Transitional Characteristics of Phase Shift in Lock-in Phenomena of an Oscillating Cylinder*

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Abstract
In this study, characteristics of phase lag between displacement of forcibly oscillating test body and velocity fluctuation caused by vortex formation behind the test body were investigated. A right isosceles triangular cross-sectioned prism was used as the test body. The base of the isosceles triangle is 100mm and span-wise length of the prism is 300mm. The test body was rotationally oscillated at seven different frequencies from 0.205Hz to 0.235Hz in 0.005Hz interval providing the natural vortex shedding frequency is 0.220Hz. As the results, the behavior of the phase lag is summarized as followings. i) In general sense, it is well known that the phase lag is shifted by $\pi$ within the lock-in region. Besides that, intermittent slip of phase lag was observed. ii) The velocity fluctuation does not synchronize perfectly with the forced oscillation even when the frequency of the forced oscillation is set to natural vortex shedding frequency of the test body. iii) For the case of forced oscillation frequency is set to the natural vortex shedding frequency, synchronization rate of the velocity fluctuation with the oscillation was found to be 80% at the maximum.

Key words: Oscillation, Phase Lag, Synchronization, Lock-In, Phase Slip, Triangular Prism, Vortex

1. Introduction
Structures such as suspension bridges are sometimes swung or vibrated by fluctuating fluid forces originated in the formation of Karman vortices behind a bluff body placed in a flow. When the fluctuating frequency of fluid force and the natural frequency of a body are matched, the body may be destroyed as a result of resonance. From these aspects, research on the Karman vortices formed in the wake of a body is extremely important, and hence, there are many reports in international literature on the subject matter in variety of fields. For instance, Haniu et al. (1~3), Ongoren et al.(4,5), Kim et al. (6), Griffin et al. (7), Williamson et al. (8), Gu et al. (9) and Fujisawa et al. (10) studied or controlled the wake characteristics of circular cylinders under still and vortex induced vibration conditions. In this research area, the lock-in phenomena, where the frequency of linear or rotational oscillation with small amplitude was set to near the natural vortex shedding frequency of the body were studied by several research groups.

According to Haniu et al.(1), the enhancement of irregularity in vortex shedding within the lock-in region when amplitude of rotational oscillation exceeds a certain range was
likely to be caused by intermittent switching of vortex shedding phase. Such an irregularity in vortex shedding phase would cause difficulty in making use of the lock-in phenomena to improve accuracy of quantitative flow measurements such as by phase ensemble averaging technique. Therefore, in order to study the unsteady flow structure in a wake behind a body, it is very important to clarify the mechanism of irregularity in vortex shedding phase fluctuations. As to the vortex shedding phase, Ongoren et al.\textsuperscript{(4)} linearly oscillated a triangular prism in cross-flow direction at an amplitude \( \pm 0.065 \) times of its major dimension, and reported that the vortex shedding phase is shifted by \( \pi \) while the forced oscillation frequency was varied from approximately 0.9 to 1.1 times of its natural vortex shedding frequency. However, they did not give detailed explanation on the phase shift, and they treated the phase that was shifted gradually while the forced oscillation frequency was varied. On the other hand, when the phase shifts drastically in oscillating frequency band near the natural vortex shedding frequency, as reported by Haniu et al.\textsuperscript{(2)} and Ongoren et al.\textsuperscript{(5)}, there will be high possibility of intermittent switching of flow patterns between the two different states of vortex shedding phase. In the earlier report of Ongoren et al.\textsuperscript{(4)}, phase of the velocity fluctuation with respect to the displacement of the oscillating body was measured by means of cross-spectra, and hence the reported phase shift while the oscillating frequency was varied from approximately 0.9 to 1.1 times of the natural vortex shedding frequency was obtained as time averaged values. Therefore, even if the flow switching was occurring, their analyzing method would have failed to detect it.

