Low-energy effective Lagrangian in unified theories with nonuniversal supersymmetry-breaking terms

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Supersymmetric grand unified theories with nonuniversal soft supersymmetry- (SUSY-) breaking terms are studied. By integrating out the superheavy fields at a unification scale, we compute their low-energy effective Lagrangian. We find new contributions to the scalar potential specific to the nonuniversal supersymmetry breaking. A D-term contribution to the scalar masses is one example. The gauge hierarchy achieved by a fine-tuning in the superpotential would be violated in general due to the nonuniversal SUSY-breaking terms. We show, however, it is preserved for a certain class of the soft terms derived from a *hidden* ansatz. We also discuss some phenomenological implications of the nonuniversal supersymmetry breaking, including predictions of the radiative electroweak symmetry-breaking scenario and of no-scale-type models.

I. INTRODUCTION

Supersymmetry (SUSY) has been regarded as a beautiful mechanism which ensures the stability of the hierarchy between the weak scale and the grand unified theory (GUT) [1] or Planck scale against radiative corrections. There is, however, another virtue of SUSY which is not often quoted: the SUSY-breaking terms at the weak scale directly reflect the physics at very high energies thanks to the moderate renormalization effects, i.e., the absence of the quadratic divergences. For instance, one can study experimentally whether the weak scale SUSY spectrum is consistent with the GUT by measuring the gaugino masses [2]. Also, the scalar mass spectrum has certain "sum rules" specific to symmetry-breaking patterns [3,4]. Therefore, it is important to know the predictions of the various models on the SUSY-breaking terms at low energy.

Many efforts have been devoted to the low-energy predictions under the assumption of the universal SUSY-breaking terms, which have led to remarkable progress in recent years [5,6]. The idea of the universal SUSY-breaking terms was led by the nonobservation of the large flavor-changing neutral current processes due to the SUSY particle loops [7]. And the hidden sector SUSY breaking [8] combined with the minimal supergravity indeed leads to the universal SUSY-breaking terms, with or without grand unification [9]. However, a decade after these works, there are increasing interests in the nonuniversal form of the SUSY-breaking terms, at least due to the following two reasons: (1) The unification scale is now believed to be substantially lower than the grav-

itational scale[1] $M_{\text{Pl}}/\sqrt{8\pi}$, and the radiative correction changes the form of the supersymmetry breaking terms; (2) the superstring theory implies a highly nonuniversal form of the Kähler potential in general [10]. Therefore, it is an important task to see what low-energy theory results from the unified theories with a nonuniversal form of the supersymmetry-breaking terms.

There are some indications that the nonminimal SUSY-breaking terms give potentially important consequences in the low-energy effective Lagrangian. First, they give rise to the so-called D-term contribution to the scalar masses when the rank of the gauge group is reduced [11]. We pointed out in the previous Letter [3] that they can be generated even at the GUT scale and give observable consequences on the weak scale scalar mass spectrum, which are important to distinguish the various symmetry-breaking patterns [3,4]. Second, the nonuniversal SUSY-breaking terms may ruin the fine-tuning in SUSY-GUT models such that the Higgs doublet may acquire a mass of intermediate scale in general. Third, the nonminimal initial conditions at the GUT scale modify the phenomenological conclusions made in the literature, in predictions on neutralino cosmic abundance, the radiative breaking scenario, and so on.

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[1]Note that the difference between the GUT scale $\simeq 10^{16}$ GeV and the gravitational scale $\simeq 10^{18}$ GeV is one-seventh that between the GUT scale and the weak scale if measured in log scale.

In this paper, we derive the low-energy effective Lagrangian starting from the grand unified theories with nonuniversal SUSY-breaking terms. In fact, we find new contributions to the SUSY-breaking terms at the low energy, after integrating out the superheavy fields at the GUT scale. It of course reduces to that obtained in Ref. [9] in the case with the universal SUSY-breaking terms. Furthermore, the nonminimality of the SUSY-breaking terms at the GUT scale can lead to many interesting phenomenological consequences. We point out, for instance, that the scalar lepton always becomes heavier than the B-ino due to the radiative correction between Planck and GUT scales, and the upper bound on the slepton mass in the no-scale type models [12,13] becomes invalid. Possible effects on the radiative breaking scenario are also discussed.

The paper is organized as follows. In Sec. II, we first review the low-energy Lagrangian with universal SUSY-breaking terms. Then we point out the importance of studying the case of the nonuniversal SUSY-breaking terms and demonstrate its important outcomes in explicit examples. We give the low-energy effective Lagrangian with the nonuniversal SUSY-breaking terms in Sec. III. Here we also give an ansatz of the SUSY-breaking terms based on the assumption that the hidden sector is hidden; this ansatz is stable under the renormalization effects, and we show that the fine-tuning in SUSY-GUT is not ruined in this ansatz of the SUSY-breaking terms. We point out phenomenological implications of the nonuniversal SUSY-breaking terms in Sec. IV. Section V is devoted to conclusions.

II. WHY NONUNIVERSAL SOFT SUSY-BREAKING TERMS?

In this section, we briefly explain the typical effects of the nonuniversal soft SUSY-breaking terms, to demonstrate the potential importance of their low-energy consequences. First we review the basic results by Hall $et\ al.$ [9], where the low-energy effective Lagrangian is derived in the case with the universal SUSY-breaking terms. We then point out that the universal form of the SUSY-breaking terms is not preserved by the radiative corrections. As examples of the potential importance of the nonuniversal SUSY-breaking terms, we show that the nonuniversal scalar mass of the superheavy fields gives rise to D-term contributions to the scalar masses in the low-energy effective Lagrangian. We also give an example where the nonuniversal soft SUSY-breaking terms destabilizes the hierarchy between the unification and weak scales. These observations give us motivation to study the low-energy consequence of the nonuniversal SUSY-breaking terms in a more general framework.

A. Minimal supergravity

The low-energy effective Lagrangian of the minimal supergravity was shown to be extremely simple in Ref. [9]. Although the discussion in that paper was based on the

supergravity Lagrangian, we rephrase their result in the flat limit, i.e., in the context of the global SUSY Lagrangian with soft SUSY-breaking terms.[2]

The minimal supergravity suggests the following form of the scalar potential in the observable sector:

$$V = V_{\text{SUSY}} + V_{\text{SUSY}} \,, \tag{2.1}$$

$$V_{\text{SUSY}} = -F^\kappa F_\kappa^* - \frac{1}{2} D^\alpha D^\alpha + F^\kappa \frac{\partial W}{\partial z^\kappa} + \text{H.c.}$$
$$+ g_\alpha D^\alpha z_\kappa^* (T^\alpha)_\lambda^\kappa z^\lambda, \tag{2.2}$$

$$V_{\text{SUSY}} = AW + Bz^\kappa \frac{\partial W}{\partial z^\kappa} + \text{H.c.} + |B|^2 z_\kappa^* z^\kappa, \tag{2.3}$$

where W is the superpotential, and z^κ and F^κ are the scalar component and the F component of a chiral multiplet, respectively. D^α is the D component of a gauge multiplet and T^α is a gauge generator. V_{SUSY} stands for the supersymmetric part, while V_{SUSY} contains the SUSY-breaking terms, in which A and B are soft SUSY-breaking parameters.[3] This form of the SUSY-breaking terms is referred to as "universal," because all the scalar masses are equal.

The main result in Ref. [9] is that the following form of the SUSY-breaking terms results in the low-energy Lagrangian after integrating out the superheavy fields in the above Lagrangian:

$$V_{\text{SUSY}} = -2AW_{\text{eff}} + (A + B)z^k \frac{\partial W_{\text{eff}}}{\partial z^k}$$
$$+ \text{H.c.} + |B|^2 z_k^* z^k, \tag{2.4}$$

and it still has the same form as the original one by suitable redefinition of the A and B parameters except for the mass-squared terms. Here z^k are the light scalar fields and W_{eff} is the superpotential W with the extremum values for superheavy fields plugged in. Moreover, the scalar mass terms are still universal with the same mass, B. It is noteworthy that the authors of Ref. [9] did the analysis including the hidden sector fields, and proved that they do not shift when the light fields fluctuate at $O(m_S)$ where m_S is the SUSY-breaking scale ~ 1 TeV, while the constant term in the superpotential should be shifted to cancel the cosmological constant.

The remarkable simplicity of the low-energy Lagrangian led to a number of strong conclusions, like the natural absence of the flavor-changing neutral currents [7] or radiative breaking scenario due to the heavy top quark [14]. Because of these successes, it became like a dogma in the phenomenological analysis of the SUSY

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[2]This treatment can be actually justified from their analysis, that the fields in the hidden sector do not shift as the light fields are varied at $O(m_S)$.

[3]The definition here is related to that in Ref. [9] by $m_g = B^*$ and $m_g' = (A+3B)^*$. It is noteworthy that they also discussed a modification of the minimal supergravity based on $U(n)$ invariance in the kinetic function to ensure the universality of the scalar masses.

models. However, it becomes increasingly apparent that the supergravity Lagrangian may not have the minimal form, and it may lead to important consequences on the low-energy effective Lagrangian, as will be discussed in the next sections.

B. Naturalness of the universal SUSY-breaking terms

It was pointed out that the higher order corrections in general destroy the minimal form of the Kähler potential [15,16]. This poses a question on the naturalness of the minimal supergravity Lagrangian. However, this discussion is based on the one-loop corrections with a naive cut-off set at the Planck scale, and such higher order corrections may be absent in specific dynamics beyond the Planck scale. Note that even when the Planck scale dynamics satisfy certain symmetry to ensure the minimal form of the SUSY-breaking terms, we still expect that it will be modified by the radiative corrections below the Planck scale.

