The Cost of Dishonesty on Optimal Distributed Frequency Control of Power Networks

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Abstract— **Optimal frequency controllers for power networks based on distributed averaging have previously been shown to be an effective means of distributing control authority among agents while maintaining a globally optimal operating point. Distributed control architectures however require an implicit trust between participating agents, in that each must faithfully communicate the appropriate control variables to neighboring agents. Here we study the case where some agents attempt to "cheat the system" by adding a bias to the averaging controller in order to lower their generation cost. We quantify the effect of this dishonesty on the resource allocation problem and introduce a "cost graph" whose weights measure the effect of the bias on the optimal equilibrium. Moreover, we propose an "honesty-enforcing" controller which counteracts the dishonest agents, and restores the optimal setpoint of the network.**

I. INTRODUCTION

The modern power system is currently evolving at a rapid pace, from a centrally-managed hierarchical system based on bulk generation, to a more decentralized system based on distributed energy resources. The most basic challenge in operating the grid is to continuously balance electrical generation and load, and the AC frequency of the grid provides a globally-available measurement signal indicating the supply-demand balance. Balancing is typically accomplished through a combination of decentralized frequency feedback control at bulk generators, along with centralized optimization by system operators using forecasts. However, as bulk generation is slowly replaced by distributed generation, control authority is more dispersed, and small-scale power electronic devices will shoulder an increasing amount of the responsibility for grid-wide frequency control and online optimization.

As a method for achieving the performance of centralized optimization while distributing frequency control responsibility among many small-scale devices, distributed frequency controllers based on distributed averaging (consensus) have been proposed by several authors [1]–[4]. These controllers fuse decentralized integral control with inter-unit cooperation: the integral action ensures that the grid frequency is

regulated, while cooperation ensures that the units converge to a common marginal economic cost, thereby minimizing the total cost of generation.

For global optimality to be achieved, each participating unit must cooperate with the other participants in good faith. It may however be beneficial for any particular unit to break this pact, and potentially gain an economic advantage over the other, more honest units. Global optimality — and even convergence — would no longer be guaranteed. Among the many possible scenarios, we consider in this paper the case in which the controllers broadcast biased marginal costs, and study the effect of this corrupted information on the system's equilibria and their stability.

This problem is related with consensus problems where malicious agents update their states *arbitrarily*, and a control must be designed for the honest agents that guarantees consensus; see [5] for first-order and [6] for second-order multi-agent systems. The control for each agent proposed in [5], [6] discards information from the neighbors which sensibly deviates from the current state of the agent. Similarities can also be established with the problem of stealthy attacks on cooperative systems investigated in [7]. There, uncertainties generated by stable dynamical systems are injected into the cooperative system to disrupt consensus. A hidden network is then deployed to compete against the attacker.

The role of corrupted measurements in control algorithms for power networks has been investigated in [8], where the impact of various measurements attack on voltage controllers are studied. Corrupting measurements or injecting perturbative terms in the dynamics are only two of the many possible ways to force a control system to mis-behave. Another possibility in the context of power networks could be to change the cost coefficients to enable power savings, a scenario that is related to the problem in [9]. In parallel, a number of studies have been devoted to the problem of detecting attacks or malicious behaviors in power networks. Among these many studies, we mention the geometric approach of [10], [11], and refer the interested reader to [12] for a comprehensive introduction to problems of security in network systems with emphasis on power networks.

Contribution. This paper investigates the effects of biased marginal cost broadcasts on the performance of distributed averaging integral controllers for optimal frequency regulation in structure-preserving models of power networks. Using established tools such as incremental passivity and energy functions, it is shown how controllers that broadcast biased information can take advantage of their dishonest action to

decrease their costs by lowering their power production and inducing the other honest participating units to compensate for the lack of generated power. In doing so, the overall demand-supply power balance is not altered — and thus frequency regulation is ultimately guaranteed — but the dispatch of power production between units is no longer optimal. To counteract this dishonesty, we propose a higher-level honesty-enforcing controller which neutralizes the action of dishonest units and restores the previous optimality condition in which all the agents, including the dishonest ones, fairly contribute to the optimal power dispatch. The honesty-enforcing controller can be implemented in a completely decentralized manner, using only known parameters. We illustrate our higher-level controller through a simple simulation. Due to lack of space, the proofs are omitted and will be presented elsewhere.

