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Pursuing an evader through cooperative relaying in multi-agent surveillance networks [*]

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Abstract

We provide a distributed control strategy for each mobile agent in a surveillance network in the plane to cooperatively pursue an evader. The pursuit task is relayed from one agent to another when the evader crosses the boundary of the Voronoi regions divided according to the agents' positions. The dynamics of the resulted cooperative relay-pursuit network are described by a novel model of impulsive systems. As a result, to guarantee the stability of the closed-loop network system, the controllers' gains are chosen effectively using the solution of an algebraic Riccati equation. The proof of the stability is based on the construction of a switched Lyapunov function. We also show that the proposed controller is able to deal with delays if some sufficient conditions in the form of a set of linear inequalities are satisfied. A numerical example is provided to validate the performance of the proposed controller.

Key words: Multi-agent systems, cooperative relay pursuit, network access and computational delays, impulsive systems

1 Introduction

Distributed coordination of mobile agents has attracted increasing attention in recent years due to its wide range of applications, such as distributed tracking, cooperative surveillance, and intrusion detection [2–4, 6, 12, 13, 19, 22]. In particular, tracking and surveillance have given rise to especially important research problems for distributed cooperative control for multi-agent systems. In this context, the agents are usually required to move to their desired positions within a given deployment area where a mobile target of interest is moving around [4, 8, 11, 25]. As a result of the growing research interest, a number of papers have appeared addressing the multi-agent tracking and surveillance problem from different angles. The distributed consensus tracking control of multi-agent systems is studied in [13] and [18] for first-order agent dynamics under a dynamically changing environment. The authors of [19] prove that consensus tracking for such multi-agent systems can

be achieved if and only if the time-varying network topology contains a directed spanning tree jointly as the network evolves over time. The distributed relay pursuit of a maneuvering target in the plane is investigated in [1], where the Voronoi-like partition approach is used to solve such a relay-pursuit problem. In [8], [10], [11], the tracking control for second-order agent dynamics is investigated by using Lyapunov-like functions to check the systems' invariant sets. Distributed controllers for general linear agent dynamics are designed in [15] and [23], where the network topologies are assumed to be fixed, while in comparison the network topologies considered in [13], [18], [19], [8], [10], [11] are changing. Delays have not been taken into account in all of the above listed papers except for [18]. To deal with delays, Lyapunov-Krasovskii functionals and inequality techniques are used in [16] and [25], respectively. In addition, multi-agent surveillance and distributed environmental monitoring are investigated in [22], [21] and [5]. In [4], with the help of Voronoi partitions, Cortés et al. study the coverage control problem for sensor networks.

It is important to note that the majority of the existing literature on tracking control for multi-agent systems is exclusively devoted to smooth agent dynamics. However, in practice, the closed-loop system dynamics are very likely not to be smooth due to different reasons. For instance, the state of an agent may change abruptly when its interacting agents

drop out or an intruding target enters its field of view. Thus, impulsive system models become promising to describe precisely such scenarios that are challenging to be delineated otherwise. It is also worth noting that some works have been done along this line of research. In [14], [24], impulsive control strategies for multi-agent systems with second-order agent dynamics are considered, while in [7], the first-order agent dynamics are investigated. However, those results cannot be applied directly to systems with general linear agent dynamics. In addition, the information used for control may be constrained by several factors, e.g. local information determined by the Voronoi diagram, possible delays in communication and computation etc. Hence, it is in urgent need to design new control strategies for multi-agent tracking and surveillance when modeling the overall closed-loop non-smooth dynamical systems under information constraints.

In this paper, a distributed control strategy is designed for each agent in a surveillance multi-agent network in the plane to cooperatively pursue an evader. The pursuing task is relayed from one agent to another. A novel switched impulsive system is built to describe such a relay-pursuit problem. Although similar problem settings have been considered before in [1], we consider more general linear agent dynamics in this paper than those of the single integrator type used in [1]. The controller gain in our paper is obtained by solving an algebraic Riccati equation (ARE), which is much easier to implement in practice. In addition, network access and computational delays are also taken into account in this paper. Note that the delay phenomenon is still not considered in most of the related literature [14], [7] and [24], and that in [24], the controller design method is not provided. So, compared with the existing literature, our main contributions can be summarized as follows: First, a novel cooperative relay pursuit strategy for agents with general linear dynamics is proposed using Voronoi partitioning, under which only a preset number of agents need to track the evader. Second, a novel impulsive system model is built for the closed-loop network system, for which the relay pursuit control is obtained by solving an ARE.

The rest of this paper is organized as follows. In Section 2, the problem is formulated. In Section 3 and Section 4, the cooperative relay pursuit problem without and with delays are investigated, respectively. A simulation example is presented in Section 5 followed by the section of concluding remarks.

2 Problem formulation

In this paper, we study the cooperative relay pursuit problem in a bounded area in the plane. We distribute N_a mobile agents in this area at distinct locations by partitioning the area into N_a Voronoi cells, each containing an agent monitoring this cell. This can be done after solving the dynamic Voronoi-like partition problem [1], where the positions of the agents are taken as the corresponding Voronoi sites. We call all these agents *monitoring agents*. We consider the

problem, when an evader intrudes this area, how to guide a preset number of agents to cooperate with each other to catch such an intruder. We call those agents carrying out the pursuing task the *pursuers*.