Let us give an example of the minimal SU(5) model with vanishing scalar masses at the Planck scale [17]. The scalar fields acquire their masses through renormalization group equations, which are different for $\bar{5}$ and $\mathbf{10}$ representations. Their masses at the SU(5)-breaking scale $\simeq 2 \times 10^{16}$ GeV are

$$m_{\bar{5}}^2 = 0.30 M^2, \tag{2.5}$$

$$m_{10}^2 = 0.45 M^2, \tag{2.6}$$

where M refers to the SU(5) gaugino mass at the supergravity scale $M_{\mathrm{Pl}}/\sqrt{8\pi}$. Apparently these contributions cannot be neglected in the phenomenological analyses and are also nonuniversal. They become even larger in the nonminimal GUT models because the gauge coupling constant tends to be larger than that in the minimal SU(5) model. This demonstrates that the running between the Planck scale and the SU(5)-breaking scale is not negligible [18] (a similar point was also made recently in Ref. [19]). Then the SUSY-breaking terms do not have a minimal form any more at the GUT scale, where the superheavy fields should be integrated out.

Instead of the universal SUSY-breaking terms, we will take the following ansatz for the SUSY-breaking terms:

$$V_{\mathrm{SUSY}} = m_S A W + m_S B^\kappa(z)\frac{\partial W}{\partial z^\kappa} + \mathrm{H.c.} + O(m_S^2). \tag{2.7}$$

(See Sec. III A 2 for the notation.[4]) It will be shown that this form of the SUSY-breaking terms is stable under renormalization, or in other words, natural in the weak sense. Since the minimal form is not stable under the renormalization, we believe this is the framework to work out the low-energy effective Lagrangian. More-

over, from the viewpoint of supergravity, this form is the most general form of the SUSY-breaking terms induced by super Higgs mechanism with the assumption that the hidden sector is "hidden," i.e., that the superpotential is a sum of two independent pieces consisting of observable and hidden fields, respectively. We leave the detail to Sec. III A 2.

Recall, also, that the superstring theory suggests nonminimal forms of the SUSY-breaking terms in general [20]. This is especially true when the SUSY is broken by the F component of the moduli fields. The scalar masses depend on their modular weights. The gaugino masses are also nonuniversal.

These observations give us strong motivation to study the low-energy effective Lagrangian from the unified theories with nonuniversal SUSY-breaking terms. In the next two sections, we will point out potentially important effects of the nonuniversality.

One remark on the squark degeneracy is in order. It has been often stated that one needs high degeneracy of the scalar masses to ensure the potentially large contribution of the squark loop diagrams to the K^0-$\overline{K}^0$ mixing. However, this does not require the degeneracy of all scalar masses at the GUT or Planck scale. The only requirement is the degeneracy of the first- and second-generation squarks with the same quantum numbers. Note that neither the D-term contributions nor the renormalization group evolution due to the gauge interactions destroy the degeneracy as long as they do not distinguish the generations of the light quarks. One may have highly degenerate squark masses due to the gluino mass contribution even with the different masses as the initial conditions at the Planck scale. Though the absence of flavor-changing neutral current processes puts a strong constraint on the nonminimality, it does not diminish our interest to study its low-energy consequences.

C. Possible D-term contributions

Though the low-energy Lagrangian is surprisingly simple when it has a universal form at the unification scale, there arise different contributions to V_{SUSY} when one integrates out the superheavy fields from the Lagrangian with a nonuniversal form of the SUSY-breaking terms. This has an important consequence on the scalar masses when one probes the symmetry-breaking pattern from the weak-scale measurements of the masses [3,4] (see Sec. IV A). In fact, we pointed out that the D-term contribution may arise at the GUT scale [3], and there have appeared papers which discuss the effect of the D-term contributions to the Higgs boson masses in the radiative breaking scenario [21,22].

In this section, we give a simple example which generates a D-term contribution to the low-energy scalar masses. Take a simple superpotential $W = h(\phi_1\phi_2 - \mu^2)\chi$, with U(1) charges $Q = +1$ for ϕ_1, $Q = -1$ for ϕ_2, and $Q = 0$ for χ. We also have light fields z^k which couple to the U(1) gauge fields but do not couple to the heavy fields ϕ_1, ϕ_2, and χ in the superpotential. Then the full potential reads as

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[4]B^κ are holomorphic functions of z^λ independent for each κ, and *not* the derivative of a single function B.

$$V = -|F_1|^2 - |F_2|^2 - |F_\chi|^2 - |F^k|^2 - \frac{1}{2}D^2 + [F_1 h\phi_2\chi + F_2 h\phi_1\chi + F_\chi h(\phi_1\phi_2 - \mu^2) + \text{H.c.}]$$

$$+ gD\left(|\phi_1|^2 - |\phi_2|^2 + \sum_k Q_k|z^k|^2\right) + (Ah\phi_1\phi_2\chi - Ch\mu^2\chi + \text{H.c.})$$

$$+ m_1^2|\phi_1|^2 + m_2^2|\phi_2|^2 + m_\chi^2|\chi|^2 + m_k^2|z^k|^2, \tag{2.8}$$

where F_1, F_2, F_χ, F^k are the auxiliary fields of ϕ_1, ϕ_2, χ, z^k, respectively, and D is the auxiliary field of the U(1) gauge field.[5] The SUSY-breaking terms A, C, m_κ^2 are $O(m_S)$, $O(m_S)$, $O(m_S^2)$, respectively.

We integrate out the heavy fields ϕ_1, ϕ_2, and χ from this potential, taking light fields z^k fluctuating at $O(m_S)$. The stationary solutions to the heavy fields are solved by expanding in powers of m_S/μ, and one finds

$$\phi_1 = \mu + \left[\frac{-A^2 + C^2}{8h^2\mu} - \frac{m_1^2 + m_2^2}{4h^2\mu} - \frac{m_1^2 - m_2^2}{8g^2\mu} - \frac{\sum_k Q_k|z^k|^2}{4g\mu}\right] + O\left(\frac{m_S^3}{\mu^2}\right), \tag{2.9}$$

$$\phi_2 = \mu + \left[\frac{-A^2 + C^2}{8h^2\mu} - \frac{m_1^2 + m_2^2}{4h^2\mu} + \frac{m_1^2 - m_2^2}{8g^2\mu} + \frac{\sum_k Q_k|z^k|^2}{4g\mu}\right] + O\left(\frac{m_S^3}{\mu^2}\right), \tag{2.10}$$

$$\chi = -\frac{A - C}{2h} + O\left(\frac{m_S^3}{\mu^2}\right), \tag{2.11}$$

and, for the auxiliary fields,

$$F_1 = h\phi_2\chi = -\frac{(A - C)}{2}\mu + O\left(\frac{m_S^3}{\mu}\right), \tag{2.12}$$

$$F_2 = h\phi_1\chi = -\frac{(A - C)}{2}\mu + O\left(\frac{m_S^3}{\mu}\right), \tag{2.13}$$

$$F_\chi = h(\phi_1\phi_2 - \mu^2)$$
$$= \frac{-A^2 + C^2}{4h} - \frac{m_1^2 + m_2^2}{2h} + O\left(\frac{m_S^3}{\mu}\right), \tag{2.14}$$

$$D = g\left(|\phi_1|^2 - |\phi_2|^2 + \sum_k Q_k|z^k|^2\right)$$
$$= -\frac{m_1^2 - m_2^2}{2g} + O\left(\frac{m_S^3}{\mu}\right). \tag{2.15}$$

The low-energy effective Lagrangian can be obtained by plugging these solutions into the original Lagrangian, giving

$$V_{\text{eff}} = [z^k\text{-independent terms of } O(m_S^2\mu^2)]$$
$$+ m_k^2|z^k|^2 - \frac{1}{2}(m_1^2 - m_2^2)\sum_i Q_k|z^k|^2$$
$$+ [\text{terms of } O(m_S^5/\mu)]. \tag{2.16}$$

The latter terms are the contributions from the nonvanishing value of the D term, giving rise to different masses for different quantum numbers. One also sees that the D-term contributions vanish if the scalar masses are universal, consistent with the analysis in Ref. [9] that was reviewed in the previous section.

The D-term contribution exists in general when the rank of the gauge group is reduced by the symmetry breaking [11], and the SUSY-breaking terms are nonuniversal. They give rise to observable effects at the weak scale.

D. Instability of the hierarchy

The nonuniversal SUSY-breaking terms have a dramatic consequence when there is fine-tuning to keep light fields at the weak scale. The light fields acquire masses of the order of the intermediate scale in general. It seems to us that this problem is not widely recognized in the literature (see, however, [23,24]).

In the SUSY standard model, we need (at least) two Higgs doublets with opposite hypercharges, and these doublet scalars form a SUSY-breaking mixing mass term. Indeed, the problem we now discuss is related to the fine-tuning problem of the Higgs doublet mass (the gauge hierarchy problem), which is inevitable in a wide class of SUSY-GUT models in order to obtain the light Higgs doublets of the SUSY standard model. We will explain the problem in some detail. The discussions given below are based on the observation in an unpublished work [25].

To exemplify the problem, let us consider the minimal SUSY SU(5) model [1] whose superpotential is

$$W = \lambda \text{tr}\Sigma^3 + M_\Sigma \text{tr}\Sigma^2 + H_u(f\Sigma + M_H)H_d. \tag{2.17}$$

Here Σ, H_u, and H_d are the fields of $\mathbf{24}$, $\mathbf{5}$, and $\bar{\mathbf{5}}$ representations of SU(5), respectively. λ and f are dimensionless coupling constants, while M_Σ, M_H are GUT scale mass parameters. If we add the SUSY-breaking terms arbitrarily, the potential reads

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[5]One can also study the same potential after integrating out the auxiliary fields, as one usually does. However, we keep the auxiliary fields as independent variables for the later convenience.

$$V = \left|\frac{\partial W}{\partial \Sigma}\right|^2 + \left|\frac{\partial W}{\partial H_u}\right|^2 + \left|\frac{\partial W}{\partial H_d}\right|^2 + g^2 \sum_\alpha \{\Sigma^\dagger[T^\alpha,\Sigma] + H_u^\dagger T^\alpha H_u + H_d^\dagger(-T^{\alpha*})H_d\}^2$$

$$+\{\lambda A_\Sigma \mathrm{tr}\Sigma^3 + B_\Sigma M_\Sigma \mathrm{tr}\Sigma^2 + f A_H H_u \Sigma H_d + B_H M_H H_u H_d + \mathrm{H.c.}\}, \tag{2.18}$$

where A_Σ, B_Σ, A_H, and B_H are the SUSY-breaking parameters of order m_S and we have omitted the SUSY-breaking scalar masses which are irrelevant to the following discussions. The SU(5) gauge coupling constant is denoted by g. Taking

$$\Sigma = \frac{1}{\sqrt{60}} \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} \sigma, \tag{2.19}$$

the minimal of the potential is located at $\sigma_0 = \frac{2\sqrt{60}}{3\lambda} M_\Sigma$ in the SUSY limit, which is shifted by $\delta\sigma = \frac{\sqrt{60}}{3\lambda}(A_\Sigma - B_\Sigma)$ in the presence of the SUSY-breaking terms. The mixing mass of the two doublet Higgs bosons $m_{12}^2 H_u H_d$ is given by

$$m_{12}^2 = \frac{3f}{\sqrt{60}}\sigma_0(A_\Sigma - B_\Sigma - A_H + B_H) + O(m_S^2), \tag{2.20}$$

where we have used that the supersymmetric mass of the Higgs doublets is fine-tuned to be

$$M_H - \frac{3f}{\sqrt{60}}\sigma_0 = O(m_S) \tag{2.21}$$

at the SUSY limit. Clearly for a class of the SUSY-breaking parameters where the combination $A_\Sigma - B_\Sigma - A_H + B_H$ does not vanish, m_{12}^2 lies at an intermediate scale $\sim m_S M_X$ and the gauge hierarchy is destabilized.