II. POWER NETWORK MODEL AND OPTIMAL DISTRIBUTED FREQUENCY CONTROL

This section provides the power network models and controllers, and reviews the convergence and optimality for the resulting closed-loop system *without* biased information.

A. Dynamical model of the power grid

We consider the Bergen-Hill structure-preserving model of a quasi-synchronous power network consisting of generators and frequency-dependent loads [13]. The topology of the grid is described by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V}$ is the set of nodes (buses) and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of edges (transmission lines). We partition the buses of $\mathcal{G}$ into two sets $\mathcal{V}_G$ and $\mathcal{V}_L$, corresponding to the set of generators and loads, respectively. The power generators can be either inverter-interfaced or synchronous generator units. The network operates around a nominal angular frequency ω^*, and to each bus we associate a phasor voltage $V_i \angle \theta_i$ with magnitude $V_i > 0$ and phase angle $\theta_i \in \mathbb{S}$. At each generator, the phase θ_i evolves according to the so-called *swing equation*

$$M_i \ddot{\theta}_i = -D_i \dot{\theta}_i - P_i(\theta) + u_i, \qquad i \in \mathcal{V}_G, \tag{1}$$

where

$$P_i(\theta) = \sum_{\{i,j\} \in \mathcal{E}} |\operatorname{Im}(Y_{ij})| V_i V_j \sin(\theta_i - \theta_j) \tag{2}$$

is the active power injection at bus i. Here, $\dot{\theta}_i$ is the frequency deviation from nominal, $M_i > 0$ is the inertia constant, $D_i > 0$ is the damping constant, and $u_i \in \mathbb{R}$ is the local controllable power generation. The value $Y_{ij} \in \mathbb{C}$ is the admittance of the branch $\{i,j\} \in \mathcal{E}$. Voltage magnitudes V_i are assumed to be constant, following the standard decoupling assumption [1], [2], [14], [15]. Relaxing this assumption requires a more sophisticated analysis [16]. The dynamics (1) can both model synchronous generators [17] and droop-controlled inverters with virtual inertia [18], [19] or power measurement delays [20]. In the case of inverters, M_i is the virtual inertia or power measurement time constant, and D_i is a tuneable droop control gain. For simplicity, in what follows we use the term "generator" for either case.

To model loads, we consider frequency-dependent loads governed by the first-order system [13]

$$D_i \dot{\theta}_i = P_i^* - P_i(\theta), \qquad i \in \mathcal{V}_L. \tag{3}$$

Here $D_i > 0$ is the damping coefficient, $P_i(\theta)$ is given by (2), and $P_i^* < 0$ is the constant power consumption at the nominal frequency. Technical extensions to the case of $D_i = 0$ — representing constant power loads — are also possible. In the desired synchronous steady-state where $\dot{\theta}_i = 0$, $\theta_i(t) = \bar{\theta}_i, i \in \mathcal{V}$, where $\{\bar{\theta}_i\}_{i \in \mathcal{V}}$ are a set of constants such that

$$|\bar{\theta}_i - \bar{\theta}_j| < \frac{\pi}{2}, \qquad \text{for all } \{i,j\} \in \mathcal{E}. \tag{4}$$

In this situation, from (1) and (3) we have that

$$0 = -\overline{P}_i + \bar{u}_i, \qquad i \in \mathcal{V}_G \tag{5a}$$
$$0 = -\overline{P}_i + P_i^*, \qquad i \in \mathcal{V}_L \tag{5b}$$

where

$$\overline{P}_i = \sum_{\{i,j\} \in \mathcal{E}} |\operatorname{Im}(Y_{ij})| V_i V_j \sin(\bar{\theta}_i - \bar{\theta}_j),$$

and $\{\bar{u}_i\}_{i \in \mathcal{V}_G}$ are the steady-state generator injections.

