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Twistor-like formulation of super p-branes

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Abstract

Closed super $(p+2)$-forms in target superspace are relevant for the construction of the usual super p-brane actions. Here we construct closed super $(p+1)$-forms on a *world-volume superspace*. They are built out of the pull-backs of the Kalb–Ramond super $(p+1)$-form and its curvature. We propose a twistor-like formulation of a class of super p-branes which crucially depends on the existence of these closed super $(p+1)$-forms.

1. Introduction

The manifestly spacetime supersymmetric formulation of string theory à la Green and Schwarz has a fermionic gauge symmetry, known as the κ-symmetry [1]. This symmetry is of crucial importance for the model, but also gives rise to formidable problems in its quantization. A few years ago a geometrical understanding of κ-symmetry has emerged after the work of ref. [2] which also holds the promise of better prospects for a quantization of the model [3]. It should be emphasized, however, that a fully covariant quantization scheme has not yet emerged.

In the simple situation of a superparticle in $d=3$ dimensions, the theory was reformulated in such a way that the κ-symmetry can be interpreted as an $N=1$ local worldline supersymmetry [2]. The key to this formulation is the introduction of twistor-like variables, λ, which are commuting spinors arising as the superpart-

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ners of the target superspace fermionic coordinates †. The idea is essentially to make the change of variable $P^\mu \to \lambda^\alpha \gamma^\mu_{\alpha\beta} \lambda^\beta$ such that the mass shell constraint $P^\mu P_\mu = 0$ is satisfied, and that local supersymmetry is now formulated with the help of the new variable λ. This construction was later generalized to superparticles in higher dimensions [6–10], type-I superstrings in $d = 3, 4, 6$ [11] and $d = 10$ [12–14], type-II superstrings in $d = 3$ [15] and supermembranes in $d = 11$ [16]. After these works it became clear that there exists a closed super $(p + 1)$-form $(p = 0, 1, 2)$ on the worldvolume superspace which plays a central role. This should be contrasted with the crucial role the super $(p + 2)$-forms in target superspace play for the existence of the usual super p-branes. In fact, the closed super $(p + 1)$-form on the worldvolume superspace is built out of the pull-backs of the super $(p + 1)$-form and its curvature in target superspace.

The purpose of this paper is to investigate twistor-like formulations of other super p-branes as well. Beyond the cases discussed above, there are four more cases in the usual brane-scan: $(p = 3, d = 6, 8)$, $(p = 4, d = 9)$ and $(p = 5, d = 10)$, where d is the dimension of the target spacetime. We construct closed super $(p + 1)$-forms for $(p = 3, d = 6)$ and $(p = 5, d = 10)$, and using these forms we propose an action for the twistor-like formulation of these theories, thereby generalizing previous results mentioned above (whether the obstacles encountered for the cases of $(p = 3, d = 8)$ and $(p = 4, d = 9)$ are circumventable remains to be seen). We hope that, among other things, this formulation will be useful in search of the so far elusive heterotic 5-brane action.

The case of the superstring is somewhat special due to the extra worldsheet Weyl symmetry. This case has been treated in great detail in ref. [13]. Here we shall focus on super p-branes with $p \neq 1$, of which the massive superparticle $(p = 0)$ is the simplest, and therefore we begin with its description. The massive particle in $d = 2$ with worldline $n = 1$ local supersymmetry has been considered in refs. [17,18], and in $d = 3$ with $n = 2$, in ref. [18]. The massive particle action which will be presented here has the maximal $n = 8$ local worldline supersymmetry.

2. The massive superparticle $(p = 0)$

Consider a superspace $\underline{\mathscr{M}}$ in d-dimensional spacetime with coordinates $Z^{\underline{M}} = (X^{\underline{m}}, \theta^{\underline{\mu}})$. Following the notation and conventions of ref. [13], we shall always use underlined indices for target superspace quantities. Let us define the pulled-back supervielbein as

$$E_\tau^{\underline{A}} = \partial_\tau Z^{\underline{M}} E_{\underline{M}}^{\underline{A}}, \tag{2.1}$$

† The word *twistor-like* is used to avoid confusion with the supertwistor which consists of a multiplet of fields forming a multiplet of superconformal groups which are known to exists in dimensions $d \leqslant 6$. In fact such variables have been used previously in a twistor formulation of superparticles and superstrings in $d = 3, 4, 6$ [4]. A similar, but not quite the same, multiplet of variables was introduced in ref. [5] to give a twistor-like formulation of these models in $d = 10$. The twistor-like formulation of ref. [2], which we will be following in this paper, differs from both.