Within this region $\mathcal{S}$, we now have two types of agents: pursuers and monitoring agents. The roles of the pursuers and monitoring agents may switch back and forth. This can be thought of as the police-thief game, in which the monitoring agents are mobile police stations, a Voronoi cell reflects the effective range of a police station, and the pursuers are the policemen with the mission to capture the thief. Once an evader enters this monitoring region $\mathcal{S}$, a predefined number of agents which are nearest to it will cooperate to catch it.

Suppose that the agents placed in the monitoring region $\mathcal{S}$ are identical and the kinematic equation of agent i is given by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \ i \in \mathcal{N}_a := \{1, \ldots, N_a\}, \qquad (1)$$

where $x_i(t) \in \mathbb{R}^2$ and $u_i(t) \in \mathbb{R}^m$ represent the position and the control input of agent i, respectively, and $A \in \mathbb{R}^{2 \times 2}$, $B \in \mathbb{R}^{2 \times m}$ are constant matrices with $\mathbb{R}$ denoting the set of real numbers. The motion of the evader is described by

$$\dot{x}_0(t) = Ax_0(t), \qquad (2)$$

where $x_0(t) \in \mathbb{R}^2$ denotes the position of the evader.

We make the following standard assumption.

Assumption 1 *The pair (A, B) is stabilizable.*

The precise goal of this paper is then to design a cooperative relay pursuit strategy that can ensure the tracking agents to effectively pursue the evader in the monitoring region. In such a strategy, only a preset number $N < N_a$ of pursuers need to track the evader, while the others are kept stationary.

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ be an undirected graph, where $\mathcal{V} = \{1, \ldots, N\}$ is the set of all the indices of N vertices, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of the edges. The graph is used to describe the neighbor relationships of the N pursuers in the cooperative pursuit network where each vertex corresponds to a pursuer. The set of neighbors of vertex i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}, j \neq i\}$. An edge of $\mathcal{G}$ is denoted by e_{ij}, which in our context of the multi-agent cooperative pursuit problem means that pursuers i and j can exchange information with each other. $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ denotes the adjacency matrix. The evader is labeled by vertex 0 and the neighbor of this evader can sense the target in real time. Then, we have a graph $\bar{\mathcal{G}}$, for both of the N pursuers and the moving evader. A diagonal matrix $\mathcal{D} = \{d_1, \ldots, d_N\}$, is specified by its diagonal elements $d_i = \sum_{j \in \mathcal{N}_i} a_{ij}$. The Laplacian of the graph is defined by $\mathcal{L} = \mathcal{D} - \mathcal{A}$.

Apparently, in such a relay cooperative pursuit control problem, the pursuers are not fixed. Thus, results from graph

theory on fixed graphs will not be applicable, and one has to deal with the switching topologies. The dependence of the graphs upon time can be characterized by a left piecewise continuous function $\sigma(t) : [0, \infty) \to \mathcal{P} = \{1, \ldots, m\}$. Here, m denotes the total number of all possible topologies of the multi-agent system during the pursuit. The relationship $\sigma(t_s) = i$ and $\sigma(t_s^+) = j$ implies that the topology switches from the ith to the jth at the time instant t_s.

Now, we define a set of piecewise continuous functions indicating at time t, the mapping to the indices of the pursuers

$$\alpha_i(t) : [0, +\infty) \to \mathcal{N}_a, i = 1, \ldots, N, \qquad (3)$$

where $\alpha_i(t) \neq \alpha_j(t), \forall i \neq j$ at any time instant t.

Research problem: design a cooperative relay pursuit control law $u_{\alpha_i(t)}(t)$ such that N pursuers among the monitoring agents will pursue the evader successfully.

Formally, we say the relay pursuit strategy is successful if, there exist a time-varying subset $\Omega_t \subset \mathcal{N}_a$ of N pursuers and a control input $u_{\alpha_i(t)}(t)$ such that

$$\lim_{t \to \infty} \|x_{\alpha_i(t)}(t) - x_0(t)\| = 0, i = 1, \ldots, N, \qquad (4)$$

for the pursuers and the evader in this monitoring region.

The control protocol is designed to be

$$
\begin{aligned}
u_{\alpha_i(t)}(t) = K \Big\{ & \sum_{\alpha_j(t) \in \mathcal{N}_{\alpha_i(t)}} a_{ij}(t)(x_{\alpha_j(t)}(t) - x_{\alpha_i(t)}(t)) \\
& + d_{\alpha_i(t)}(t)(x_0(t) - x_{\alpha_i(t)}(t)) \Big\},
\end{aligned}
\qquad (5)
$$

where K is the feedback gain to be designed later, $i, j \in \mathcal{V}$, and $a_{ij}(t) = 1$ if $\alpha_j(t) \in \mathcal{N}_{\alpha_i(t)}$, otherwise $a_{ij}(t) = 0$; $d_{\alpha_i(t)}(t) = 1$ if $\alpha_i(t)$ is the group leader, otherwise $d_{\alpha_i(t)}(t) = 0$.

In this paper, we will consider the performance of the proposed cooperative relay pursuit controller without and with network access and computational delays. And we report the obtained results in two separate sections.

3 Cooperative relay pursuit without delay

In this section, we will discuss the cooperative relay pursuit problem without delays. To simplify the notation, sometimes α_i and a_{ij} without explicitly indicating the time are used to refer to $\alpha_i(t)$ and $a_{ij}(t)$, respectively.