B. Optimal Distributed Frequency Control

We now review how the control inputs u_i in (1) are selected. When u_i is constant for all $i \in \mathcal{V}_G$, under some technical assumptions the dynamics (1)–(3) converge from every initial condition to a common steady-state frequency $\dot{\theta}_i \to \omega_{ss}$ which can be easily calculated to be $\omega_{ss} = (\sum_{i \in \mathcal{V}_G} u_i + \sum_{i \in \mathcal{V}_L} P_i^*)/(\sum_{i \in \mathcal{V}} D_i)$. When $\omega_{ss} \neq 0$, this represents a static deviation from nominal, which must be eliminated. To determine the the steady-state values for u_i, an *optimal frequency regulation problem* is formulated as

$$\underset{u \in \mathbb{R}^{|\mathcal{V}_G|}}{\text{minimize}} \qquad \sum_{i \in \mathcal{V}_G} \frac{1}{2} q_i u_i^2 \tag{6a}$$

$$\text{subject to} \qquad 0 = \sum_{i \in \mathcal{V}_G} u_i + \sum_{i \in \mathcal{V}_L} P_i^*. \tag{6b}$$

where we minimize the total quadratic cost of generation (6a) subject to the balance of power (6b). Here, $q_i > 0$ is the cost coefficient and $\frac{1}{2} q_i u_i^2$ is the local generation cost at the ith generator. In a synchronous state where $\ddot{\theta}_i = 0$ for all $i \in \mathcal{V}$, the constraint (6b) in fact implies that $\dot{\theta}_i = 0$ for all $i \in \mathcal{V}$, as one may verify by summing the equations (1),(3), and hence the frequency is regulated to its nominal value. Following the standard Lagrange multipliers method, the optimal control $u_i^\star$ that minimizes (6a) subject to the constraint (6b) is computed as

$$u_i^\star = -\lambda q_i^{-1} \tag{7}$$

where

$$\lambda = \frac{\sum_{i \in \mathcal{V}_L} P_i^*}{\sum_{i \in \mathcal{V}_G} q_i^{-1}} \tag{8}$$

is the multiplier of the constraint (6b), and can be interpreted as the "price" per unit of generation. The equality (7) implies that $u_i q_i = u_j q_j$ for all $i, j \in \mathcal{V}_G$. That is, the generators should provide power at *identical marginal costs*.

The calculation of (8) requires centralized information however. To distribute the solution of this problem in real-time, distributed averaging integral controllers have been proposed in the literature [1]–[4]. These controllers are defined on a communication graph $\mathcal{G}_c = (\mathcal{V}_c, \mathcal{E}_c)$ and have the form

$$\dot{\xi}_i = -\sum_{\{i,j\}\in\mathcal{E}_c}(\xi_i - \xi_j) - q_i^{-1}\omega_i \tag{9a}$$

$$u_i = q_i^{-1}\xi_i, \tag{9b}$$

where $\omega_i = \dot{\theta}_i$ and $i \in \mathcal{V}_G$. Here, the state ξ_i acts as a local copy of the multiplier λ for each unit: the term $q_i^{-1}\omega_i$ attempts to regulate the frequency deviation to zero while the consensus based algorithm $\sum(\xi_i - \xi_j)$ ensures that the marginal costs ξ_i are identical at steady-state.

Next, we write the system in a compact form. To this end, we need the following nomenclature: For each $k = 1, 2, \ldots, \ell$, with ℓ the number of links of the physical network, let γ_k be defined as $\gamma_k = |\operatorname{Im} Y_{ij}|V_iV_j$ with $\{i, j\}$ being the kth edge of the graph, where the edge numbers are in accordance with the *incidence matrix* A of the graph $\mathcal{G}$. Recall that the incidence matrix $A \in \mathbb{R}^{n\times\ell}$ is obtained by assigning arbitrary orientation to the edges of $\mathcal{G}$, and define component-wise $A_{vk} = 1$ if vertex v is the source node of edge $k \sim (i, j)$, and $A_{vk} = -1$ if vertex v is the sink node of edge k, with all other elements being zero.

We define the diagonal matrix Γ as $\Gamma = \operatorname{diag}(\gamma_k)$, with $k = 1, 2, \ldots, \ell$. After potentially reordering the nodes, we can partition A as $A = (\begin{bmatrix} A_G^{\mathsf{T}} & A_L^{\mathsf{T}} \end{bmatrix})^{\mathsf{T}}$, corresponding to generator nodes $\mathcal{V}_G$ and load nodes $\mathcal{V}_L$. It is easy to observe that the dynamics of the generators and the loads can be written compactly as

$$M_G\ddot{\theta}_G + D_G\dot{\theta}_G = -A_G\Gamma\underline{\sin}(A^{\top}\theta) + u \tag{10a}$$

