atkinson 1970 and Sen 1973).

Three indices meeting the first three requirements are the coefficient of varia-

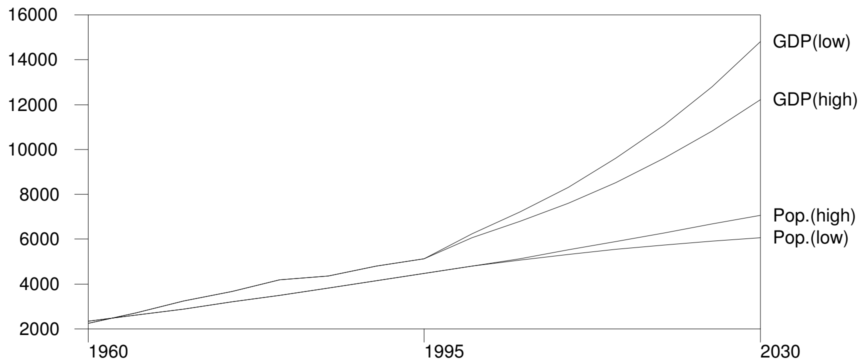

Figure 9: Total population and per capita GDP under Low and High Variants.

tion, the Theil index, and the Gini coefficient. In the context of income division between countries, the Gini coefficient, G, is the average absolute difference between all pairs of incomes y_i and y_j of countries relative to the mean of the sample income, μ:

$$\begin{aligned} G &= \frac{\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}|y_i - y_j|}{2\mu} \qquad (7)\\ &= \frac{2}{n^2\mu}\sum_{i=1}^{n} i \cdot y_i - \frac{n+1}{n}, \end{aligned}$$

in which n is the number of the countries and i is an index running from the poorest to the richest country. The Gini coefficient, however, does not meet the requirement of the sensitivity of transfers. To see this, assume that we give a transfer of h dollars from country m with income y_m to country k with income y_k , such that initially $y_k < y_m$. Let G and G' be the Gini coefficient before and after the transfer, and let $l = 1, 2, \cdots, n$ be an index such that it includes all numbers but m and k. In this case

$$G' = \frac{2}{n^2\mu}[\sum_{l=1}^{n} l \cdot y_l + k(y_k + h) + m(y_m - h)] - \frac{n+1}{n}.$$

Then $G' - G = \frac{2}{n^2\mu}(k - m)h$. Therefore, the Gini coefficient only depends on the difference of the ranking between countries. For international comparisons, however, the linearity of the implied welfare function seems to be merely a desirable property because it is not clear what should be the interpretation for such a function at the international level. For this reason Gini coefficient is used in this paper.

Year	1960	1965	1970	1975	1980	1985	1990	1995
10%	0.5559	0.5756	0.5747	0.5755	0.5704	0.5553	0.5568	0.5281
5%	0.5611	0.5786	0.5795	0.5796	0.5766	0.5607	0.5627	0.5338
Year	**2000**	**2005**	**2010**	**2015**	**2020**	**2025**	**2030**	
Low								
10%	0.5292	0.5309	0.5333	0.5362	0.5398	0.5441	0.5491	
5%	0.5353	0.5372	0.5398	0.5429	0.5466	0.5509	0.5558	
Medium								
10%	0.5300	0.5320	0.5342	0.5369	0.5402	0.5442	0.5488	
5%	0.5360	0.5383	0.5408	0.5438	0.5474	0.5513	0.5557	
High								
10%	0.5215	0.5160	0.5118	0.5103	0.5093	0.5088	0.5092	
5%	0.5269	0.5213	0.5182	0.5172	0.5165	0.5165	0.5172	

Table 4: The Gini coefficients.

We calculate Gini coefficients for 110 countries in the sample to evaluate the inequality of incomes in the past (from 1960 to 1995), and in the future (from 2000 to 2030). The values are given for every fifth year. The future values are calculated for Low, Medium, and High Variants.

We assume that all inhabitants in a country earn the average per capita income in that country. Because the size of the countries in the sample varies, the population of the sample is divided into fractiles. Both 10 % and 5 % fractiles are used. The country specific incomes y_i in equation (7) are replaced by income shares of the fractiles so that n refers to the number of fractiles (10 or 20). To calculate the income shares of the fractiles, locate the poorest 10 % (5 %) of the population to the first fractile and address the income earned by these people to that fractile. Continue in this way from fractile to fractile. Note that some fractiles contain several countries but large countries (USA, China, and India) are divided between several fractiles. Table 4 reports the Gini coefficients for both 10 % and 5 % fractiles.

