Conformal or confining

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We present a lattice study of the zero-temperature line of chiral symmetry breaking phase transitions in the conformal window of QCD for varying number of flavors. We argue that it is consistent with a lower edge of the conformal window between $N_f = 8$ and $N_f = 6$, and remarkably in agreement with perturbation theory and recent large-N arguments. We add a theoretical analysis of the anomalous dimension of the scalar glueball operator and show that its agreement with perturbation theory and large-N arguments would be sufficient to exclude an ultraviolet-infrared fixed-point merging as a mechanism for the loss of conformality.

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The possibility of an emergent conformal symmetry underlying the interactions of fundamental constituents between the electroweak symmetry breaking scale and the Planck scale has recently attained strong theoretical and experimental appeal. From the theoretical point of view, we have one successful quantum gauge theory, Quantum Chromodynamics with massless fermions, where conformal symmetry is lost in a highly non trivial way, leading to two manifestations of one single breaking phenomenon: asymptotic freedom and confinement [1] — one cannot exist without the other. This concept is strikingly clear in the recently proposed solution [2] for the scalar glueball correlator in the ‘t Hooft limit of large-N QCD. It has also become clear that a wide class of gauge field theories arising from applying present-day AdS/CFT correspondence are fundamentally different from QCD, i.e., they cannot become QCD by perturbing the conformal field theory neither in the ultraviolet nor in the infrared. The family of theories called the conformal window falls into this category — these theories in the continuum live at a non-trivial infrared fixed point (IRFP) and are separated from QCD or supersymmetric-QCD (SQCD) by a phase transition whose nature has yet to be established. We are likely to learn more about the deep differences between confining and conformal gauge theories and the applicability of the AdS/CFT correspondence by better understanding which mechanism(s) separate the conformal window from QCD and SQCD; as a byproduct, we may hope to shed light on the detailed consequences of removing supersymmetry.

This letter is an attempt to establish to what extent the emergence of the conformal window realizes the predictions of perturbation theory or the implications of large-N arguments. We address this question by means of a lattice field theory study of the order parameter of chiral symmetry breaking. Subsequently, we offer a theoretical analysis of the anomalous dimension of the scalar glueball operator, probe of the deconfinement transition. It allows us to show how this quantity discriminates among different mechanisms of disappearance of the conformal window. The combined use of observables sensitive to chiral symmetry and observables sensitive to confinement is also the way to understand the interplay of confinement and chiral symmetry breaking at the lower edge of the conformal window.

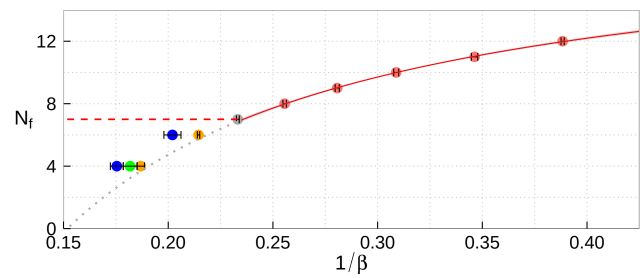

FIG. 1. Lattice phase diagram $N_f - 1/\beta$, β the inverse squared lattice coupling. Data points for $N_f = 7, \ldots, 12$ are midpoints of sharp zero-temperature crossovers in Fig. 2. Best fit to data is linear in β and extrapolated down to $N_f = 4$ (dotted gray). Flattening of bulk line at endpoint is sketched (dashed red); chiral symmetry is broken below the line. Thermal transitions for $N_f = 4$, 6 occur away from the bulk line, at weaker coupling, with volumes $24^3 \times N_t$, $N_t = 6$ (orange), 8 (green) and 12 (blue), right to left.

The lower edge of the conformal window: a lattice study. A numerical determination of the lower edge of the conformal window of QCD is one way to establish how far large-N QCD or perturbation theory to a given loop order are from the complete theory. We have numerically mapped out the line of zero-temperature (bulk) chiral symmetry breaking phase transitions for $N_f \leq 12$, aiming to identify the critical number of flavors N_f^c that signals the disappearance of the conformal window. The setup of the simulations is the one of [3]. The results are summarized in Fig. 1. We have also extrapolated the line to $N_f = 16$, obtaining $\beta_c \sim 1.2$ consistent with previous studies [4] and [5] carried out with different lattice actions and heavier fermions. This line, i.e., the critical coupling as a function of N_f in any given renormalization scheme, should manifest the fact that fermion screening is increasingly effective for increasing N_f; this