Let us define the position error between the ith pursuer and the evader by $\varepsilon_{\alpha_i}(t) = x_{\alpha_i}(t) - x_0(t)$. Then the error

dynamics of agent α_i under (5) can be described by

$$
\begin{aligned}
\dot{\varepsilon}_{\alpha_i}(t) =& A\varepsilon_{\alpha_i}(t) + Bu_{\alpha_i}(t) \\
=& A\varepsilon_{\alpha_i}(t) + BK \sum_{\alpha_j \in \mathcal{N}_{\alpha_i}} a_{ij}(\varepsilon_{\alpha_j}(t) - \varepsilon_{\alpha_i}(t)) \\
& - BKd_{\alpha_i}(t)\varepsilon_{\alpha_i}(t)
\end{aligned}
\qquad (6)
$$

By introducing $\varepsilon(t) = (\varepsilon_{\alpha_1}^T(t), \ldots, \varepsilon_{\alpha_N}^T(t))^T$, which is the collection of position disagreement vector between the pursuers and the evader, we rewrite the closed-loop overall system dynamics into

$$\dot{\varepsilon}(t) = (I_N \otimes A)\varepsilon(t) - (H_{\sigma(t)} \otimes BK)\varepsilon(t), t \neq t_s \quad (7a)$$

$$\|\varepsilon(t^+)\| = \gamma_s \|\varepsilon(t)\|, t = t_s \qquad (7b)$$

where $H_{\sigma(t)} = \mathcal{L}_{\sigma(t)} + \mathcal{D}_{\sigma(t)}, \mathcal{D}_{\sigma(t)} = \text{diag}\{d_{\alpha_1}(t), \ldots, d_{\alpha_N}(t)\}$, and $\gamma_s \in (0, 1]$ is a positive scalar characterizing the relationship between the norms of $\varepsilon(t)$ before and after the switching instant t_s. We use a set Θ to denote all the possible values of γ_s. Here, "$\otimes$" denotes the Kronecker product, $\varepsilon(t^-) = \lim_{h \to 0^+}(t - h), \varepsilon(t^+) = \lim_{h \to 0^+} \varepsilon(t + h)$, and $\varepsilon(t_s^-) = \lim_{h \to 0^+} \varepsilon(t_s - h) = \varepsilon(t_s)$, which follows from the fact that system (7) is left continuous, $t_0 < t_1 < \cdots < t_s < \cdots < \infty, t_s \to \infty$ as $s \to \infty$.

It is assumed that only the nearest pursuer can measure the position of the evader. This agent is called the *group leader*. All the other pursers only know the position of this group leader. When the evader enters a new Voronoi cell, it is natural to relay the role of the group leader to the new nearest pursuer, namely that agent associated with the new Voronoi cell. Fig. 1 gives an illustration of such a role changing, where we have four agents monitoring the region, and three pursuers pursuing the evader. Figure a shows the initial topology, and Figures $b-d$ show the possible changing topologies in the relay pursuit process. It is assumed that the topology does not change before the evader enters a new Voronoi cell, i.e., the topology is fixed in the time interval $[t_i, t_{i+1})$.
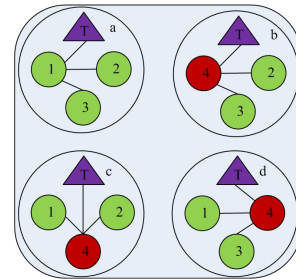

Fig. 1. Illustration of the changing topology in the relay pursuit process

Remark 1 *Note that the distance between the "group leader" and the evader usually changes with time in the*

time interval $[t_i, t_{i+1})$, and may become bigger than those between other tracking agents and the evader. Taking sub-figure (a) for example, for some time $t \in [t_i, t_{i+1})$, the distance between agent 2 (or 3) and the evader may be smaller than the distance between agent 1 and the evader. However, we enforce that the role of group leader is fixed during such a time interval $[t_i, t_{i+1})$ and consequently the topology of the multi-agent system remains fixed within each interval.

Remark 2 *Setting N=1, one can recover the single-pursuer case in our setting. Note that although in practical applications, sometimes more than one pursuer is needed, e.g. to conquer the evader, having more pursuers does not necessarily lead to faster capturing.*

To analyze the properties of system (7), we first review some results on the H_i matrices that have been established in the existing literature.

Lemma 3 ([17]) *(1) The matrices $H_i, i \in \mathcal{P}$, have non-negative eigenvalues.*
(2) The matrices $H_i, i \in \mathcal{P}$, are positive definite if and only if the graph $\bar{\mathcal{G}}$ is connected.

The following assumption is adopted throughout this paper.

Assumption 2 *The graph $\bar{\mathcal{G}}$ is connected.*

With this additional assumption, some stronger properties of the H_i matrices have been proved in the literature.

