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Lepton flavor violation and seesaw symmetries

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Abstract When the standard model is extended with right-handed neutrinos the symmetries of the resulting Lagrangian are enlarged with a new global $U(1)_R$ Abelian factor. In the context of minimal seesaw models we analyze the implications of a slightly broken $U(1)_R$ symmetry on charged lepton flavor violating decays. We find, depending on the R-charge assignments, models where charged lepton flavor violating rates can be within measurable ranges. In particular, we show that in the resulting models due to the structure of the light neutrino mass matrix muon flavor violating decays are entirely determined by neutrino data (up to a normalization factor) and can be sizable in a wide right-handed neutrino mass range.

Keywords Neutrino mass and mixings · Right handed neutrinos · Decays of leptons

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1 Introduction

Apart from demonstrating that neutrinos are massive and have non-vanishing mixing angles among the different generations [1,2], neutrino oscillation experiments have also proved that lepton flavor is not conserved in the neutral lepton sector. Once the standard model is extended to account for neutrino masses—unavoidably—lepton flavor violation (LFV) also takes place in the charged lepton sector. This, however, not necessarily implies that these effects are sizable, so whether these processes can or not have measurable rates depends to a large extent on the details of the corresponding model. Despite this fact, from a general point of view, charged lepton flavor violating processes are expected to have large decay branching fractions as long as the LFV mediators

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have $\mathcal{O}(\text{TeV})$ masses and their couplings to the standard model leptons are about $\gtrsim 10^{-2}$.

Majorana neutrino masses can be generated in a model independent way by adding the dimension-five effective operator $\mathcal{O}_5 \sim LLHH$ to the standard model Lagrangian [3]. And in turn the different concrete realizations of this operator constitute a model for neutrino masses [1]. Among the tree-level realizations the type-I seesaw is certainly the most popular one [14,15,16,17, 18,19]. In this model, light neutrino masses are generated via the exchange of electroweak fermionic singlets (right-handed (RH) neutrinos for brevity). Consistency with neutrino data then requires either heavy RH neutrino masses ($\mathcal{O}(M_N) \sim \Lambda_{\text{GUT}}$) or tiny Yukawa couplings ($\mathcal{O} \sim 10^{-6}$), thus implying negligibly small charged lepton flavor violating effects.

In addition to the standard model gauge symmetry the seesaw Lagrangian features a global Abelian $U(1)_R$ symmetry, typically related with phase rotations of the standard model lepton $SU(2)$ singlets, and thus broken by the charged lepton Yukawa couplings. However, relating this symmetry with phase rotations of the RH leptons fields is not the only possibility, and another approach in which rotations of the left-handed lepton fields and RH neutrinos are allowed is feasible as well. In that case one is left with (at least) two choices: (i) slightly broken $U(1)_R$; (ii) $\mathbb{Z}_n \subset U(1)_R$ invariance of the Lagrangian.

In what follows we will consider possibility (i). In the context of minimal seesaw models (models featuring only 2 RH neutrinos) we will classify the viable scenarios arising from different R-charge assignments, that as we already discussed are not anymore limited to the RH leptons, and identify those models for which charged lepton flavor violating processes have sizable decay branching ratios. For these models we will analyze the μ flavor violating phenomenology. The results presented here are entirely based on ref. [21].

2 The models

Depending on the R-charge assignments of the different standard model and RH neutrino fields different models can be constructed. In order to restrict the discussion only to the lepton sector we start by setting $R(H) = 0$. Requiring the charged lepton Yukawa couplings to be $U(1)_R$ invariant allows to fix $R(e) = R(\ell)$ (e, ℓ being the lepton electroweak singlets and doublets). We are thus left with the lepton doublets and RH neutrinos R-charge assignments. Large lepton flavor violating rates require (at least) the RH neutrino mass terms to be $U(1)_R$ breaking suppressed (the suppression factor denoted by $\epsilon \ll 1$), implying $R(N_{1,2}) \neq 0$ and one of the following three possibilities: (A) $R(N_1) = R(N_2)$; (B) $R(N_1) = -R(N_2)$ or (C) $R(N_1) \neq R(N_2)$. In practice possibilities (A) and (C) turn out to be equivalent as they both lead to models with $N_1 - N_2$ suppressed mixing and therefore to suppressed LFV effects. In contrast in case (B) the $N_1 - N_2$ mixing is maximal and a set of Yukawa couplings can be large