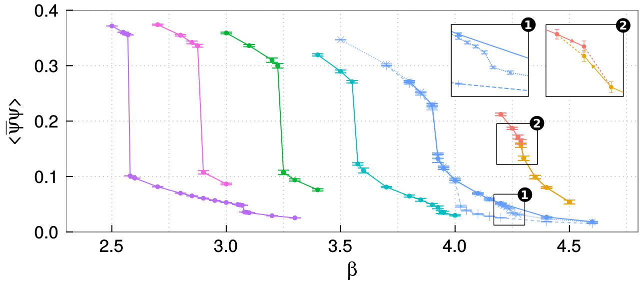

FIG. 2. Sequence of sharp crossovers of the chiral condensate for $N_f = 12$ to 7 (left to right). $N_f = 12$ from [4], $N_f = 11, 10, 9$, $V = 12^3 \times 24$, $N_f = 8$ $16^3 \times 32$ (circles, solid line), $24^3 \times 12$ (triangles, dotted line), $24^3 \times 6$ (squares, dashed line), and $N_f = 7$ $16^3 \times 32$, all with bare lattice mass $am = 0.01$. Edge of exotic phase displayed for $N_f = 12$, 9 and $N_f = 8$, $N_t = 6$ and $N_t = 12$ (inset 1). Inset 2 shows data for two branches of possible hysteresis loop for $N_f = 7$.

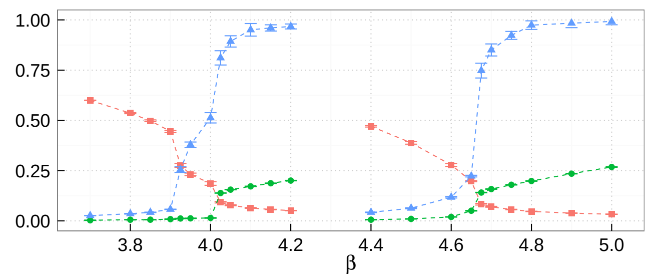

FIG. 3. Chiral condensate (red squares), (connected) R_π (blue triangles) and rescaled 2×Polyakov loop (green circles) for $N_f = 8$ (left) and $N_f = 6$ (right), $24^3 \times 6$, $am = 0.01$. Points in Fig. 1 correspond to the combined observation of the maximum derivative of the chiral condensate, of R_π and, when appropriate, of the Polyakov loop.

N_f dependence is a leading order effect separating two phases with different symmetries for $N_f \geq N_f^c$, different in nature from the subleading N_f dependence carried by (fake) zero-temperature transitions possibly occurring within the chirally broken phase at $N_f < N_f^c$ on coarse lattices. The critical bulk line should flatten at N_f^c in the massless limit, as sketched in Fig. 1; at finite lattice spacing a and temperature $T = 1/(aN_t)$ the critical bulk line is the $N_t = 1/(aT) \to \infty$ limit of an N_t-finite family of thermal phase transitions that exists for all $N_f > 0$.

To illustrate this concept and provide a determination of N_f^c we show in Fig. 1 the location of thermal phase transitions for varying $N_t < N_l$, with spatial volume $L = aN_l$, for the $N_f = 6$ and $N_f = 4$ theories. The locations have been determined with a study analogous in spirit to [6] and contained in a forthcoming paper [7]. As expected for theories below the conformal window, where chiral symmetry restoration occurs at a critical temperature $T_c = 1/(a(\beta_c)N_t)$, the locations move to weaker coupling for increasing N_t. Fig. 1 also shows that already at the smallest $N_t = 6$ the transitions occur at a coupling weaker than the one predicted by the linear in β extrapolation of the bulk line. At the same time, no zero-temperature phase transition occurs for $N_f = 6, 4$ along the extrapolated line, nor it is observed at weaker coupling [8], consistently with the fact that chiral symmetry is all the way broken for these systems. This behavior has to be contrasted with the one of the $N_f = 8$ system, for which we have carried out the same study.

Fig. 2 illustrates the sequence of sharp crossovers that nicely align to form the critical bulk line in Fig. 1. Notably for $N_f = 8$, the transitions with $N_t = 6$ and $N_t = 12$ nicely overlap with the zero-temperature transition [9], which is indeed their limiting point on the bulk line in Fig. 1. The edge of the chirally symmetric exotic phase, a genuine lattice artifact studied in [4, 10] is also shown for $N_f = 12$, 9 and $N_f = 8$ with $N_t = 6$, 12; this edge is observed to be N_t dependent up to some N_t [11], see also [12], and it disappears in the massless limit consistently with exact chiral symmetry [4]. At nonzero mass, it serves as a useful discriminator on the lattice of theories inside and outside the conformal window.