Lemma 4 ([20]) *Under Assumption 2, for any positive constant $0 < \epsilon < 2 \min_{i \in \mathcal{P}} \lambda_{min}(H_i)$, there exist positive definite matrices $\bar{P}_i \in \mathbb{R}^{N \times N}, \bar{Q}_i \in \mathbb{R}^{N \times N}(i \in \mathcal{P})$ such that*

$$\bar{P}_i H_i + H_i^T \bar{P}_i - \epsilon \bar{P}_i = \bar{Q}_i > 0, i \in \mathcal{P} \tag{8}$$

Now, for convenience, we define some notations. Define β_1, β_2 by

$$\beta_1 = \min_{i \in \mathcal{P}} \lambda_{\min}(\bar{P}_i), \beta_2 = \max_{i \in \mathcal{P}} \lambda_{\max}(\bar{P}_i), \tag{9}$$

where $\lambda_{\min}(\bar{P}_i)$ and $\lambda_{\max}(\bar{P}_i)$ are the minimum and maximum eigenvalues of the matrix $\bar{P}_i$, respectively.

During the pursuit, the partitions of such a monitoring region are dynamic. We use $\chi_t^{i,j} \subseteq \mathbb{R}^2$ to denote the moving cell boundary line in the plane, where $\chi_t^{i,j} := \{x \in \mathcal{S} : |x - a_i(t)| = |x - a_j(t)|\}(i \neq j, i, j \in \mathcal{N}_a)$, for $t \geq 0$. Thus, at time t, the line $\chi_t^{i,j}$ divides $\mathcal{S}$ into two open half-planes, namely, $P_t^i(a_i(t), a_j(t)) := \{x \in \mathcal{S} : |x - a_i(t)| < |x - a_j(t)|\}$ and $P_t^j(a_i(t), a_j(t)) := \{x \in \mathcal{S} : |x - a_i(t)| > |x - a_j(t)|\}$. If the moving evader lies on the boundary line at some time t, then the above mentioned role switching will be difficult to be determined. This may lead to the well known

undesirable Zeno behavior. In order to avoid the occurrence of Zeno behavior, we redesign the moving line as in [1] to be $\chi_{t,\varpi}^{i,j} := \{x \in \mathcal{S} : \big||x - a_i(t)| - |x - a_j(t)|\big| \leq \varpi\}$, where $\varpi > 0$ is a hysteresis constant. Then, $P_t^i(a_i(t), a_j(t))$ and $P_t^j(a_i(t), a_j(t))$ are modified as $P_{t,\varpi}^i(a_i(t), a_j(t)) := \{x \in \mathcal{S} : |x - a_i(t)| < |x - a_j(t)| - \varpi\}$ and $P_{t,\varpi}^j(a_i(t), a_j(t)) := \{x \in \mathcal{S} : |x - a_i(t)| > |x - a_j(t)| + \varpi\}$, respectively. In such a scheme, the farthest pursuer will not be replaced as long as the evader remains inside the set $\bigcup_{i = \alpha_1, \dots, \alpha_N} P_{t,\varpi}^i \cup \chi_{t,\varpi}^{i,j}$.

Remark 5 *With the help of the hysteresis constant ϖ, if the proposed cooperative pursuit strategy is successful, then there exists $T < \infty$, such that $\forall t \geq T$, the elements of Ω_t do not change with time anymore.*

Since (A, B) is stabilizable, there exist positive definite matrices $R_0 > 0$ and $Q_0 > 0$ such that the following ARE has a symmetric positive solution P_0 to

$$P_0 A + A^T P_0 - P_0 B R_0^{-1} B^T P_0 = -Q_0. \tag{10}$$

The matrix K in (5) is given by

$$K = \frac{1}{\epsilon} R_0^{-1} B^T P_0. \tag{11}$$

Then, for system (7), we have the following result.

Theorem 6 *Suppose Assumptions 1 and 2 are satisfied. Let $\bar{P}_i > 0, P_0 > 0$ and K be the solutions to (8), (10) and (11), respectively. Assume further the following inequality holds,*

$$\lambda_{\max}(\bar{P}_i \otimes P_0)\gamma_s^2 \leq \lambda_{\min}(\bar{P}_j \otimes P_0), i, j \in \mathcal{P}, \gamma_s \in \Theta \tag{12}$$

Then, under the control law (5), the cooperative relay pursuit problem of system (7) is solved.

Proof: We rewrite system (7) as

$$\dot{\varepsilon}(t) = \bar{A}\varepsilon(t) - \bar{B}_{\sigma(t)}\varepsilon(t), t \neq t_s \tag{13a}$$
$$\|\varepsilon(t^+)\| = \gamma_s \|\varepsilon(t)\|, t = t_s \tag{13b}$$

where $\bar{A} = I_N \otimes (A - \frac{1}{2}\epsilon BK)$ and $\bar{B}_{\sigma(t)} = (H_{\sigma(t)} - \frac{1}{2}\epsilon I_N) \otimes BK$.

Then, consider the following switched Lyapunov function

$$V(t) = \varepsilon^T(t)(\bar{P}_{\sigma(t)} \otimes P_0)\varepsilon(t). \tag{14}$$

The Dini derivative of $V(t)$ along the trajectories of system (13) is given by

$$\begin{aligned} D^+ V(t) =& \varepsilon^T(t)\bar{P}_{\sigma(t)} \otimes (P_0 A + A^T P_0)\varepsilon(t) \\ &- \varepsilon^T(t)\bar{P}_{\sigma(t)} \otimes P_0 B R_0^{-1} B^T P_0 \varepsilon(t) \\ &- \frac{1}{\epsilon}\varepsilon^T(t)\bar{Q}_{\sigma(t)} \otimes P_0 B R_0^{-1} B^T P_0 \varepsilon(t), \end{aligned} \tag{15}$$

