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A Jamming-resilient Algorithm for Self-triggered Network Coordination

Danial Senejohnny, Pietro Tesi, and Claudio De Persis

Abstract—The issue of cyber-security has become ever more prevalent in the analysis and design of cyber-physical systems. In this paper, we investigate self-triggered consensus networks in the presence of communication failures caused by Denial-of-Service (DoS) attacks. A general framework is considered in which the network links can fail independent of each other. By introducing a notion of Persistency-of-Communication (PoC), we provide an explicit characterization of DoS frequency and duration under which consensus can be preserved by suitably designing time-varying control and communication policies. An explicit characterization of the effects of DoS on the consensus time is also provided. The considered notion of PoC is compared with classic average connectivity conditions that are found in pure continuous-time consensus networks. Finally, examples are given to substantiate the analysis.

Index Terms—Consensus networks; Self-triggered control; Denial-of-Service.

I. INTRODUCTION

Recent years have witnessed a growing interest towards Cyber-Physical systems (CPSs), namely systems that exhibit a tight conjoining of communication, computational and physical units. The fact that breaches in the cyber-space can have consequences in the physical domain has triggered considerable attention towards the issue of cyber-physical security [1], [2]. In CPSs, attacks to the communication links can be classified as either deception attacks or Denial-of-Service (DoS) attacks. The former affect the trustworthiness of data by manipulating the packets transmitted over the network; see [3]-[4] and the references therein. DoS attacks are instead primarily intended to affect the timeliness of the information exchange, *i.e.*, to cause packet losses. This paper is concerned with DoS attacks, and, in particular, with *jamming* attacks [5], [6], although in this paper we shall use these two terms interchangeably.

In the literature, the issues of securing robustness of CPSs against DoS has been widely investigated only for centralized architectures [7]-[14]. On the other hand, very little is known about DoS for distributed coordination problems. In this paper, we investigate the issue of DoS with respect to consensus-like networks. Specifically, inspired by [15], we consider a *self-triggered* consensus network, in which communication and control actions are planned ahead in time, depending on the information currently available at each agent. The attacker objective is to prevent consensus by denying communication

among the network agents. Consensus is a prototypical problem in distributed settings with a huge range of applications, spanning from formation and cooperative robotics to surveillance and distributed computing; see for instance [15]-[16]. On the other hand, self-triggered coordination turns out to be of major interest when consensus has to be achieved in spite of possibly severe communication constraints. In this respect, a remarkable feature of self-triggered coordination lies in the possibility of ensuring consensus properties in the absence of any global information on the graph topology and with no need to synchronize the agents local clocks.

A basic question in the analysis of distributed coordination in the presence of DoS is concerned with the modeling of DoS attacks. In [12], [13], a general model is considered that only constrains DoS attacks in terms of their average frequency and duration, which makes it possible to capture many different types of DoS attacks, including trivial, periodic, random and protocol-aware jamming attacks [5], [6], [17], [18]. Building on [13], a preliminary analysis of consensus networks in the presence of DoS is presented in [19] under the simplifying assumption that the occurrence of DoS cause all the network links to fail simultaneously. This scenario is representative of networks operating through a single access point, in the so-called “infrastructure” mode. In this paper, we consider the more general scenario in which the network communication links can fail independent of each other, thereby extending the analysis to “ad-hoc” (peer-to-peer) networks. One contribution of this paper is an explicit characterization of the frequency and duration of DoS at the various network links under which consensus can be preserved by suitably designing time-varying control and communication policies. Moreover, an explicit characterization of the effects of DoS on the consensus time is provided.

Since DoS induces communication failures, the problem of achieving consensus under DoS can be naturally cast as a consensus problem for networks with switching topologies. This approach is certainly not new in the literature. In [20], for instance, it is shown that consensus can be reached whenever graph connectivity is preserved point-wise in time; [21] considers a notion of *Persistency-of-Excitation* (PoE), which stipulates that graph connectivity should be established over a period of time, rather than point-wise in time, which is similar to the joint connectivity assumption in [22]. In CPSs, however, the situation is different. In CPSs, one needs to deal with the fact that networked communication is inherently digital, which means that the rate at which the transmissions are scheduled cannot be arbitrarily large. Under such circumstances, the aforementioned tools turn out to be ineffective. In order to cope

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with this situation, we introduce a notion of *Persistency-of-Communication* (PoC), which naturally extends the PoE condition to a digital networked setting by requiring graph (link) connectivity over periods of time that are consistent with the constraints imposed by the communication medium. A characterization of DoS frequency and duration under which consensus properties can be preserved is then obtained by exploiting the PoC condition.

The remainder of this paper is as follows. In Section II, we formulate the control problem and provide prototypical results for self-triggered consensus. In Section III, we describe the considered class of DoS signals. The main results of this paper are presented in Section IV. In Section V, we provide a detailed discussion of the results, and show how the analysis can be extended so as to account for genuine (non-malicious) transmission failures. A numerical example is presented in Section VI. Section VII ends the paper with concluding remarks.

II. SELF-TRIGGERED CONSENSUS NETWORK

A. System definition

We consider a consensus network, which is represented by an undirected graph $\mathcal{G} = (\mathcal{I}, \mathcal{E})$, where $\mathcal{I} = \{1, \dots, n\}$ denotes the node set and $\mathcal{E} \subseteq \mathcal{I} \times \mathcal{I}$ denotes the edge set. Specifically, we denote by D and L the incidence and Laplacian matrix of \mathcal{G} , respectively. For each node $i \in \mathcal{I}$, we denote by \mathcal{N}_i the set of its neighbors, and by $d^i = |\mathcal{N}_i|$, i.e., the cardinality of \mathcal{N}_i . Throughout the paper, we shall refer to \mathcal{G} as the “nominal” network, and we shall assume that \mathcal{G} is connected.

The consensus network of interest employs *self-triggered* communication [15], defined via hybrid dynamics, with state variables $(x, u, \theta) \in \mathbb{R}^n \times \mathbb{R}^d \times \mathbb{R}^d$, where x is the vector of nodes states, u is the vector of controls, θ is the vector of clock variables, and d is the sum of the neighbors of all the nodes, i.e., $d := \sum_{i=1}^n d^i$. The control signals are assumed to belong to $\mathcal{T} := \{-1, 0, +1\}$. The specific quantizer of choice is $\text{sign}_\varepsilon : \mathbb{R} \rightarrow \mathcal{T}$, which is given by

$$\text{sign}_\varepsilon(z) := \begin{cases} \text{sign}(z) & \text{if } |z| \geq \varepsilon \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where $\varepsilon > 0$ is a sensitivity parameter, which can be used at the design stage for trading-off frequency of the transmissions vs. accuracy of the consensus region.