Table 4 shows that the values of 5 % fractile Gini coefficients are larger than 10 % fractile values. This is in line with results found by Park (1997). Figure 10 shows the trends of the Gini coefficient for 5 % fractiles. Measured by Gini coefficients, inequality increased from 1960 to 1965, is constant between 1965-1980 and decreased by 1995.

Proposition 2 argues that income inequality decreases as the population growth rate in developing countries starts to decrease. All countries in clubs A, C, and D were developing countries at the beginning of the period (see Appendix A). The population growth in these countries in years 1960-1995 is given in Figure 11. The resemblances to the Gini coefficients in Figure 10 are apparent. This gives some support to the inverted-U hypothesis as expressed in Proposition 2. Unfortunately, even if a pair of Gini coefficients can be compared by calculating their confidence intervals (see below), no testing procedure for

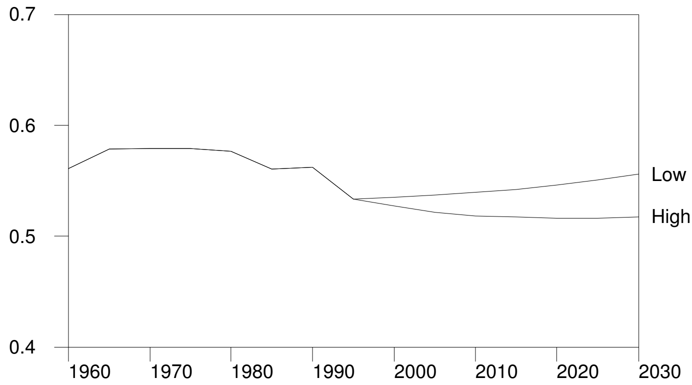

Figure 10: Trends of Gini coefficient for Low and High Variant, 5% fractiles.

a series of such coefficients is known to us, and we are bound to base our affirmative conclusions in relation to Proposition 2 on the graphical information in Figures 10 and 11.

From 1995 to 2030 instead, the results given in Table 4, and illustrated in Figure 10, confront the inverted-U hypothesis formulated in Proposition 3. Low, not high, population growth rate leads to greater inequality of the incomes in the sample. This is an unexpected result. To have a deeper insight, consider countries that were in clubs A, C, and D during period 1960-1995 as developing countries, and countries in club B as rich countries. To see which was the income inequality within developing countries and within rich countries under Low and High Variants, calculate the appropriate Gini coefficients within these sub-samples. Within developing countries (clubs A, C, and D), the coefficients where 0.3980 and 0.4107, so that inequality within developing countries was larger under High Variant. Within rich countries (club B), however, the numbers are 0.4488 and 0.3981, and the income inequality within rich countries is larger under Low than under High Variant. The reason is that USA and Luxembourg, the two richest countries in the sample, stay in club B (growth rate 2.7810 %) under Low Variant, but move to club D (growth rate 0.5623 %) under High Variant, and the inequality within B decreases because of this move.

The move of (at least some of) the richest countries to unfavorable demographic clubs might be really true in the future. The problem is that the clubs, created on the basis of information from 1960 to 1995, might not capture all the demographic clubs, that may be present in the future. The economic growth rate and the borderlines of club D, to which USA and Luxembourg are classified under the High Variant, was identified among initially poor (developing) countries (see Appendix A) with ever increasing cohorts of young people (see

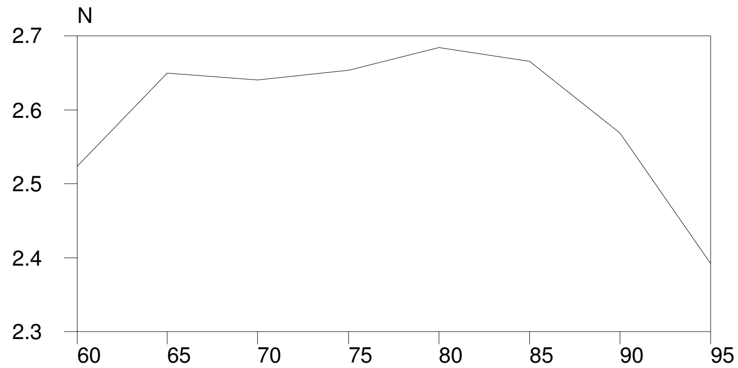

Figure 11: Population growth rate in clubs A, C, and D from 1960 to 1995.