4

where we have used the properties $(A + B) \otimes C = A \otimes C + B \otimes C, (A \otimes B)(C \otimes D) = (AC) \otimes (BD)$ and $(A \otimes B)^T = A^T \otimes B^T$ of the Kronecker product [9], for A, B, C with appropriate dimensions. Since $\bar{Q}_{\sigma(t)} > 0$ and $P_0 B R_0^{-1} B^T P_0 \geq 0$, we have that

$$D^+ V(t) \leq -\delta \varepsilon^T(t) \bar{P}_{\sigma(t)} \otimes P_0 \varepsilon(t) = -\delta V(t), \quad (16)$$

where $\delta = \lambda_{min}(Q_0)/\lambda_{max}(P_0)$. The (16) holds due to the fact that

$$-Q_0 \leq -\lambda_{min}(Q_0) I = -\delta \lambda_{max}(P_0) I \leq -\delta P_0.$$

Then, from (12), when $t = t_s$ we have

$$\begin{aligned} V(t^+) - V(t) \leq &\lambda_{\max}(\bar{P}_{\sigma(t^+)} \otimes P_0) \varepsilon^2(t^+) \\ &- \lambda_{\min}(\bar{P}_{\sigma(t)} \otimes P_0) \varepsilon^2(t) \leq 0 \end{aligned} \quad (17)$$

Suppose that the topology of system (7) switches s times at $t_1 < t_2 \cdots < t_s \leq t$ in the time interval $[t_0, t)$. From the above computation, we have

$$\begin{aligned} D^+ V(q) &\leq -\delta V(q), t_{i-1} < q < t_i, \\ V(q^+) &\leq V(q), q = t_{i-1}. \end{aligned}$$

Then, it follows that

$$\begin{aligned} V(t) &\leq V(t_s) e^{-\delta(t-t_s)} \\ &\leq V(t_{s-1}^+) e^{-\delta[(t-t_s)+(t_s-t_{s-1})]} \\ &\vdots \\ &\leq V(t_0) e^{-\delta[(t-t_s)+(t_s-t_{s-1})+\cdots+(t_1-t_0)]} \\ &= V(t_0) e^{-\delta(t-t_0)} \end{aligned} \quad (18)$$

From the definition of $V(t)$ in (14), it is clear that

$$\kappa_1 \|\varepsilon(t)\|^2 \leq V(t) \leq \kappa_2 \|\varepsilon(t)\|^2 \quad (19)$$

where $\kappa_1 = \beta_1 \lambda_{\min}(P_0)$ and $\kappa_2 = \beta_2 \lambda_{\max}(P_0)$. According to (18) and (19), we obtain that

$$\|\varepsilon(t)\| \leq \sqrt{\kappa_2/\kappa_1} \|\varepsilon(t_0)\| e^{-\frac{1}{2}\delta(t-t_0)}, \quad (20)$$

which implies that the cooperative relay pursuit problem of system (7) is solved. The proof is completed. ■

Remark 7 *It is worth mentioning that in Theorem 6, only an asymptotic result is obtained. Such a result is rather conservative for the relay pursuit problem because the evader will only be guaranteed to be tracked as time goes to infinity. Apparently, it would be more desirable if one can obtain a finite-time result; however, such finite-time convergence result will require stronger conditions that deserve to be looked into in the future.*

Remark 8 *From (20), if we choose $T = (2\ln\|\varepsilon(t_0)\| - 2\ln(\sqrt{N}\kappa_1/\kappa_2\varpi))/\delta$, then, $\forall t \geq T, \|x_{\alpha_i}(t)-x_0(t)\|_{t\geq T} \leq \varpi(i \in \mathcal{V})$. This indicates that the set Ω_t will not change $\forall t \geq T$.*

Remark 9 *System (7) is actually an impulsive switched system. In order to analyze such a system, we have constructed a new switched Lyapunov function that dependents on the solution of an ARE and the topology of the multi-agent system. Based on such an ARE, a design method to properly select the controller gain has been proposed.*

Remark 10 *When an evader moves into a new Voronoi cell, the pursuer that is the farthest away from it will be replaced by a new agent and becomes a monitoring agent. This means that at this time instant, the distances between the pursuers and the evader change abruptly. In order to describe such a situation, an impulsive system has been introduced. As a matter of fact, at the jumping point the tracking error $\|\varepsilon(t)\|$ decreases, which indicates $\gamma_s \leq 1$. This can also be thought of as one kind of "event trigger", because the time instants are determined by the distance change rather than the time.*

According to *Remark 10*, if we choose $\bar{P} = \bar{P}_i(i \in \mathcal{P})$ and in addition if $\gamma_s \in (0, \delta_0), \delta_0 = \sqrt{\frac{\lambda_{\min}(\bar{P} \otimes P_0)}{\lambda_{\max}(\bar{P} \otimes P_0)}}$, we readily have the following corollary.