The system $(x, u, \theta) \in \mathbb{R}^n \times \mathbb{R}^d \times \mathbb{R}^d$ satisfies the continuous evolution

$$\begin{cases} \dot{x}^i = \sum_{j \in \mathcal{N}_i} u^{ij} \\ \dot{u}^{ij} = 0 \\ \dot{\theta}^{ij} = -1 \end{cases} \quad (2)$$

where $i \in \mathcal{I}$ and $j \in \mathcal{N}_i$. The system satisfies the differential equation above for all t except for those values of the time at which the set

$$\mathcal{J}(\theta, t) = \{(i, j) \in \mathcal{I} \times \mathcal{I} : j \in \mathcal{N}_i \text{ and } \theta^{ij}(t^-) = 0\} \quad (3)$$

is non-empty. At these times, in the “nominal” operating mode (when communication between nodes is always possible), a discrete transition occurs, which is governed by the following discrete update:

$$\begin{cases} x^i(t) = x^i(t^-) & \forall i \in \mathcal{I} \\ u^{ij}(t) = \begin{cases} \text{sign}_\varepsilon(\mathcal{D}^{ij}(t)) & \text{if } (i, j) \in \mathcal{J}(\theta, t) \\ u^{ij}(t^-) & \text{otherwise} \end{cases} \\ \theta^{ij}(t) = \begin{cases} f^{ij}(x(t)) & \text{if } (i, j) \in \mathcal{J}(\theta, t) \\ \theta^{ij}(t^-) & \text{otherwise} \end{cases} \end{cases} \quad (4)$$

where for every $i \in \mathcal{I}$ and $j \in \mathcal{N}_i$, the map $f^{ij} : \mathbb{R}^n \rightarrow \mathbb{R}_{>0}$ is defined by

$$f^{ij}(x(t)) := \begin{cases} \frac{|\mathcal{D}^{ij}(t)|}{2(d^i + d^j)} & \text{if } |\mathcal{D}^{ij}(t)| \geq \varepsilon \\ \frac{|\mathcal{D}^{ij}(t)|}{2(d^i + d^j)} & \text{if } |\mathcal{D}^{ij}(t)| < \varepsilon \end{cases} \quad (5)$$

and

$$\mathcal{D}^{ij}(t) = x^j(t) - x^i(t) \quad (6)$$

Notice that for all $\{i, j\} \in \mathcal{E}$ we have $\theta^{ij}(t) = \theta^{ji}(t)$ and $u^{ij}(t) = -u^{ji}(t)$ for all $t \in \mathbb{R}_{\geq 0}$. As such, the system (2)-(4) can be regarded as an edge-based consensus protocol. Here, the term “self-triggered”, first adopted in the context of real-time systems [23], expresses the property that the data exchange between nodes is driven by local clocks, which avoids the need for a common global clock.

B. Prototypical result for self-triggered consensus

The following result characterizes the limiting behavior of the system (2)-(4).

Theorem 1: [15] Let x be the solution to (2)-(4). Then, for every initial condition, x converges in finite time to a point $x^* \in \mathbb{R}^n$ belonging to the set

$$\mathcal{E} = \{x \in \mathbb{R}^n : |x^i(t) - x^j(t)| < \delta \quad \forall (i, j) \in \mathcal{I} \times \mathcal{I}\} \quad (7)$$

where $\delta = \varepsilon(n-1)$. ■

Theorem 1 will be used as a reference frame for the analysis of Section IV and V. This theorem is prototypical in the sense that it serves to illustrate the salient features of the problem of consensus/coordination in the presence of communication interruptions. Following [15], the analysis of this paper could be extended to include important aspects such as *quantized communication*, *delays* and *asymptotic consensus* (rather than practical consensus as in (7)). While important, these aspects do not add much to the present investigation and will be therefore omitted. We refer the interested reader to [15] for a discussion on how these aspects can be dealt with.

III. NETWORK DENIAL-OF-SERVICE

We shall refer to Denial-of-Service (DoS, in short) as the phenomenon by which communication between the network nodes is interrupted. We shall consider the very general scenario in which the network communication links can fail independent of each other. From the perspective of modeling, this amounts to considering multiple DoS signals, one for each network communication link.

A. DoS characterization

Let $\{h_n^{ij}\}_{n \in \mathbb{Z}_{\geq 0}}$ with $h_0^{ij} \geq 0$ denote the sequence of DoS off/on transitions affecting the link $\{i, j\}$, namely the sequence of time instants at which the DoS status on the link $\{i, j\}$ exhibits a transition from zero (communication is possible) to one (communication is interrupted). Then

$$H_n^{ij} := \{h_n^{ij}\} \cup [h_n^{ij}, h_n^{ij} + \tau_n^{ij}] \quad (8)$$

represents the n -th DoS time-interval, of a length $\tau_n^{ij} \in \mathbb{R}_{\geq 0}$, during which communication on the link $\{i, j\}$ is not possible.

Given $t, \tau \in \mathbb{R}_{\geq 0}$, with $t \geq \tau$, let

$$\Xi^{ij}(\tau, t) := \bigcup_{n \in \mathbb{Z}_{\geq 0}} H_n^{ij} \cap [\tau, t] \quad (9)$$

and

$$\Theta^{ij}(\tau, t) := [\tau, t] \setminus \Xi^{ij}(\tau, t) \quad (10)$$

where \setminus denotes relative complement. In words, for each interval $[\tau, t]$, $\Xi^{ij}(\tau, t)$ and $\Theta^{ij}(\tau, t)$ represent the sets of time instants where communication on the link $\{i, j\}$ is denied and allowed, respectively.

The first question to be addressed is that of determining a suitable modeling framework for DoS. Following [13], we consider a general model that only constrains DoS attacks in terms of their average frequency and duration. Let $n^{ij}(\tau, t)$ denote the number of DoS off/on transitions on the link $\{i, j\}$ occurring on the interval $[\tau, t]$.