Fig. 3 displays the differences between $N_f = 8$ and $N_f = 6$. The chiral condensate, the (connected) chiral cumulant $R_\pi = \chi_{\rm conn}/\chi_\pi = (\partial\langle\bar\psi\psi\rangle/\partial m_{\rm valence})/(\langle\bar\psi\psi\rangle/m)$ ratio of the connected scalar and the pseudoscalar chiral susceptibilities, and the real part of the Polyakov loop are shown for $N_t = 6$. For the $N_f = 8$ theory, a second sharp crossover of the chiral condensate and a dip in the chiral cumulant mark the edge of the exotic phase, analogously to our $N_f = 12$ study [4]. These are absent for $N_f = 6$. Another contrast between the two theories is in the Polyakov loop: for $N_f = 6$ it becomes nonzero at the thermal transition, signaling the onset of the high-temperature deconfined phase, while for $N_f = 8$ it only becomes nonzero at the edge of the exotic phase, where the system is deconfined and chiral symmetry exact. We observe the $N_f = 4$ theory to behave as $N_f = 6$. Throughout this study, we have not observed an anomalous behavior that could hint at consequences of the fourth-root of the fermion determinant for staggered lattice fermions.

The most striking result is the extension of the bulk line in Fig. 1, and therefore the conformal window, below $N_f = 8$. The fate of $N_f = 7$ is less certain at this stage, though we observe a reduced crossover, an almost closing gap, and signs of hysteresis in the interval $\beta = [4.275, 4.3]$ right on the bulk line; a study analogous to $N_f = 8$ and 6, possibly at a lighter fermion mass, is likely to clarify the situation. These results also clearly suggest that the $N_f = 6$ theory is below the conformal window. The sharp crossovers in Fig. 3 do not leave space for a behavior analogous to $N_f = 8$ at a lower mass.

We suspect that the transition observed for $N_f = 8$ and interpreted as a thermal transition in [6, 13] — with a different improvement of the fermion action — may instead be the edge of the exotic phase, later on discovered by the same authors [14] and extensively studied [4, 10, 12]. This result does not contradict the upper bounds on the chiral phase boundary in, e.g., [6, 15], but it shows the delicacy of this study and how the gradual understanding of many aspects of the conformal window finally seems to lead to a consistent picture.

The strategy of this study is motivated by the observation that most quantities will plausibly evolve in a nonsingular way from the conformal to the confining phase, rendering the determination of the endpoint numerically uncertain if not corroborated by the signature of a phase transition. Interestingly, a lower edge below $N_f = 8$ and close to $N_f = 7$ is not far from the prediction of two-loop perturbation theory and intriguingly in good agreement with the large-N result $N_f/N = 5/2$, for $N = 3$ [16] and four-loop perturbation theory. It is also in agreement with the perturbatively small value of the fermion mass anomalous dimension for $N_f = 12$ [3, 17], because $N_f = 12$ is not close to the lower edge of the conformal window.

The anomalous dimension of the scalar glueball operator in perturbation theory and large-N. It is well known that the anomalous dimension of the scalar glueball operator $\mathrm{Tr}(G^2)$ is constrained by the trace anomaly, i.e., the nonzero contribution to the trace of the energy-momentum tensor. The trace anomaly of QCD that enters the matrix elements of renormalized gauge invariant operators is [18]

$$T^\mu_\mu = \frac{\beta(\alpha)}{16\pi\alpha^2}\,\mathrm{Tr}(G^2) + \text{fermion mass contribution}\,, \quad (1)$$

with the beta-function

$$\beta(\alpha) \equiv \frac{d\alpha(\mu)}{d\ln\mu} \qquad \alpha \equiv \frac{g^2}{4\pi}\,. \quad (2)$$