Gabon). In the future, the variables N and DN in USA and Luxembourg might be within the limits of club D, but the age structure is still different from that in the countries in club D from 1960 to 1995. The number of the old, not the number of the young, is large in USA and Luxembourg in the future. What will be the economic growth rate in such conditions may not be correctly predicted by the number 0.5623 %, typical for countries in club D from 1960 to 1995. However, a large number of the young and a large number of the old both lead to low worker/dependent ratio, and — in the presence of the data for the past alone — we take the predictions based on clubs $A - D$ as reliable.[7]

3.2.1 Testing Gini Coefficients, Proposition 3

A graphical device to illustrate income inequality is the Lorenz curve (see Lorenz 1905), which depicts the cumulative proportional share of population against that of income. The diagonal line gives perfect equality ($G = 0$). The lower the curve, the higher the inequality. Figure 12 depicts the Lorenz curves for Low and High Variants in 2030. The Lorenz curve for the Low Variant lies everywhere below that for the High Variant. This confirms the information given in Figure 10.

Proposition 3 argues that low population growth leads low inequality of income, but the Gini coefficients show that income inequality is lager if the Low Variant is realized. We test this controversial result for the year 2030 by deriving the confidence intervals of the coefficients.

Wolter (1985) shows that the jackknife variance estimator for the Gini coefficient takes the formula

[7] Kelley and Schmidt (1996) argue, however, that the young and the old might be different types of dependents.

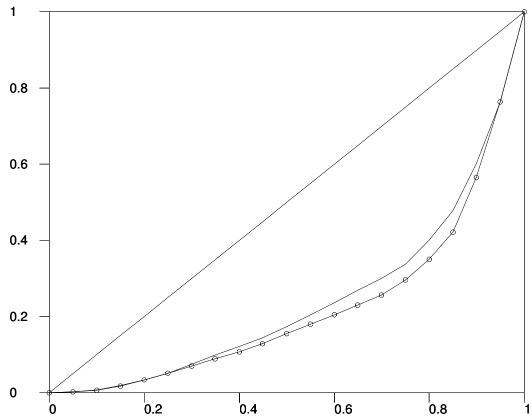

Figure 12: The Lorenz Curves in 5% fractiles for 2030. Low Variant (dotted curve) and High Variant (solid line).

$$s^2 = \frac{n-1}{n}\sum_{i=1}^{n}(G_i - G)^2,$$

in which n is the size of the sample (20 for 5 % fractiles), G is the Gini coefficient for the whole sample, and G_i the Gini coefficient for the sub-sample without i_{th} observation. Recently, Ogwang (2000) has shown how to calculate the values for G_i without resampling the data. Assuming that the population of Gini coefficients is approximately normally distributed, we can derive the 90 % confidence limits for the Low and High Variant Gini coefficients in 2030. These limits are $(0.4544 - 0.6573)$ and $(0.3968 - 0.6377)$. Even at the 90 % level, the confidence intervals for Low and High Variants are overlapping and the difference in these coefficients is not statistically significant.[8] Even if the results derived contradict Proposition 3, this controversy is not confirmed by a statistical test.

4 Conclusions

Approach I derives four demographic clubs by applying the regression tree technique to levels and changes of population growth rate. The time series unit root test suggested by Evans and Karras (1996) shows that three of the four

[8] Allison (1978) suggests an alternative way to estimate the confidence intervalls for Gini coefficient based on the maximum likelihood estimation of the coefficient. To be able to derive the maximum likelihood estimates, Allison assumes that income is lognormally distributed.

demographic clubs exhibit conditional convergence. However, in failing to give the final evidence on the role of demography in convergence, the exercise should be seen as preliminary attempt to deal with the question

In the future, population growth rate decreases, and countries move from one club to another. Approach II provides forecasts for future growth rates showing that low population growth tends to increase economic growth in the future. The inverted-U hypothesis suggests that low population growth would be favorable to income division as well. This hypothesis gets some support during the period 1960-1995; the timing of the population growth decrease in developing countries is identical to the timing of the decrease in inequality as measured by the Gini coefficient. The results for future income inequality are not confirmed by test statistics, however. Instead some light is shed on the possibility that the deterioration of worker/dependent ratio, due to an increase in the share of the elderly might lead to low economic growth rates in some currently rich countries.