A crucial observation is, however, that the mixing mass m_{12}^2 becomes of order m_S^2 if we assume the form of the soft terms as in Eq. (2.7). Indeed for the special form of the SUSY-breaking terms

$$AW + B_1 \Sigma \frac{\partial W}{\partial \Sigma} + B_2 H_u \frac{\partial W}{\partial H_u} + B_3 H_d \frac{\partial W}{\partial H_d}, \tag{2.22}$$

which follows from the ansatz in Sec. II B, the coefficients A_Σ, etc., in Eq. (2.18) are written as

$$A_\Sigma = A + 3B_1, \tag{2.23}$$
$$B_\Sigma = A + 2B_1, \tag{2.24}$$
$$A_H = A + B_1 + B_2 + B_3, \tag{2.25}$$
$$B_H = A + B_2 + B_3. \tag{2.26}$$

Then we find

$$A_\Sigma - B_\Sigma - A_H + B_H = 0, \tag{2.27}$$

which guarantees the lightness of the Higgs doublets in the minimal SU(5) model.

III. SCALAR POTENTIAL IN THE EFFECTIVE THEORY

In the preceding section, we examined some examples to demonstrate that the nonuniversal SUSY-breaking terms can lead to important consequences. In this section, we will give a more general discussion. We consider a softly broken supersymmetric unified theory[6] whose gauge group is broken at an energy scale M_X. We will derive the scalar potential V_{eff} in the low-energy effective theory by integrating out the heavy sector.

A. General discussion

We first list the basic assumptions in the following discussion: (1) The unified theory is described as a renormalizable supersymmetric theory with soft SUSY-breaking terms; (2) the unified gauge symmetry is broken at a scale M_X which is much higher than the SUSY-breaking scale m_S (~ 1 TeV); (3) SUSY is not spontaneously broken in the absence of the soft SUSY-breaking terms in the Lagrangian; (4) all the particles can be classified as heavy [with mass $O(M_X)$] or light [with mass $O(m_S)$] in the absence of the SUSY-breaking terms; (5) the light scalar fields have vacuum expectation values (VEV's) as well as fluctuations only of $O(m_S)$.

The assumptions here are basically the same as in Ref. [9].

We would like to integrate out the heavy particle at the tree level. For this purpose, we solve the stationary conditions of the potential of the full theory for the heavy scalar fields while keeping the light scalar fields arbitrary. We then "integrate out" the heavy fields by inserting the solutions of the stationary conditions into the potential. The potential obtained in this way should be regarded as the potential of the low-energy effective theory renormalized at the scale M_X. This potential serves a matching condition when we solve one-loop renormalization group equations above and below the scale M_X. In deriving the effective scalar potential, we fully utilize the equalities from the gauge invariance given in the Appendix.

The procedure to obtain the low-energy effective potential in this paper is quite similar to that of Ref. [9], though they started from the (minimal) supergravity Lagrangian whereas our starting point is the softly broken global SUSY. As for the SUSY breaking, we consider a

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[6]The gauge group of the theory is not necessarily grand-unified into a simple group.

nonuniversal form which is expected to be realized in realistic models as we discussed in the preceding section.

1. SUSY limit

First we review the basic properties of the scalar potential in the absence of the SUSY-breaking terms.

The superpotential is a holomorphic function of chiral superfields Φ:

$$W = W(\Phi^\kappa). \tag{3.1}$$

The supersymmetric scalar potential is given as

$$V_{\text{SUSY}} = -\frac{1}{2}(D^\alpha)^2 - F_\kappa^* F^\kappa + D^\alpha(z^\dagger T^\alpha z)$$
$$+ F_\kappa^* \frac{\partial W^*}{\partial z_\kappa^*} + \frac{\partial W}{\partial z^\kappa} F^\kappa, \tag{3.2}$$

where z^κ are the scalar components of the chiral superfields Φ^κ. F^κ is the auxiliary field of Φ^κ, T^α stands for a gauge generator and D^α is its D term. $z^\dagger T^\alpha z$ is an abbreviation of $z_\lambda^*(T^\alpha)_\kappa^\lambda z^\kappa$. Summation over α, κ, and λ is implied.

The stationary conditions of the potential are simply

$$F_\kappa^* = \frac{\partial W}{\partial z^\kappa} = 0, \tag{3.3}$$

$$D^\alpha = z^\dagger T^\alpha z = 0 \tag{3.4}$$

under the condition that SUSY is unbroken. We denote the solutions to the stationary conditions as $z^\kappa = z_0^\kappa$.

We can always take a basis of z^κ to diagonalize a part of the fermion mass matrix coming from the superpotential:

$$\mu_{\kappa\lambda} = \left.\frac{\partial^2 W}{\partial z^\kappa \partial z^\lambda}\right|_{z=z_0}. \tag{3.5}$$

Then the scalar fields are classified [9] either as "heavy" fields $z^K, z^L, \ldots$, "light" fields $z^k, z^l, \ldots$, or Nambu-Goldstone fields which will be discussed just below.

The mass matrix of the gauge bosons is

$$(M_V^2)_{\alpha\beta} = z_{0\lambda}^*\{T^\alpha, T^\beta\}_\kappa^\lambda z_0^\kappa = 2(z_0^\dagger T^\alpha)_\kappa (T^\beta z_0)^\kappa, \tag{3.6}$$

where we used Eq. (3.4) in the last equality, and it can be diagonalized so that the gauge generators are classified into "heavy" (those broken at M_X) $T^A, T^B, \ldots$ and "light" (which remain unbroken above m_S) $T^a, T^b, \ldots$. For the heavy generators, the fields $(T^A z_0)^\kappa$ correspond to the direction of the Nambu-Goldstone fields in the field space, which span a vector space with the same dimension as the number of heavy generators. We can take a basis of the Nambu-Goldstone multiplets, $z^A, z^B, \ldots$ so that

$$\sqrt{2}(T^A z_0)^B = M_V^{AB}. \tag{3.7}$$

Here the Nambu-Goldstone fields are taken to be orthogonal to the heavy and light fields such as $(T^A z_0)^K = 0$, $(T^A z_0)^k = 0$. Also, the vanishing of the D terms Eq. (3.4) implies

$$z_0^A = 0. \tag{3.8}$$

2. Soft SUSY-breaking terms

The soft SUSY-breaking terms can be classified by spurion insertions as[7]

$$\int d^2\theta(m_S\theta^2)U(\Phi) = m_S U(z), \tag{3.9}$$

$$\int d^4\theta(m_S\theta^2)B(\Phi, \Phi^\dagger) = m_S F_\lambda^* \frac{\partial B}{\partial z_\lambda^*}, \tag{3.10}$$

$$\int d^4\theta(m_S^2\theta^2\bar\theta^2)C(\Phi, \Phi^\dagger) = m_S^2 C(z, z^*). \tag{3.11}$$

$U(\Phi)$ is a holomorphic function of the chiral superfields. B is expressed as

$$B(\Phi, \Phi^\dagger) = B_{2\lambda}^\kappa \Phi_\kappa^\dagger \Phi^\lambda \tag{3.12}$$

for renormalizable theories and thus $\partial B/\partial z_\lambda^*$ is a function of z, not of z^*. A term which depends only on $\Phi^\dagger$ does not appear in Eq. (3.12), since it can be absorbed into the superpotential as [26]

$$\int d^4\theta(m_S\theta^2)H(\Phi^\dagger) = \int d^2\bar\theta m_S H(\Phi^\dagger). \tag{3.13}$$

$C(\Phi, \Phi^\dagger)$ is a bilinear polynomial of the chiral and antichiral fields.

The nonrenormalization theorem [27] implies that the form $U(z)$ is preserved from radiative corrections since Eq. (3.9) is an F term. On the other hand, the functions B and C are generally renormalized, as they are the D terms. For example, the minimal supergravity induces the soft terms such as $U = AW(\Phi)$, $B = \Phi^\dagger\Phi$, and $C = 0$ in the flat limit with A being a constant. When we take radiative corrections into account, U remains the same, but B and C suffer from the renormalization and in general become nonuniversal. This observation leads us to investigate nonuniversal soft terms.

In the rest of this paper, we take the ansatz

$$U(\Phi) = AW(\Phi) \tag{3.14}$$

as in the case of the minimal supergravity, while we admit nonuniversal structure for B and C. We can show that Eq. (3.14) is derived from nonminimal supergravity Lagrangian in the flat limit, provided that the observed sector does not have couplings in the superpotential to the sector which is responsible for the spontaneous breaking of the local supersymmetry (the "hidden" sector) [28,29].[8] On phenomenological grounds this "hidden" assumption is widely accepted, since otherwise the

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[7]We owe this classification to the discussions with K. Inoue, Y. Okada, and T. Yanagida.

