

University of Groningen

Robust Synchronization and Model Reduction of Multi-Agent Systems

Jongsma, Hidde-Jan

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version

Publisher's PDF, also known as Version of record

Publication date:

2017

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Jongsma, H-J. (2017). Robust Synchronization and Model Reduction of Multi-Agent Systems [Goningen]: Rijksuniversiteit Groningen

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

ROBUST SYNCHRONIZATION
AND MODEL REDUCTION
OF MULTI-AGENT SYSTEMS

HIDDE-JAN JONGSMA



university of
 groningen

The research for this doctoral dissertation has been carried out at the Faculty of Mathematics and Natural Sciences, University of Groningen, The Netherlands, within the Johann Bernoulli Institute for Mathematics and Computer Science.



This work is part of the research programme 'DISC Graduate Programme' with project number 022.002.025, which is financed by the Netherlands Organisation for Scientific Research (NWO).



The research reported in this dissertation is part of the research program of the Dutch Institute of Systems and Control (DISC). The author has successfully completed the educational program of the Graduate School DISC.

Robust Synchronization and Model Reduction of Multi-Agent Systems

Hidde-Jan Jongsma

PhD Thesis University of Groningen

Printed by Ipskamp Printing

ISBN 978-90-367-9553-1 (printed version)

ISBN 978-90-367-9552-4 (electronic version)



rijksuniversiteit
groningen

Robust Synchronization and Model Reduction of Multi-Agent Systems

Proefschrift

ter verkrijging van de graad van doctor aan de
Rijksuniversiteit Groningen
op gezag van de
rector magnificus prof. dr. E. Sterken
en volgens besluit van het College voor Promoties.

De openbare verdediging zal plaatsvinden op
vrijdag 17 februari 2016 om 14.30 uur

door

Hidde-Jan Jongsma

geboren op 11 januari 1989
te Heerenveen

Promotores

Prof. dr. H. L. Trentelman

Prof. dr. M. K. Camlibel

Beoordelingscommissie

Prof. dr. H. Sandberg

Prof. dr. ir. J. M. A. Scherpen

Prof. dr. K. Takaba

ACKNOWLEDGMENTS

This thesis is the culmination of a journey that started roughly ten years ago, when I first set foot on that hallowed ground that is called the Zernike campus. These past ten years I have lived, learned and worked in the wonderful city of Groningen. While the city itself is beautiful, the greatest thing about it is that here, I have had the opportunity to meet and collaborate with amazing people. These people have cared for me, motivated me and educated me. Without them, I would have never been able to make this thesis a reality.

I will forever be thankful for everything that my promoters Harry Trentelman and Kanat Camlibel have done for me. I met Harry and Kanat already during the first years of my Bachelor. They introduced me to linear algebra and control theory. Immediately, these subject captured my interest and attention. However, I could not have imagined then, that I would work so closely for four years with these two wonderful gentlemen.

Harry, you encouraged me to apply for the DISC Graduate Programme, which enabled me to pursue this PhD in the first place. You have taught me how to write and think like a mathematician. I am grateful for all our daily conversations on scientific and nonscientific topics. I will miss your humor and the intensity of your personality, but I hope that a bit of your admirable passion and decisiveness has rubbed off on me in all these years.

Kanat, you are one of the warmest people and one of the sharpest minds that I have ever had the pleasure to meet. You were always able to accommodate me and my quirks. You are an extremely hard worker and are full of brilliant and crazy ideas. I enjoyed all the moments that you, Harry, and I were staring at and writing things on the whiteboard. That, to me, is what research is all about. I wish you the best of luck, together with your wonderful wife Berfu.

I would like to thank the members of my assessment committee: Hendrik Sandberg, Jacquélien Scherpen, and Kiyotsugu Takaba, for their valuable comments. I thank Petar Mlinarić, Sara Grundel, and Peter Benner for the pleasant collaboration that resulted in Chapter 3 of this thesis. I am grateful for the people of DISC and the NWO for the funding that enabled this PhD. In particular, I would like to thank all the teachers of the excellent DISC courses.

Dear colleagues from the third and fourth floor of the Bernoulliborg, you have provided a wonderful environment for me during the past four years. I am thankful for all our day-to-day interactions and the support and liveliness that you have provided during my time here. Many thanks to our wonderful secretaries: Esmee, Helga, Ineke, and 'myn pop' Desiree.