On the other hand, Kim et al.\textsuperscript{(6)} reported the state of flow induced vibration (lock-in) of an elastically supported circular cylinder with forced vibration apparatus. They reported that the phase lag between the displacement of the vibrating cylinder and pressure fluctuation on the surface of the cylinder is shifted from 0 to \( \pi \) where the flow velocity was varied within the conditions from the starting to the ending of the vibration. However, the details of the phase shift and the flow switching were not clarified. In regard to the characteristics of the phase shift in vortex shedding, there are limited number of reports for instance Ongoren et al.\textsuperscript{(4,5)} and Kim et al.\textsuperscript{(6)}, and there is no report on the characteristics of local time phase fluctuation.

Accordingly, clarification about the local time characteristics of fluctuating vortex shedding in lock-in region is the major objective of this study with an interest of phase switching in the flow field. In the present experiments, a triangular prism is placed in the flow as a test body, and it was rotationally oscillated with small amplitude. Velocity fluctuation around the separated shear layer generated in the wake was measured by Laser Doppler Velocimetry (LDV). Phase lag between the rotational oscillation and the velocity fluctuation was calculated for every rotational oscillation of the prism. Furthermore, local time behavior of the phase lag was compared with time averaged results obtained by Ongoren et al.\textsuperscript{(4)} and Kim et al.\textsuperscript{(6)}.

2. Experimental Apparatus and Methods

In this experiment, a water circulating open channel of 4,500mm in full length with 400mm width and 890mm depth measuring cross-section was used. In order to fix the separation point of the separated shear layer from the test body and to enhance flow two-dimensionality, a right isosceles triangular cross-sectioned prism was used. The prism is made of acrylic material with its base \( h \) is 100mm, and its span is 300mm. As shown in Fig.1(b), the test body was placed in such manner that the time averaged orientation of the base is right angle to the free stream. A stainless steel pipe of 6mm diameter is installed at the center of the test body along the span as shown in Fig.1(a), and the test body is rotationally oscillated about the pipe by induction motor with crank mechanism. LDV is used to measure the velocity fluctuation in the shear layer separated from the test body.
Its measuring point is, as shown in Fig.1(b), at 3h downstream from the base of the test body and 1h above the center of the wake (x axis) where velocity fluctuation is found sufficiently significant and phase variation is small in y direction. The body was oscillated periodically with an amplitude of $\pm 4^\circ$. The oscillation frequencies were varied for 7 steps from 0.205Hz to 0.235Hz with an interval 0.005Hz taking the natural vortex shedding frequency of 0.220Hz at the middle of the variation, and the phase lag between the oscillation displacement of the body and velocity fluctuation was investigated.

In the present investigation, free stream velocity and temperature were kept constant at 11.5cm/s and $20^\circ C$, respectively. The phase information of the rotational oscillation of the test body is obtained by a photo micro sensor placed at outer rim of a half circle plate attached to the rotational axis of the induction motor. Where, on-off signal is obtained when the rotating half circle crosses the optical path of the photo micro sensor intermittently at an equal time interval. By taking the on-off signal from the photo micro sensor as a reference signal, phase lag was calculated for every oscillation from the time lag between the zero crossing points of the on-off signal and the velocity signal after digital filtering to eliminate high frequency noise. In this study, such a phase lag for each oscillation is termed as “local time phase lag”.

Fig.1 Experimental arrangement and definitions of symbols.

3. Results and Discussion

3.1 Rearrangement of probability density distribution of local time phase lag

Figure 2 shows the procedure to obtain probability density distribution (PDD) of the phase lag between oscillatory displacement of the test body and velocity fluctuation in the wake by means of LDV. PDD of measured local time phase lag in a range of phase 0 to $2\pi$ is presented in Fig.2(a), in which the continuing distribution from $2\pi$ to 0 is separated. Therefore, another one period (0 to $2\pi$) of the distribution is attached to both side (see Fig.2(b)), and a section of the distribution for the length of $2\pi$ well showing the phase variation is extracted (see Fig.2(c)). In the present study, the distributions those obtained for each experimental condition were investigated. Where, data for approximately 65 oscillations were used to obtain one distribution.