[8]Our usage of the term "hidden sector" is somewhat different from that in the second paper of Ref. [28], where the moduli fields whose F components break SUSY are not included in the hidden sector.

large SUSY-breaking would directly be transmitted to the observed sector and the SUSY is badly broken in the low-energy effective theory as a consequence. For example, the Yukawa coupling of the "hidden" sector field to the Higgs doublets induces a large mass term of the Higgs bosons of order $m_S M_{\text{Pl}}$ and destabilizes the weak scale. Furthermore we believe that the assumption allows us to integrate the hidden sector first to leave the softly broken global SUSY theory.[9]

In the preceding section, we demonstrated that the fine-tuning of the Higgs doublet masses in the SUSY-GUT is not preserved if the most general soft terms are switched on, and there arises a large mixing mass term for the scalars. As will be seen later, our ansatz (3.14) avoids the emergence of the dangerous terms from the heavy sector. On the other hand, if we take $U \neq AW$ the gauge hierarchy achieved by the fine-tuning will be generally violated by the soft terms. We will discuss this in Sec. III C.

The scalar potential we consider is summarized as

$$V = V_{\text{SUSY}} + V_{\overline{\text{SUSY}}} \tag{3.15}$$
$$V_{\overline{\text{SUSY}}} = \{m_S A W(z) + m_S F^\kappa B_\kappa^*(z^*) + \text{H.c.}\}$$
$$+ m_S^2 C(z, z^*). \tag{3.16}$$

Note that the scalar potential (3.15) is rewritten as

$$V = -F_\kappa^* F^\kappa + \left\{ F^\kappa \frac{\partial W}{\partial z^\kappa} + \text{H.c.} \right\}$$
$$-\frac{1}{2} D^\alpha D^\alpha + D^\alpha (z^\dagger T^\alpha z)$$
$$+ \left\{ m_S A W(z) + m_S B^\kappa(z) \frac{\partial W}{\partial z^\kappa} + \text{H.c.} \right\}$$
$$+ m_S^2 B_\kappa^*(z^*) B^\kappa(z) + m_S^2 C(z, z^*) \tag{3.17}$$

by shifting the auxiliary field F^κ as $F^\kappa \to F^\kappa + m_S B^\kappa(z)$. This is the form given in the preceding section.

B. Calculation of the effective potential

In this section we compute the scalar potential of the effective low-energy theory by substituting the heavy fields with the solutions to the stationary conditions of the full potential. With this aim, it is convenient to write both the superpotential and the soft SUSY-breaking terms in terms of the variations $\Delta z^\kappa = z^\kappa - z_0^\kappa$ in

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place of the scalar fields z^κ themselves. For the renormalizable theories, the superpotential can always be written as

$$W = \frac{1}{2!} \mu_{\kappa\lambda} \Delta z^\kappa \Delta z^\lambda + \frac{1}{3!} h_{\kappa\lambda\mu} \Delta z^\kappa \Delta z^\lambda \Delta z^\mu. \tag{3.18}$$

The functions $B^\kappa(z)$, $B_\kappa^*(z^*)$, and $C(z, z^*)$ are also expanded as

$$B^\kappa(z) = B_1^\kappa + B_{2\lambda}^\kappa \Delta z^\lambda, \tag{3.19}$$
$$B_\kappa^*(z^*) = B_{1\kappa}^* + B_{2\kappa}^{*\lambda} \Delta z_\lambda^*, \tag{3.20}$$
$$C(z, z^*) = C_{1\kappa} \Delta z^\kappa + C_1^\kappa \Delta z_\kappa^* + C_{2\lambda}^\kappa \Delta z_\kappa^* \Delta z^\lambda \tag{3.21}$$
$$+ \frac{1}{2} C_{2\kappa\lambda} \Delta z^\kappa \Delta z^\lambda + \frac{1}{2} C_2^{\kappa\lambda} \Delta z_\kappa^* \Delta z_\lambda^*. \tag{3.22}$$

From Eq. (3.12), it follows that $B_1^\kappa = B_{2\lambda}^\kappa z_0^\lambda$.

The variations of the potential (3.15) with respect to the auxiliary fields F, D, and the scalar fields z are given as

$$\frac{\partial V}{\partial F^\kappa} = -F_\kappa^* + \frac{\partial W}{\partial z^\kappa} + m_S B_\kappa^*(z^*)$$
$$= -F_\kappa^* + \mu_{\kappa\lambda} \Delta z^\lambda + \frac{1}{2} h_{\kappa\lambda\mu} \Delta z^\lambda \Delta z^\mu$$
$$+ m_S (B_{1\kappa}^* + B_{2\kappa}^{*\lambda} \Delta z_\lambda^*), \tag{3.23}$$
$$\frac{\partial V}{\partial D^\alpha} = -D^\alpha + z^\dagger T^\alpha z, \tag{3.24}$$
$$\frac{\partial V}{\partial z^\kappa} = D^\alpha (z^\dagger T^\alpha)_\kappa + \frac{\partial^2 W}{\partial z^\kappa \partial z^\lambda} F^\lambda$$
$$+ m_S A \frac{\partial W}{\partial z^\kappa} + m_S \frac{\partial B^\lambda}{\partial z^\kappa} F_\lambda^* + m_S^2 \frac{\partial C}{\partial z^\kappa}$$
$$= D^\alpha (z^\dagger T^\alpha)_\kappa + (\mu_{\kappa\lambda} + h_{\kappa\lambda\mu} \Delta z^\mu) F^\lambda$$
$$+ m_S A (\mu_{\kappa\lambda} \Delta z^\lambda + h_{\kappa\lambda\mu} \Delta z^\lambda \Delta z^\mu)$$
$$+ m_S B_{2\kappa}^\lambda F_\lambda^* + m_S^2 (C_{1\kappa} + C_{2\kappa}^\lambda \Delta z_\lambda^* + C_{2\kappa\lambda} \Delta z^\lambda). \tag{3.25}$$

Once the SUSY-breaking terms are turned on, the F^κ or D^α may be nonvanishing, but should be at most $O(m_S M_X)$ since they have to vanish in the absence of the SUSY-breaking terms. We expand the F^κ, D^α, and z^κ in powers of m_S such as

$$F^\kappa = F_0^\kappa + \delta F^\kappa + \delta^2 F^\kappa + \cdots, \tag{3.26}$$
$$D^\alpha = D_0^\alpha + \delta D^\alpha + \delta^2 D^\alpha + \cdots, \tag{3.27}$$
$$z^\kappa = z_0^\kappa + \Delta z = z_0^\kappa + \delta z^\kappa + \delta^2 z^\kappa + \cdots, \tag{3.28}$$

with $\delta^n F^\kappa, \delta^n D^\alpha = O(m_S^n / M_X^{n-2})$, and $\delta^n z^\kappa = O(m_S^n / M_X^{n-1})$. Here F_0^κ and D_0^α are defined as the vacuum expectation values (VEV's) in the absence of the SUSY-breaking terms and are exactly zero as discussed in Sec. II A 1, and the higher order terms are defined as the shifts of their VEV's due to the presence of the SUSY-breaking terms. We assume $z^k = O(m_S)$ for the light fields, e.g., $z_0^k = O(m_S)$ and $\delta^2 z^k = \delta^3 z^k = \cdots = 0$.

We can solve the stationary conditions (3.23)–(3.25) by using the above expansions (3.26)–(3.28) order by order. Here we list the equations which are to be used to obtain the scalar potential of the effective theory. For simplicity

we assume

$$B_1^k = O(m_S) \tag{3.29}$$

for the time being. This is automatically satisfied if there is no light singlet field. The stationary conditions $\partial V/\partial F^K = 0$ and $\partial V/\partial F^A = 0$ imply

$$\delta F_K^* = \mu_{KL}\delta z^L + m_S B_{1K}^* \tag{3.30}$$

and

$$\delta F_A^* = m_S B_{1A}^*, \tag{3.31}$$

$$\delta^2 F_A^* = \frac{1}{2}h_{A\lambda\mu}\delta z^\lambda \delta z^\mu + m_S B_{2A}^\lambda \delta z_\lambda^*$$

$$= \frac{1}{2}h_{AKL}\delta z^K \delta z^L + h_{AKk}\delta z^K \delta z^k + m_S B_{2A}^\lambda \delta z_\lambda^*, \tag{3.32}$$

respectively,[10] while $\partial V/\partial D^A = 0$ gives

$$\delta D^A = (z_0^\dagger T^A)_B \delta z^B + \delta z_B^*(T^A z_0)^B. \tag{3.33}$$

From the conditions $\partial V/\partial z^K = 0$ and $\partial V/\partial z^A = 0$, we find

$$-\mu_{KL}\delta F^L = 0, \tag{3.34}$$

$$-\mu_{KL}\delta^2 F^L = h_{K\lambda\mu}\delta F^\lambda \delta z^\mu + \delta D^A(\delta z^\dagger T^A)_K$$

$$+ m_S A\mu_{KL}\delta z^L + m_S B_{2K}^\lambda \delta F_\lambda^*$$

$$+ m_S^2 C_{1K}, \tag{3.35}$$

and

$$\delta D^B(z_0^\dagger T^B)_A = 0, \tag{3.36}$$

$$-\delta^2 D^B(z_0^\dagger T^B)_A = h_{A\lambda\mu}\delta F^\lambda \delta z^\mu + m_S B_{2A}^\lambda \delta F_\lambda^*$$

$$+ m_S^2 C_{1A}, \tag{3.37}$$

respectively.