Assumption 1 (DoS frequency): For each $\{i, j\} \in \mathcal{E}$, there exist $\eta^{ij} \in \mathbb{R}_{\geq 1}$ and $\tau_f^{ij} \in \mathbb{R}_{> 0}$ such that

$$n^{ij}(\tau, t) \leq \eta^{ij} + \frac{t - \tau}{\tau_f^{ij}} \quad (11)$$

for all $t, \tau \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$. ■

Assumption 2 (DoS duration): For each $\{i, j\} \in \mathcal{E}$, there exist $\kappa^{ij} \in \mathbb{R}_{\geq 0}$ and $\tau_d^{ij} \in \mathbb{R}_{> 1}$ such that

$$|\Xi^{ij}(\tau, t)| \leq \kappa^{ij} + \frac{t - \tau}{\tau_d^{ij}} \quad (12)$$

for all $t, \tau \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$. ■

In Assumption 1, the term ‘‘frequency’’ stems from the fact that τ_f^{ij} provides a measure of the ‘‘dwell-time’’ between any two consecutive DoS intervals on the link $\{i, j\}$. The quantity η^{ij} is needed to render (11) self-consistent when $t = \tau = h_n^{ij}$ for some $n \in \mathbb{Z}_{\geq 0}$, in which case $n^{ij}(\tau, t) = 1$. Likewise, in Assumption 2, the term ‘‘duration’’ is motivated by the fact that τ_d^{ij} provides a measure of the fraction of time ($\tau_d^{ij} > 1$) the link $\{i, j\}$ is under DoS. Like η^{ij} , the constant κ^{ij} plays the role of a regularization term. It is needed because during a DoS interval, one has $|\Xi(h_n^{ij}, h_n^{ij} + \tau_n^{ij})| = \tau_n^{ij} \geq \tau_n^{ij} / \tau_d^{ij}$ since $\tau_d^{ij} > 1$, with $\tau_n^{ij} = \tau_n^{ij} / \tau_d^{ij}$ if and only if $\tau_n^{ij} = 0$. Hence, κ^{ij} serves to make (12) self-consistent. Thanks to the quantities η^{ij} and κ^{ij} , DoS frequency and duration are both average quantities.

Remark 1: Throughout this paper, we will mostly focus on the case where DoS is caused by malicious attacks. Of course, DoS might also result from a ‘‘genuine’’ network congestion. We shall briefly address this case in Section V-C. ■

B. Examples

The considered assumptions only pose limitations on the frequency of the DoS status and its duration. As such, this characterization can capture many different scenarios, including trivial, periodic, random and protocol-aware jamming attacks [5], [6], [17], [18]. For the sake of simplicity, we limit out discussion to the case of radio frequency (RF) jammers, although similar considerations can be made with respect to spoofing-like threats [24].

Consider for instance the case of *constant jamming*, which is one of the most common threats that may occur in a wireless network [5], [25]. By continuously emitting RF signals on the wireless medium, this type of jamming can lower the Packet Send Ratio (PSR) for transmitters employing carrier sensing as medium access policy as well as lower the Packet Delivery Ratio (PDR) by corrupting packets at the receiver. In general, the percentage of packet losses caused by this type of jammer depends on the Jamming-to-Signal Ratio and can be difficult to quantify as it depends, among many things, on the type of anti-jamming devices, the possibility to adapt the signal strength threshold for carrier sensing, and the interference signal power, which may vary with time. In fact, there are several provisions that can be taken in order to *mitigate* DoS attacks, including spreading techniques, high-pass filtering and encoding [26], [18]. These provisions decrease the chance that a DoS attack will be successful, and, as such, limit in practice the frequency and duration of the time intervals over which communication is effectively denied. This is nicely captured by the considered formulation.

As another example, consider the case of *reactive jamming* [5], [25]. By exploiting the knowledge of the 802.11 MAC layer protocols, a jammer may restrict the RF signal to the packet transmissions. The collision period need not be long since with many CRC error checks a single bit error can corrupt an entire frame. Accordingly, jamming takes the form of a (high-power) burst of noise, whose duration is determined by the length of the symbols to corrupt [26], [27]. Also this case can be nicely accounted for via the considered assumptions.

IV. DOS-RESILIENT CONSENSUS

A. Modified communication protocol

In order to achieve robustness against DoS, the nominal discrete evolution (4) is modified as follows:

$$\begin{cases} x^i(t) = x^i(t^-) & \forall i \in \mathcal{I} \\ u^{ij}(t) = \begin{cases} \text{sign}_\varepsilon(\mathcal{D}^{ij}(t)) & \text{if } (i, j) \in \mathcal{J}(\theta, t) \wedge t \in \Theta^{ij}(0, t) \\ 0 & \text{if } (i, j) \in \mathcal{J}(\theta, t) \wedge t \in \Xi^{ij}(0, t) \\ u^{ij}(t^-) & \text{otherwise} \end{cases} \\ \theta^{ij}(t) = \begin{cases} f^{ij}(x(t)) & \text{if } (i, j) \in \mathcal{J}(\theta, t) \wedge t \in \Theta^{ij}(0, t) \\ \frac{\varepsilon}{2(d^i + d^j)} & \text{if } (i, j) \in \mathcal{J}(\theta, t) \wedge t \in \Xi^{ij}(0, t) \\ \theta^{ij}(t^-) & \text{otherwise} \end{cases} \end{cases} \quad (13)$$

In words, the control action u^{ij} is reset to zero whenever the link $\{i, j\}$ is in DoS status. Notice that this requires that the

nodes are able to detect the occurrence of DoS. This is the case, for instance, with transmitters employing carrier sensing as medium access policy. Under such circumstances, a DoS signal in the form of *constant jamming* (cf. Section III-B) can be detected. Another example is when transceivers use TCP acknowledgment and DoS takes the form of *reactive jamming* (cf. Section III-B). In addition to u , also the local clocks are modified upon DoS, yielding a *two-mode* sampling logic. In particular, for each $\{i, j\} \in \mathcal{E}$, let $\{t_k^{ij}\}_{k \in \mathbb{Z}_{\geq 0}}$ denote the sequence of transmission attempts. Then, each θ^{ij} satisfies

$$t_{k+1}^{ij} = t_k^{ij} + \begin{cases} f^{ij}(x(t_k^{ij})) & \text{if } t_k^{ij} \in \Theta^{ij}(0, t) \\ \frac{\varepsilon}{2(d^i + d^j)} & \text{otherwise} \end{cases} \quad (14)$$

As it will become clear later on, this is in order to maximize the robustness of the consensus protocol against DoS. By (14), it is an easy matter to see that for each $\{i, j\} \in \mathcal{E}$ the sequences $\{t_k^{ij}\}_{k \in \mathbb{Z}_{\geq 0}}$ satisfy a ‘‘dwell-time’’ property, since

$$\Delta_k^{ij} := t_{k+1}^{ij} - t_k^{ij} \geq \frac{\varepsilon}{4d_{\max}} \quad (15)$$

for all $k \in \mathbb{R}_{\geq 0}$, where $d_{\max} = \max_{i \in \mathcal{I}} d^i$. This ensures that all the sequences of transmission times are Zeno-free.

For the sake of clarity, the DoS-resilient consensus protocol is summarized below.