A Appendix: Demographic Clubs

A

Mauritius, Trinidad&Tobago, Hong Kong, Korea Rep., Singapore.

B

Barbados, Canada, United States, Argentina, Uruguay, China, Israel, Japan, Sri Lanka, Thailand, Austria, Belgium, Denmark, Finland, France, Greece, Hungary, Iceland, Ireland, Italy, Luxembourg, Malta, Netherlands, Norway, Portugal, Romania, Spain, Sweden, Switzerland, U.K., Australia, New Zealand.

C

Botswana, Cape Verde, Egypt, Ivory Coast, Kenya, Morocco, South Africa, Tunisia, Uganda, Zimbabwe, Bahamas, Belize, Costa Rica, Dominican Rep., El Salvador, Jamaica, Mexico, Panama, Brazil, Chile, Colombia, Ecuador,Guyana, Paraguay, Peru, Suriname, Bangladesh, India, Philippines, Turkey, Fiji.

D

Angola, Benin, Burkina Faso, Burundi, Cameroon, Central Africa, Chad, Comoros, Congo, Ethiopia, Gabon, Gambia, Ghana, Guinea, Guinea-Bissau, Lesotho, Madagascar, Malawi, Mali, Mauritania, Mozambique, Namibia, Niger, Senegal, Sierra Leone, Sudan, Swaziland, Tanzania, Togo, Zaire, Zambia, Guatemala, Haiti, Honduras, Nicaragua, Bolivia, Jordan, Nepal, Oman, Pakistan, Syria, Papua New Guinea.

B Appendix: Assumptions Underlying the Population Projections

The data for three variants (Low Variant, Medium Variant, and High Variant) for future population growth rates is supplied by the United Nations (2000). The data is supplied until 2050, but the time span of Exercise II only extends to 2030. The three variants are based on identical assumptions about mortality and migration trends, but on different assumptions about fertility. Detailed description of data sources used and methods applied in volume III of World Population Prospects: The 2000 Revision (forthcoming). The assumptions include:

A. FERTILITY ASSUMPTIONS

Fertility assumptions are described in terms of the following groups of countries:

- *High-fertility countries*: Countries that until 2000 have had no fertility reduction or only an incipient decline;
- *Medium-fertility countries*: Countries where fertility has been declining but whose level is still above replacement level (2.1 children per woman);
- *Low-fertility countries*: Those countries with fertility at or below replacement level (2.1 children per woman), plus a few with levels very close to replacement levels that are expected to fall below replacement level in the near future.

Medium-fertility assumptions:

- Fertility in high fertility countries is generally assumed to decline at an average pace of nearly one child per decade starting in 2005 or later. Consequently, some of these countries do not reach replacement level by 2050.
- Fertility in medium-fertility countries is assumed to reach replacement level before 2050.
- Fertility in low-fertility countries is generally assumed to remain below replacement level during most of the projection period, reaching by 2045-2050 the fertility of the cohort of women born in the early 1960s or, if that information is lacking, reaching 1.7 children per woman if current fertility is below 1.5 children per woman, or 1.9 children per woman if current fertility is equal to or higher than 1.5 children per woman.

High-fertility assumptions:

- Fertility in high-fertility and medium-fertility countries remains above the fertility in the medium fertility assumption and eventually reaches a value 0.5 children above that reached by the medium-fertility assumption in 2054-2050.
- For low-fertility countries, the value eventually reached is 0.4 children per woman above that reached by the medium-fertility assumption in 2045-2050.

Low-fertility assumptions:

- Fertility in high-fertility and medium-fertility countries remains below the fertility in the medium-fertility assumption and eventually reaches a value 0.5 children below that reached by the medium-fertility assumption in 2045-2050.

- For low-fertility countries, the value eventually reached is 0.4 children per woman below that reached by the medium-fertility assumption in 2045-2050.

B. MORTALITY ASSUMPTION

Mortality is projected on the basis of the models of change of life expectancy produced by the United Nations. In countries highly affected by the HIV/AIDS epidemic, estimates of the impact of the disease are made explicitly through assumptions about the future course of the epidemic, that is, by projecting the yearly incidence of HIV infection.

C. INTERNATIONAL MIGRATION ASSUMPTION

The future path of international migration is set on the basis of past international migration estimates and an assessment of the policy stance of countries with regard to future international migration flows.