where ∂_τ denotes differentiation with respect to the worldline time variable. The tangent space index splits as $\underline{A} = (\underline{a}, \underline{\alpha' r})$, where $\underline{a} = 0, 1, \ldots, d-1$ is the Lorentz vector index, $\underline{\alpha} = 1, \ldots, M$ labels the spinor irrep of the Lorentz group and $\underline{r} = 1, \ldots, N$ labels the defining representation of the automorphism group of the super-Poincaré algebra in d dimensions. For the sake of definiteness we shall consider the cases listed in Table 1. In fact, they essentially correspond to the cases suggested by the super p-brane theories. Furthermore, to simplify the notation, we shall denote the pairs of indices $\underline{\alpha' r}$ by a single index $\underline{\alpha}$, e.g. $C_{\underline{\alpha\beta}} = C_{\underline{\alpha' r, \beta' s}} = C_{\underline{\alpha' \beta'}} \eta_{\underline{rs}}$.

We next introduce the super one-form $B = \mathrm{d} z^{\underline{M}} B_{\underline{M}}$ whose tangent space components are defined with the help of the inverse supervielbein as follows: $B_{\underline{A}} = E_{\underline{A}}^{\underline{M}} B_{\underline{M}}$. The action for a massive superparticle, whose mass we shall set equal to one, can then be written as

$$S = \int \mathrm{d}\tau \left(\tfrac{1}{2} e^{-1} E_\tau^{\underline{a}} E_\tau^{\underline{a}} + \tfrac{1}{2} e + E_\tau^{\underline{A}} B_{\underline{A}} \right), \tag{2.2}$$

where e is the einbein on the worldline. This action is invariant under the following κ-symmetry transformations:

$$\delta Z^{\underline{M}} E_{\underline{M}}^{\underline{a}} = 0, \quad \delta Z^{\underline{M}} E_{\underline{M}}^{\underline{\alpha}} = (1 + \Gamma)^{\underline{\alpha\beta}} \kappa_{\underline{\beta}}, \quad \delta e = S^{\underline{\alpha}} \kappa_{\underline{\alpha}}, \tag{2.3}$$

where

$$\Gamma^{\underline{\alpha\beta}} = -\frac{1}{e} E_\tau^{\underline{a}} (\Gamma_{\underline{a}})^{\underline{\alpha\beta}}, \quad S^{\underline{\alpha}} = 4i E_\tau^{\underline{\alpha}} + 2 E_\tau^{\underline{a}} \left(u^{\underline{\alpha}}{}_{\underline{a}} + \Gamma_{\underline{a}}^{\underline{\alpha\beta}} v_{\underline{\beta}} \right) \tag{2.4}$$

and $u^{\underline{\alpha}}{}_{\underline{a}}$ is an arbitrary *Γ-traceless* vector-spinor superfield, $v_{\underline{\alpha}}$ is an arbitrary

Table 1
In this table d indicates the dimension of spacetime, M is the dimension of the spinor irrep of $SO(d-1, 1)$, N is the dimension of the defining representation of the automorphism group $\underline{G}$ of the super-Poincaré algebra in d dimensions, $C_{\underline{\alpha'\beta'}}$ is the charge conjugation matrix, $\Gamma^{\underline{a}}_{\underline{\alpha'\beta'}}$ are the Dirac matrices $(\Gamma^{\underline{a}} C)_{\underline{\alpha'\beta'}}$ and $\eta_{\underline{rs}}$ is the invariant tensor of $\underline{G}$. We often use the notation in which a pair of indices $(\underline{\alpha' r})$ is replaced by a single index $\underline{\alpha}$. Furthermore, in $d = 6, 10$ the matrices $\Gamma^{\underline{a}}_{\underline{\alpha\beta}}$ are chirally projected Dirac matrices and $\Gamma_{\underline{a}}^{\underline{\alpha\beta}}$ are projected with opposite chirality. In this notation raising or lowering of the spinor indices is not needed. The types of spinors are characterized according to the reality and chirality conditions imposed on them, namely Majorana (M), pseudo-Majorana (PM), symplectic Majorana (SM), Majorana–Weyl (MW) and symplectic Majorana–Weyl (SMW). Corresponding quantities are listed for the super p-branes that arise in target space dimension of d