Fig.2 Processing of probability density distributions of phase lag between forced oscillation and velocity fluctuation ($f_f = 0.220Hz$)
3.2 Probability density distribution of local time phase lag

Figure 3 shows PDD of phase lag mentioned in the previous section for 7 different oscillation frequencies of the test body from 0.205Hz to 0.235Hz. In particular, after (b) in figures, two representative distributions are shown for each condition. For clarity, it should be noted that the natural vortex shedding frequency is 0.220Hz. In Figure 3(a), PDD of the local time phase lag of the oscillation frequencies of 0.205Hz(left) and 0.235Hz(right) exhibit gently sloping hills, and there is no particular phase of concentration. Therefore, in these cases, there is no relation between the reference signal comes from rotational oscillation of the body and the velocity signal. In consequence, it can be concluded that the two signals are not synchronized. Thus, these two regions of forced oscillation frequency are said to be out of lock-in region.

Fig.3 Probability density distributions of phase lag for each forced oscillation frequency
Figure 3(b) shows PDD for the oscillation frequency of 0.210Hz. As denoted by the symbol ▽ in Fig.3(b) (ii), unlike in the case of outside the lock-in region, there is a concentration of the distribution at a particular phase lag near π. Therefore, probability of reference signal and velocity signal are synchronized would be higher near the phase lag π. However, its peak is not as distinctive as line spectrum, and also the distribution near the peak is broad to show a scatter of phase lag. On the other hand, as shown in Fig.3(b)(i), similar to the case of outside the lock-in region, there are cases that show no concentration in the distribution at a particular phase lag. Therefore, the case 0.210Hz of the forced oscillation frequency is considered to be in transition region as both the synchronization and non-synchronization of reference signal with velocity signal coexist.

Figure 3(c) shows PDD for oscillation frequency of 0.215Hz, and as shown in (i) in the figure, similar to the case of 0.210Hz, concentration of the distribution is found near the phase lag π. However, in (ii) of the figure, except at phase lag π as denoted by the symbol ▽, another concentration of the distribution is appeared near phase lag 2π as denoted by the symbol ▼. This phenomenon indicating an occurrence of phase lag switching between π and 2π.

Figure 3(d) shows PDD for oscillation frequency of 0.220Hz which is same as natural vortex shedding frequency, and in (ii) of the figure, high concentration of phase lag is observed near phase lag 2π as denoted by the symbol ▼. From these facts, a probability of the reference signal obtained from oscillatory displacement synchronizing with velocity signal becomes higher, when the forced oscillation frequency is closing to natural vortex shedding frequency, starting from 0.210Hz, increasing towards 0.215Hz and more increasing towards 0.220Hz. However, as shown in Fig.3(d)(i), even the oscillation frequency is 0.220Hz, the distributions are presented without concentration. The phenomenon exhibits various PDD at the same oscillation frequency is discussed later in detail in the section 3.5.

Figure 3(e) shows PDD for the oscillation frequency of 0.225Hz. Similar to the case of forced oscillation frequency of 0.220Hz, concentration of the distribution is appeared near phase lag 2π as denoted by the symbol ▼. Compare to the case of 0.220Hz, peak value of concentration is smaller and the phase lag is more spreading. This is indicating the probability that the reference signal synchronize with the velocity signal is reduced.

Figure 3(f) shows PDD for the oscillation frequency of 0.230Hz. Although (i) of the figure is showing slight concentration of phase, as shown in (ii) of the figure, distribution of gently sloping hill is observed in many cases, and the probability that the reference signal synchronizes with the velocity signal is further reduced.

3.3 Mode classification of probability density distributions

Figure 4 shows the phase lag values of peak appeared in PDD for each forced oscillation frequency as presented in Fig.3. In Fig.4, phase lag shifts from π to 2π via transition associated with switching between the two states while the forced oscillation frequency varies from 0.205Hz to 0.235Hz. Here, we can classify the PDD in accordance with its different shapes of the distribution which appeared when the phase lag shifted from π to 2π as following.

Mode1: Similar to the case of forced oscillation frequencies 0.205Hz and 0.235Hz, and as well as the feature of outside of the lock-in region, no peak appears in PDD of the phase lag.

Mode2: Similar to the case of forced oscillation frequencies 0.210Hz and 0.230Hz, a smaller peak appears in PDD, and the reference signal and the velocity signal likely to synchronize intermittently.

Mode3: Similar to the case of forced oscillation frequency 0.220Hz, a distinctive peak appears in PDD, and the reference signal and velocity signal are almost completely
synchronized.