From Eqs. (3.34) and (3.30), we find the shift of z^K is

$$\delta z^K = -m_S(\mu^{-1})^{KL}B_{1L}^*. \tag{3.38}$$

On the other hand, Eqs. (3.33) and (3.36) imply

$$\delta z^A = 0. \tag{3.39}$$

Equation (3.35) gives the solution for $\delta^2 F^K$ as

$$\delta^2 F^K = \langle F^K \rangle - m_S(\mu^{-1})^{KL}h_{LAl}B_1^A \delta z^l, \tag{3.40}$$

$$\langle F^K \rangle = -(\mu^{-1})^{KL}\{m_S h_{LAM}B_1^A \delta z^M + m_S A\mu_{LM}\delta z^M$$

$$+ m_S^2 B_{2L}^A B_{1A}^* + m_S^2 C_{1L}\}. \tag{3.41}$$

From Eq. (3.37),

$$-\delta^2 D^B(z_0^\dagger T^B)_A = m_S h_{ABK}B_1^B \delta z^K + m_S^2 B_{2A}^B B_{1B}^*$$

$$+ m_S^2 C_{1A}, \tag{3.42}$$

where we have used $h_{ABk} = O(m_S/M_X)$, a consequence of the gauge invariance. Note that $(z_0^\dagger T^B)_A = \frac{1}{\sqrt{2}}(M_V^*)_{BA}$ can be inverted to obtain $\delta^2 D^B$. Equation (3.42) shows that $\delta^2 D^A$ is a constant independent of the light fields. Therefore we will denote it by $\langle D^A \rangle$.

Now it is straightforward to calculate the scalar potential of the low-energy effective theory V_{eff} by substituting the solutions to the stationary conditions for the heavy fields. The result can be compactly expressed if we define

$$\widetilde{W}(z) = W(z^k, z^K = z_0^K + \delta z^K, z^A = z_0^A + \delta z^A), \tag{3.43}$$

$$\widetilde{B}^k(z) = B^k(z^l, z^K = z_0^K + \delta z^K, z^A = z_0^A + \delta z^A), \tag{3.44}$$

$$\widetilde{C}(z, z^*) = C(z^k, z^K = z_0^K + \delta z^K, z^A = z_0^A + \delta z^A, \ldots). \tag{3.45}$$

Note that the above are the functions of only light fields. In particular, the $\widetilde{W}$ is the superpotential of the effective theory. Then we can write down the effective potential as

$$V_{\text{eff}} = -F_k^* F^k + F^k \left(\frac{\partial \widetilde{W}}{\partial z^k} + m_S \widetilde{B}_k^*\right) + \text{H.c.}$$

$$- \frac{1}{2}D^a D^a + D^a(z^\dagger T^a z)$$

$$+ m_S A\widetilde{W}(z) + \text{H.c.} + m_S^2 \widetilde{C}(z, z^*)$$

$$+ \Delta V, \tag{3.46}$$

where the new contribution ΔV is

$$\Delta V = -\left|\langle F^K \rangle - m_S(\mu^{-1})^{KL}h_{LAk}B_1^A \delta z^k\right|^2$$

$$+ \left\{\langle F^K \rangle \left(\frac{1}{2}h_{K\lambda\mu}\delta z^\lambda \delta z^\mu + m_S B_{2K}^\lambda \delta z_\lambda^*\right) - m_S(\mu^{-1})^{KL}h_{LAk}B_1^A \delta z^k \left(\frac{1}{2}h_{K\lambda\mu}\delta z^\lambda \delta z^\mu + m_S B_{2K}^\lambda \delta z_\lambda^*\right) + \text{H.c.}\right\}$$

$$+ \left|m_S B_{1A}^* + m_S B_{2A}^{*K}\delta z_K^* + \frac{1}{2}h_{AKL}\delta z^K \delta z^L + m_S B_{2A}^{*k}\delta z_k^* + h_{AKl}\delta z^K \delta z^l\right|^2 + \langle D^A \rangle \delta z^\dagger T^A \delta z. \tag{3.47}$$

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[10] Gauge invariance implies $h_{Akl} = O(m_S/M_X)$; see Appendix. We have also used $\delta z^A = 0$, which will be derived below.

Recall that $\langle D^A \rangle$ stands for $\delta^2 D^A$ [see Eq. (3.42)]. The last term in Eq. (3.47) comes from the D term of the heavy gauge sector and is referred to as the D-term contribution, while the other contributions are called the F-term contributions. This new contribution ΔV should be included in any analysis of one-loop renormalization group equations with tree-level matching.

Eliminating the auxiliary fields F^k and D^a by using the equations of motion, we obtain the SUSY-breaking part of the effective potential

$$V_{\text{eff,SUSY}} = m_S A \widetilde{W}(z) + m_S \widetilde{B}^k(z)\frac{\partial \widetilde{W}}{\partial z^k} + \text{H.c.}$$

$$+ m_S^2 \{\widetilde{B}_k^*(z^*)\widetilde{B}^k(z) + \widetilde{C}(z,z^*)\} + \Delta V.$$

$$(3.48)$$

C. Stability of the weak scale

In this section, we will investigate whether the effective potential is of the order of magnitude m_S^4.[11] The occurrence of the terms of $O(m_S^3 M_X)$ or even larger is very dangerous since it would destabilize the weak scale. Such dangerous terms may appear for a mixing mass term (proportional to $\delta z^k \delta z^l$) and for a linear term (proportional to δz^k). The latter is related to the notorious difficulty in the presence of the light singlet [30]. Note that the linear term cannot exist for a nonsinglet field. Whether the linear term is actually large or not is highly model dependent. We will not discuss this problem further. In the rest of this section we will concentrate on the mixing mass term.

From Eqs. (3.48) and (3.47), it follows that the mixing mass terms are

$$m_S A\mu_{kl} + m_S h_{klm}(B_1^m + B_{2M}^m \delta z^M) + m_S(B_{2k}^m \mu_{ml} + B_{2l}^m \mu_{mk}) + m_S^2 C_{2kl} + \langle F^K \rangle h_{Kkl}$$
$$- m_S(\mu^{-1})^{KL} h_{KAk} B_1^A h_{LMl}\delta z^M - m_S(\mu^{-1})^{KL} h_{KAl} B_1^A h_{LMk}\delta z^M$$
$$+ m_S h_{KAk}\delta z^K B_{2l}^{*A} + m_S h_{KAl}\delta z^K B_{2k}^{*A},$$

$$(3.49)$$

which are of order m_S^2. This is due to our assumptions (1) $U = AW$ and (2) $B_1^k = O(m_S)$. We will show that larger terms arise when we relax the assumptions.

First consider the case of $U \neq AW$. An inspection similar to the preceding section shows that there exist terms of order $m_S M_X$ for the mixing mass

$$m_S \left\langle \frac{\partial^2 U}{\partial z^k \partial z^l} \right\rangle + \langle F^K \rangle h_{Kkl} + O(m_S^2), \qquad (3.50)$$

where

$$\langle F^K \rangle = -m_S(\mu^{-1})^{KL} \left\langle \frac{\partial U}{\partial z^L} \right\rangle + O(m_S^2). \qquad (3.51)$$

The first term in Eq. (3.50) can be $O(m_S M_X)$, since there is no *a priori* reason that the fine-tuning of the Higgs boson mass achieves simultaneously both in W and U. The second term remains large if the conditions $\partial W/\partial z^K = 0$ and $\partial U/\partial z^K = 0$ do not hold simultaneously. Our ansatz $U = AW$ automatically makes each term of Eq. (3.50) small. We may fine-tune the soft term U so that two terms of (3.50) cancel each other. Since U is an F term and not renormalized, this fine-tuning will not be violated by radiative corrections. However, such a fine-tuning can be done only in a model-independent manner, and is clearly beyond our context of general analysis of low-energy effective Lagrangian. We take our ansatz, which guarantees the hierarchy irrespective of the details of each model, to discuss the general consequence of the nonminimality. In short, our ansatz is minimal, which is stable against radiative corrections, but also maximal, which guarantees the stability of the hierarchy automat-

ically.

Next consider the effects of B_1^m of order M_X. Obviously B_1^m is nonzero only for a (light) singlet. Then we find an additional contribution to the mixing mass term:

$$m_S B_1^m h_{klm}. \qquad (3.52)$$

This again reflects the well-known difficulty related to the light singlet. Indeed a large B term can appear at the tree level or may be induced by radiative corrections [30,31]. To proceed further we need a model-dependent analysis which is beyond the scope of the paper.[12]

To conclude, the large mixing mass terms do not arise if (1) one takes the hidden assumption and thus $U = AW$, and (2) there is no light singlet.

D. Mass terms

We now discuss a *chirality-conserving* mass term, namely, the coefficient of $\delta z^k \delta z_l^*$. They are easily extracted from Eqs. (3.46) and (3.47), given by

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[11]Here we disregard the constant terms independent of the light fields.

[12]For example, in a flipped SU(5) model [32], it is known that the light singlet which couples to the Higgs doublets does not induce the large mixing mass.

$$m_S^2(B_{2k}^m B_{2m}^{*l} + C_{2k}^l) - m_S^2(\mu^{-1})^{MK} h_{KAk} B_1^A (\mu^{*-1})_{ML} h^{*LBl} B_{1B}^*$$
$$-m_S^2(\mu^{-1})^{KL} h_{KAk} B_1^A B_{2L}^l - m_S^2(\mu^{*-1})_{KL} h^{*LAl} B_{1B}^* B_{2k}^{*K}$$
$$+m_S^2 B_{2k}^A B_{2A}^{*l} + h_{AKk} h^{*ALl} \delta z^K \delta z_L^* + \langle D^A \rangle (T^A)_k^l. \tag{3.53}$$

The term $m_S^2(B_{2k}^m B_{2m}^{*l} + C_{2k}^l)$ is present before the heavy sector is integrated out. Therefore it respects the large gauge symmetry of the unified group. On the other hand, other terms coming from ΔV can pick up effects of the symmetry breaking.

The last term in Eq. (3.53) is the D-term contribution. Phenomenologically it is important because it gives an additional contribution to squarks and sleptons [3,4]. The $\langle D \rangle$ may not vanish only for broken generators in the absence of the Fayet-Iliopoulos D term. Furthermore, they must be singlet under the unbroken gauge group at low energies. One can choose a basis of the broken gauge generators such that only diagonal ones have the VEV's. Thus the nonzero D term can arise when the rank of the gauge group is reduced by the gauge symmetry breaking. The D-term contribution is proportional to the charge of the broken U(1) factor, i.e., the diagonal generator, and gives mass splittings within the same multiplet in the full theory.