Target space data							Worldvolume data ($p \geqslant 2$)						
d	(M, N)	$\underline{G}$	$C_{\underline{\alpha'\beta'}}$	$\Gamma^{\underline{a}}_{\underline{\alpha'\beta'}}$	$\eta_{\underline{rs}}$	Type	p	(m, n)	G	$C_{\alpha'\beta'}$	$\Gamma^a_{\alpha'\beta'}$	η_{rs}	Type
11	(32, 1)	–	A	S	–	M	2	(2, 8)	SO(8)	A	S	S	M
10	(16, 1)	–	A	S	–	MW	5	(4, 2)	USp(2)	S	A	A	SMW
9	(16, 1)	–	S	S	–	PM	4	(4, 2)	USp(2)	S	A	A	SM
8	(16, 1)	–	S	S	–	PM	3	(4, 2)	SO(2)	A	S	S	M
7	(8, 2)	USp(2)	S	A	A	SM	2	(2, 4)	SO(4)	A	S	S	M
6	(4, 2)	USp(4)	S	A	A	SMW	3	(4, 1)	–	A	S	–	M
5	(4, 2)	Usp(4)	S	A	A	SM	2	(2, 2)	SO(2)	A	S	S	M
4	(4, 1)	SO(4)	A	S	S	M	2	(2, 1)	–	A	S	–	M

spinor superfield and $C_{\alpha\beta}$ is the charge conjugation matrix [19]. The invariance of the action imposes the following torsion T and $H = \mathrm{d}B$ constraints:

$$\begin{aligned} &T_{\underline{\alpha\beta}}{}^{\underline{c}} = -2i(\Gamma^{\underline{c}})_{\underline{\alpha\beta}}, \quad T_{\alpha(\underline{bc})} = u^{\beta}{}_{(\underline{b}}\Gamma_{\underline{c})\beta\alpha} + \eta_{\underline{bc}}v_{\underline{\alpha}}, \\ &H_{\underline{\alpha\beta}} = -2iC_{\alpha\beta}, \quad H_{\underline{\alpha a}} = (\Gamma_{\underline{a}})_{\underline{\alpha\beta}}v^{\underline{b}} + u_{\underline{\alpha a}}. \end{aligned} \tag{2.5}$$

Notice that the RHS of the equation involving $H_{\underline{\alpha\beta}}$ must be symmetric. Therefore, $d = 5, 8, 9$ are singled out in Table 1. Lower values of N can allow other dimensions (e.g. $d = 3$) which can be easily incorporated to the present scheme by ammending our notation slightly. The physical interpretation of the constraints (2.5) requires a lengthy analysis of the Bianchi identities, which we hope to return to elsewhere. The expected results is that they will be consistent with supergravity theories, possibly coupled to a matter/Maxwell sector in appropriate dimensions.

For future use, we also write down the Nambu–Goto form of the action, which can be obtained from (2.2) by substituting the field equation of the einbein:

$$S = \int \mathrm{d}\tau \left[(E_\tau^{\underline{a}} E_\tau^{\underline{a}})^{1/2} + E_\tau^{\underline{A}} B_{\underline{A}} \right]. \tag{2.6}$$

Our purpose now is to reformulate the above theory in such a way that the κ-symmetry is traded for worldline local supersymmetry. Since the κ-symmetry parameter has MN real components, and due to the usual argument that only half of them count as true gauge transformation parameters, it follows that the maximum worldline extended symmetry to expect is $\frac{1}{2}MN$. From Table 1, we see that for $d = 9, 8, 5$ we have $n = 8$. Thus let us elevate the worldline to a super worldline $\mathscr{M}$ with coordinates $Z^M = (\tau, \theta^\mu)$, $\mu = 1, \ldots, 8$. Following refs. [10,13], we shall take $\mathscr{M}$ to be superconformally flat. (We refer to refs. [10,13] for a detailed geometrical description of such a space). In particular, the components of the super torsion T_{AB}^{C} will be those of a flat n-extended worldline superspace:

$$T_{rs}{}^{0} = -2i\delta_{rs}, \quad T_{0r}{}^{0} = 0, \quad T_{s0}{}^{r} = 0, \quad T_{rs}{}^{q} = 0. \tag{2.7}$$

The super worldline tangent space index A splits as $A = (0, r)$, $r = 1, \ldots, 8$. As shown in refs. [10,13], the superdiffeomorphisms which preserve these constraints take the form

$$\delta\tau = \lambda - \tfrac{1}{2}\theta^r D_r \lambda, \quad \delta\theta_r = -\tfrac{1}{2} i D_r \lambda, \tag{2.8}$$

where λ is an arbitrary superfield. These transformations contain the worldline diffeomorphisms and the $n = 8$ local worldline supersymmetry. Under these transformations, the covariant derivative D_r transforms homogeneously.