$$D_L\dot{\theta}_L = -A_L\Gamma\underline{\sin}(A^{\top}\theta) + P_L^*, \tag{10b}$$

where the indices G and L are used to distinguish generator with load parameters, respectively. Notice that, if $\theta = \operatorname{col}(\theta_G, \theta_L)$ is a solution to (10) for a given input u, then $\theta + \mathbb{1}\alpha$ is also a solution to this system for any constant $\alpha \in \mathbb{R}$. To exclude this rotational invariance in the analysis, we change to the angular difference variables $\eta = A^{\mathsf{T}}\theta$. In addition, let $\omega_G = \dot{\theta}_G$, $\omega_L = \dot{\theta}_L$, and $\dot{\theta} = \omega = \operatorname{col}(\omega_G, \omega_L)$. Then the network dynamics (10), admits the following representation

$$\dot{\eta} = A^{\mathsf{T}}\omega = A_G^{\mathsf{T}}\omega_G + A_L^{\mathsf{T}}\omega_L \tag{11a}$$

$$M_G\dot{\omega}_G + D_G\omega_G = -A_G\Gamma\underline{\sin}(\eta) + u \tag{11b}$$

$$D_L\omega_L = -A_L\Gamma\underline{\sin}(\eta) + P_L^*. \tag{11c}$$

Note that $(\eta, \omega, u) \in \operatorname{im} A^{\mathsf{T}} \times \mathbb{R}^{|\mathcal{V}|} \times \mathbb{R}^{|\mathcal{V}_G|}$. The controller (9) can be written in the vector from as

$$\dot{\xi} = -L_c\xi - Q^{-1}\omega_G \tag{12a}$$

$$u = Q^{-1}\xi, \tag{12b}$$

where L_c is the *Laplacian matrix* of the communication graph $\mathcal{G}_c$, and $Q = \operatorname{diag}(q_i)$ with $i \in \mathcal{V}_G$. In vector notation,

the equilibrium/optimality results of the previous subsection now read for (11)-(12) as

$$\eta = A^{\mathsf{T}}\bar{\theta} := \bar{\eta} \tag{13a}$$

$$\omega = 0 \tag{13b}$$

$$\xi = -\frac{\mathbb{1}^{\mathsf{T}}P_L^*}{\mathbb{1}^{\mathsf{T}}Q^{-1}\mathbb{1}}\mathbb{1} \tag{13c}$$

and thus

$$u = u^\star = -\left(\frac{\mathbb{1}^{\mathsf{T}}P_L^*}{\mathbb{1}^{\mathsf{T}}Q^{-1}\mathbb{1}}\right)Q^{-1}\mathbb{1} \tag{14}$$

where $\bar{\theta} = \operatorname{col}(\bar{\theta}_i)$, and $u^\star = \operatorname{col}(u_i^\star)$ given by (7). We also refer to the above as an *optimal synchronous motion* of the power network. Finally, notice from (11) that the vector $\bar{\eta}$ must satisfy

$$0 = -A\Gamma\underline{\sin}(\bar{\eta}) + \begin{bmatrix} u^\star \\ P_L^* \end{bmatrix}. \tag{15}$$

By (4) and the definition of the incidence matrix, the vector $\bar{\eta}$ is in the image of A^{T}, and its elements belong to the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$. The following assumption guarantees the existence of a set of equilibrium angles.

Assumption 1: There exists $\bar{\eta} \in (-\frac{\pi}{2}, \frac{\pi}{2})^\ell \cap \operatorname{im} A^{\mathsf{T}}$ such that (15) holds.

The condition in Assumption 1 can be verified using the results in [1], [21].

III. THE COST OF DISHONESTY

First, we note the following result based on [14], [22]:

Proposition 1: Under Assumption 1, solutions of the system (11)-(12) locally converge to an optimal synchronous motion (13)-(14).

By the proposition above, the dynamic controller (12) achieves both frequency regulation and an optimal resource allocation in steady-state for the power network (11). As evident from (9), this requires a cooperative scheme where agents broadcast the state of their controllers in order to reach a consensus on the marginal costs.

We now depart from this fully cooperative scenario and consider the case where some agents *cheat* the system to get some advantage over other agents. Formulating the way of cheating as well as the corresponding payoff is by no means unique. Among all the possibilities, we fix one scenario to be examined in detail — other variations are listed in Section VI, the investigation of which we defer to future work.