Corollary 11 *Suppose Assumptions 1 and 2 are satisfied. Let $\bar{P} = \bar{P}_i(i \in \mathcal{P}) > 0, P_0 > 0$ and K be the solutions to (8), (10) and (11), respectively. Then, under the control law (5), if in addition $\gamma_s \in (0, \delta_0)$, the cooperative relay pursuit problem of system (7) is solved.*

4 Cooperative relay pursuit with delay

In the previous section, we have discussed the cooperative relay pursuit problem for the multi-agent system without delays. It should be noted that the *group leader* communicates the position of the evader to the other pursuers through the network and this will bring network access delays. In addition, each agent needs time to compute the local control algorithm, which will lead to computational delays. Thus, in the tracking process, delays must be considered in the analysis of such systems. In this paper, for conciseness of discussion, the delays are considered to be identical and fixed. If the network access and computational delays are taken into account, the control input of system (1) is given as

$$\begin{aligned} u_{\alpha_i}(t) = K\Big\{ &\sum_{\alpha_j \in \mathcal{N}_{\alpha_i}} a_{ij}(x_{\alpha_j}(t-\tau) - x_{\alpha_i}(t-\tau)) \\ &+ d_{\alpha_i}(x_0(t-\tau) - x_{\alpha_i}(t-\tau))\Big\}, \end{aligned} \quad (21)$$

where the controller gain has been designed in Section 3. Then, the overall system (1) can be described by the follow-

ing system

$$\dot{\varepsilon}(t) = (I_N \otimes A)\varepsilon(t) - \bar{H}_{\sigma(t)}\varepsilon(t-\tau), t \neq t_s \quad (22a)$$

$$\|\varepsilon(t^+)\| = \gamma_s \|\varepsilon(t)\|, t = t_s \quad (22b)$$

$$\varepsilon(\theta) = \varphi(\theta), \theta \in [-\tau, 0] \quad (22c)$$

where $\bar{H}_{\sigma(t)} = (H_{\sigma(t)} \otimes BK)$, and $\varphi(\theta)$ is the initial condition defined in time interval $[-\tau, 0]$.

Then, for system (22), we have the following result.

Theorem 12 *Suppose Assumptions 1 and 2 are satisfied. Let K be the solution of* (11)*, and assume there exist positive definite matrices $P_i > 0, Q_i > 0, R_i > 0 (i \in \mathcal{P})$ and matrices $N_i = [I_N \otimes N_1^{iT} \ I_N \otimes N_2^{iT}]^T, i \in \mathcal{P}$, such that the following conditions satisfied*

$$\begin{bmatrix} \Xi_{11}^i & \Xi_{12}^i & \tau I_N \otimes N_1^i \\ \Xi_{12}^{iT} & \Xi_{22}^i & \tau I_N \otimes N_2^i \\ \tau I_N \otimes N_1^{iT} & \tau I_N \otimes N_2^{iT} & -\tau I_N \otimes R_i \end{bmatrix} < 0, i \in \mathcal{P}; \quad (23)$$

$$\lambda_{\max}(P_i \otimes P_0))\gamma_s^2 \leq \lambda_{\min}(P_j \otimes P_0), i, j \in \mathcal{P}, \gamma_s \in \Theta \quad (24)$$

where

$$\Xi_{11}^i = I_N \otimes (P_i A + A^T P_i + Q_i + N_1^i + N_1^{iT} + \tau A^T R_i A),$$
$$\Xi_{12}^i = -H_i \otimes P_i BK - \tau H_i \otimes A^T R_i BK + I_N \otimes (N_2^{iT} - N_1^i),$$
$$\Xi_{22}^i = -I_N \otimes (Q_i + N_2^i + N_2^{iT}) + \tau H_i^T H_i \otimes K^T B^T R_i BK.$$

Then, the cooperative relay pursuit problem of system (22) *is solved.*

Proof: Consider the following Lyapunov-Krasovskii functional candidate

$$V(t) = \varepsilon^T(t)(I_N \otimes P_{\sigma(t)})\varepsilon(t)$$
$$+ \int_{t-\tau}^t \varepsilon^T(s)(I_N \otimes Q_{\sigma(t)})\varepsilon(s)ds$$
$$+ \int_{-\tau}^0 \int_{t+\theta}^t \dot{\varepsilon}^T(s)(I_N \otimes R_{\sigma(t)})\dot{\varepsilon}(s)dsd\theta \quad (25)$$

The Dini derivative of (25) along the trajectory of system (22) is given by

$$D^+V(t) = \varepsilon^T(t)(I_N \otimes (P_{\sigma(t)}A + A^T P_{\sigma(t)} + Q_{\sigma(t)}))\varepsilon(t)$$
$$- 2\varepsilon^T(t)(H_{\sigma(t)} \otimes BK)\varepsilon(t-\tau)$$
$$- \varepsilon^T(t-\tau)(I_N \otimes Q_{\sigma(t)})\varepsilon(t-\tau)$$
$$+ \tau\varepsilon^T(t)(I_N \otimes A^T R_{\sigma(t)}A)\varepsilon(t)$$
$$- \tau\varepsilon^T(t)(H_{\sigma(t)} \otimes A^T BK)\varepsilon(t-\tau)$$
$$- \tau\varepsilon^T(t-\tau)(H_{\sigma(t)} \otimes K^T B^T R_{\sigma(t)}A)\varepsilon(t)$$
$$+ \tau\varepsilon^T(t-\tau)(H_{\sigma(t)}^2 \otimes K^T B^T R_{\sigma(t)}BK)\varepsilon(t-\tau)$$
$$- \int_{t-\tau}^t \dot{\varepsilon}^T(s)(I_N \otimes R_{\sigma(t)})\dot{\varepsilon}(s)ds \quad (26)$$