The change of variable, which is sometimes referred to as the twistor constraint, which is needed to pass from the κ-symmetric formulation to the worldline supersymmetric one, is as follows:

$$\lambda_r^{\underline{\alpha}} \Gamma_{\underline{\alpha\beta}}^{\underline{a}} \lambda_s^{\underline{\beta}} = \delta_{rs} E_0^{\underline{a}}, \tag{2.9}$$

where $\lambda_r^{\underline{\alpha}}$ are *commuting* spinors referred to as the twistor variables and $E_0^{\underline{a}} = E_0^M \partial_M Z^{\underline{M}} E_{\underline{M}}^{\underline{a}}$. The strategy is to arrange that this equation arises as the $\theta_r = 0$

component of an appropriate superfield equation. To this end, it is convenient to define the matrix

$$E_A^{\underline{A}} = E_A^M (\partial_M Z^{\underline{M}}) E_{\underline{M}}^{\underline{A}}. \tag{2.10}$$

Using these matrices, we can write the desired superfield equation as

$$(E_r \Gamma^{\underline{a}} E_s) = \delta_{rs} E_0^{\underline{a}}, \tag{2.11}$$

with the identifications

$$E_r^{\underline{\alpha}}|_{\theta=0} = \lambda_r^{\underline{\alpha}}, \quad E_0^{\underline{a}}|_{\theta=0} = E_0^{\underline{a}}. \tag{2.12}$$

We use a notation in which the contracted $\underline{\alpha}$ indices are suppressed, and the parentheses in (2.11) indicate that such contractions are made. In flat superspace, these identifications mean that $\theta^{\underline{\alpha}}(\tau, \theta) = \theta^{\underline{\alpha}}(\tau) + \lambda_r^{\underline{\alpha}}(\tau)\theta^r + \dots$, i.e. the twistor variable $\lambda_r^{\underline{\alpha}}$ is the superpartner of the target superspace fermionic coordinate $\theta^{\underline{\alpha}}(\theta)$.

From the identity (2.11) it follows that

$$(E_r \Gamma^{\underline{a}} E_s) = \tfrac{1}{8}\delta_{rs}(E_q \Gamma^{\underline{a}} E_q). \tag{2.13}$$

This identity has arisen in the twistor formulation of string theory in $d = 10$, and its group theoretic interpretation has been given in ref. [8]. Its dimensional reduction from $d = 10$ down to $d = 9, 8, 5$ yields, in addition to the corresponding twistor identities of the form (2.13), other identities as well. In particular, the following identity will arise:

$$(E_r E_s) + \tfrac{1}{8}\delta_{rs}(E_q E_q). \tag{2.14}$$

Our task is to write an action in $n = 8$ worldline superspace which will
(a) give rise to the constraints (2.11) and (2.14), and,
(b) given the constraints (2.5) and (2.7), will possess a worldline $n = 8$ local supersymmetry.

To this end, we propose the following action which is the appropriate generalization for a massive superparticle of the action given for the massless superparticle in refs. [10,13]:

$$S = \int d\tau\, d^8\theta \left[P_{\underline{a}}^r E_r^{\underline{a}} + P^M (\tilde{B}_M - \partial_M Q) \right], \tag{2.15}$$

where $P_{\underline{a}}^r$, P^M and Q are Lagrange multiplier superfields and $\tilde{B}_M$ is defined by

$$\tilde{B}_M = \partial_M Z^{\underline{M}} B_{\underline{M}} - \tfrac{1}{16} i E_M^0 H_{rr}, \tag{2.16}$$

where $H_{rr} = E_r^{\underline{A}} E_r^{\underline{B}} H_{\underline{BA}}$ and $H_{\underline{BA}}$ are the tangent space components of the field strength $H = dB$:

$$H_{\underline{AB}} = (-1)^{\underline{A}(\underline{B}+\underline{N})} E_{\underline{B}}^{\underline{N}} E_{\underline{A}}^{\underline{M}} H_{\underline{MN}}, \tag{2.17}$$

where the indices in the exponent indicate grassmannian parities. Recall that $M = (\tau, \mu)$, $A = (0, r)$, $\underline{M} = (\underline{m}, \underline{\mu})$ and $\underline{A} = (\underline{a}, \underline{\alpha})$. The indices of the bosonic (fermionic) coordinates have the parity 0 (1). This is consistent with the fact that the twistor variables $\lambda_r^{\underline{\alpha}}$ are commuting variables.