Note that there are two objectives in the cooperative scenario. The first objective, which is vital for a properly functioning network, is to serve the loads by matching the supply and demand. This is captured by (6b), and leads to a zero frequency deviation in steady-state. The second goal amounts to optimizing the dispatch of power injections, with the optimal dispatch given in (7). This is achieved in a distributed fashion by reaching a consensus on marginal costs via the distributed averaging algorithm in (9). The cheating scenario we consider here is when one or more agents may broadcast and implement a *biased* version of the state of their local controller ξ_i, rather than the actual one,

in order to lower their local generation cost. It turns out that this amounts to reporting a *fake* load by a "dishonest" agent in an attempt to push the other agents toward more production. Consequently, the dishonest agent produces less power (while pretending to produce more) and this will lead to a drop in its local generation cost. On the other hand, as we will show, the aforementioned cheating scenario does not jeopardize frequency regulation.

Let the bias terms be collected in a vector $b = \mathrm{col}(b_i)$, for all generators $i \in \mathcal{V}_G$; if agent i is honest, then $b_i = 0$. The controller dynamics (12) with bias become

$$\dot{\xi} = -L_c(\xi + b) - Q^{-1}\omega_G \tag{16a}$$
$$u = Q^{-1}\xi. \tag{16b}$$

Note that the agents both broadcast and implement biased versions of the state of their controllers, yet the controller output u uses the (possibly lower) unbiased state ξ. Despite the exchange of biased controller states, we now show that the frequency is still regulated to its nominal value (under an appropriate feasibility condition). However, the resulting synchronous motion will not be the optimal one, as u does not converge to $u^\star$. To state this formally, we need to tailor Assumption 1 to the controller (16). By fairly straightforward calculations, it follows that a (non-optimal) synchronous solution $(\overline{\eta}, \overline{\omega}, \overline{\xi})$ of (11),(16) with $\overline{\omega} = 0$ and constant vectors $\overline{\eta}$ and $\overline{\xi}$ needs to satisfy

$$0 = -A\Gamma\underline{\sin}(\overline{\eta}) + \begin{bmatrix} Q^{-1}\overline{\xi} \\ P_L^* \end{bmatrix} \tag{17a}$$
$$\overline{\xi} = Qu^\star - \left(I - \frac{\mathbb{1}\mathbb{1}^\mathsf{T}Q^{-1}}{\mathbb{1}^\mathsf{T}Q^{-1}\mathbb{1}} \right) b, \tag{17b}$$

which brings us to the following assumption:

Assumption 2: There exists $\overline{\eta} \in (-\frac{\pi}{2}, \frac{\pi}{2})^\ell \cap \mathrm{im}\, A^\mathsf{T}$ such that (17a) holds.

Proposition 2: Under Assumption 2, solutions of the system (11),(16) locally converge to $(\overline{\eta}, \overline{\omega}, \overline{\xi})$ with $\overline{\eta}$ satisfying (17a), $\overline{\omega} = 0$, and $\overline{\xi}$ given by (17b).

By the proposition above, the vector u at steady-state is obtained as

$$\overline{u} = Q^{-1}\overline{\xi} = u^\star - \Pi b \tag{18}$$

where

$$\Pi = Q^{-1} - \frac{Q^{-1}\mathbb{1}\mathbb{1}^\mathsf{T}Q^{-1}}{\mathbb{1}^\mathsf{T}Q^{-1}\mathbb{1}}. \tag{19}$$

Therefore, the effect of biased information exchange is to perturb the optimal steady-state control $u^\star$ by the term $-\Pi b$. We now investigate the properties of this matrix Π. First, notice that Π is a symmetric matrix and $\Pi\mathbb{1} = 0$. In addition, the ijth element of Π is given by

$$\pi_{ij} = -\frac{q_i^{-1}q_j^{-1}}{\sum_{k \in \mathcal{V}_G} q_k^{-1}}, \qquad \text{for } i, j \in \mathcal{V}_G, i \neq j,$$
$$\pi_{ii} = -\sum_{j \neq i,\, j \in \mathcal{V}_G} \pi_{ij}.$$

Therefore, the matrix Π can be viewed as the Laplacian matrix of a complete undirected graph with edge weights