From the Newton-Leibnitz formula, for matrices N^i, we have

$$2[\varepsilon^T(t)(I_N \otimes N_1^i) + \varepsilon^T(t-\tau)(I_N \otimes N_2^i)]$$
$$\times [\varepsilon(t) - \int_{t-\tau}^t \dot{\varepsilon}(s)ds - \varepsilon(t-\tau)] = 0 \quad (27)$$

From (26) and (27), it holds that

$$D^+V(t) \leq \zeta^T(t)[\Xi_{\sigma(t)} + \tau N^{\sigma(t)T}(I_N \otimes R_{\sigma(t)}^{-1})N^{\sigma(t)}]\zeta(t), \quad (28)$$

where $\zeta(t) = [\varepsilon^T(t) \ \varepsilon^T(t-\tau)]^T$ and

$$\Xi_{\sigma(t)} = \begin{bmatrix} \Xi_{11}^{\sigma(t)} & \Xi_{12}^{\sigma(t)} \\ \Xi_{12}^{\sigma(t)T} & \Xi_{22}^{\sigma(t)T} \end{bmatrix}. \quad (29)$$

Then, from (23), it is easy to see $D^+V(t) < 0$. In the above, we have investigated the property of $V(t)$ in the whole timeline except for the switching time instants. Now, let us look at these time instants. In order to ensure the asymptotic stability, the following condition is required to be satisfied

$$V(t_s^+) - V(t_s) = \varepsilon^T(t_s^+)(I_N \otimes P_{\sigma(t_s^+)})\varepsilon(t_s^+)$$
$$- \varepsilon^T(t_s)(I_N \otimes P_{\sigma(t_s)})\varepsilon(t_s) \leq 0 \quad (30)$$

Then, from inequality (24), we have that (30) holds. The proof is completed. ∎

According to *Remark 10*, for system (22), we readily have the following corollary, which is similar to *Corollary 11*.

Corollary 13 *Suppose Assumptions 1 and 2 are satisfied. Let K be the solution of* (11)*, for a given constant $\tau > 0$, if there exist positive definite matrices $P > 0, Q > 0, R > 0$ and matrices $N = [I_N \otimes N_1^T \ I_N \otimes N_2^T]^T$, and in addition $\gamma_s \in (0, \bar{\delta}_0), \bar{\delta}_0 = \sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}$, such that the following condition is satisfied*

$$\begin{bmatrix} \hat{\Xi}_{11} & \hat{\Xi}_{12} & \tau I_N \otimes N_1 \\ \hat{\Xi}_{12}^T & \hat{\Xi}_{22} & \tau I_N \otimes N_2 \\ \tau I_N \otimes N_1^T & \tau I_N \otimes N_2^T & -\tau I_N \otimes R \end{bmatrix} < 0, i \in \mathcal{P} \quad (31)$$

where

$$\hat{\Xi}_{11} = I_N \otimes (PA + A^T P + Q + N_1 + N_1^T + \tau A^T TA),$$
$$\hat{\Xi}_{12} = -H_i \otimes PBK - \tau H_i \otimes A^T RBK + I_N \otimes (N_2^T - N_1),$$
$$\hat{\Xi}_{22} = -I_N \otimes (Q + N_2 + N_2^T) + \tau H_i^T H_i \otimes K^T B^T RBK.$$

Then, the cooperative relay pursuit problem of system (22) *is solved.*

Remark 14 *It should be noted that in the pursuit process, γ_s is not a constant, and its variation is usually hard to know in real time. Thus, although Theorem 12 provides a general method to solve the cooperative relay pursuit problem, it is in principle difficult to solve for the bound of the communication delay. In order to obtain the upper bound of the allowable communication delay,* Corollary 13 *can be utilized.*

5 Numerical Simulation

In this section, a numerical example will be given to show the effectiveness of the proposed method. We just consider the scenario with network access and computational delays; if there are no delays, similar results can also be obtained. The dynamics of the agents are describe by the following equation

$$\dot{x}_i(t) = \begin{bmatrix} 0 & 0.3 \\ 0.02 & 0.28 \end{bmatrix} x_i(t) + \begin{bmatrix} 1 \\ 1.2 \end{bmatrix} u_i(t), i \in \mathcal{N}_a \qquad (32)$$

Suppose there are 36 agents monitoring an area of $24 \times 24m^2$. The initial deployment of the agents is shown in Fig. 4. When an evader intrudes this area, it is assumed that 3 agents are assigned to track the evader. In Fig. 4, the agents are labeled in proper order from the left to right and from the bottom to top as $1, 2, \ldots, 36$. The value of ϵ in *Lemma 4* is given by $\min \lambda_{\min}(H_i)(i \in \mathcal{P}) = 0.2679$ and is computed according to all the possible topologies in the relay pursuit process. The $\bar{Q}_1, \bar{Q}_2, \bar{Q}_3, Q_0$ are selected as

$$\bar{Q}_1 = \begin{bmatrix} 5.7321 & -2 & -2 \\ -2 & 1.7321 & 0 \\ -2 & 0 & 1.7321 \end{bmatrix}, \bar{Q}_2 = \begin{bmatrix} 1.7321 & -2 & -2 \\ -2 & 5.7321 & -2 \\ -2 & 0 & 1.7321 \end{bmatrix},$$

$$\bar{Q}_3 = \begin{bmatrix} 1.7321 & -2 & -2 \\ -2 & 1.7321 & 0 \\ -2 & 0 & 5.7321 \end{bmatrix}, Q_0 = \begin{bmatrix} 18.9392 & 15.5126 \\ 15.5126 & 14.1297 \end{bmatrix},$$

and $R = 14$. Then, according to *Corollary 11*, the parameter δ_0 can be chosen as 0.9452, and the controller gain is designed as $K = [4.3432 \ 4.7290]$. In order to avoid the Zeno behavior, we choose $\varpi = 0.12$. As mentioned in *Remark 14*, we will use *Corollary 13* to solve for the maximum delay bound. Applying *Corollary 13*, it is obtained that the maximum delay bound is $\tau = 0.026$.