DoS-resilient consensus protocol

- 1: **initialization:** For all $i \in \mathcal{I}$ and $j \in \mathcal{N}_i$, set $\theta^{ij}(0^-) = 0$, $u^{ij}(0^-) \in \{-1, 0, +1\}$, and $u^i(0^-) = \sum_{j \in \mathcal{N}_i} u^{ij}(0^-)$;
 - 2: **for all** $i \in \mathcal{I}$ **do**
 - 3: **for all** $j \in \mathcal{N}_i$ **do**
 - 4: **while** $\theta^{ij}(t) > 0$ **do**
 - 5: *i* applies the control $u^i(t) = \sum_{j \in \mathcal{N}_i} u^{ij}(t)$;
 - 6: **end while**
 - 7: **if** $\theta^{ij}(t^-) = 0 \wedge t \in \Theta^{ij}(0, t)$ **then**
 - 8: *i* updates $u^{ij}(t) = \text{sign}_\varepsilon(x^j(t) - x^i(t))$;
 - 9: *i* updates $\theta^{ij}(t) = f^{ij}(x(t))$;
 - 10: **else**
 - 11: **if** $\theta^{ij}(t^-) = 0 \wedge t \in \Xi^{ij}(0, t)$ **then**
 - 12: *i* updates $u^{ij}(t) = 0$;
 - 13: *i* updates $\theta^{ij}(t) = \frac{\varepsilon}{2(d^i + d^j)}$;
 - 14: **end if**
 - 15: **end if**
 - 16: **end for**
 - 17: **end for**
-

B. Convergence of the solutions and δ -consensus

We are now in position to characterize the overall network behavior in the presence of DoS. In this respect, the analysis is subdivided into two main steps: i) we first prove that all the network nodes eventually stop to update their local controls; and ii) we then provide conditions on the DoS frequency and duration such that consensus, in the sense of (7), is preserved. The latter property is achieved by resorting to a notion of *Persistency-of-Communication*, which determines the amount of DoS (frequency and duration) under which consensus can be preserved.

As for i), the following result holds true.

Proposition 1: (Convergence of the solutions) Let x be the solution to (2) and (13). Then, for every initial condition, there exists a finite time T_* such that, for any $i \in \mathcal{I}$, it holds that $u^i(t) = 0$ for all $t \geq T_*$.

Proof. Consider the Lyapunov function

$$V(x) = \frac{1}{2} x^\top x \quad (16)$$

Let $t_k^{ij} := \max\{t_\ell^{ij} : t_\ell^{ij} \leq t, \ell \in \mathbb{Z}_{\geq 0}\}$. First notice that the derivative of V along the solutions to (2) satisfies

$$\begin{aligned} \dot{V}(x(t)) &= \sum_{i=1}^n x^i(t) \dot{x}^i(t) \\ &= \sum_{i=1}^n [x^i(t) \sum_{j \in \mathcal{N}_i} u^{ij}(t)] \\ &= - \sum_{\substack{\{i,j\} \in \mathcal{E}: \\ |\mathcal{D}^{ij}(t_k^{ij})| \geq \varepsilon \wedge t_k^{ij} \in \Theta^{ij}(0, t)}} \mathcal{D}^{ij}(t) \text{sign}_\varepsilon(\mathcal{D}^{ij}(t_k^{ij})) \\ &\leq - \sum_{\substack{\{i,j\} \in \mathcal{E}: \\ |\mathcal{D}^{ij}(t_k^{ij})| \geq \varepsilon \wedge t_k^{ij} \in \Theta^{ij}(0, t)}} \frac{|\mathcal{D}^{ij}(t_k^{ij})|}{2} \end{aligned} \quad (17)$$

In words, the derivative of V decreases whenever, for some $\{i, j\} \in \mathcal{E}$, two conditions are met: i) $|\mathcal{D}^{ij}(t_k^{ij})| \geq \varepsilon$, which means that i and j are not ε -close; and ii) communication on the link that connects i and j is possible. The third equality follows from the fact that for any $\{i, j\} \in \mathcal{E}$ for which $|\mathcal{D}^{ij}(t_k^{ij})| < \varepsilon$ or $t_k^{ij} \in \Xi^{ij}(0, t)$ we have $u^{ij}(t) = 0$ for all $[t_k^{ij}, t_{k+1}^{ij}[$, and the fact that $u^{ij}(t) = \text{sign}_\varepsilon(\mathcal{D}^{ij}(t_k^{ij}))$ where $\mathcal{D}^{ij}(t) = x^j(t) - x^i(t)$. The inequality follows from the fact that, during the continuous evolution $|\dot{\mathcal{D}}^{ij}(t)| \leq d^i + d^j$ and at the jumps $\mathcal{D}^{ij}(t)$ does not change its value. This implies that $\mathcal{D}^{ij}(t)$ cannot differ from $\mathcal{D}^{ij}(t_k^{ij})$ in absolute value for more than $(d^i + d^j)(t - t_k^{ij})$. Exploiting this fact, if communication is allowed and $|\mathcal{D}^{ij}(t_k^{ij})| \geq \varepsilon$ then by (5) and (14) we have

$$|\mathcal{D}^{ij}(t)| \geq |\mathcal{D}^{ij}(t_k^{ij})|/2 \quad (18)$$

and

$$\text{sign}_\varepsilon(\mathcal{D}^{ij}(t)) = \text{sign}_\varepsilon(\mathcal{D}^{ij}(t_k^{ij})) \quad (19)$$

for all $[t_k^{ij}, t_{k+1}^{ij}[$.

From (17) there must exist a finite time T_* such that, for every $\{i, j\} \in \mathcal{E}$ and every k with $t_k^{ij} \geq T_*$, it holds that $|\mathcal{D}^{ij}(t_k^{ij})| < \varepsilon$ or $t_k^{ij} \in \Xi^{ij}(0, t)$. This is because, otherwise, V would become negative. The proof follows recalling that in both the cases $|\mathcal{D}^{ij}(t_k^{ij})| < \varepsilon$ and $t_k^{ij} \in \Xi^{ij}(0, t)$ the control $u^{ij}(t)$ is set equal to zero. ■

The above result does not allow one to conclude anything about the final disagreement vector in the sense that given a pair of nodes (i, j) the asymptotic value of $|x^j(t) - x^i(t)|$ can be arbitrarily large. As an example, if node i is never allowed to communicate then $x^i(t) = x^i(0)$ for all $t \in \mathbb{R}_{\geq 0}$. In order to recover the same conclusions as in Theorem 1, bounds on DoS frequency and duration have to be enforced. The result which follows provides one such characterization.