We can rewrite $\delta^2 D^A = \langle D^A \rangle$ by using the gauge invariance of W and B as

$$\langle D^A \rangle = -2m_S^2 (M_V^2)^{-1AB} \{ B_{1\kappa}^* (T^B)_\lambda^\kappa B_1^\lambda - B_{1K}^* (T^B)_L^K B_1^L$$
$$+ C_{2\kappa}^\lambda z_{0\lambda}^* (T^B z_0)^\kappa + C_{2\kappa\lambda} z_0^\lambda (T^B z_0)^\kappa \}. \tag{3.54}$$

We will see in the next section that the VEV of the D term (3.54) vanishes when the SUSY-breaking terms are universal and hence the D-term contribution to the sfermion masses is a characteristic of the nonuniversal SUSY breaking.

E. Case of the universal soft SUSY-breaking terms

Let us now discuss the effective potential for the case of the universal soft terms. In addition to our ansatz (3.14), we assume that

$$B(\Phi, \Phi^\dagger) = B\Phi^\dagger \Phi, \tag{3.55}$$
$$C(\Phi, \Phi^\dagger) = C\Phi^\dagger \Phi, \tag{3.56}$$

with dimensionless constants B and C. Then we find

$$B_1^\kappa = B z_0^\kappa, \quad B_{2\lambda}^\kappa = B\delta_\lambda^\kappa \tag{3.57}$$

and

$$C_{1\kappa} = C z_{0\kappa}^*, \quad C_1^\kappa = C z_0^\kappa,$$
$$\tag{3.58}$$
$$C_{2\lambda}^\kappa = C\delta_\lambda^\kappa, C_{2\kappa\lambda} = C_2^{\kappa\lambda} = 0.$$

In particular, $B_1^A = C_1^A = 0$ because $z_0^A = 0$. Furthermore, our assumption $B_1^k = O(m_S)$ is automatically satisfied since we postulate $z_0^k = O(m_S)$, which allows us to use the effective potential (3.46) and (3.47).

We first evaluate ΔV in Eq. (3.47). Equation (3.38) now reads

$$\delta z^K = -m_S(\mu^{-1})^{KL} B z_{0L}^*. \tag{3.59}$$

The VEV's of the auxiliary fields become

$$\langle F^K \rangle = -m_S A \delta z^K - m_S^2 (\mu^{-1})^{KL} C z_{0L}^*, \tag{3.60}$$
$$\langle D^A \rangle = 2m_S^2 (M_V^2)^{-1AB} C(z_0^\dagger T^A z_0) = 0, \tag{3.61}$$

where the last equality is due to Eq. (3.4). Noting that the gauge invariance of the superpotential shows

$$h_{BKl} \delta z^K (T^A z_0)^B = -\mu_{KL} \delta z^L (T^A)_l^K + O(m_S^2)$$
$$= m_S B(z_0^\dagger T^A)_l + O(m_S^2)$$
$$= 0, \tag{3.62}$$

we find ΔV becomes simply

$$\Delta V = \frac{1}{2} \langle F^K \rangle h_{K\lambda\mu} \delta z^\lambda \delta z^\mu. \tag{3.63}$$

Hence all contributions to the *chirality-conserving* mass terms disappear with the universal soft terms. The SUSY-breaking part of the potential is found to be

$$V_{\text{eff,SUSY}} = m_S A\widetilde{W} + m_S B z^k \frac{\partial \widetilde{W}}{\partial z^k}$$
$$+ \frac{1}{2} \langle F^K \rangle h_{K\lambda\mu} \delta z^\lambda \delta z^\mu + \text{H.c.}$$
$$+ m_S^2 (B^2 + C) z_k^* z^k. \tag{3.64}$$

An important conclusion is that the scalar mass is common in this case.

To compare our results with those of Ref. [9], we further take

$$B = 1, \quad C = 0, \tag{3.65}$$

as well as $\mu_{kl} = 0$, $z_0^k = 0$. Then after a little algebra, we find

$$\Delta V = -\frac{1}{2} m_S A h_{K\lambda\mu} \delta z^K \delta z^\lambda \delta z^\mu$$
$$= -m_S A \left(3\widetilde{W} - z^k \frac{\partial \widetilde{W}}{\partial z^k} \right), \tag{3.66}$$

and the SUSY-breaking part is

$$V_{\text{eff,SUSY}} = -2m_S A\widetilde{W} + m_S(A+1) z^k \frac{\partial \widetilde{W}}{\partial z^k} + \text{H.c.}$$
$$+ m_S^2 z_k^* z^k, \tag{3.67}$$

which is in agreement with the result of Ref. [9].

IV. PHENOMENOLOGICAL IMPLICATIONS

In this section, we point out phenomenological implications of the general discussion in the preceding sections. The main new feature is that one may have nonuniversal scalar masses at the GUT scale, both due to the D-term and F-term contributions. For the first two generation slepton and/or squark fields, we expect that the superpotential coupling is weak enough to be neglected, and the term $m_S^2 B_{2k}^A B_{2A}^{*l}$ in Eq. (3.53) vanishes under the assumption of the R-parity conservation. Therefore we deal with only the D-term contributions. It was pointed out in our previous paper [3,4] that the scalar masses satisfy "sum rules" corresponding to the symmetry-breaking pattern of the grand unified theory, which can be tested at future collider experiments. On the other hand, the Higgs fields and third generation slepton/squark fields are likely to acquire both D- and F-term contributions, which may drastically change the analysis of the radiative breaking scenario.

A. Squark and slepton masses

For the first two generation matter fields, the superpotential coupling is small and can be neglected. Then the masses of their scalar components are determined solely by the initial conditions and their gauge quantum numbers. As a consequence, they have a definite pattern in the mass spectrum once one has a specific symmetry-breaking pattern from the grand-unified group down to the standard model gauge group, $G_{\rm SM} = {\rm SU}(3)_C \times {\rm SU}(2)_L \times {\rm U}(1)_Y$.

In our previous paper [3], we showed that the squark and slepton masses satisfy certain "sum rules" for various examples of the symmetry-breaking patterns. Let us briefly review the results below, as an example where the D-term contributions to the scalar masses play a major phenomenological role.

Let us take the following symmetry-breaking pattern, for instance:

$$ {\rm SO}(10) \xrightarrow{M_U} {\rm SU}(4)_{\rm PS} \times {\rm SU}(2)_L \times {\rm SU}(2)_R \xrightarrow{M_{\rm PS}} G_{\rm SM}. $$
(4.1)

There appear D-term contributions to the scalar masses when the rank of the gauge group reduces from 5 to 4 at the intermediate Pati-Salam symmetry-breaking scale $M_{\rm PS}$. The matter multiplets belong either to $L = (\mathbf{4}, \mathbf{2}, \mathbf{1})$ or $R = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ representations, with masses m_L^2 and m_R^2, respectively, above $M_{\rm PS}$. When the Pati-Salam group breaks to $G_{\rm SM}$, we obtain the masses

$$ m_{\tilde{q}}^2 = m_L^2 + g_4^2 D, $$
(4.2)

$$ m_{\tilde{u}}^2 = m_R^2 - (g_4^2 - 2g_{2R}^2)D, $$
(4.3)

$$ m_{\tilde{e}}^2 = m_R^2 + (3g_4^2 - 2g_{2R}^2)D, $$
(4.4)

$$ m_{\tilde{l}}^2 = m_L^2 - 3g_4^2 D, $$
(4.5)

$$ m_{\tilde{d}}^2 = m_R^2 - (g_4^2 + 2g_{2R}^2)D, $$
(4.6)

where D represents the D-term contributions whose nor-

malization is taken arbitrarily. These expressions do not depend on a particular choice of the Higgs representation which breaks Pati-Salam group, and hence fixed completely by the symmetry-breaking pattern.[13] Note that the gauge coupling constants g_4^2, g_{2R}^2 can be determined from the low-energy gauge coupling constants $\alpha_i(m_Z)$ ($i = 1, 2, 3$) as a function of $M_{\rm PS}$ alone. On the other hand, one can eliminate D, m_L^2, and m_R^2 from the above formulas, to obtain

$$ m_{\tilde{q}}^2(M_{\rm PS}) - m_{\tilde{l}}^2(M_{\rm PS}) = m_{\tilde{e}}^2(M_{\rm PS}) - m_{\tilde{d}}^2(M_{\rm PS}), $$
(4.7)

$$ g_{2R}^2(M_{\rm PS})(m_{\tilde{q}}^2 - m_{\tilde{l}}^2)(M_{\rm PS}) = g_4^2(M_{\rm PS})(m_{\tilde{u}}^2 - m_{\tilde{d}}^2)(M_{\rm PS}). $$
(4.8)

Once we measure the gaugino and scalar masses at low energy, we can calculate the scalar masses at $M_{\rm PS}$ as a function of $M_{\rm PS}$ alone. Since we have two relations for one free parameter $M_{\rm PS}$, one can solve for $M_{\rm PS}$ and further make a consistency check.

There exist more relations when SO(10) is broken directly into $G_{\rm SM}$:

$$ m_{\tilde{q}}^2 = m_{16}^2 + g_{10}^2 D, $$
(4.9)

$$ m_{\tilde{u}}^2 = m_{16}^2 + g_{10}^2 D, $$
(4.10)

$$ m_{\tilde{e}}^2 = m_{16}^2 + g_{10}^2 D, $$
(4.11)

$$ m_{\tilde{l}}^2 = m_{16}^2 - 3g_{10}^2 D, $$
(4.12)

$$ m_{\tilde{d}}^2 = m_{16}^2 - 3g_{10}^2 D, $$
(4.13)

where the only unknown parameters are m_{16}^2 and D after the measurements of the SUSY-breaking masses. We use one relation to fix D, one for m_{16}^2, and there remain three relations for the consistency check. On the other hand, we have more free parameters when the symmetry is smaller: for instance,

$$ {\rm SO}(10) \xrightarrow{M_U} {\rm SU}(3)_C \times {\rm SU}(2)_L \times {\rm SU}(2)_R \times {\rm U}(1)_{B\text{-}L} $$
$$ \xrightarrow{M_{3221}} G_{\rm SM}. $$
(4.14)

In this case we cannot even determine the parameters in the original model.