Mode4: Similar to the case of forced oscillation frequencies 0.215Hz and 0.225Hz, two peaks appear in PDD.

Appearance time rate of PDD modes for each forced oscillation frequency are summarized in Fig.5. One can see from the figure that time rate of each mode varies with the oscillation frequency, and hence the state of flow will be changing gradually within the lock-in region. In the range of oscillation frequency from 0.210Hz to 0.230Hz, where flow is considered to be in lock-in state, appearance time rates of Mode2 and Mode3 are significant. Therefore, the probability of synchronizing the velocity signal with the reference signal may be considered to be high. When the oscillation frequency deviated from 0.220Hz, which is considered in the complete lock-in region, appearance time rate of the Mode3 tending to decrease but that of Mode2 tending to increase. Therefore, within the lock-in region where in general sense the velocity signal is considered to be completely synchronized with the reference signal, probability of synchronization becomes the highest.

Fig.4 Peak phase in probability density Distributions for each forced oscillation frequency

Fig.5 Appearance time rate of probability distributions for each forced oscillation frequency
when the oscillation frequency is set to natural vortex shedding frequency, and the probability decreases as the oscillation frequency deviated from the natural vortex shedding frequency. On the other hand, the Mode4 is found for the forced oscillation frequencies from 0.215Hz to 0.225Hz, and its time rate is higher at the oscillation frequency of 0.215Hz. Therefore, the phase lag shifts from $\pi$ to $2\pi$ is considered to be occurring in the range of oscillation frequencies 0.215Hz to 0.225Hz. The forced oscillation frequency 0.215Hz is approximately 0.977 times the natural vortex shedding frequency, and 0.225Hz is approximately 1.023 times. Therefore, this fact agrees with the report by Ongoren et al.$^{14}$ that vortex shedding phase shifts by $\pi$ while the forced vibration frequency changed from approximately 0.9 to 1.1 times the natural vortex shedding frequency. The above fact also agree with the report by Kim et al.$^{6}$ that the phase lag between the displacement of the vibrating cylinder and pressure fluctuation on the surface of the cylinder shift by $\pi$ while the flow velocity varies within the condition from the starting to the ending of the flow induced vibration.

3.4 Revision of local time phase lag variation with time

Figure 6 shows the time variation of local time phase lag for the forced oscillation frequency $f_f = 0.225$Hz. Figure 6(a) depicts the trend that phase lag increases with time which is repeatedly appeared. In particular, one can see near the center of the figure the local time phase lag suddenly back to 0 after it reaches to about $2\pi$. However, there should be continuity of phase lag variation from $2\pi$ to 0. Therefore in this experiment, when the phase lag suddenly changed from $2\pi$ to 0, phase lag data is added by $2\pi$. In contrast, for the case of the phase lag reduction with time, when the phase lag suddenly changed from $-2\pi$ to 0, phase lag data is deducted by $2\pi$. Figure 6(b) is the revised time variation of the phase lag, and the similar revisions were made for other forced oscillation frequencies.

![Fig.6 Revision of phase lag variation with time ($f_f = 0.225$Hz)](image_url)
3.5 Time variation of local time phase lag

Figure 7 shows the variation of local time phase lag for each oscillation frequency. As one can see from the figure, time traces of the phase lag are constituted of flat part, increasing part and decreasing part. Among them, the flat parts are showing the local time phase lags are almost constant, and hence the reference signal obtained from the driving mechanism of the forced oscillation is considered to be synchronized with velocity signal in this zone. On the other hand, vortex shedding phases are delaying in the increasing parts.