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Summary

Demographic transition — a shift of both fertility and mortality to a lower level — took place in Europe and its offshoots together with Industrial Revolution. The decrease in mortality started in end of the eighteenth century and that of fertility around 1900. Because the decrease in fertility much lagged that in mortality, population growth accelerated temporarily. The transition escalated to developing countries during the twentieth century and the peak population growth in these countries took place in the mid of 1960s.

In Chapter 1, a survey of demographic theories dealing with demographic transition is given. The traditional theory of transition, written by Notestein (1945) sees the transition as an outcome of the Industrial Revolution and the socio-economic change accompanied by it in general. The subsequent theories are more limited in scope. The economic theories given by Becker (1960), Easterlin (1978), and Caldwell (1982) concentrate on the role of the incomes, the wages, the costs of contraceptives, and the directions of wealth flows in fertility determination. The cultural theories delivered by Lesthaege (1983), Cleland and Wilson (1987), and Rosero-Bixby and Casterline (1993) emphasize the role of idealistic movements, changes in tastes, and diffusion of values between countries. The homeostatic theory discusses the role of carrying capacity of the environment in the population growth rate.

Mason (1997) has suggested that these alternative theories of transition complementary rather than competitive. They all provide a partial explanation of demographic transition. This implies that empirical studies should be based on a large number of explanatory variables suggested by these theories.

In the "Demographic Introduction" we use ten regressors suggested by the theories above to explain the level and change of total fertility rate. A panel data analysis shows that, according to Notestein's idea, the role of mortality decrease is important in fertility decline. Other important explanatory variables are the share of agricultural labor force, the level of per capita income, and the rate of economic growth. The result that population growth, or at least fertility, is endogenous to economic variables is both favorable and unfavorable from the point of view of this thesis. In theoretical studies in Chapter 2 and 3, endogenous population growth makes the modelling of demographic transition possible and interesting, but the empirical results provided in Chapter 4 might be biased due to this endogeneity, and they should be interpreted with some reservation.

Chapter 2, "Demographic Transition in the Ramsey Model" provides the basic theoretical results of the thesis. It takes as its starting point the controversy between empirical facts and neoclassical models. Empirics shows that

population growth is variable due to demographic transition, whereas growth models take it as a constant. The essay introduces the population function as $n = n[k(t)]$, with the properties

$$\begin{aligned} n'[k(t)] > 0 &\Leftrightarrow k(t) < \mu, \\ n'[k(t)] = 0 &\Leftrightarrow k(t) = \mu, \\ n'[k(t)] < 0 &\Leftrightarrow k(t) > \mu. \end{aligned}$$

The introduction of the population function to the Ramsey model leads to objective functional

$$U = \int_0^\infty u[c(t)] \cdot \exp\left\{ -\int_0^t \{\rho - n[k(\tau)]\} d\tau \right\} dt,$$

in which the discount rate is variable. Following the idea of Uzawa (1968) we suggest in the essay that writing $\Delta(t) = \int_0^t \{\rho - n[k(\tau)]\} d\tau$ and having then the objective functional

$$U = \int_0^\infty \frac{u[c(t)]}{\rho - n[k(t)]} e^{-\Delta(t)} d\Delta(t),$$

the problem becomes regular, i.e., the discount rate is constant, and both the utility function $u[c(t)]$ and per capita production function $y = f[k(t)]$ are strictly concave. Therefore, in virtual time the planner's problem can be solved by using standard methods.

The result of the solution is that the model may have multiple steady states. The local stability analysis shows that in this case an unstable focus or node is surrounded by two saddle stable steady states. Two alternative types of global dynamics appears in this case. First, the saddle path towards the high income steady state can spiral from the focus between saddles. Then, for some initial capital stock, two saddle paths can be reached. Skiba (1978) has shown that the optimal path can be found by comparing the optimized Hamiltonian at the initial point of each of these saddle paths. The essay shows, that Skiba's result is applicable to the case of variable discount rate as well. The second type of global dynamics, in which the saddle path towards the high income steady state runs from the origin is also discussed.