The above form of $\tilde{B}$ is engineered such that $\mathrm{d}\tilde{B}=0$ modulo the constraints (2.5), (2.7), (2.11) and (2.14), as we shall show below. Note that the independent worldline *superfields* are: $P_{\underline{a}}^r$, P^M, Q, E_M^A and $Z^{\underline{M}}$. An important property of the action (2.15) is that it is invariant under $n=8$ local worldline supersymmetry, as opposed to the κ-symmetry. (The latter emerges as a special case of the former in a certain gauge.) The supersymmetry of the second and third terms in the action is manifest (everything transforms like supertensors), while the supersymmetry of the first term is due to the fact that $E_r^{\underline{a}}$ transforms homogeneously like D_r does, and this can be compensated by a suitable transformation of the Lagrange multiplier.

At this stage, to simplify matters, we shall set the inconsequential superfields $u^{\underline{\alpha}}{}_{\underline{a}}$ and $v_{\underline{\alpha}}$ in (2.5) equal to zero, and take the resulting constraints and their Bianchi consequences to characterize the target space background. Thus we have the constraints

$$
\begin{aligned}
&T_{\underline{\alpha\beta}}{}^{\underline{c}} = -2i(\Gamma^{\underline{c}})_{\underline{\alpha\beta}}, \quad T_{\underline{b\alpha}}{}^{\underline{a}}=0, \quad T_{\underline{\alpha\beta}}{}^{\underline{\gamma}}=0, \\
&H_{\underline{\alpha\beta}} = -2iC_{\underline{\alpha\beta}}, \quad H_{\underline{a\alpha}}=0.
\end{aligned} \tag{2.18}
$$

With Eqs. (2.7) and (2.18) at hand, we can now analyze the content of the superfield equations that follow from the action (2.15). Firstly, the equation of motion for $P_{\underline{a}}^r$ is simply

$$E_r^{\underline{a}}=0. \tag{2.19}$$

The supercovariant derivative of this equation in the spinorial direction evaluated at $\theta^r=0$ gives the desired constraint (2.11). To see this, it is useful first to evaluate the curl of $E_A^{\underline{A}}$ defined in (2.10). We find

$$D_A E_B^{\underline{c}} - (-1)^{AB} D_B E_A^{\underline{C}} = -T_{AB}{}^{C} E_C^{\underline{C}} + (-1)^{A(B+\underline{D})} E_B^{\underline{D}} E_A^{\underline{E}} T_{\underline{ED}}{}^{\underline{C}}, \tag{2.20}$$

where the covariant derivative $D_A = E_A^M D_M$ rotates the indices A and $\underline{A}$ and the tangent space components of the supertorsion $T_{MN}{}^{C} = \partial_M E_N^C + \Omega_M^{CD} E_{ND} - (-1)^{MN}(M \leftrightarrow N)$ are defined as follows:

$$T_{AB}{}^{C} = (-1)^{A(B+N)} E_B^N E_A^M T_{MN}{}^{C}, \tag{2.21}$$

and similarly for $T_{AB}{}^{\underline{C}}$. Taking the spinor–spinor component of (2.20) and using the constraints (2.7), (2.14) and (2.19) we indeed obtain the twistor constraint equation (2.11). The $\theta_r=0$ component of the equation gives (2.9) and one can show that there is no further information coming from the higher order θ expansion.

We next consider the equation of motion for the Lagrange multiplier P^M which simply reads

$$H_{MN} = \partial_M \tilde{B}_N - (-1)^{MN} \partial_N \tilde{B}_M = 0. \tag{2.22}$$

This equation, together with (2.19), is at the center of the construction of the model. Defining $\tilde{H} = \mathrm{d}\tilde{B}$, and referring to its tangent space components, we obtain

$$\begin{aligned}\tilde{H}_{AB} &= (-1)^{A(B+\underline{B})} E_B^{\underline{B}} E_A^{\underline{A}} H_{\underline{AB}} - \tfrac{1}{16} i (-1)^{A(B+N)} E_B^N E_A^M \\ &\quad \times \left[\partial_M \left(E_N^0 H_{rr} \right) - (-1)^{MN} \partial_N \left(E_M^0 H_{rr} \right) \right] \\ &= 0. \end{aligned} \tag{2.23}$$

where the (worldsheet) tangent space components H_{AB} are related to the (target space) tangent space components $H_{\underline{AB}}$ according to

$$H_{AB} = (-1)^{A(B+\underline{B})} E_B^{\underline{B}} E_A^{\underline{A}} H_{\underline{AB}}. \tag{2.24}$$