Fig. 7 Phase lag variation with time at each forced oscillation frequency of the test body
of the phase lag and advancing in the decreasing parts of the phase lag. As shown in Fig.7(a) where the forced oscillation frequency is 0.205Hz and vortex shedding is outside the lock-in region, there are almost no flat parts and the phase lag is decreasing continuously, and hence the reference signal and the velocity signal are not synchronizing. Such a trend is also seen in the case (g) of the figure where the forced oscillation frequency is 0.235Hz excluding the fact that the phase lag is increasing. In other 5 cases of forced oscillation frequency, since flat parts indicate presence of intermittent synchronization between the oscillation and vortex shedding, these 5 oscillation frequencies are considered to be within the lock-in region. In the case of the oscillation frequencies 0.210Hz as shown in Fig.7(b) and 0.215Hz as shown in Fig.7(c), flat parts are found near the phase lag \(-\pi\) and \(-\pi\) respectively. Therefore, a simple phase analysis such as evaluating from maximum value of cross correlation coefficient would yields phase lag value near \(\pi\) or \(-\pi\). Now let’s observe more in detail the time varying data presenting longer flat part in Fig.7(c). The phase lag is about \(2\pi\) within the vortex shedding numbers around 0 to 30, and is about \(\pi\) near the vortex shedding number 50. Then it becomes \(-\pi\) after the vortex shedding number around 80. These facts are supporting the phase lag switching in between \(\pi\) and \(2\pi\) as mentioned in Fig.3(c) of the section 3.2 of this article. Moreover, in these oscillation frequencies, there also are cases of phase lag continuously decreases, and there are periods of no synchronization between the reference signal and the velocity signal. As shown in Fig.7(f), similar phenomenon is occurring in the case of \(f_f = 0.230\text{Hz}\) excluding the fact that the phase lag is increasing.

On the other hand, when the forced oscillation frequency is 0.220Hz which is same as the natural vortex shedding frequency shown in Fig.7(d) and when it is 0.225Hz shown in (e) of the figure, presenting results with no change in phase lag. Therefore, at these forced oscillation frequencies, synchronization of the forced oscillation with the vortex shedding is said to be very strong. Moreover, at these forced oscillation frequencies, flat parts are appeared near the phase lag 0, \(2\pi\) and \(4\pi\), and hence a simple phase lag analysis will yields phase lag value near 0 (or \(2\pi\)). From these facts, it can be concluded that the phase lag is shifted by \(\pi\) when the forced oscillation frequency is varied from 0.215Hz to 0.220Hz. Also in these forced oscillation frequency range, the phase lag variation change from decreasing trend to increasing trend. Furthermore, although at the complete lock-in condition namely the forced oscillation frequency is set at 0.220Hz of the natural vortex shedding frequency, forced oscillation and vortex shedding are considered to completely synchronize in general sense, phase lag shifts (slips) intermittently by \(2\pi\) as apparent from the Fig.7(d).

![Fig.8 Occurrence rate of phase synchronization between forced oscillation frequency and vortex shedding frequency](image-url)
3.6 Appearance time rate of phase synchronization

Fig. 8 shows the appearance time rate of the flat portion (time zone of the synchronization) as shown in Fig. 7 of phase lag time variation for each forced oscillation frequency. From the figure, the maximum time rate of synchronization between the reference signal and velocity signal is around 80% at the forced oscillation frequency of 0.220 Hz. And taking this frequency at the middle, the time rate is distributed like a normal distribution. Therefore, it can be highlighted again that the probability of synchronization between the forced oscillation and the vortex shedding becomes higher as the forced oscillation frequency is closer to the natural vortex shedding frequency.

4. Conclusions

From present experiment, following facts are disclosed in regard to the phase lag between the oscillation of the test body and the velocity fluctuation due to the vortex shedding.

(1) Phase shift of the vortex shedding while the forced oscillation frequency is varied from about 0.9 to 1.1 times the natural vortex shedding frequency goes by transition stage associated with switching phenomenon and was found to be about $\pi$.

(2) Even in the lock-in region, forced oscillation and vortex shedding do not completely synchronize.

(3) As forced oscillation is closer to the complete lock-in condition, the oscillation and vortex shedding are more synchronized, and the maximum rate of complete synchronization is around 80% at the complete lock-in condition.

(4) Within a lock-in region, phase shift exhibits decreasing trend when the forced oscillation frequency is lower than the natural vortex shedding frequency, but exhibits increasing trend when the forced oscillation frequency is higher than the natural vortex shedding frequency.

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Nomenclature

$f_f$: Forced oscillation frequency
$h$: Base dimension of the right isosceles triangular cross-sectioned prism
$PDD$: Probability density distribution
$x$: Main stream wise (longitudinal) coordinate
$y$: Transverse coordinate

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