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[1] Examples range from tree-level up to three-loop induced neutrino mass models [4,5,6, 7,8,9,10,11,12,13]

provided the $R(\ell)$ charges are chosen appropriately. In that sense models of type (B) are much more interesting as they might yield large LFV effects.

With the purpose of studying the implications for LFV of type B models we fix the R-charges as $R(N_1, \ell_i, e_i) = +1$ and $R(N_2) = -1$. With this charge assignment the seesaw Lagrangian becomes [2]

$$\mathcal{L} = -\bar{\ell}\,\boldsymbol{\lambda_1}^*\,N_1\tilde{H} - \epsilon_\lambda\,\bar{\ell}\,\boldsymbol{\lambda_2}^*\,N_2\tilde{H} - \frac{1}{2}N_1^T\,CM\,N_2 - \frac{1}{2}\epsilon_N N_a^T\,CM_{aa}\,N_a + \text{h.c.}\,. \tag{1}$$

Here $\tilde{H} = i\sigma_2 H^*$, C is the charge conjugation operator, $\boldsymbol{\lambda_a}^\dagger = (\lambda_{1a}^*, \lambda_{2a}^*, \lambda_{3a}^*)$ with $a = 1, 2$ (matrices are denoted in bold-face) and $\epsilon_{\lambda,N}$ are dimensionless parameters that slightly break $U(1)_R$. Diagonalization of the RH neutrino mass matrix yields two quasi-degenerate states with masses

$$M_{N_{1,2}} = M \mp \frac{M_{11} + M_{22}}{2}\epsilon_N\,. \tag{2}$$

After diagonalization the Yukawa couplings read

$$\lambda_{ka} \to -\frac{(i)^a}{\sqrt{2}}\left[\lambda_{k1} + (-1)^a\epsilon_\lambda\lambda_{k2}\right]\,, \qquad (k = e, \mu, \tau \ \text{ and } \ a = 1, 2)\,, \tag{3}$$

and thus the light neutrino mass matrix is determined to be

$$\boldsymbol{m_\nu^{\text{eff}}} = -\frac{v^2\epsilon_\lambda}{M}|\boldsymbol{\lambda_1}||\boldsymbol{\Lambda}|\left(\hat{\boldsymbol{\lambda}}_{\boldsymbol{1}}^* \otimes \hat{\boldsymbol{\Lambda}}^* + \hat{\boldsymbol{\Lambda}}^* \otimes \hat{\boldsymbol{\lambda}}_{\boldsymbol{1}}^*\right)\,, \tag{4}$$

with

$$\hat{\boldsymbol{\Lambda}}^* = \hat{\boldsymbol{\lambda}}_{\boldsymbol{2}}^* - \frac{M_{11} + M_{22}}{4M}\frac{\epsilon_\lambda}{\epsilon_N}\hat{\boldsymbol{\lambda}}_{\boldsymbol{1}}^*\,. \tag{5}$$

Note that the parameter space vectors have been expressed according to $\boldsymbol{\lambda_1} = |\boldsymbol{\lambda_1}|\hat{\boldsymbol{\lambda}}_{\boldsymbol{1}}$, $\boldsymbol{\Lambda} = |\boldsymbol{\Lambda}|\hat{\boldsymbol{\Lambda}}$, where $\hat{\boldsymbol{\lambda}}_{\boldsymbol{1}}, \hat{\boldsymbol{\Lambda}}$ are unitary vectors along the $\boldsymbol{\lambda_1}, \boldsymbol{\Lambda}$ directions. Due to the structure of the light neutrino matrix the parameter space vectors are—up to normalization factors—completely determined by neutrino mixing angles and masses. For the normal hierarchical spectrum they can be written according to [22]:

$$\boldsymbol{\lambda_1} = |\boldsymbol{\lambda_1}|\,\hat{\boldsymbol{\lambda}}_{\boldsymbol{1}} = \frac{|\boldsymbol{\lambda_1}|}{\sqrt{2}}\left(\sqrt{1+\rho}\,\boldsymbol{U_3}^* + \sqrt{1-\rho}\,\boldsymbol{U_2}^*\right)\,, \tag{6}$$

$$\boldsymbol{\Lambda} = |\boldsymbol{\Lambda}|\,\hat{\boldsymbol{\Lambda}} = \frac{|\boldsymbol{\Lambda}|}{\sqrt{2}}\left(\sqrt{1+\rho}\,\boldsymbol{U_3}^* - \sqrt{1-\rho}\,\boldsymbol{U_2}^*\right)\,, \tag{7}$$

where the $\boldsymbol{U_i}$'s correspond to the columns of the leptonic mixing matrix and

$$\rho = \frac{\sqrt{1+r} - \sqrt{r}}{\sqrt{1+r} + \sqrt{r}}\,, \qquad r = \frac{m_{\nu_2}^2}{m_{\nu_3}^2 - m_{\nu_2}^2}\,. \tag{8}$$

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[2] Phenomenologically these models are similar to models where lepton number is slightly broken (see e.g. references [22,23,24,25,26,27,28])

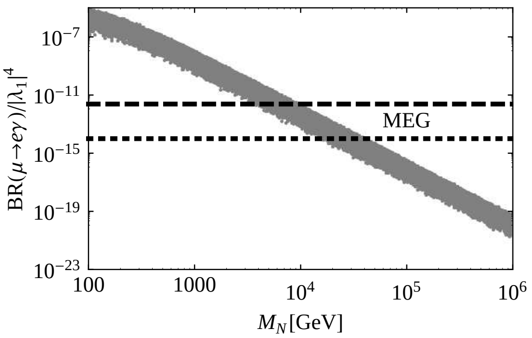

Fig. 1 Decay branching ratio $\mathrm{BR}(\mu \to e\gamma)$ normalized to $|\boldsymbol{\lambda_1}|^4$ for the normal light neutrino mass spectrum as a function of the common RH neutrino mass. The upper horizontal dashed line indicates the current experimental upper limit from the MEG experiment [34], whereas the lower dotted one marks prospective future experimental sensitivities [30].

3 Charged lepton flavor violating decays

Currently the most competitive bounds on charged lepton flavor violating processes are placed for μ decays, being $\mu \to e\gamma$, $\mu \to 3e$ and $\mu - e$ conversion in nuclei the ones with the most stringent upper limits [29]. In addition it is for these processes that the most tight bounds are expected in near-future experimental proposals: MEG [30], *Mu3e* [31] PRISM/PRIME [32]. So henceforth we will focus on μ decays, in particular on the reactions $\mu \to e\gamma$ and $\mu \to 3e$ (for $\mu - e$ conversion in nuclei see ref. [21]).

3.1 $\mu \to e\gamma$ process

In the limit $M_W/M \ll 1$ the decay branching ratio for this decay can be written as [33]

$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{\alpha}{1024\pi^4}\frac{m_\mu^5}{M^4}\frac{|\boldsymbol{\lambda_1}|^4}{\Gamma^\mu_{\mathrm{Tot}}}\left|\hat{\lambda}_{21}\,\hat{\lambda}^*_{11}\right|^2 . \tag{9}$$

Thus showing that apart from the parameters M and $|\boldsymbol{\lambda_1}|$ this branching fraction is entirely determined by low-energy data (see eqs. (6), (7) and (8)). Figure 1 shows the results obtained from the full formula involving the complete one-loop function (see ref. [21] for details) and by randomly generating the low-energy observables in their 2σ allowed range [1,2] (normal hierarchical mass spectrum), the parameters $|\boldsymbol{\lambda_1}|$ and M in the intervals $[10^{-5}, 1]$ and $[10^2, 10^6]$ GeV and the $N_{1,2}$ mass splittings in the range $[10^{-8}, 10^{-6}]$ GeV. As can be realized from eqs. (6), (7), (8) and (9) the width of the band is due to neutrino data uncertainties.