$-\pi_{ij}$ given above. We refer to this graph as the *cost graph*, denoted by $\mathcal{G}_\pi$, versus the physical graph $\mathcal{G}$ and the communication graph $\mathcal{G}_c$. Notice that the edge weights of $\mathcal{G}_\pi$ are completely determined by the cost coefficients q_i. The equality (18) can be then written component-wise as

$$\overline{u}_i = u_i^\star - \sum_{j \in \mathcal{V}_G} \pi_{ij} b_j, \tag{20}$$

or equivalently as

$$\overline{u}_i = u_i^\star - \sum_{j \in \mathcal{V}_G} |\pi_{ij}|(b_i - b_j) \tag{21}$$

by using the structure of the Laplacian matrix Π. The equality (21) shows how dishonest agents must compete to decrease their power injections $\overline{u}_i$ and consequently decrease their local generation costs $\frac{1}{2}q_i\overline{u}_i^2$.

Now let the nodes in $\mathcal{V}_G$ be partitioned into two subsets of *dishonest* agents $\mathcal{V}_\mathrm{d}$ and *honest* agents $\mathcal{V}_\mathrm{h}$, where we call an agent i dishonest whenever $b_i \neq 0$, and honest otherwise. Without loss of generality, we assume that the first d nodes of $\mathcal{V}_G$ are dishonest, and b is partitioned as $b = \mathrm{col}(b_d, 0)$ with $b_d \in \mathbb{R}^{|\mathcal{V}_\mathrm{d}|}$ containing no zero entries. The case of only one dishonest agent, say the first one, is illustrative. In that case, $b = \mathrm{col}(b_1, 0)$ with $b_1 \in \mathbb{R} \setminus \{0\}$. Then, (20) writes as

$$\overline{u}_1 = u_1^\star - \pi_{11} b_1,$$

which means that agent 1 can lower $\overline{u}_1$ and thus its generation cost by applying a positive bias $b_1 > 0$. Notice that the resulting benefit of this dishonest action depends on the weighted degree π_{11} of agent 1 in the cost graph.

Recall that $\overline{u}_i < u_i^\star$ implies a lower generation cost for agent i compared with the fair/optimal situation (7). In the general case of multiple dishonest agents, lower generation costs can be guaranteed under the following conditions.

Proposition 3: For every $i \in \mathcal{V}_\mathrm{d}$, the following conditions are equivalent:

(i) $$\overline{u}_i < u_i^\star;$$

(ii) $$\sum_{j \in \mathcal{V}_\mathrm{d}} \pi_{ij} b_j > 0 \tag{22}$$

(iii) $$b_i \sum_{j \in \mathcal{V}, j \neq i} |\pi_{ij}| > \sum_{j \in \mathcal{V}_\mathrm{d}, j \neq i} |\pi_{ij}| b_j;$$

(iv) $$b_i \sum_{j \in \mathcal{V}, j \neq i} q_j^{-1} > \sum_{j \in \mathcal{V}_\mathrm{d}, j \neq i} q_j^{-1} b_j.$$

Note that by (21), we find that

$$\overline{u}_i = u_i^\star - \sum_{j \in \mathcal{V}_d} |\pi_{ij}|(b_i - b_j) - b_i \sum_{j \in \mathcal{V}_h} |\pi_{ij}|.$$

The term $b_i - b_j$ in the above indicates the competition between two dishonest agents i and j to apply a possibly larger bias, and the third term is the benefit that the dishonest agent i gets with respect to its honest neighbors.

On the other hand, for each honest agent $i \in \mathcal{V}_\mathrm{h}$ we have

$$\overline{u}_i = u_i^\star + \sum_{j \in \mathcal{V}_\mathrm{d}} |\pi_{ij}| b_j. \tag{23}$$

This results in an increase in the local generation cost of honest agents when dishonest agents apply a positive bias $b_j > 0$, $j \in \mathcal{V}_\mathrm{d}$.

In the special case where all dishonest agents apply the same bias $b_i = b^*$ for all $i \in \mathcal{V}_\mathrm{d}$, then

$$\overline{u}_i = \begin{cases} u_i^\star - b^* \sum_{j \in \mathcal{V}_\mathrm{h}} |\pi_{ij}| & i \in \mathcal{V}_\mathrm{d} \\ u_i^\star + b^* \sum_{j \in \mathcal{V}_\mathrm{d}} |\pi_{ij}| & i \in \mathcal{V}_\mathrm{h} \,, \end{cases}$$

which means a lower generation cost for all dishonest agents, and a higher one for the honest ones.

