# On the alleged breakdown of the Kirchhoff theory in the case of diffraction and interference by a two-dimensional array

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# Letters to the Editor

## On the alleged breakdown of the Kirchhoff theory in the case of diffraction and interference by a two-dimensional array

By treating a two-dimensional diffraction array Chapman and Butland (1977) encounter the following problem with respect to application of the standard result to the measurement of wire-cloth screens. In many cases sieve screens are so constructed that the filaments are smaller than the widths of the apertures.

Confusion may arise as to whether the filament or the aperture dimensions determine the diffraction pattern. The authors then state that this uncertainty points out a major defect in the Kirchhoff integral approach to interference–diffraction problems.

Although we agree with their final results we should like to correct their statements about an alleged breakdown of the Kirchhoff theory. According to the Babinet principle we have in the Kirchhoff approximation

$$\int_{-\infty}^{+\infty} \exp(ikx \sin\alpha) H_1(x)\,dx + \int_{-\infty}^{+\infty} \exp(ikx\sin\alpha)(1-H_1(x))\,dx = \delta(k\sin\alpha), \quad (1)$$

where $H_1(x)$ denotes the transmission function of the grating multiplied by the incoming plane wave. The first integral on the left-hand side of (1) leads to the following interference–diffraction pattern of a grating with an aperture of width $b_x$, a filament of width $w_x$ and a pitch $p_x$:

$$I_x(\alpha) = \left(\frac{b_x \sin\beta_x}{\beta_x}\frac{\sin N\delta_x}{\sin\delta_x}\right)^2 \quad (2)$$

with

$$\beta_x = (b_x\pi/\lambda)\sin\alpha,$$
$$\delta_x = (p_x\pi/\lambda)\sin\alpha$$

and

$$p_x = b_x + w_x.$$

The second integral on the left-hand side of (1) denotes the interference–diffraction pattern of the complementary grating with apertures where the other grating has opaque areas and vice versa. The Babinet principle therefore shows that the interference–diffraction patterns of a grating and the complementary grating add up to the image of the incoming plane wave.

We now compare equation (2) for the two complementary arrays. This simplifies to a comparison between $\sin[(b_x\pi/\lambda) \times \sin\alpha]$ and $\sin[(w_x\pi/\lambda)\sin\alpha]$ or between

$$\sin[(b_x\pi/\lambda)\sin\alpha] \quad (3)$$

and

$$\sin[(p_x\pi/\lambda)\sin\alpha]\cos[(b_x\pi/\lambda)\sin\alpha] - \cos[(p_x\pi/\lambda)\sin\alpha]\sin[b_x\pi/\lambda)\sin\alpha]. \quad (4)$$

Only for $\sin[(p_x\pi/\lambda)\sin\alpha]=0$ and $\cos[(p_x\pi/\lambda)\sin\alpha]=1$ are both diffraction patterns identical and that is just the condition for a principal interference maximum, or the locations in the diffraction pattern where there is a non-zero intensity for large $N$.

However, this means that one cannot distinguish the diffraction patterns of the two complementary arrays, either for $b_x = w_x$, or for $b_x < w_x$ or $b_x > w_x$.

This ambiguity can be removed by taking into consideration the fact that for $k \sin\alpha = 0$ Babinet's principle still holds but results in a different value for the zeroth order in the two complementary arrays. We use the expressions given in the original paper for the $m$th order of interference in the normal array,

$$I_m(\alpha) = [(b_xN\sin\beta_m)/\beta_m]^2 \text{ with } \beta_m = b_x\pi m/p_x \quad (5)$$

and in the complementary array,

$$I_m(\alpha) = [(w_xN\sin\beta_m)/\beta_m]^2 \text{ with } \beta_m = w_x\pi m/p_x. \quad (6)$$

We then divide equations (5) and (6) by the zeroth order of the normal array,

$$I_0(0) = (b_xN)^2, \quad (7)$$

resulting in the two ratios

$$(\sin\beta_m/\beta_m)^2 \text{ and } [\sin\beta_m/(m\pi-\beta_m)]^2.$$

When plotted they produce figure 3 of the original paper implying a unique relation between the measured intensity ratio and $b_x$, $w_x$ and $p_x$.

This has nothing to do with the alleged breakdown of the Kirchhoff theory because all the formulae used satisfy this theory.

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### Reference

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