Let $\{i, j\} \in \mathcal{E}$ be a generic network link, and consider a DoS sequence on $\{i, j\}$, which satisfies Assumption 1 and 2. Define

$$\alpha^{ij} := \frac{1}{\tau_d^{ij}} + \frac{\Delta_*^{ij}}{\tau_f^{ij}} \quad (20)$$

where

$$\Delta_*^{ij} := \frac{\varepsilon}{2(d^i + d^j)} \quad (21)$$

Proposition 2 (Link Persistency-of-Communication (PoC)): Consider any link $\{i, j\} \in \mathcal{E}$ employing the transmission protocol (13). Also consider any DoS sequence on $\{i, j\}$, which satisfies Assumption 1 and 2 with η^{ij} and κ^{ij} arbitrary, and τ_d^{ij} and τ_f^{ij} such that $\alpha^{ij} < 1$. Let

$$\Phi^{ij} := \frac{\kappa^{ij} + (\eta^{ij} + 1)\Delta_*^{ij}}{1 - \alpha^{ij}} \quad (22)$$

Then, for any given unsuccessful transmission attempt t_k^{ij} , at least one successful transmission occurs over the link $\{i, j\}$ within the interval $[t_k^{ij}, t_k^{ij} + \Phi^{ij}]$.

Proof. In order to maintain continuity, a proof of this result is reported in Appendix. ■

We refer to the property above as a PoC condition since this property guarantees that DoS does not permanently destroy communication. Combining Proposition 1 and 2, the main result of this section can be stated.

Theorem 2 (δ -consensus): Let x be the solution to (2) and (13). For each $\{i, j\} \in \mathcal{E}$, consider any DoS sequence that satisfies Assumption 1 and 2 with η^{ij} and κ^{ij} arbitrary, and τ_d^{ij} and τ_f^{ij} such that $\alpha^{ij} < 1$. Then, for every initial condition, x converges in finite time to a point x^* belonging to the set \mathcal{E} as in (7).

Proof. By Proposition 1, all the local controls become zero in a finite time T_* . In turns, Proposition 2 excludes that this is due to the persistence of a DoS status. This means that, for all $\{i, j\} \in \mathcal{E}$, $|\mathcal{D}^{ij}(t)| = |x^j(t) - x^i(t)| < \varepsilon$ for all $t \geq T_*$. Since each pair of neighboring nodes differs by a most ε and the nominal graph is connected, we conclude that each pair of network nodes can differ by at most $\delta = \varepsilon(n-1)$. ■

C. Convergence time

The above theorem shows that convergence is reached in a finite time. The following result characterizes the effect of DoS on the convergence time.

Lemma 1 (Bound on the convergence time): Consider the same assumptions as in Theorem 1. Then,

$$T_* \leq \left[\frac{1}{\varepsilon} + \frac{d_{\max}}{\varepsilon d_{\min}} + \frac{4d_{\max}}{\varepsilon^2} \Phi \right] \sum_{i \in \mathcal{I}} (x^i(0))^2 \quad (23)$$

where $d_{\min} := \min_{i \in \mathcal{I}} d^i$ and $\Phi := \max_{\{i, j\} \in \mathcal{E}} \Phi^{ij}$.

Proof. Consider the same Lyapunov function V as in the proof of Proposition 1. Notice that, by construction of the control law and the scheduling policy, for every successful transmission t_k^{ij} characterized by $|\mathcal{D}^{ij}(t_k^{ij})| \geq \varepsilon$, the function V decreases with rate not less than $\varepsilon/2$ for at least $\varepsilon/(4d_{\max})$

units of time. Hence, V decreases by a least $\varepsilon^2/(8d_{\max}) := \varepsilon_*$. Considering all the network links, such transmissions are in total no more than $\lfloor V(0)/\varepsilon_* \rfloor$ since, otherwise, the function V would become negative. Hence, it only remains to compute the time needed to have $\lfloor V(0)/\varepsilon_* \rfloor$ of such transmissions. In this respect, pick any $t_* \geq 0$ such that consensus has still not been reached. Note that we can have $u^{ij}(t_*) = 0$ for all $\{i, j\} \in \mathcal{E}$. However, this condition can last only for a limited amount of time. In fact, if $u^{ij}(t_*) = 0$ then the next transmission attempt, say ℓ^{ij} , over the link $\{i, j\}$ will necessarily occur at a time less than or equal to $t_* + \Delta_*^{ij}$ with $\Delta_*^{ij} \leq \varepsilon/(4d_{\min})$. Let $\mathcal{Q} := [t_*, t_* + \varepsilon/(4d_{\min})]$, and suppose that over \mathcal{Q} all the controls u^{ij} have remained equal to zero. This implies that for some $\{i, j\} \in \mathcal{E}$ we necessarily have that ℓ^{ij} is unsuccessful. This is because if $u^{ij}(t) = 0$ for all $\{i, j\} \in \mathcal{E}$ and all $t \in \mathcal{Q}$ then $x^i(t) = x^i(t_*)$ for all $i \in \mathcal{I}$ and all $t \in \mathcal{Q}$. Hence, if all the ℓ^{ij} were successful, we should also have $u^{ij}(\ell^{ij}) \neq 0$ for some $\{i, j\} \in \mathcal{E}$ since, by hypothesis, consensus is not reached at time t_* . Hence, applying Proposition 2 we conclude that at least one of the controls u^{ij} will become non zero before $\ell^{ij} + \Phi^{ij}$ units of time have elapsed. Overall, this implies that at least one control will become nonzero before $\varepsilon/(4d_{\min}) + \Phi$ units of time have elapsed. Since t_* is generic, we conclude that V decreases by at least ε_* every $\varepsilon/(4d_{\max}) + \varepsilon/(4d_{\min}) + \Phi$ units of time, which implies that

$$T_* \leq \left[\frac{\varepsilon}{4d_{\max}} + \frac{\varepsilon}{4d_{\min}} + \Phi \right] \frac{V(0)}{\varepsilon_*} \quad (24)$$

The thesis follows by recalling that $V(0)$ can be rewritten as $V(0) = \frac{1}{2} \sum_{i \in \mathcal{I}} (x^i(0))^2$. ■

V. DISCUSSION AND EXTENSIONS

A. Persistency-of-Communication and consensus under permanent link disconnections

As it follows from the foregoing analysis, consensus is achieved whenever for each link $\{i, j\} \in \mathcal{E}$, the DoS signal satisfies $\alpha^{ij} < 1$. This condition poses limitations on both DoS frequency and duration. It is worth noting that this condition is in a wide sense also necessary in order to achieve consensus. To see this, consider a network for which removing the link $\{i, j\}$ causes the network underlying graph to be disconnected. Of course, if communication over $\{i, j\}$ is always denied then consensus cannot be achieved for arbitrary initial conditions. In this respect, it is an easy matter to see that condition $\alpha^{ij} < 1$ becomes necessary to achieve consensus. In fact, denote by $\mathcal{S}(\tau_f^{ij}, \tau_d^{ij})$ the class of all DoS signals for which $\alpha^{ij} \geq 1$. Then, $\mathcal{S}(\tau_f^{ij}, \tau_d^{ij})$ does always contain DoS signals for which communication over the link $\{i, j\}$ can be permanently denied. As an example, consider the DoS signal characterized by $(h_n^{ij}, \tau_n^{ij}) = (t_k^{ij}, 0)$. This DoS signal satisfies Assumption 1 and 2 with $(\eta^{ij}, \kappa^{ij}, \tau_f^{ij}, \tau_d^{ij}) = (1, 0, \Delta_*^{ij}, \infty)$, but destroys any communication attempt over the link $\{i, j\}$. As another example, consider the DoS signal characterized by $(h_0^{ij}, \tau_0^{ij}) = (0, \infty)$. This signal satisfies Assumption 1 and 2 with $(\eta^{ij}, \kappa^{ij}, \tau_f^{ij}, \tau_d^{ij}) = (1, 0, \infty, 1)$, but, as before, destroys any communication attempt over the link $\{i, j\}$. In both the examples, $\alpha^{ij} = 1$.