The dimension of a quantum operator O is dictated by the scaling equation

$$\frac{dO}{d\ln\mu} = d_O O \qquad O(\mu) \sim \mu^{d_O}\,, \quad (3)$$

$d_O = d_c + \gamma_O$, with canonical dimension d_c and anomalous dimension γ_O. The nonrenormalization of T^μ_μ implies that it scales classically, i.e., $d_{T^\mu_\mu} = 4$ in four dimensions, and the scaling equation (3) applied to Eq. (1) gives for $\mathrm{Tr}(G^2)$

$$d_G = 4 - \beta'(\alpha) + \frac{2}{\alpha}\beta(\alpha)\,, \quad (4)$$

with $\beta'(\alpha)$ the derivative of the beta-function w.r.t. α. At a nontrivial stable IRFP, $\beta(\alpha_*) = 0$ and the anomalous dimension

$$\gamma_G = -\beta'(\alpha_*) \quad (5)$$

is a physical property of the system, renormalization scheme independent. It is instructive to determine γ_G in perturbation theory to a given loop order inside the conformal window, and compare this result with the Veneziano limit of large-N QCD ($N \to \infty$, $N_f/N = \text{const}$) [16]. Given the loop expansion $\beta(\alpha) = -\alpha\sum_{l=1}^{\infty} b_l\alpha^l$, with universal coefficients $b_{1,2}$ [19] and $b_{3,4}$ in the $\overline{MS}$ scheme [20], the IRFP coupling to n-loop order $\alpha_{\mathrm{IR},n}$ is solution of $\beta(\alpha) = 0$ to that order, while $\beta'(\alpha) = -\sum_{l=1}^{n}(l+1)\bar{b}_l\alpha^l$. However, the zero of the beta-function is in general a necessary, but not sufficient condition for the existence of a stable IRFP. At two loops $\alpha_{\mathrm{IR},2} = -b_1/b_2$ and $\beta'(\alpha_{\mathrm{IR},2}) = -b_1^2/b_2$. The four-loop beta-function is a cubic equation with three zeros, one of which is negative [21]. The smallest positive zero is listed in Table I, and values agree with [21]. In all cases,

	$n=2$		$n=3$		$n=4$	
N_f	$\alpha_{\mathrm{IR},n}$	$\beta'(\alpha_{\mathrm{IR},n})$	$\alpha_{\mathrm{IR},n}$	$\beta'(\alpha_{\mathrm{IR},n})$	$\alpha_{\mathrm{IR},n}$	$\beta'(\alpha_{\mathrm{IR},n})$
6	-	-	12.992	84.646	-	-
7	-	-	2.453	5.956	-	-
8	-	-	1.464	2.654	1.552	1.784
9	5.237	4.169	1.027	1.472	1.070	1.460
10	2.21	1.522	0.764	0.869	0.815	0.851
11	1.23	0.706	0.578	0.513	0.626	0.496
12	0.754	0.360	0.435	0.296	0.470	0.281

TABLE I. Smallest positive zero of the QCD beta-function $\alpha_{\mathrm{IR},n}$, $n = 2, 3, 4$ loops in the $\overline{MS}$ scheme, and derivative $\beta'(\alpha_{\mathrm{IR},n})$ at the zero, in the interesting range $N_f \leq 12$.

the derivative $\beta'(\alpha_*)$ is positive and increases along the IRFP line for decreasing N_f. The disappearance of the zero occurs for $N_f > 8$ at two loops, while it shifts to lower N_f at three and four loops, suggesting a lower endpoint of the conformal window in the range $7 < N_f < 8$ at four loops — provided the zero can be taken as sufficient condition. At two loops the disappearance of the zero is determined by the change of sign of b_2, implying that the fixed point disappears at infinite coupling $\alpha_{\mathrm{IR},2} \to \infty$. This behavior, however, is likely to be an artifact of the truncated perturbative expansion; the same singularity occurs in $\beta'(\alpha_{\mathrm{IR},2})$.

It is most interesting to compare these results with the implications of the exact beta-function of large-N QCD in the Veneziano limit derived in [16], which manifests salient analogies and differences with the exact beta-function of SQCD [22–24]. Writing in short the large-N QCD beta-function for the canonical ‘t Hooft coupling as

$$\beta(g) = \frac{g^3\,c(g)}{1 - \frac{4}{(4\pi)^2}g^2}\,, \quad (6)$$

the absence of supersymmetry generates a new anomalous dimension contribution, not present in SQCD, in $c(g)$. The fate of a fixed point depends on the numerator and denominator of Eq. (6). The pole generates a cusp in the flow of g, unless the numerator has a zero before the pole is hit — and a zero associated to an IRFP inside the conformal window must be shown to be renormalization scheme independent. Remarkably, this has been shown to be true [16] for the large-N QCD beta-function in the Veneziano limit at the lower edge of the conformal window, identified at $N_f/N = 5/2$ where the stability of the glueball kinetic term is lost. Below the conformal window, zeros of the beta-function may still occur. One example is the saturation of the coupling g_{phys} entering the static inter-quark potential in large-N Yang-Mills