Of course, I thank you, Tjerk, for agreeing to act as my paronymph, for being such a great office mate, and for our great time in Japan. I wish you all the best in your own life, marriage, and PhD.

Anneroos, I cannot thank you enough for being such a lovely colleague, friend, and travel mate. You were a joy to talk to and your office couch was always available when I couldn't see forest for the tree graphs. Thank you for granting me the honor of being your paronymph, and now for returning the favor.

I am extremely grateful for my friends and family. For my own personal gang of five and my pre-PhD colleagues. For my parents, Dirk and Froukje, who have always encouraged me to think for myself and be considerate of others. For my sister Jacomien and my brother Gerwolt, who always made me feel loved unconditionally, and for the wonderful people they have brought into my life. Finally, I am grateful for my 'new' family, who have embraced me since I met the love of my life.

Fleur, there isn't a person in the world that means as much to me as you do. I will forever be thankful that you said yes when I asked you if you were willing to share your life with me. I love what we have together and am eagerly awaiting everything the future still holds for us. I love you.

Hidde-Jan

Groningen, January 11th 2017

CONTENTS

1	INTRODUCTION	1
1.1	Synchronization	2
1.2	Model reduction	4
1.3	Outline of this thesis	7
1.4	Origin of the chapters	8
1.5	Notation	8
I	ROBUST SYNCHRONIZATION	
2	ROBUST SYNCHRONIZATION OF UNCERTAIN MULTI-AGENT SYSTEMS	13
2.1	Introduction	13
2.2	Preliminaries	14
2.3	Graphs	16
2.4	Multi-Agent Systems	17
2.5	Robust synchronization	18
2.6	Robustly synchronizing protocols	28
2.7	Numerical example	38
2.8	Conclusions	42
II	MODEL REDUCTION	
3	MODEL REDUCTION BY CLUSTERING	45
3.1	Introduction	45
3.2	Preliminaries	47
3.3	Problem formulation	51
3.4	Graph partitions and reduction by clustering	53
3.5	\mathcal{H}_2 -error bounds	59
3.6	\mathcal{H}_∞ -error bounds	68
3.6.1	The single integrator case	69
3.6.2	The general case with symmetric agent dynamics	73

3.7	Towards a priori error bounds for general graph partitions	77
3.8	Conclusions	83
4	MODEL SIMPLIFICATION BY CYCLE-REMOVAL	85
4.1	Introduction	85
4.2	Preliminaries	87
4.3	Consensus model	91
4.4	Model reduction error	97
4.5	Star graphs	111
4.6	Single integrator dynamics	116
4.7	Numerical examples	118
4.8	Conclusions	124
5	CONCLUSIONS	127
5.1	Contributions	127
5.2	Outlook	129
	BIBLIOGRAPHY	131
	LIST OF SYMBOLS	141
	SUMMARY	143
	SAMENVATTING	147

LIST OF FIGURES

Figure 2.1	Block diagram of the coprime factor perturbed transfer matrix of each agent.	20
Figure 2.2	Block diagram of a coprime factor perturbed agent.	20
Figure 2.3	Mass-spring system with two masses.	39
Figure 2.4	Undirected graph with $N = 10$ nodes.	41
Figure 2.5	Output of the first five agents.	41
Figure 3.1	Graph G with $N = 11$ nodes and a nontrivial almost equitable partition.	57
Figure 3.2	Reduced order graph \hat{G} with $N = 7$ nodes.	57
Figure 3.3	Modifying the line graph on 5 nodes to make the partition $\{\{1, 2, 3\}, \{4, 5\}\}$ almost equitable.	81
Figure 4.1	Graph on 7 nodes with star graph S_6 as spanning tree.	90
Figure 4.2	RL circuit with three inductors and four resistors.	118
Figure 4.3	Eigenvalues $\sigma_1, \sigma_2, \sigma_3$ of $A - BB^T$ for increasing R_4 .	121
Figure 4.4	Approximation error $\ S - \hat{S}\ _2^2$ per removed edge.	121
Figure 4.5	Graph on $N = 20$ nodes with 5 edge-disjoint cycles.	123
Figure 4.6	Relative approximation error versus $\omega_2(\mathcal{T})$ of all possible spanning trees.	123