Requiring $\alpha^{ij} < 1$ is not surprising. In fact, the fulfillment of this condition requires that

$$\tau_f^{ij} > \Delta_*^{ij} \quad \text{and} \quad \tau_d^{ij} > 1 \quad (25)$$

The first requirement, $\tau_f^{ij} > \Delta_*^{ij}$, simply means that DoS can occasionally occur at a rate faster than the highest transmission rate of the link $\{i, j\}$. However, on the average, the frequency at which DoS can occur must be sufficiently small compared to sampling rate of the network link. Likewise, the second requirement, $\tau_d^{ij} > 1$, simply means that, on the average, the amount of DoS affecting link $\{i, j\}$ must necessarily be a fraction of the total time. PoC can be therefore regarded as an average connectivity property.

It is worth noting that in some cases consensus can be preserved even if $\alpha^{ij} \geq 1$ for certain network links. This happens whenever removing such links does not cause the graph to be disconnected. More precisely, let \mathcal{X} be any set of links such that $\mathcal{G}_{\mathcal{X}} := (\mathcal{I}, \mathcal{E} \setminus \mathcal{X})$ remains connected. From the foregoing analysis, it is immediate to conclude that consensus is preserved whenever $\alpha^{ij} < 1$ for all $\{i, j\} \in \mathcal{E} \setminus \mathcal{X}$, even if communication over the links $\{i, j\} \in \mathcal{X}$ is permanently denied.

B. Comparison with classic connectivity conditions

As previously noted, PoC can be regarded as an average connectivity property as it does not require graph connectivity point-wise in time. In this sense, it is reminiscent of *Persistency-of-Excitation* conditions that are found in the literature on consensus under switching topologies (e.g., see [21]). There are, however, noticeable differences. To see this, consider the simple situation in which the DoS pattern is the same for all the links, i.e., $(h_n^{ij}, \tau_n^{ij}) = (h_n, \tau_n)$ for all $\{i, j\} \in \mathcal{E}$ and all $n \in \mathbb{Z}_{\geq 0}$. Under such circumstances, the incidence matrix of the graph is a time-varying matrix satisfying: i) $D(t) = 0$ in the presence of DoS; and ii) $D(t) = D$ in the absence of DoS, where D represents the incidence matrix related to the nominal graph configuration. Consider now a DoS pattern consisting of countable number of singletons, i.e., $H_n = \{h_n\}$ for all $n \in \mathbb{Z}_{\geq 0}$. In a classic continuous-time setting, such a DoS pattern does not destroy consensus. In fact, it is trivial to conclude that there exist constants $c_1, c_2 \in \mathbb{R}_{>0}$ such that (cf. [21])

$$\int_{t_0}^{t_0+c_1} QD(t)D^\top(t)Q^\top dt = QDD^\top Q^\top c_1 > c_2 I \quad (26)$$

for all $t_0 \in \mathbb{R}_{\geq 0}$, where Q is a suitable projection matrix such that $QD(t)D^\top(t)Q^\top$ is nonsingular if and only if the graph induced by $D(t)$ is connected. In the present case, in accordance with the previous discussion, consensus can instead be destroyed. The subtle, yet important, difference is due to the constraint on the frequency of the information exchange that is imposed by the network. In this sense, the notion of PoC naturally extends the *Persistency-of-Excitation* condition to digital networked settings by requiring that the graph connectivity be established over periods of time that are consistent with the maximum transmission rate imposed by the communication protocol.

C. Accounting for genuine DoS

In the foregoing analysis, we focused on the case where DoS is caused by malicious attacks. Of course, DoS might also result from a “genuine” network congestion. Hereafter, we will briefly discuss how the case of genuine DoS can be incorporated into the present framework. We shall focus on a deterministic formulation of the problem. A probabilistic characterization of the problem, though restricted to a centralized setting, has been proposed in [28].

Let $\beta^{ij} \in [0, 1]$ be an upperbound on the average percentage of transmission failures that can occur over the link $\{i, j\}$. This bound can be chosen as representative of the situation where all the network nodes exchange information at the highest transmission rate (according to (14), this is equal to $4d_{\max}/\varepsilon$ for each link). Here, by “average” we mean that, denoting by $T_A^{ij}(\tau, t)$ and $T_F^{ij}(\tau, t)$ the number of transmission attempts and transmission failures for the link $\{i, j\}$ on the interval $[\tau, t]$, it holds that

$$\frac{T_F^{ij}(\tau, t)}{T_A^{ij}(\tau, t)} \leq \beta^{ij} \quad (27)$$

as $T_A^{ij}(\tau, t) \rightarrow \infty$.

This condition can be suitably rearranged. To this end, first notice that the above condition is equivalent to the existence of a positive constant a^{ij} such that

$$T_F^{ij}(\tau, t) \leq a^{ij} + \beta^{ij} T_A^{ij}(\tau, t) \quad (28)$$

for all $t, \tau \in \mathbb{R}_{\geq 0}$ with $t \geq \tau$. Moreover, it holds that $T_A^{ij}(\tau, t) \leq \lceil (t - \tau)/\Delta_*^{ij} \rceil$ since, by construction, Δ_*^{ij} is the smallest inter-transmission time for the link $\{i, j\}$. Letting $b^{ij} := a^{ij} + 1$, we then have

$$T_F^{ij}(\tau, t) \leq b^{ij} + \frac{t - \tau}{(\Delta_*^{ij}/\beta^{ij})} \quad (29)$$

