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Alternating-offers bargaining, dynamic matching and threat of Bertrand competition

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Abstract

We consider alternating-offers bargaining between a buyer and a seller in a dynamic matching model where the appearance of a third party triggers a Bertrand -type competition between the identical agents. In the continuous-time limit, Nash' axiomatic bargaining solution holds only if buyers' and sellers' matching rates are equal. Otherwise the equilibrium sharing rule assigns a greater fraction of the surplus to the party facing less immediate threat of competition. This suggests that not only the reservation values but also the partition rule according to which the surplus is divided should be treated as endogenous in bilateral matching models.

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1   Introduction

We consider the alternating-offers model à la Rubinstein (1982) in a dynamic matching framework where a buyer and a seller bargain over the terms of trade. The novel feature in the model is that the arrival of a third party may trigger a Bertrand-type competition on the congested side of the negotiation table. If the new contact is another buyer, a bidding game among the buyers ends the transaction. If another seller interrupts the bilateral bargaining, the competing sellers lower their price offers until they are indifferent between selling the good and remaining unmatched. Given this setup, we derive the equilibrium partition rule according to which the matching surplus is shared immediately upon a newly formed match. In the limiting equilibrium where the length of one bargaining period is arbitrarily, the outcome coincides with Nash's axiomatic bargaining solution (symmetric) only in the special case where buyers and sellers contact alternative trading partners with equal probabilities. Otherwise, the possibility of Bertrand competition gives rise to asymmetric surplus division. When potential demand and supply in the market do not balance, the bargaining solution favors the short-side of the market in a sense that a greater fraction of the matching surplus goes to the party facing less immediate threat of competition. The analysis thus suggests that in dynamic matching markets not only the parties' reservation values but also the partition rule according to which the matching surplus is divided should be endogenous and based on market fundamentals. This result is in contrast with the fixed sharing rule practice widely used in many popular applications of search and matching theory.[1] Since the bargaining solution obtained here is about as simple as the symmetric Nash sharing rule, it should be readily applicable in various search theoretic models.

In a closely related study, Rubinstein and Wolinsky (1985) also consider a matching market where the possibility of meeting alternative trading partners creates a potential risk of breakdown of the alternating-offers bargaining. They do not, however, allow for competition between identical agents but assume that the agent with two alternative partners always abandons the previous partner and starts bargaining with the new one. The limiting equilibrium of Rubinstein-Wolinsky (1985) model always yields the symmetric Nash solution, regardless of the underlying matching rates. In

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[1] E.g. the Mortensen-Pissarides model (Mortensen and Pissarides, 1994; Pissarides, 2000).

our model, however, fierce competition between the alternative partners fortifies the effect of the matching rates on traders' relative bargaining positions: the party with better matching prospects is able to claim a greater share of the surplus.

Kultti (2000) and Kultti and Virrankoski (2004) were the first to introduce the possibility of Bertrand competition into a pairwise matching framework. These papers show that the option to wait for alternative partners can be used to determine a unique transaction price in a random matching model with 'take-it-or-leave-it' bargaining. Such an extreme version of bilateral bargaining gives, however, a substantial advantage to the party that has the right to propose the take-it-or-leave-it offer so that the outcome is sensitive to the probability at which a trader gets this opportunity. In the alternating-offers bargaining, in turn, the advantage of the agent who proposes the first offer becomes negligible when the length of one bargaining period is small. Therefore the limiting equilibrium partition rule is not dependent on any arbitrary elements (e.g. the probability distribution according to which the right for the first offer is allocated).

2 The model

2.1 The market environment

Consider a dynamic search market where buyers and sellers randomly match and trade. Each seller has one indivisible and homogeneous good for sale. All buyers have a fixed demand for one unit of the good. Both buyers and sellers have linear preferences. Buyers' valuation for the good is denoted by y, sellers' valuation is normalized to zero. The divisible rent in any trading opportunity is thus y.

In order to become matched in the market, agents must search for trading partners. Search as such is costless but the coordination failure in the market extends the time span an agent remains unmatched. Time is discrete and extends over infinity, $t = 0, 1, ..., \infty$. Each time interval is of length Δ. The discount factor between two periods is $\delta = e^{-r\Delta}$ where $r > 0$ is the discount rate common to all agents. Poisson arrival rates are used to measure the probabilities of locating a trading partner. The probability that a buyer meets a seller in any given time interval is $\alpha\Delta$ while the probability that a seller is matched with a buyer is $\beta\Delta$. After a successful match, both the

buyer and the seller exit the market. We only consider a stationary state where the agents who leave the market are replaced by the same measure of new agents so that the relative population sizes of buyers and sellers, and thereby the meeting rates $\alpha\Delta$ and $\beta\Delta$, remain constant over time.