We can write (2.23) as

$$\tilde{H}_{AB} = H_{AB} - \tfrac{1}{16} i T_{AB}{}^0 H_{rr} + \tfrac{1}{16} i \left[\delta_A^0 D_B H_{rr} - (-1)^{AB} \delta_B^0 D_A H_{rr} \right] = 0. \tag{2.25}$$

Taking the spinor–spinor component of this equation gives

$$H_{rs} - \tfrac{1}{8} \delta_{rs} H_{qq} = 0. \tag{2.26}$$

From (2.24) and (2.18), we see that $H_{rs} = -2i(E_r E_s)$, and hence (2.14) follows from (2.26). Thus, we shall consider (2.14) to follow from the integrability condition of the P^M equation of motion.

Next, we consider the time–spinor projection of (2.25). It yields,

$$H_{0r} + \tfrac{1}{16} i D_r H_{qq} = 0. \tag{2.27}$$

This equation is precisely what one obtains by considering the Bianchi identity $D_{(r} H_{st)} - T^0_{(rs} H_{t)0} = 0$ and using Eqs. (2.7) and (2.26). Therefore, (2.27) is satisfied as well without implying new constraints. This concludes the proof that indeed $\mathrm{d}\tilde{B} = 0$. As a consequence of this property, the action (2.15) has also the gauge invariance

$$\delta P^M = \partial_N \Lambda^{NM}, \tag{2.28}$$

where Λ^{MN} is an arbitrary graded antisymmetric superfield. In showing this invariance we need to use (2.22), which in turn involves the use of the constraint (2.19). This constraint follows as the field equation of the Lagrange multiplier $P_r^{\underline{a}}$. Such terms can be cancelled by an appropriate variation of the Lagrange multiplier $P_r^{\underline{a}}$. Therefore, (2.28) is indeed a symmetry of the action.

Next, we consider the equation of motion for Q which reads

$$\partial_M P^M = 0. \tag{2.29}$$

This equation has the solution [13]

$$P^M = \partial_N \Sigma^{NM} + \theta^8 \delta_\tau^M T, \tag{2.30}$$

where T is a constant and Σ^{MN} is an arbitrary graded antisymmetric superfield. Substituting $P^M = \theta^8 \delta_\tau^M T$ into the action (2.15) yields (setting $T = 1$)

$$
\begin{aligned}
S &= \int d\tau\, d^8\theta\, P_{\underline{a}}^r E_r^{\underline{a}} + \int d\tau\, \tilde{B}_\tau\big|_{\theta=0} \\
&= \int d\tau\, d^8\theta\, P_{\underline{a}}^r E_r^{\underline{a}} + \int d\tau\left[\partial_\tau Z^{\underline{M}} B_{\underline{M}} - \tfrac{1}{8} E_\tau^0 (\lambda_r \lambda_r)\right].
\end{aligned} \tag{2.31}
$$

To simplify this further, consider the following Dirac matrix identity which holds in $d = 5, 9$:

$$
\Gamma^{\underline{a}}{}_{\underline{\alpha\beta}} \Gamma^{\underline{a}}{}_{\underline{\gamma\delta}} + C_{\underline{\alpha\beta}} C_{\underline{\gamma\delta}} + \text{cyclic}(\underline{\alpha\beta\gamma}) = 0. \tag{2.32}
$$

Multiplying this equation by $E_r^{\underline{\alpha}} E_r^{\underline{\beta}} E_s^{\underline{\gamma}} E_s^{\underline{\delta}}$ and using the identities (2.13) and (2.14), we find that $(E_r E_r) = -8(E_0^{\underline{a}} E_0^{\underline{a}})^{1/2}$. Evaluating this at $\theta = 0$ and substituting the result into (2.31), we find

$$
S = \int d\tau\, d^8\theta\, P_{\underline{a}}^r E_r^{\underline{a}} + \int d\tau\left[\partial_\tau Z^{\underline{M}} B_{\underline{M}} + E_\tau^0 (E_0^{\underline{a}} E_0^{\underline{a}})^{1/2}\right]. \tag{2.33}
$$