Therefore, we can regard genuine transmission failures as the result of a DoS signal in the form of a train of pulses that are superimposed to the transmission instants, where $T_F^{ij}(\tau, t)$ coincides with the number $n^{ij}(\tau, t)$ of DoS off/on transitions occurring on the interval $[\tau, t]$. Thus, Assumption 1 and 2 are satisfied with $(\eta^{ij}, \kappa^{ij}, \tau_f^{ij}, \tau_d^{ij}) = (b^{ij}, 0, \Delta_*^{ij}/\beta^{ij}, \infty)$. According to the analysis of Section IV, one can conclude the following: i) if only genuine transmission failures are present (no malicious DoS), *Persistency-of-Communication* is preserved as long as

$$\frac{1}{\tau_d^{ij}} + \frac{\Delta_*^{ij}}{\tau_f^{ij}} = \beta^{ij} < 1 \quad (30)$$

This is consistent with intuition and, in fact, simply means that communication over the link $\{i, j\}$ is not permanently destroyed if and only if $T_F^{ij}(\tau, t) < T_A^{ij}(\tau, t)$ on the average; ii) in case of genuine and malicious transmission failures, one can simply consider two independent DoS signals acting on the same link, each one characterized by its own 4-tuple $(\eta^{ij}, \kappa^{ij}, \tau_f^{ij}, \tau_d^{ij})$. It is immediate to see that the analysis of Section IV carries over to the present case by replacing condition $\alpha^{ij} < 1$ with $\alpha^{ij} + \beta^{ij} < 1$.

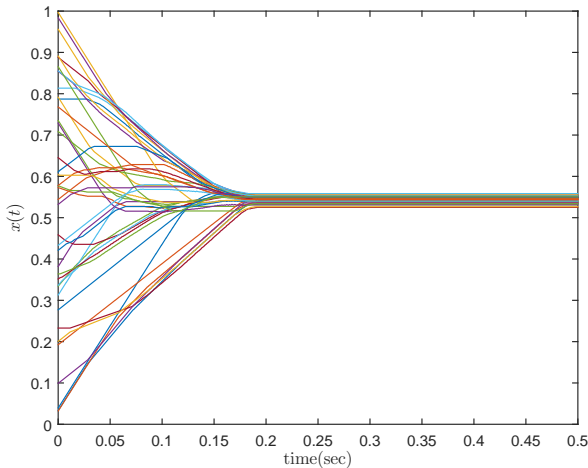


Fig. 1. Evolution of x , corresponding to the solution to (2) and (13) for a random graph with $n = 40$ nodes in the absence of DoS.

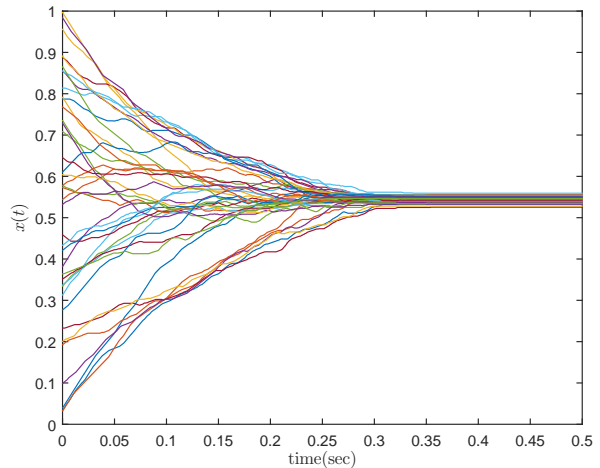


Fig. 2. Evolution of x , corresponding to the solution to (2) and (13) for a random graph with $n = 40$ nodes in the presence of DoS.

TABLE I
DoS AVERAGE DUTY CYCLE OVER SOME LINKS

| Link $\{i, j\}$ | Duty cycle (%) | Link $\{i, j\}$ | Duty cycle (%) |
|-----------------|----------------|-----------------|----------------|
| $\{13, 14\}$ | 49 % | $\{6, 34\}$ | 44.78 % |
| $\{34, 39\}$ | 55.96 % | $\{9, 26\}$ | 47.3 % |
| $\{9, 21\}$ | 52.76 % | $\{33, 38\}$ | 58.96 % |

VI. A NUMERICAL EXAMPLE

We consider a random connected undirected graph with $n = 40$ nodes and with $d^i = 4$ for all $i \in \mathcal{I}$. Nodes and control initial values are generated randomly within the interval $[0, 1]$ and the set $\{-1, 0, 1\}$, respectively.

We consider the behavior of (2) and (13) with $\varepsilon = 0.005$. Figure 1 depicts simulation results for the nominal case in which DoS is absent. Notice that in this case (13) coincides with (4). We next consider the case in which DoS is present. Simulation results are reported in Figure 2. In the simulation, we considered DoS attacks which affect each of the network links independently. For each link, the corresponding DoS pattern takes the form of a pulse-width modulated signal with variable period and duty cycle (maximum period of 0.15sec and maximum duty cycle equal to 100%), both generated randomly. These patterns are reported in Table I and depicted in Figure 3 for a few number of network links. Notice that, for each DoS pattern, one can compute corresponding values for $(\eta^{ij}, \kappa^{ij}, \tau_f^{ij}, \tau_d^{ij})$. They can be determined by computing the values $n^{ij}(\tau, t)$ and $|\Xi^{ij}(\tau, t)|$ of each DoS pattern (*cf.* Assumption 1 and 2) over the considered simulation horizon. Figure 4 depicts the obtained values of τ_f^{ij} and τ_d^{ij} for each $\{i, j\} \in \mathcal{E}$. One sees that these values are consistent with the requirements imposed by the PoC condition.

VII. CONCLUDING REMARKS

We investigated self-triggered coordination for distributed network systems in the presence of Denial-of-Service at the communication links, of both genuine and malicious nature. We considered a general framework in which DoS can affect each of the network links independently, which is relevant for

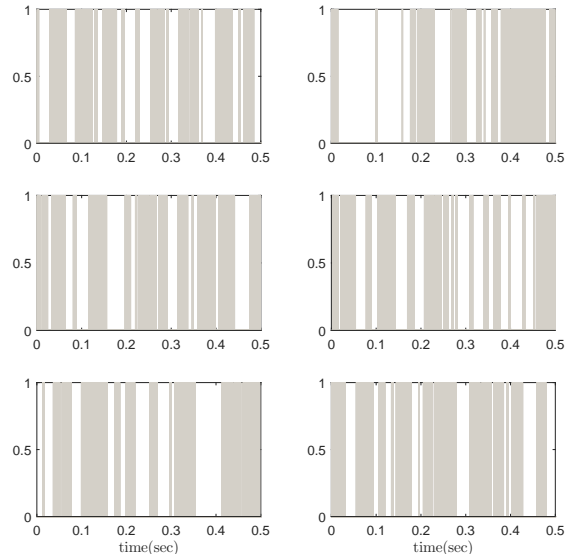


Fig. 3. DoS pattern for the network links $\{13, 14\}$, $\{6, 34\}$, $\{34, 39\}$, $\{9, 26\}$, $\{9, 21\}$ and $\{33, 38\}$. The vertical gray stripes represent the time-intervals over which DoS is active.

networks operating in peer-to-peer mode. By introducing a notion of Persistency-of-Communication (PoC), we provided an explicit characterization of DoS frequency and duration under which consensus can be preserved by suitably designing time-varying control and communication policies. An explicit characterization of the effects of DoS on the consensus time has also been provided. We compared the notion of PoC with classic average connectivity conditions that are found in pure continuous-time consensus networks. The analysis reveals that PoC naturally extends such classic conditions to a digital networked setting by requiring graph connectivity over periods of time that are consistent with the constraints imposed by the communication medium.