2.2 Bargaining process

Upon a new match, say at time t, the agents immediately start bargaining over the division of the rent y. The bargaining process obeys the rules of the alternating-offers procedure by Rubinstein (1982), except that the bargaining process may become interrupted by the arrival of a third agent. If a third agent appears, a Bertrand-type competition is triggered between the identical agents.

We assume that the seller always delivers the first offer. We could equally well have that right to be allocated randomly, as in Binmore (1987), but this would not affect the results as the first-mover advantage becomes negligible in the continuous-time limit, which is the ultimate goal of our analysis. If the buyer accepts seller's initial offer, transaction concludes. If the buyer rejects, she starts preparing her counter-offer that will be delivered after one bargaining period Δ at time $t + 1$. However, the bargaining process may never reach that point. Within the first bargaining period Δ, another seller interrupts the bargaining process with probability $\alpha\Delta$, in which case trade is conducted between the buyer and one of the sellers at a price that drives the seller to her reservation utility. With probability $\beta\Delta$, in turn, a rival buyer candidate shows up and the competing buyers raise their bids until they are indifferent between buying the good and remaining unmatched. As we will focus on the limiting equilibrium where Δ is arbitrarily small, the possibility of several arriving agents within a single time interval is not taken into account; i.e. the events with probabilities of order $(x\Delta)^2, (x\Delta)^3 ...$ are ignored. Hence, the buyer will have the chance to deliver her counter-offer at $t + 1$ with probability $1 - \alpha\Delta - \beta\Delta$. If this offer is rejected by the seller then the seller proposes yet another offer at $t + 2$, unless the bilateral meeting becomes interrupted by a third agent in the meantime.

2.3 Value functions

The expected utility of a seller proposing (receiving) an offer is denoted by $\hat{U}^s$ (U^s). Similarly, the corresponding utilities of a buyer are denoted by $\hat{U}^b$ and U^b. A partition rule $\mathcal{P}_t^i$ amounts to saying that the rule is proposed at time t by the agent $i = b, s$ (where b stands for buyer and s for seller) and it specifies the utilities $\hat{U}^i$ and $U^j = y - \hat{U}^i$ obtained by agent i and j $(i \neq j)$ in a completed transaction.

Consider a match formed at $t = 0$. The seller proposes the first offer $\mathcal{P}_0^s$. If the buyer rejects this offer, she expects to earn

$$R_{t=0}^b = \delta \left\{ \alpha \Delta \left(y - \bar{U}^s \right) + \beta \Delta \bar{U}^b + (1 - \alpha \Delta - \beta \Delta)\hat{U}_{t=1}^b \right\}. \tag{1}$$

The first term inside the curly brackets represents the probability that another seller appears and there will be Bertrand competition between the sellers. As a result, the sellers are driven to their reservation utility level $\bar{U}^s$ and the buyer earns $y - \bar{U}^s$. $\bar{U}^s$ equals with the value of being an unmatched seller in the market. Since upon every new match the seller gets to propose the first offer, $\bar{U}^s$ is determined by the following asset pricing equation:

$$\bar{U}^s = \delta \left\{ \beta \Delta \hat{U}^s + (1 - \beta \Delta) \bar{U}^s \right\},$$

Solving for $\bar{U}^s$ obtains

$$\bar{U}^s = \frac{\delta \beta \Delta}{1 - \delta (1 - \beta \Delta)} \hat{U}^s. \tag{2}$$

The second term in (1) captures the probability of competition between two buyers. In this case the buyer is driven to her reservation utility $\bar{U}^b$. Since the buyer always acts as the receiver of the first offer, $\bar{U}^b$ is given by

$$\bar{U}^b = \frac{\delta \alpha \Delta}{1 - \delta (1 - \alpha \Delta)} U^b. \tag{3}$$

Finally, the last term in (1) represents the possibility that there will be no interruption in the bilateral bargaining and the buyer gets to propose her counter-offer $\mathcal{P}_{t=1}^b$ at $t = 1$. Rejection of this offer would provide the seller with the following expected utility:

$$R_{t=1}^s = \delta \left\{ \alpha \Delta \bar{U}^s + \beta \Delta \left(y - \bar{U}^b \right) + (1 - \alpha \Delta - \beta \Delta)\hat{U}_{t=2}^s \right\}. \tag{4}$$

2.4 Equilibrium

A well-established result in the literature (Rubinstein, 1982; Binmore, 1987; Rubinstein and Wolinsky, 1985) is that when one assumes a stationary configuration where identical agents use the same bargaining tactics against all the partners they might meet, the alternating-offers game has a unique perfect equilibrium with the following properties: $\mathcal{P}_t^i = \mathcal{P}^i \ \forall t \in [0, \infty)$, $\forall i = b, s$ such that

$$\hat{U}^i = y - R^j \text{ and } U^i = R^i \ i, j = b, s, \ i \neq j. \tag{5}$$

