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## Total variation error bounds for convex approximations of two-stage mixed-integer recourse models

Romeijnders, Ward

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*Document Version*

Publisher's PDF, also known as Version of record

*Publication date:*

2015

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Romeijnders, W. (2015). Total variation error bounds for convex approximations of two-stage mixed-integer recourse models [Groningen]: University of Groningen, SOM research school

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Ward Romeijnders

Publisher: University of Groningen  
Groningen, The Netherlands

Printed by: Ipskamp Drukkers B.V.  
Enschede, The Netherlands

ISBN: 978-90-367-7893-0 (printed version)  
978-90-367-7892-3 (electronic version)

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university of  
 groningen

# Total variation error bounds for convex approximations of two-stage mixed-integer recourse models

## PhD thesis

to obtain the degree of PhD at the  
 University of Groningen  
 on the authority of the  
 Rector Magnificus Prof. E. Sterken  
 and in accordance with  
 the decision by the College of Deans.

This thesis will be defended in public on

Monday 7 September 2015 at 12.45 hours

by

**Ward Romeijnders**

born on 5 September 1988  
 in Groningen

## **Supervisors**

Prof. M.H. van der Vlerk

Prof. W.K. Klein Haneveld

## **Assessment committee**

Prof. D. den Hertog

Prof. R.T. Rockafellar

Prof. I.F.A. Vis

# Preface

This thesis is the result of four years of research at the Department of Operations at the University of Groningen. I want to take the opportunity to thank everyone who has contributed to this thesis or who has supported me during this period. The Netherlands Organisation for Scientific Research (NWO) is gratefully acknowledged for its financial support.

I wish to express my sincere gratitude to my supervisors Maarten van der Vlerk and Wim Klein Haneveld for their excellent and patient guidance. I have enjoyed our research discussions and I greatly value our collaboration.

I want to thank Professors Dick den Hertog, Terry Rockafellar, and Iris Vis for their willingness to be part of the Assessment committee and for their time and effort spent reading this thesis. Furthermore, I wish to thank the coauthors who contributed to parts of this thesis or to any of our joint journal publications. Special thanks are due to David Morton and Rüdiger Schultz for their hospitality during research visits to the University of Texas at Austin (in the fall of 2013) and to the University of Duisburg-Essen (on several occasions).

I am grateful to my colleagues at the Department of Operations for providing a pleasant work environment. In particular, I want to mention Bram de Jonge and Arjan Dijkstra, with whom many lengthy discussions finally resulted in a journal publication, and Gerlach van der Heide, with whom I shared an office these past four years. I am happy that Gerlach and Arjan have agreed to be my paranymphs.

I want to thank my family and friends, whose interest in my work I greatly appreciate. I apologize if I have not been able to explain what I am exactly doing. Finally, I wish to thank my parents Johan and Elma for always being there for me, and my wife Anouk for her unconditional love and support.

Groningen, 2015.



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