$$V(r) = \sigma r - \frac{g^2_{phys}(1/r)}{4\pi r}\,, \tag{7}$$

with nonzero string tension σ. The effective charge g_{phys} in the Coulomb potential is observed to saturate to a constant at large distances in lattice $SU(3)$ Yang-Mills [25] and provides agreement with the effective bosonic string theory prediction and open-close string duality [26]. In other words, the beta-function of g_{phys} develops a zero, while conformal symmetry remains broken due to the linear confining contribution to the potential (non-zero string tension) dominating the large distance behavior. From [27] we learn an important lesson. In [27] a renormalization scheme for the large-N Yang-Mills exact beta-function was constructed, where the canonical 't Hooft coupling coincides with the physical effective charge entering the static inter-quark potential in Eq. (7), and it develops a zero at the Landau pole of the Wilsonian coupling, at $r^{-1} = \Lambda_W$. Hence, it actually predicts that for $r > \Lambda_W^{-1}$ the effective charge g_{phys} saturates to a constant.

If the same behavior is realized beyond large-N in the presence of $N_f < N_f^c$ massless fundamental flavors, then the QCD beta-function below the conformal window in a given renormalization scheme can develop zeros without implying conformal symmetry. The only probes of the absence of conformal symmetry remain the n-point functions that involve the string-tension term of the inter-quark potential, the observables sensitive to chiral symmetry breaking and, importantly, topological quantities.

A fundamental difference between SQCD and QCD is the presence in SQCD of a phase just below the lower edge, for $N_c + 2 \leq N_f \leq 3N_c/2$, where the only description that makes physical sense is in terms of the dual variables that describe the free non-Abelian magnetic phase [28]; this property is consistent with the occurrence of the cusp in the flow just below the conformal window for SQCD. The absence of this phase in QCD calls instead for a differentiable flow, thus without cusps, across and below the lower edge of the conformal window. It is rewarding that the beta-function of large-N QCD can realize this property [16]. It also supports that the lower edge singularity for $b_2 = 0$ at two loops arises as an artifact of n-loop perturbation theory. Finally, since the large-N beta-function reproduces the two-loop one up to $O(1/N^2)$ contributions [16], the predicted γ_G also reproduces the two-loop result up to $O(1/N^2)$; however, it is expected to remove the singularity for $b_2 = 0$ at the lower edge of the conformal window.

Perturbation theory, as well as the large-N solution thus predict an increasing anomalous dimension $|\gamma_G|$ ($|\beta'(\alpha_*|$) for $N_f \searrow N_f^c$. We observe that this behavior is opposite to the one implied by an UV-IR fixed-point merging phenomenon at N_f^c. In fact, assuming in some generality

$$\beta(\alpha, N_f) = (N_f - N_f^c) - (\alpha - \alpha^c)^2 \tag{8}$$

close to N_f^c where the merging would occur, see also [29], the beta-function develops a local maximum at N_f^c, i.e., $\beta'(\alpha^c) = 0$, with decreasing $|\beta'|$ along the IRFP line $N_f \searrow N_f^c$. A measurement of γ_G for varying N_f inside the conformal window and in agreement with perturbation theory and large-N would therefore exclude a fixed-point merging mechanism for the conformal window of QCD. We recall that the UV-IR fixed-point merging is one simple way to realize preconformal Miransky-BKT scaling [29, 30] just below the conformal window. If the fixed-point merging turns out not to occur in QCD, it becomes interesting to understand from a renormalization group point of view if preconformal scaling can at all be realized with a single IRFP, and if it can reproduce the trend of the anomalous dimensions implied by perturbation theory and large-N arguments.

To conclude, our numerical study of the order parameter of chiral symmetry breaking points at a lower edge of the conformal window for QCD between $N_f = 8$ and $N_f = 6$, in remarkably close agreement with perturbation theory and large-N arguments, the latter based on the behavior of glueball operators sensitive to confinement. A lattice determination of the anomalous dimension of the scalar glueball operator at the conformal IRFP is desirable. We also recognize that the Wilson flow proposed in [31, 32] can be a useful tool in this context, in order to discriminate between a conformal and a confining behavior in the theory formulated on a lattice. Finally, a conformal window for non-supersymmetric QCD extending to flavors as low as $N_f = 7$ would make these theories more appealing for phenomenology beyond the standard model, realizing the emergence of conformal symmetry right above QCD at the electroweak symmetry breaking scale.