IV. Counteracting biased information via a higher level control scheme

We have now seen that the presence of biased information exchange shifts the system away from its optimal steady-state equilibrium. Our goal now is to design a control scheme which counteracts the cheating effect of dishonest agents, and restores the optimal steady-state equilibrium point. To this end, we add a *higher level* control layer generating a "neutralizing" signal, denoted by v_i, $i \in \mathcal{V}_G$. The term "higher level" reflects the fact the agents are not *authorized* to access this control layer. We do not decide a priori whether an agent is honest or dishonest. With this new control layer in place, the power network model (11) becomes

$$\dot{\eta} = A^\mathsf{T} \omega \tag{24a}$$
$$M_G \dot{\omega}_G + D_G \omega_G = -A_G \Gamma \underline{\sin}(\eta) + u + v \tag{24b}$$
$$D_L \omega_L = -A_L \Gamma \underline{\sin}(\eta) + P_L^* \tag{24c}$$

with $v = \mathrm{col}(v_i)$, $i \in \mathcal{V}_G$. To dynamically compensate for possible biased information broadcasted by dishonest agents in the distributed controller (16), for each $i \in \mathcal{V}_G$ we propose the following honesty-enforcing controller:

$$M_i \dot{\chi}_i = -(D_i + F_i)\omega_i - P_i(\theta) + Q_i^{-1}(\xi_i + b_i) \tag{25a}$$
$$v_i = \lambda_i(\chi_i - \omega_i)\,, \tag{25b}$$

where $\lambda_i, F_i > 0$ are design parameters, and $P_i(\theta)$ is the active power as given by (2).

A few points are in order regarding the controller above. First, the proposed controller is assumed to have access to the nodal active power injection $P_i = (A_G \Gamma \underline{\sin}(\eta))_i$, the frequency ω_i, the parameters M_i and Q_i, and the biased transmitted information $\xi_i + b_i$, $i \in \mathcal{V}_G$. Notice that the controller does not exploit the actual unbiased information ξ_i. Moreover, observe that the parameter D_i needs not to be exactly known and an upper bound is sufficient thanks to the design parameter F_i. Finally, note that the controller above is fully decentralized.

The dynamics (25) can be written compactly as

$$M\dot{\chi} = -(D_G + F)\omega_G - P_G + Q^{-1}(\xi + b) \tag{26a}$$
$$v = \Lambda(\chi - \omega_G) \tag{26b}$$

where $P_G = A_G \Gamma \underline{\sin}(\eta)$, $F = \mathrm{diag}(F_i)$, $\Lambda = \mathrm{diag}(\lambda_i)$, and $\chi = \mathrm{col}(\chi_i)$.
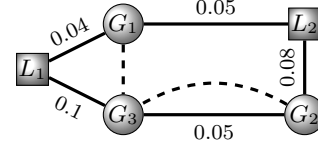

Fig. 1. The solid lines denote the transmission lines in $\mathcal{G}$, and the dashed lines depict the communication links in $\mathcal{G}_c$.

An optimal synchronous motion of (24) with the distributed averaging controller (16) and the decentralized controller (26) is any solution with a zero frequency deviation, $\overline{\eta}$ satisfying Assumption 1, and the optimality condition defined by

$$\overline{u} + \overline{v} = u^\star, \tag{27}$$

with $u^\star$ given by (14). By straightforward calculations, such a solution satisfies

$$\overline{\omega} = 0 \tag{28a}$$
$$\overline{\xi} = -b - \frac{\mathbb{1}^\mathsf{T} P_L^*}{\mathbb{1}^\mathsf{T} Q^{-1} \mathbb{1}} \mathbb{1} \tag{28b}$$
$$\overline{\chi} = \Lambda^{-1} Q^{-1} b \tag{28c}$$

and (15). The main result of this section is provided in the following theorem.

Theorem 1: (Dynamic compensation for biased information) Consider the power network model (24) with the distributed averaging controller (16) and the decentralized honesty-enforcing controller (26). Then, under Assumption 1, solutions of the closed-loop system locally converge to an optimal synchronous motion defined by (27), (28), and $\eta = \overline{\eta}$ satisfying (15).