The tracking errors in x- and y-coordinates are given in Fig. 2. The norm of the tracking error is shown in Fig. 3. The tracking trajectory is shown in Fig. 4. In Fig. 5, the coordinates of each agents are given. From Fig. 5, it can be seen that three agents capture the evader successfully, and the three tracking agents overlap at the same point, which is indicated by a "+" in the figure. The replacements during the pursuit course

are shown in Fig. 6. At the time instants $4.464s, 6.178s$ and $7.302s$, the replacement occurs. In the time intervals $[0, 4.464s], (0.464s, 6.178s], (6.178s, 7.302s], [7.302s, 9s]$, the *group leader* during the pursuit are agents $1, 8, 15, 22$ respectively.
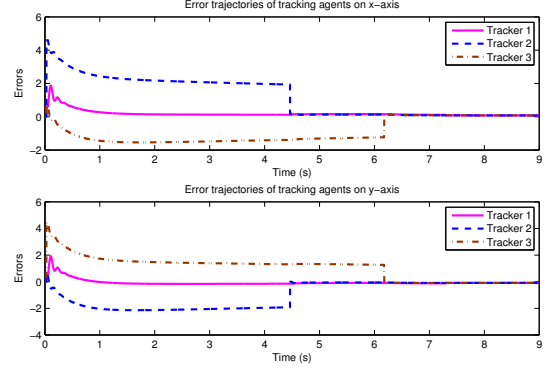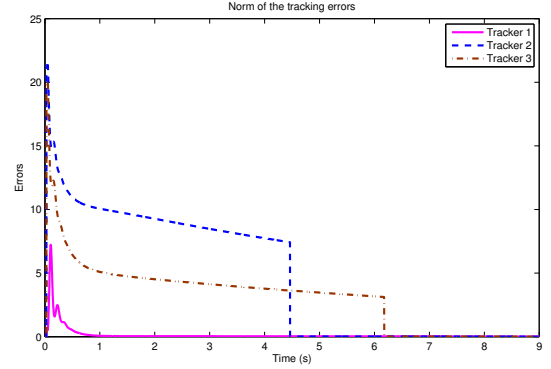

Fig. 2. Tracking errors on x-axis and y-axis

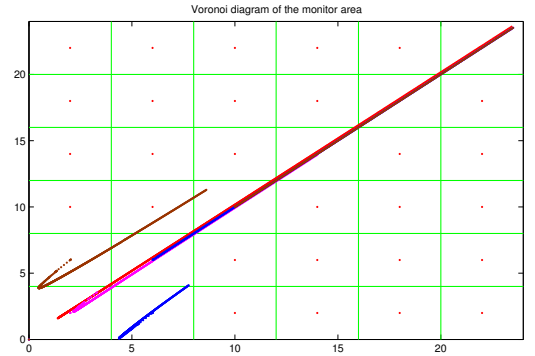Fig. 3. Norm of the tracking errors

Fig. 4. Initial deployment of agents and tracking trajectories

6 Conclusions

We have investigated the cooperative relay pursuit problem for distributed mutli-agent systems in the plane. With the help of Voronoi diagram, this plane has been divided into
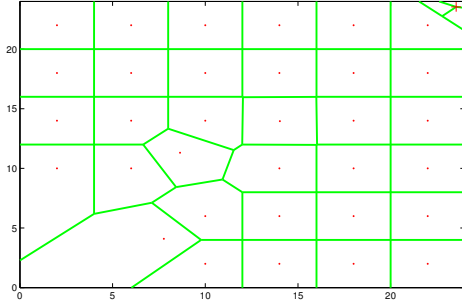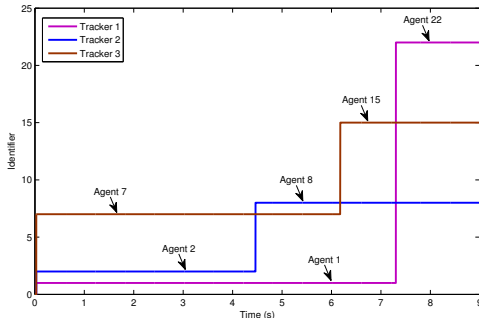
Fig. 5. Voronoi diagram after tracking

Fig. 6. Switching during the pursuit process

several "capture zones". A novel impulsive system model has been proposed in order to design controller for such a cooperative relay pursuit problem. The controller gain can be obtained by solving a Riccati equation. Sufficient conditions in terms of LMIs have been derived for the case with delays. An example has been given to show the effectiveness of the proposed method. For the ongoing work, we are looking into different nonlinear models for agent dynamics and considering to reformulate the control design using the event-triggered mechanism.

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