Note that

$$
E_\tau^0 E_0^{\underline{a}} = E_\tau^A E_A^{\underline{a}} = E_\tau^{\underline{a}}, \tag{2.34}
$$

modulo the constraint (2.19). The effect of using the constraint (2.19) in the action amounts to a redefinition of the Lagrangian multiplier $P_{\underline{a}}^r$. Therefore, using (2.34) we can simplify the last term in the action and obtain

$$
S = \int d\tau\, d^8\theta\, P_{\underline{a}}^r E_r^{\underline{a}} + \int d\tau\left[\partial_\tau Z^{\underline{M}} B_{\underline{M}} + (E_\tau^{\underline{a}} E_\tau^{\underline{a}})^{1/2}\right]. \tag{2.35}
$$

We see now that the second integral in (2.35) agrees with the κ-symmetric action (2.6). Finally, following the same arguments in ref. [13], the component form of the first term in the action can also be computed and one finds the following component action:

$$
S = \int d\tau \left\{ p_{\underline{a}}\left[E_0^{\underline{a}} - \tfrac{1}{8}(\lambda_r \Gamma^{\underline{a}} \lambda_r)\right] + \partial_\tau Z^{\underline{M}} B_{\underline{M}} + (E_\tau^{\underline{a}} E_\tau^{\underline{a}})^{1/2} \right\}, \tag{2.36}
$$

where $p_{\underline{a}} = (D^7)_r P_{\underline{a}}^r|_{\theta=0}$. With arguments parallel to those of refs. [10,13], we expect that the Lagrange multiplier $p_{\underline{a}}$ does not describe any new degree of freedom, and the field equations of (2.6) and (2.36) are classically equivalent. In the case of the massless superparticle, showing this equivalence requires the use of an important abelian gauge symmetry [10]. A generalized version of this symmetry is also present in the massive superparticle case. We find that the action (2.15) is invariant under the gauge transformations

$$
\delta P_{\underline{a}}^r = D_q(\xi^{qrs} \Gamma_{\underline{a}} E_s), \quad \delta P^M = -E_r^M D_q(\xi^{qrs} E_s), \tag{2.37}
$$

where the parameter $\xi_{\underline{\alpha}}^{qrs}(\tau, \theta)$ is totally symmetric and traceless in its worldline indices. Note that, unlike in the massless particle case, both of the lagrange multipliers transform here. To show that this is an invariance of the action, we need to use the Dirac matrix identity (2.33) and the constraints (2.7) and (2.18), which imply the target space equations of motion.

The rest of the paper will be devoted to a discussion of the twistor-like formulation of super p-branes with $p \geqslant 2$.

3. Super p-branes ($p \geqslant 2$)

The κ-symmetric formulation of super p-branes is well known [20–22]. Here, we shall directly investigate the construction of a worldvolume locally supersymmetric version along the lines of the massive superparticle case described in detail above. The relevant target spaces are listed in Table 1. As before, the maximum number of real components of the worldvolume supersymmetry parameter is $\frac{1}{2}MN$. This translates into $n = 2, 4, 8$ supersymmetry in various cases as indicated in Table 1.

The coordinates of the worldvolume superspace $\mathscr{M}$ are $Z^M = (X^m, \theta^\mu)$, $m = 1, \ldots, p+1$, $\mu = 1, \ldots, \frac{1}{2}MN$. The supervielbein is again denoted by E_M^A with the tangent space indices splitting as $A = (a, \alpha' r)$, $a = 1, \ldots, p+1$, $\alpha' = 1, \ldots, m$, $r = 1, \ldots, n$ (see Table 1). For simplicity in notation, *we will indicate the pair of indices $\alpha' r$ by a single index α*. Following ref. [16], we shall take $\mathscr{M}$ to be characterized by the following super torsion constraints:

$$T_{\alpha\beta}{}^{a} = -2i(\Gamma^a)_{\alpha\beta}, \quad T_{b\alpha}{}^{a} = 0, \quad T_{bc}{}^{a} = 0, \quad T_{\alpha\beta}{}^{\gamma} = 0. \tag{3.1}$$

See Table 1 for the symmetry properties of the gamma matrices. In particular note that $(\Gamma^a)_{\alpha\beta} = (\Gamma^a)_{\alpha'\beta'}\eta_{rs}$, where η_{rs} is the invariant tensor of the automorphism group G. Thus, η_{rs} is the unit matrix δ_{rs} when G is an orthogonal group, and the constant antisymmetric matrix Ω_{rs} when G is a symplectic group. From Table 1 we see that $\eta_{rs} = -\epsilon\eta_{sr}$ and $\Gamma_{\alpha'\beta'} = -\epsilon\Gamma_{\beta'\alpha'}$, with $\epsilon = -1$ for orthogonal G and $\epsilon = 1$ for symplectic G. Similar properties hold for the corresponding target space quantities.