By Theorem 1, the higher level controller (26) asymptotically compensates for the mismatch $u - u^\star$ to achieve the optimality condition (27). By (24b), the latter yields $\overline{P}_G = A_G \Gamma \underline{\sin}(\overline{\eta}) = u^\star$ which coincides with the original optimality condition (14).

V. A numerical example

We illustrate the proposed results by a numerical example of a power network consisting of three power sources and two loads. The interconnection topology is depicted in Figure 1. The network parameters are chosen as: $M_{G_1} = 4.49$, $M_{G_2} = 4.22$, $M_{G_3} = 0.50$, $D_{G_1} = 1.38$, $D_{G_2} = 1.42$, $D_{G_3} = 0.50$, $D_{L_1} = 1.00$, $D_{L_2} = 1.00$. The (negative of the) line susceptances are chosen as shown in Figure 1.

The system is initially at steady-state with a constant loading. At time $t = 5$, loads L_1 and L_2 are increased by 10 percent of their original values. The frequency evolution and the active power injections are depicted in Figure 2(a) and 2(b). It is observed that the system is regulating the frequency at 50 Hz (the frequencies at the various nodes are so similar to each other that no difference can be noticed in the plot). The active power is shared according to the ratios $q_{G_1}^{-1} = 3$, $q_{G_2}^{-1} = 2$, and $q_{G_3}^{-1} = 1$. Now, at time $t = 45$, agents 1 and 2 attempt to cheat the system by applying a bias $b_1 = 5$ and
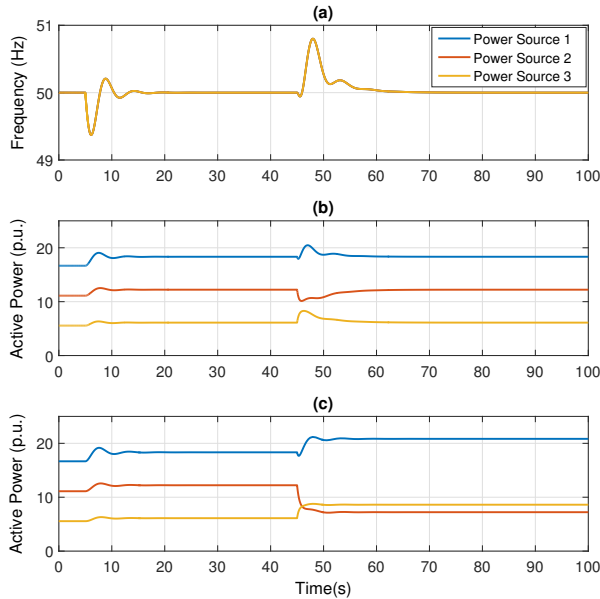
Fig. 2. Frequency behavior and optimal resource allocation: (a) frequency regulation (b) active power sharing in the presence of both biased information and the higher level control (c) active power sharing in the presence of biased information but in the absence of the higher level control.

$b_2 = 15$. The third agent does not apply a bias and $b_3 = 0$. Note that the higher level controller (26) is in place. As can be seen from Figure 2(a), frequency is still regulated to its nominal value. Moreover, the controller (26) foils the attempt of dishonest agents to manipulate the active power sharing. Finally, we have shown the status of active power sharing in the absence of the higher level controller in Figure 2(c). Clearly, active power sharing is affected by the cheating in this case, and consequently agent 2 (who applied the largest bias) produces much less power than expected, while agents 1 and 3 have to compensate for that.

VI. Outlook and Conclusion

The paper focuses on how dishonest agents participating in optimal distributed frequency control can transmit biased estimates of their marginal costs, achieving economic gain at the direct expense of honest agents. Many other scenarios could be considered, for instance, agents could directly bias their control actions u_i, or use both biased information and control for more sophisticated dishonest actions. Even more, one could imagine the scenario in which the dishonest agent sends different biased information to its different neighbors, a scenario which could be related to the case of Byzantine faults. In addition, instead of using an undirected communication graph, one could bias the calculation by intentionally using higher or lower weights on the incoming data from neighbors. The nature of the corrupting signal could also be substantially enriched. In this paper, we have used a constant bias term, but such term could also be generated by an intelligent adversary that uses a dynamical system possibly in feedback with the physical network to alter the overall behavior [23], [7]. The investigation of these control

scenarios is a widely open research arena.

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