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[1] Confinement in this context is the existence of a mass-gap, i.e., a non-zero glueball mass in quenched QCD.
[2] M. Bochicchio, Nucl. Phys. **B875**, 621 (2013).
[3] M. Lombardo, K. Miura, T. J. N. da Silva, and E. Pallante, JHEP **1412**, 183 (2014).
[4] A. Deuzeman, M. P. Lombardo, T. Nunes Da Silva, and E. Pallante, Phys. Lett. **B720**, 358 (2013).
[5] The transition occurs at $\beta_c = 1.675 \pm 0.025$ for $am = 0.025$, same lattice action as this work (unpublished).
[6] K. Miura and M. P. Lombardo, Nucl. Phys. **B871**, 52 (2013).
[7] T. Nunes da Silva, E. Pallante, and L. Robroek, to appear.
[8] The main discriminator between the finite-temperature $N_f = 6, 4$ systems with $N_t = 12$ and mass $am = 0.01$ and the same systems at zero temperature (volume $12^3 \times 24$) is the Polyakov loop. Only the finite temperature system undergoes a change from zero of the real part of the Polyakov loop, signaling deconfinement.
[9] We have obtained identical results at volume $16^3 \times 32$ and volume $12^3 \times 24$, barring finite volume effects for this and other N_f.
[10] A. Cheng, A. Hasenfratz, and D. Schaich, Phys. Rev. **D85**, 094509 (2012).
[11] At $N_t = 12$ the chiral condensate signal is suppressed, while a dip in the chiral cumulant and a jump in the Polyakov loop are more evident at $\beta = 4.2$.
[12] A. Deuzeman, M. P. Lombardo, T. N. da Silva, and E. Pallante, PoS **LATTICE2011**, 321 (2011).
[13] A. Deuzeman, M. P. Lombardo, and E. Pallante, Phys. Lett. **B670**, 41 (2008).
[14] A. Deuzeman, E. Pallante, M. P. Lombardo, and E. Pallante, PoS **LATTICE2008**, 056 (2008).
[15] M. P. Lombardo, K. Miura, T. N. da Silva, and E. Pallante, PoS **LATTICE2014**, 242 (2014).
[16] M. Bochicchio, (2013), arXiv:1312.1350 [hep-th].
[17] A. Cheng, A. Hasenfratz, Y. Liu, G. Petropoulos, and D. Schaich, Phys. Rev. **D90**, 014509 (2014).
[18] Here we are not interested in the most general expression, which also involves gauge-fixing and EoM operators, see [33].
[19] D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973); H. D. Politzer, *ibid.* **30**, 1346 (1973); W. Caswell, *ibid.* **33**, 244 (1974); D. Jones, Nucl. Phys. **B75**, 531 (1974).
[20] O. Tarasov, A. Vladimirov, and A. Y. Zharkov, Phys. Lett. **B93**, 429 (1980); S. Larin and J. Vermaseren, *ibid.* **B303**, 334 (1993); T. van Ritbergen, J. Vermaseren, and S. Larin, *ibid.* **B400**, 379 (1997).
[21] T. A. Ryttov and R. Shrock, Phys. Rev. **D83**, 056011 (2011).
[22] V. Novikov, M. Shifman, A. Vainshtein, and V. Zakharov, Nucl. Phys. **B229**, 381 (1983).
[23] V. Novikov, M. Shifman, A. Vainshtein, and V. Zakharov, Phys. Lett. **B166**, 329 (1986).
[24] M. Shifman and A. Vainshtein, Nucl. Phys. **B277**, 456 (1986).
[25] M. Luescher and P. Weisz, JHEP **2002**, 049 (2002).
[26] M. Luescher and P. Weisz, JHEP **2004**, 014 (2004).
[27] M. Bochicchio, JHEP **0905**, 116 (2009).
[28] N. Seiberg, Nucl. Phys. **B435** (1995).
[29] D. B. Kaplan, J.-W. Lee, D. T. Son, and M. A. Stephanov, Phys. Rev. **D80**, 125005 (2009).
[30] V. A. Miransky and K. Yamawaki, Phys. Rev. **D55**, 5051 (1997).
[31] M. Luescher, JHEP **1008**, 071 (2010).
[32] R. Narayanan and H. Neuberger, JHEP **0603**, 064 (2006).
[33] N. Nielsen, Nucl. Phys. **B120**, 212 (1977); J. C. Collins, A. Duncan, and S. D. Joglekar, Phys. Rev. **D16**, 438 (1977); S. L. Adler, J. C. Collins, and A. Duncan, *ibid.* **D15**, 1712 (1977).