The present results lend themselves to many extensions.

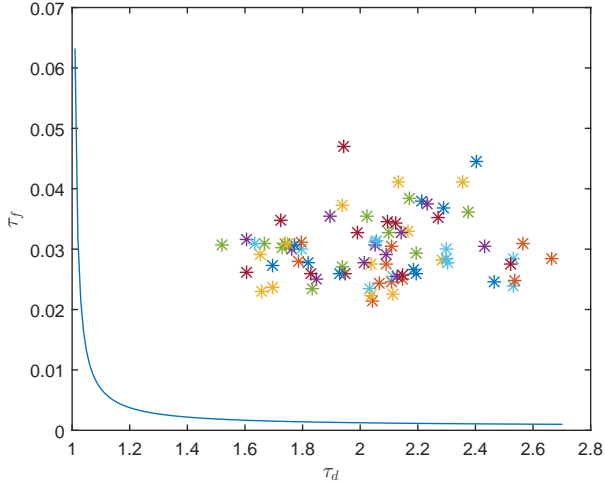


Fig. 4. Locus of the points $1/\tau_d + \Delta_*/\tau_f^{ij} = 1$ as a function of (τ_d, τ_f) with $\Delta_* = X$ (blue solid line). Notice that $\Delta_* = \Delta_*^{ij}$ for all $\{i, j\} \in \mathcal{E}$, so that the locus of point does not vary with $\{i, j\}$. The various * represent the values of $(\tau_d^{ij}, \tau_f^{ij})$ for the network links.

Most notably, it is interesting to investigate whether the present results can be extended to coordination problems involving higher-order nodes dynamics. Another interesting investigation pertains the analysis of coordination schemes in the presence of both DoS and deceptive attacks.

APPENDIX

Proof of Proposition 2. Consider any link $\{i, j\} \in \mathcal{E}$, and suppose that a certain transmission attempt t_k^{ij} is unsuccessful. We claim that a successful transmission over $\{i, j\}$ does always occur within $[t_k^{ij}, t_k^{ij} + \Phi^{ij}]$. We prove the claim by contradiction. To this end, we first introduce some auxiliary quantities. Let $\bar{H}_n^{ij} := \{h_n^{ij}\} \cup [h_n^{ij}, h_n^{ij} + \tau_n^{ij} + \Delta_*^{ij}]$, denote the n -th DoS interval over the link $\{i, j\}$ prolonged by Δ_*^{ij} units of time. Also let

$$\bar{\Xi}^{ij}(\tau, t) := \bigcup_{n \in \mathbb{Z}_{\geq 0}} \bar{H}_n^{ij} \cap [\tau, t] \quad (31)$$

$$\bar{\Theta}^{ij}(\tau, t) := [\tau, t] \setminus \bar{\Xi}^{ij}(\tau, t) \quad (32)$$

Suppose then that the claim is false, and let t_* denote the last transmission attempt over $[t_k^{ij}, t_k^{ij} + \Phi^{ij}]$. Notice that this necessarily implies $|\bar{\Theta}^{ij}(t_k^{ij}, t_*)| = 0$. To see this, first note that, in accordance with (14), the inter-sampling time over the interval $[t_k^{ij}, t_*)$ is equal to $\varepsilon/(2(d^i + d^j)) = \Delta_*^{ij}$. Hence, we cannot have $|\bar{\Theta}^{ij}(t_k^{ij}, t_*)| > 0$ since this would imply the existence of a DoS-free interval within $[t_k^{ij}, t_*)$ of length greater than Δ_*^{ij} , which is not possible since, by hypothesis, no successful transmission attempt occurs within $[t_k^{ij}, t_*)$. Thus $|\bar{\Theta}^{ij}(t_k^{ij}, t_*)| = 0$. Moreover, since t_* is unsuccessful, it must be contained in a DoS interval, say H_q^{ij} . This implies $[t_*, t_* + \Delta_*^{ij}] \subseteq \bar{H}_q^{ij}$. Hence,

$$\begin{aligned} |\bar{\Theta}(t_k^{ij}, t_* + \Delta_*^{ij})| &= |\bar{\Theta}(t_k^{ij}, t_*)| + |\bar{\Theta}(t_*, t_* + \Delta_*^{ij})| \\ &= 0 \end{aligned} \quad (33)$$

However, condition $|\bar{\Theta}(t_k^{ij}, t_* + \Delta_*^{ij})| = 0$ is not possible. To see this, simply notice that

$$\begin{aligned} |\bar{\Theta}(t_k^{ij}, t)| &= t - t_k^{ij} - |\bar{\Xi}(t_k^{ij}, t)| \\ &\geq t - t_k^{ij} - |\Xi(t_k^{ij}, t)| - (n(t_k^{ij}, t) + 1)\Delta_*^{ij} \\ &\geq (t - t_k^{ij})(1 - \alpha^{ij}) - \kappa^{ij} - (\eta^{ij} + 1)\Delta_*^{ij} \end{aligned} \quad (34)$$

for all $t \geq t_k^{ij}$ where the first inequality follows from the definition of the set $\bar{\Xi}(\tau, t)$ while the second one follows from Assumption 1 and 2. Hence, by (34), we have $|\bar{\Theta}(t_k^{ij}, t)| > 0$ for all $t > t_k^{ij} + (1 - \alpha^{ij})^{-1}(\kappa^{ij} + (\eta^{ij} + 1)\Delta_*^{ij}) = t_k^{ij} + \Phi^{ij}$. Accordingly, $|\bar{\Theta}(t_k^{ij}, t_* + \Delta_*^{ij})| = 0$ cannot occur because $t_* + \Delta_*^{ij} > t_k^{ij} + \Phi^{ij}$. In fact, by hypothesis, t_* is defined as the last unsuccessful transmission attempt within $[t_k^{ij}, t_k^{ij} + \Phi^{ij}]$, and, by (14), the next transmission attempt after t_* occurs at time $t_* + \Delta_*^{ij}$. This concludes the proof. ■

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