From fig. 1 it can be seen that $\mathrm{BR}(\mu \to e\gamma)$ can reach the current experimental upper bound [34] as long as $M_N < 0.1$ TeV, 1 TeV, 10 TeV provided $|\boldsymbol{\lambda_1}| \gtrsim 2\times 10^{-2}$, 10^{-1}, 1, respectively.

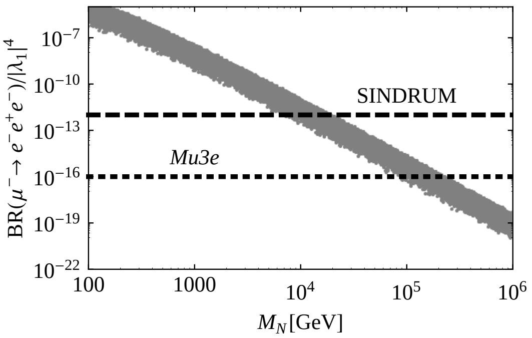

Fig. 2 Decay branching ratio $\text{BR}(\mu^- \to e^-e^+e^-)$ normalized to $|\boldsymbol{\lambda_1}|^4$ for normal light neutrino mass spectrum as a function of the common RH neutrino mass. The upper horizontal dashed line indicates the current bound on the $\mu^- \to e^+e^-e^-$ rate placed by the SINDRUM experiment [35], whereas the lower dotted one illustrates prospective future experimental sensitivities of the *Mu3e* experiment [31].

3.2 $\mu \to 3e$ process

We now turn to the discussion of the $\mu \to 3e$ process. This decay involves dipole contributions, γ and Z penguins as well as box diagrams [33], so a simple approximate formula as in the previous case does not exist. Following the same numerical procedure than in the $\mu \to e\gamma$ case we calculate the corresponding decay branching ratio. Fig. 2 shows the result for the branching fraction normalized to $|\boldsymbol{\lambda_1}|^4$ for the normal hierarchical mass spectrum.

It can be seen that $\text{BR}(\mu \to 3e)$ can exceed the experimental upper limit for RH neutrino masses $M_N < 0.1$ TeV, 1 TeV, 10 TeV provided $|\boldsymbol{\lambda_1}| \gtrsim 2 \times 10^{-2}$, 10^{-1}, 1, respectively, very similar to the $\mu \to e\gamma$ case. Mainly due to the sensitivities of the planned future experiments ($10^{-16} - 10^{-15}$) this decay has the potential to probe considerably larger values of the RH neutrino masses (compared with $\mu \to e\gamma$), reaching RH neutrino mass scales in excess of $\mathcal{O}(10^5$ GeV) for $|\boldsymbol{\lambda_1}| \sim 1$. Finally we note that due to the strong $|\boldsymbol{\lambda_1}|$ dependence, values of $|\boldsymbol{\lambda_1}|$ below 10^{-3} are not expected to yield observable rates at near future experimental facilities even for RH neutrino masses of the order 100 GeV.

4 Conclusions

We studied the implications of the seesaw global Abelian $U(1)_R$ symmetry on lepton flavor violation, in the context of minimal seesaw models. We showed that depending on the R-charge assignments—generically—two type of models can be identified. A first class where the mechanism that suppresses the light neutrino masses propagates to the lepton flavor violating observables, thus implying negligibly small LFV effects. A second class in which the mechanism "decouples" yielding in that way sizable rates for lepton flavor violating μ

decays. We discussed $\mu \to e\gamma$ and $\mu \to 3e$ and showed that these processes might have decay branching fractions within the reach of current or near-future experiments.

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