On the other hand, let us consider flipped SU(5) model [33,32] as an example of nonunified model. Its gauge group is SU(5)×U(1) and is not grand-unified into a single group. First of all, we have two independent gaugino masses $M_{\rm SU(5)}$ and $M_{\rm U(1)}$ at the GUT scale, which results in the low-energy gaugino masses which do not

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[13]One may anticipate that there are also F-term contributions since we need a superpotential $RR\chi$ to generate right-handed neutrino masses where χ transforms as $(\overline{\mathbf{15}}, \mathbf{1}, \mathbf{3})$. Indeed, a right-handed scalar neutrino may acquire contributions like $F_\chi \tilde{N}^c \tilde{N}^c$. However, other squark, slepton fields cannot acquire F-term contributions because of the $G_{\rm SM}$ invariance.

TABLE I. The "score sheet," showing how well we can distinguish among various models. The intermediate groups are defined as $G_{\rm PS}$ = $SU(4)_{\rm PS} \times SU(2)_L \times SU(2)_R$, $G_{3221} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B\text{-}L}$, $G_{3211} = SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B\text{-}L}$. The column α_i refers to the unification of the gauge coupling constants, where "natural" means that the unification is automatic, and "adjustable" employs either particular particle content or threshold corrections to reproduce the observed gauge coupling constants. The column M_i/α_i refers to the gaugino masses. The column m_i^2 states whether the model predicts a definite pattern which is testable using the low-energy scalar mass spectrum.

	α_i	M_i/α_i	m_i^2
$SU(5) \rightarrow G_{\rm SM}$	Natural	Common	Testable
$SO(10) \rightarrow G_{\rm SM}$	Natural	Common	Testable
$SO(10) \rightarrow G_{\rm PS} \rightarrow G_{\rm SM}$	Adjustable	Common	Testable
$SO(10) \rightarrow G_{3221} \rightarrow G_{\rm SM}$	Adjustable	Common	Not testable
$SO(10) \rightarrow G_{3211} \rightarrow G_{\rm SM}$	Adjustable	Common	Not testable
$SU(5) \times U(1) \rightarrow G_{\rm SM}$	Adjustable	Common only for $i = 2, 3$	Testable
Superstring with dilaton F term	Adjustable	Common	Testable
Superstring with moduli F term	Adjustable	Not common	Not testable

necessarily satisfy the GUT relation. However, one can test the scenario measuring M_2 and M_3 at the weak scale and see whether they unify at the same scale where the gauge coupling constants α_2 and α_3 unify. On the scalar masses, we have three independent masses m_{10}^2, $m_{\bar{5}}^2$, and m_1^2 at the GUT scale, and an unknown D term in addition. There are five observable scalar masses, and we know the scale where $SU(2)$ and $SU(3)$ coupling constants meet. Therefore, we are left with one additional relation which can be checked. The scalar masses satisfy the following relations at M_X where $SU(2)$ and $SU(3)$ unify to $SU(5)$:

$$m_{\bar{d}}(M_X)^2 = m_{10}{}^2 + \left(-\frac{2}{5}g_{SU(5)}{}^2 + \frac{1}{40}g_{U(1)}{}^2\right) D, \quad (4.15)$$

$$m_{\bar{l}}(M_X)^2 = m_{\bar{5}}{}^2 - \left(\frac{3}{10}g_{SU(5)}{}^2 + \frac{3}{40}g_{U(1)}{}^2\right) D, \quad (4.16)$$

$$m_{\bar{u}}(M_X)^2 = m_{\bar{5}}{}^2 + \left(\frac{1}{5}g_{SU(5)}{}^2 - \frac{3}{40}g_{U(1)}{}^2\right) D, \quad (4.17)$$

$$m_{\bar{q}}(M_X)^2 = m_{10}{}^2 + \left(\frac{1}{10}g_{SU(5)}{}^2 + \frac{1}{40}g_{U(1)}{}^2\right) D, \quad (4.18)$$

$$m_{\bar{e}}(M_X)^2 = m_1{}^2 + \frac{1}{8}g_{U(1)}{}^2 D. \quad (4.19)$$

Then the "sum rule" is obtained as

$$m_{\bar{d}}^2(M_X) - m_{\bar{l}}^2(M_X) = m_{\bar{q}}^2(M_X) - m_{\bar{u}}^2(M_X). \quad (4.20)$$

Though the "sum rules" of the scalar masses are weak when the symmetry is small, one can acquire useful information by combining the scalar mass spectrum with that of the gauginos. We pointed out that the gaugino masses satisfy the so-called GUT relation

$$\frac{M_1}{\alpha_1} = \frac{M_2}{\alpha_2} = \frac{M_3}{\alpha_3}, \quad (4.21)$$

even when the grand-unified group breaks down to $G_{\rm SM}$ in several steps. On the other hand, non-GUT models such as flipped $SU(5)$ do not necessarily have a unified

gaugino mass, and hence do not predict GUT relation of the gaugino masses. Therefore, one can draw useful information on the GUT models by measuring the scalar and gaugino masses in future experiments. We present a "score sheet" of various models in Table I. Here we refer to the paper [34] on superstring predictions.[14]

B. Radiative breaking scenario

There have been several remarkable results reported, based on the radiative-breaking scenario [14] and the universal scalar mass hypothesis at the GUT scale. One of them is that the lightest supersymmetric particle (LSP) should be gaugino dominant to correctly reproduce the weak scale [36], when the top quark is heavy. This gives a stringent constraint on the cosmic abundance of the neutralino. The other is that the proton decay in the minimal SU(5) SUSY-GUT cannot be consistent with the present bound if one further requires that the LSP does not overclose the Universe [37]. Since both analyses crucially depend on the universal scalar mass hypothesis, there may be qualitatively different consequences in the nonminimal case.

The crucial equations to determine m_Z and $\tan \beta$ in the minimal SUSY standard model (MSSM) are the following ones at the tree level:

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[14]It is noteworthy that the superstring with a dilaton F term also leads to the same relation. This is amusing because one needs rather big threshold corrections for the gauge coupling constants to reconcile the difference between the apparent GUT scale and the string scale [35]. Exactly the same correction both for the gauge coupling constants and the gaugino masses appears to give the same relation as in the (field theoretical) GUT models.

$$m_Z^2 = -\frac{m_1^2 - m_2^2}{\cos 2\beta} - (m_1^2 + m_2^2 + 2\mu^2), \qquad (4.22)$$

$$\sin 2\beta = -\frac{2m_3^2}{m_1^2 + m_2^2 + 2\mu^2}, \qquad (4.23)$$

where m_1^2, m_2^2 refer to the soft SUSY-breaking part of the Higgs boson masses, m_3^2 the off-diagonal mass, and μ the Higgsino mass parameter. When one adopts the universal scalar mass hypothesis, the large top quark Yukawa coupling drives m_2^2 much smaller than m_1^2 in general as far as $\tan\beta$ is not very large. This gives a too-large value of the first term on the right-hand side (RHS) of Eq. (4.22) in general, which should be compensated by the negative contributions of μ^2. Of course, the details depend on the renormalization group analysis; it can be studied semi-analytically as far as one can neglect the bottom quark Yukawa coupling constant [38]. In this case,

$$m_1^2 = m_f^2, \qquad (4.24)$$

$$m_2^2 = m_f^2 - 3I, \qquad (4.25)$$

$$m_3^2 = B\mu. \qquad (4.26)$$

Here, I is defined by

$$I = I_{SS}m_\infty^2 + I_{GG}M_\infty^2 + I_{GA}M_\infty A_\infty + I_{AA}A_\infty^2, \qquad (4.27)$$

where the coefficients I_{SS}, I_{GG}, I_{GA}, I_{AA} are functions of the top quark Yukawa coupling only, and m_∞, M_∞, A_∞ are the universal scalar mass, universal gaugino mass, and universal trilinear coupling at the GUT scale, respectively. One can rewrite the Eqs. (4.22) as

$$\mu^2 = 3\frac{\tan^2\beta}{\tan^2\beta - 1}(I_{SS}m_\infty^2 + I_{GG}M_\infty^2$$
$$+ I_{GA}M_\infty A_\infty + I_{AA}A_\infty^2)$$
$$- (m_\infty^2 + 0.52M_\infty^2) - m_Z^2/2. \qquad (4.28)$$

The coefficient I_{GG} varies from 0.8 to 1.2 for a top quark mass $m_t > 140$ GeV, and hence μ^2 is always an increasing function of M_∞. For small $\tan\beta$, the first term dominates, and one has a large μ^2. For moderately large $\tan\beta$, the CERN e^+e^- collider LEP bound on M_2 gives a lower bound on M_∞. Though m_∞^2 and $M_\infty A_\infty$ terms may give negative contributions to the above equation, their coefficients are in general not large, and one has to take m_∞ or $|A_\infty|$ very large to make Higgsino light. Since such parameters are not favored from the naturalness point of view, one reaches the conclusion that the Higgsino-like LSP is disfavored in the radiative breaking scenario.[15] This is especially true in the no-scale case,

where all m_∞^2, $M_\infty A_\infty$, and A_∞^2 terms vanish. It is not possible to obtain a Higgsino-like LSP within the no-scale models. Even with nonvanishing m_∞ and A_∞, it was shown that there are no solutions with a Higgsino-like LSP [36] after including the one-loop effects on the Higgs potential.

However, the situation drastically changes when one incorporates the possible D-term contributions to m_1^2 and m_2^2. Let us imagine the initial condition $m_1^2 = m_\infty^2 - \Delta m^2$, $m_2^2 = m_\infty^2 + \Delta m^2$. This gives an extra contribution $\Delta\mu^2$ to the μ^2 as

$$\Delta\mu^2 = \frac{1}{2\cos 2\beta}[2 - (1 - \cos 2\beta)I_{SS}]\Delta m^2, \qquad (4.29)$$

which allows a lighter Higgsino compared to the case of the universal scalar mass.