Effectively this means that the agent who delivers the first offer proposes a partition that makes the receiver indifferent between accepting and rejecting the offer and that the first proposal thereby always leads to agreement. Using these equilibrium conditions and the fact that terms involving Δ^2 can be ignored, $\hat{U}^s = y - R^b$ and $U^b = R^b$ imply

$$\hat{U}^s = \frac{1 - \delta(1 - \beta\Delta(1+\delta))}{1 - \delta^2(1 - 2\alpha\Delta - 2\beta\Delta)}y,$$

$$U^b = \frac{\delta(1 - \beta\Delta - \delta(1 - 2\alpha\Delta - \beta\Delta))}{1 - \delta^2(1 - 2\alpha\Delta - 2\beta\Delta)}y.$$

Consider now the limiting equilibrium where the length of one bargaining period Δ is arbitrarily small. Using l'Hospital rule, and remembering that $\delta = e^{-r\Delta}$, we obtain

$$\lim_{\Delta \to 0} \hat{U}^s = U^s = \frac{2\beta + r}{2(\alpha + \beta + r)}y, \tag{6}$$

$$\lim_{\Delta \to 0} U^b = \frac{2\alpha + r}{2(\alpha + \beta + r)}y. \tag{7}$$

Note that the first-mover advantage of the seller becomes negligible as $\Delta \to 0$; i.e. $\lim_{\Delta \to 0} \hat{U}^s = U^s$. Similarly, the limiting reservation values obtain

$$\bar{U}^b = \frac{\alpha}{\alpha + r}U^b \text{ and } \bar{U}^s = \frac{\beta}{\beta + r}U^s.$$

As the matching surplus is given by $S = y - \bar{U}^b - \bar{U}^s$, the fractions of the surplus going to the buyer and the seller yield

$$\gamma^b = \frac{U^b - \bar{U}^b}{S} = \frac{(\beta + r)(2\alpha + r)}{(\beta + r)(2\alpha + r) + (\alpha + r)(2\beta + r)} \equiv \frac{1}{1 + \Phi^b(\alpha, \beta)}, \tag{8}$$

$$\gamma^s = \frac{U^s - \bar{U}^s}{S} = \frac{(\alpha + r)(2\beta + r)}{(\beta + r)(2\alpha + r) + (\alpha + r)(2\beta + r)} \equiv \frac{1}{\Phi^s(\alpha, \beta) + 1}, \tag{9}$$

where $\Phi^b(\alpha, \beta) = \frac{(\alpha + r)(2\beta + r)}{(\beta + r)(2\alpha + r)}$ and $\Phi^s(\alpha, \beta) = \frac{(\beta + r)(2\alpha + r)}{(\alpha + r)(2\beta + r)} = [\Phi^b(\alpha, \beta)]^{-1}$.

2.5 Discussion

In the given setup, Nash's axiomatic bargaining with equal surplus division would yield the following pair of utilities for buyers and sellers respectively:

$$U_{NB}^b = \frac{\alpha + r}{\alpha + \beta + 2r}y \text{ and } U_{NB}^s = \frac{\beta + r}{\alpha + \beta + 2r}y.$$

In is an easy task to check that the surplus shares under the symmetric Nash bargaining satisfy $\gamma_{NB}^b = \gamma_{NB}^s = 1/2$. Inspection of (6)-(7) and (8)-(9) reveal that our model produces this outcome only in the special case where $\alpha = \beta$; i.e. when the potential demand and supply in the market balance.[2] Generally, we observe that Φ^b is decreasing and convex in α while it is increasing and concave in β. From this it follows that γ^b must be increasing and concave in α and decreasing and convex in β. Obviously, the converse is true for Φ^s and γ^s. Hence, the share of the matching surplus that an agent is able to claim depends positively on her own matching probability and negatively on her opponent's matching probability, albeit both effects exhibit a diminishing impact.

Compared with the model of Rubinstein and Wolinsky (1985) - which produces the symmetric Nash solution as a limiting equilibrium - threat of Bertrand competition fortifies the effect of the matching rates on the rent division and makes the equilibrium surplus partition sensitive to the market fundamentals. This finding suggests that not only the parties' reservation values but also the rule according to which the matching surplus is divided should be treated as endogenous and dependent on the relative matching rates. Moreover, as the limiting values for U^s and U^b in (6)- (7) are about as simple as the corresponding values under the symmetric Nash bargaining, the proposed sharing rule should be readily applicable in various search theoretic models.

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[2] The symmetric Nash outcome holds also if $r \to 0$, which effectively means that the market frictions become negligible.

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