The three global alternatives, single steady state and multiple steady states with the spiral saddle path from the focus or with the saddle path from the origin are studied in a calibrated version of the model to discover their connection with the type of demographic transition. It turns out that if the peak population growth rate is high, if the population function is very sensitive to income (or capital), and if the transition takes place at a high stock of productive capital, the spiralling saddle path case appears. This type of transition is labelled as *strong*. The interpretation is that an economy with strong demographic transition has a poverty trap. High income equilibrium is reached only if the stock of the productive capital is initially above some threshold value. On the other hand, if peak population function is less sensitive to income, and capital stock is lower during the transition, poverty trap is avoided even if the model has three steady states. The saddle path toward the high income steady state runs from the origin and this steady state is reached from all initial states. in this case the demographic transition is said to be *moderate*. In the *weak* type of transition, peak population growth is low, population function reacts weakly to income, and capital stock during the transition is small. In this case the model exhibits a single steady state only.

We suggest that the strong demographic transition with the poverty trap is not likely to be realized outside the model. Still, in the case of moderate transition, the time paths are affected and economic growth rate greatly varies during the demographic transition.

In the essay "Learning by Living: Early Development" we return to the role of the mortality again. In the beginning of demographic transition, i..e. during the early development (see Goodfriend and McDermott 1995), mortality decrease leads to increase in life expectancy at the micro level, and to increase of population growth rate at the macro level. Because lengthening of life increases the possibilities and demand for formal schooling and also increases the informal learning in families, we deduce that population growth (and lengthening of life) is positively related to the rate of human capital accumulation, and the human capital accumulation can be written

$$\frac{\dot{H}}{H} = h(n),\ h'(n) > 0.$$

During the Early Development, human capital manifested itself in the form of discoveries in the field of medicine. These discoveries decreased mortality and fertility, and thus affected the population growth rate. In addition, during the Early Development, increase in income increased population growth rate. We write the population function in this essay as

$$\begin{aligned} n &= n(y, H), \\ \frac{\partial n}{\partial H} &< 0;\ \forall n, \\ \frac{\partial n}{\partial y} &> 0 \text{ for } y < \bar{y};\ \forall H, \end{aligned}$$

Combining the human capital accumulation and population function above with a model of infinite horizon continuous time consumer optimization model produces a model in which multiple steady states might appear. On the contrary to the earlier results, low income steady state is related to low population growth, high mortality, low life expectancy and low rate of human capital accumulation, whereas high income steady state is related to high population growth, low mortality, long life, and high rate of human capital accumulation.

The calibrated version of the model shows that the role of the income share of capital is important: if this share is high, the economy is led to high income steady state, whereas, if this share is only slightly lower, the economy is trapped by the low income steady state.

The essay "Convergence, Income Inequality, and Demographic Clubs" discusses the demographic convergence clubs, which are derived from the data by using regression tree analysis. Four demographic clubs with greatly different economic growth rates exist. International income distribution has recently been studied in two quite different frameworks. The modern approach derives from the theory of economic growth and from the econometric theory of time series unit root tests, whereas the traditional framework derives from measures that originally were devoted to study all kinds of inequality.

If the case of decreasing returns convergence between countries should exist. Convergence clubs are based on the reasoning that convergence in large heterogenous samples is covered by great variation of many economic variables between countries, but it will be uncovered in a club of homogenous countries (Sala-i-Martin 1996). This paper argues that demography could serve as a criterion for homogeneity and that convergence should be found in demographic clubs, if such a tendency in homogenous groups were present.

The unit root time series test in the version suggested by Evans and Karras (1996) shows that three of the four demographic clubs exhibit conditional convergence. However, the final conclusion of the role of demography in the convergence must be let unsettled because we are unable to compare the results in demographic clubs to the convergence in randomly derived four clubs in the same sample.

The inverted-U hypothesis represented by Kuznets (1967) says that, in the process of economic development, income inequality first increases and then decreases. Poor people tend initially to be more fertile but have limited access to resources. Therefore, their population share initially swells, but the income share stays constant. Later, as fertility among the poor decreases, their population share decreases and income becomes less unequally distributed. If the decrease in the population growth rate first starts in rich countries, and then expands to poor ones, we might expect that, as this decrease among poor countries starts, income inequality starts to decrease. The essay tests this hypothesis by calculating the Gini coefficients for years 1960-1995. The time profile of these coefficients resembles that of population growth in poor countries giving some support to the inverted-U hypothesis.

We also calculate Gini coefficients for future incomes from 1995 to 2030 once the incomes are estimated on the basis of the four demographic clubs. The future version of inverted-U hypothesis assumes, that low population growth rate leads to low inequality of incomes in the future. This hypothesis is not supported by the results.

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