The fact that the lighter Higgsino is allowed has a very strong impact in the minimal SU(5) GUT, where it has been claimed that the nucleon decay via the dimension-five operators cannot be consistent with the longevity of the Universe [37]. The nucleon decay rate is roughly proportional to M_∞/m_∞^2 when $M_\infty \lesssim m_\infty$ [39], while gaugino-like LSP has an abundance proportional to m_∞^2/M_∞. However, the abundance becomes much smaller once the smaller Higgsino mass parameter is allowed. Therefore, one can take M_∞ to be much smaller than m_∞ without worrying about the LSP abundance, which opens a consistent region between the nucleon decay experiments and the LSP abundance.[16]

C. FCNC

The assumption of the universal scalar mass is motivated to explain the smallness of the flavor-changing neutral current (FCNC) due to the loops of SUSY particles [7]. Since we have relaxed this assumption, the readers may be worried about FCNC.

There are two classes of nonminimal effects, one which does not break the degeneracy between sfermion masses with the same quantum numbers, the other which does. The nonminimality we discussed in Sec. IV A could have been generated by the renormalization between the Planck scale and the GUT scale. The implicit assumption here is that the Yukawa interactions are small for the first two generations, even to the superheavy fields which are completely decoupled from the low-energy effective action. As long as all of their interactions in the superpotential are small, the only renormalization effects

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[15]Furthermore, for a larger top quark $m_t \gtrsim 160$ GeV, the coefficient of m_∞^2 is always positive and the contribution from m_∞^2 cannot cancel that from M_∞^2. Hence the Higgsino mass parameter becomes always large and the Higgsino-like LSP cannot be realized.

[16]The claim by Ref. [37] was criticized by the authors of Ref. [40], who pointed out the possibility of the strong annihilation of neutralinos due to the s-channel Higgs pole. Also, the constraint from the proton decay in Ref. [37] is much stronger than the conservative estimate in the second paper in Ref. [39]. However, our point in this section is the strong influence of the nonminimality on the radiative breaking scenario, and we will not go into this issue further.

arise due to the gauge interactions, and hence universal for different generations. The degeneracy of sfermion masses at the initial condition ensures the degeneracy at the weak scale. In this case, the nonminimal nature of the radiative corrections does not break the degeneracy.

On the other hand, there are many sources of radiative corrections which could break the degeneracy. For instance, there are attempts to explain the degeneracy of sfermion masses based on horizontal symmetries [41]. If the horizontal symmetry is gauged and breaks spontaneously, however, there may be D terms in the horizontal gauge group which potentially breaks the degeneracy again. Another example is when the first two generations also have ~ 1 Yukawa interactions beyond the GUT scale. In this case there are two sources of nondegeneracy: (1) renormalization due to the Yukawa interactions, and (2) F-term contributions to the scalar masses when the heavy particles decouple. There are no discussions on these effects in the literature to our knowledge.

It is noteworthy, however, that FCNC can be sufficiently suppressed even with a nondegenerate initial condition. If the scalar masses turn out to be relatively smaller than the gaugino masses, radiative corrections between the Planck or GUT scale and the weak scale tend to make squark masses universal due to the gluino contribution. Indeed, SUSY breaking via moduli F-term condensation in superstring inspired supergravity models gives nonuniversal scalar masses which depend on modular weights, and in principle can lead to large FCNC processes. However, they can be sufficiently suppressed due to the renormalization effects at least for some region of the parameter space [34].

D. Renormalization between GUT and Planck scales

As repeatedly emphasized through the text, one of the important sources of the nonminimality is the renormalization between the GUT and the Planck scales. Let us briefly comment when and how these effects can be important despite the apparent closeness of the two scales. Indeed, most of the analyses in the literature completely ignore the difference of these two scales.

One example is when the constraint is marginal. For instance, it was shown in Refs. [12,13] that one has an upper bound on the gaugino mass in a restricted class of minimal supergravity model with $m_\infty = A_\infty = 0$ at the GUT scale. The argument comes from the fact that the right-handed slepton acquires a mass only of

$$m_{\tilde{l}_R}^2 \simeq 0.87 M_1^2, \tag{4.30}$$

and hence is smaller than M_1. Then there is a danger that $\tilde{l}_R$ becomes lighter than the lightest neutralino. One is forced to use mixing in the neutralino sector to push the LSP mass down from M_1. (Higgsino LSP does not exist as a solution in the radiative breaking scenario with the universal scalar masses.) However, mixing can be substantial only when the gaugino mass is close to m_Z, and one obtains an upper bound on the gaugino mass. It was translated to an upper bound on the slepton mass,

$\lesssim 150$ GeV [12,13]. The situation completely changes when one includes the running of the slepton mass between Planck and GUT scales. Then one obtains

$$m_{\tilde{l}_R}^2 \simeq 3.1 M_1^2, \tag{4.31}$$

and one does not need a substantial mixing in the neutralino sector any more. Therefore, the upper bound on the gaugino mass becomes obsolete due to this effect.

Another example is when there is a relatively large coupling constant. If we require m_b-m_τ mass relation, one generally needs a top quark Yukawa coupling constant of $\simeq 2.0$ at the GUT scale for small $\tan\beta$, and $\simeq 0.8$ for large $\tan\beta \simeq 60$. Let us take minimal SU(5) to clarify our discussions. Then H_u has much smaller m^2 at the GUT scale compared to H_d, even when they start from the same value at the Planck scale. One has to go through the following analyses. First one solves the renormalization group running of the Higgs masses for $\mathbf{24}$, $\mathbf{5}$, $\mathbf{\bar{5}}$, and also other SUSY-breaking parameters. Then one employs the formulas presented in Sec. III to integrate out the superheavy fields to obtain the low-energy effective action. Such a nonminimality may change the parameter space of the radiative breaking scenario substantially, however, it does not affect FCNC constraint since the effects of h_t do not appear in the first- and second-generation scalar masses.

V. CONCLUSIONS

In this paper, we have derived the low-energy effective Lagrangian in the scalar sector starting from a unified theory with nonuniversal soft SUSY-breaking terms. Such nonuniversal soft terms arise if we take a flat limit of the supergravity where the Kähler potential is a nonminimal one. One should note that this is indeed the case in the string-inspired model where the moduli fields are responsible for the SUSY breaking. Even if the soft terms have the universal structure at the gravitational scale, they get renormalized and as a result become nonuniversal in general when the energy scale goes down to the GUT scale. Therefore we expect that the soft terms are nonuniversal at the GUT scale and it is important to investigate its consequences at low energies.

We have calculated the scalar potential of the low-energy theory by explicitly integrating out the heavy sector. The SUSY-breaking part of the scalar potential is summarized in Eqs. (3.48) and (3.47). We found some new contributions to the soft terms which can be nonzero only when the soft terms of the full theory are nonuniversal. In particular, the sizable D-term contributions generally exist in the chirality-conserving scalar masses when the rank of the gauge group is reduced by the gauge symmetry breaking. Its phenomenological implications were discussed in our previous papers [3,4]. Another important point is concerned with the gauge hierarchy problem. Many of the SUSY-GUT models achieve the small Higgs doublet mass by a fine-tuning of the parameters in the superpotential. If the soft terms are turned on, however, a SUSY-breaking Higgs boson mass term can

become heavy and the weak scale will be destabilized. We showed that all mass terms remain at the weak scale if the soft terms are restricted to those derived from the supergravity model where the hidden sector decouples from the observable sector in the superpotential.

We have also discussed other phenomenological implications. Recall that there are many sources which give the nonuniversal scalar masses, including the D-term and/or F-term contributions discussed in this paper. This nonuniversality changes the predictions of the radiative electroweak symmetry breaking, usually assuming the common mass for the two Higgs doublet bosons. In particular, the Higgsino can be the dominant component of the LSP even when the top quark is heavy: if we assume the universal scalar mass, the LSP is dominated by a gaugino component. This cures the apparent conflict of the nucleon lifetime and the LSP relic abundance. In the no-scale model, on the other hand, we pointed out that the upper bound on the right-handed slepton mass [12,13] disappears if we properly incorporate the renormalization group flow between the GUT scale and the Planck scale. Further study of the radiative breaking scenario without the universal scalar mass hypothesis should be encouraged.

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APPENDIX: EQUALITIES FROM GAUGE INVARIANCE

We summarize the consequences of the gauge invariance of the superpotential W.

From the gauge invariance of W, there is the equality

$$\frac{\partial W}{\partial z^\kappa}(T^\alpha)^\kappa_\lambda z^\lambda = 0, \tag{A1}$$

where summation over repeated indices κ and λ is understood. If we differentiate the above equality with respect to z^μ, we obtain

$$\frac{\partial^2 W}{\partial z^\kappa \partial z^\mu}(T^\alpha)^\kappa_\lambda z^\lambda + \frac{\partial W}{\partial z^\kappa}(T^\alpha)^\kappa_\mu = 0. \tag{A2}$$

By further differentiating the above, one finds

$$\frac{\partial^3 W}{\partial z^\kappa \partial z^\mu \partial z^\nu}(T^\alpha)^\kappa_\lambda z^\lambda + \frac{\partial^2 W}{\partial z^\kappa \partial z^\nu}(T^\alpha)^\kappa_\mu$$
$$+ \frac{\partial^2 W}{\partial z^\kappa \partial z^\mu}(T^\alpha)^\kappa_\nu = 0. \tag{A3}$$

The equalities (A2) and (A3) become

$$\mu_{AB} = 0, \tag{A4}$$
$$h_{\kappa\mu\nu}(T^\alpha)^\kappa_\lambda z^\lambda_0 + \mu_{\kappa\nu}(T^\alpha)^\kappa_\mu + \mu_{\kappa\mu}(T^\alpha)^\kappa_\nu = O(m_S), \tag{A5}$$

respectively, in the case of W of Eq. (3.18). Equation (A5) can be written as

$$h_{AKL}(T^B z_0)^A + \mu_{MK}(T^B)^M_L + \mu_{ML}(T^B)^M_K = O(m_S), \tag{A6}$$

$$h_{ACL}(T^B z_0)^A + \mu_{MK}(T^B)^M_C = O(m_S), \tag{A7}$$

$$h_{AKk}(T^B z_0)^A + \mu_{MK}(T^B)^M_k = O(m_S), \tag{A8}$$

$$h_{Akl}(T^B z_0)^A = O(m_S), \tag{A9}$$

$$h_{ACk}(T^B z_0)^A = O(m_S), \tag{A10}$$

in terms of components.

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