The coordinates of target superspace $\underline{\mathscr{M}}$ are $Z^{\underline{M}} = (X^{\underline{m}}, \theta^{\underline{\mu}})$, $\underline{m} = 0, \ldots, d-1$, $\underline{\mu} = 1, \ldots, MN$ (see Table 1). The supervielbein is $E_{\underline{M}}^{\underline{A}}$ with the tangent space index splitting as $\underline{A} = (\underline{a}, \underline{\alpha})$, $\underline{a} = 0, \ldots, d-1$, $\underline{\alpha} = 1, \ldots, MN$. The index $\underline{\alpha}$ is short for a pair of indices $(\underline{\alpha' r})$, with $\underline{\alpha}' = 1, \ldots, M$, $\underline{r} = 1, \ldots, N$. The superspace $\underline{\mathscr{M}}$ is also endowed with a super $(p+1)$-form B whose curvature is $H = \mathrm{d}B$.

The κ-symmetry of the usual super p-brane action imposes constraints on the torsion and the $(p+2)$-form H [21]. As before, arbitrary superfields $u^{\underline{\alpha}}{}_{\underline{a}}$ and $v_{\underline{\alpha}}$ arise [21,19]. As we did in the particle case, we shall set these inconsequential superfields equal to zero, and furthermore we shall fix the target space supergeometry, in a manner which is consistent with κ-symmetry, to be characterized by the following constraints:

$$\begin{aligned} &T_{\underline{\alpha\beta}}{}^{\underline{c}} = -2i(\Gamma^{\underline{c}})_{\underline{\alpha\beta}}, \quad T_{\underline{b\alpha}}{}^{\underline{a}} = 0, \quad T_{\underline{\alpha\beta}}{}^{\underline{\gamma}} = 0, \\ &H_{\underline{\alpha\beta c_1 \ldots c_p}} = i\xi^{-1}\left(\eta\Gamma_{\underline{c_1 \ldots c_p}}\right)_{\underline{\alpha\beta}}, \quad H_{\underline{\alpha b_1 \ldots b_{p+1}}} = 0, \quad H_{\underline{\alpha\beta\gamma A_1 \ldots A_{p-1}}} = 0, \end{aligned} \tag{3.2}$$

where $\xi = (-1)^{(p-2)(p-5)/4}$ and η is a matrix chosen such that $\eta\Gamma_{\underline{c_1 \ldots c_p}}$ is symmetric. $\eta = 1$ except for the following cases: $\eta = \Gamma_{d+1}$ for $(p = 3,\ d = 8)$, with the definition $\Gamma_{d+1} = \Gamma_0 \Gamma_1 \cdots \Gamma_{d-1}$, and $\eta = 1 \times \sigma_2$ for $(p = 2,\ d = 5)$. See the table for further information on the notation and properties of the Dirac matrices in diverse dimensions.

In $d = 11$ dimensions the above constraints describe the $d = 11$ supergravity theories. In other cases, a detailed analysis of the constraint remains to be carried out. Presumably, they describe supergravity theories containing $(p + 1)$-form potentials.

Having specified the geometry of the worldvolume and target superspaces, our next goal is to write down an action for twistor-like super p-branes in analogy with the action (2.15). Such an action has already been proposed in ref. [16] for the case of the supermembrane. Here we generalize that result and propose the following action for all super p-branes:

$$S = \int \mathrm{d}^{p+1}\sigma\ \mathrm{d}^{mn}\theta \Big[P^{\alpha}_{\underline{a}} E^{\underline{a}}_{\alpha} + P^{M_1 \dots M_{p+1}} \Big(\tilde{B}_{M_1 \dots M_{p+1}} - \partial_{M_1} Q_{M_2 \dots M_{p+1}} \Big) \Big], \tag{3.3}$$

where $P^{\alpha r}_{\underline{a}}$, $P^{M_1 \cdots M_{p+1}}$ and $Q_{M_1 \dots M_p}$ are Lagrange multiplier superfields (the latter two are graded totally antisymmetric) and the $(p + 1)$-form $\tilde{B}$ is given by

$$\begin{aligned} \tilde{B}_{M_1 \dots M_{p+1}} = &(-1)^{\epsilon_{p+1}(M,\underline{M})} \partial_{M_{p+1}} Z^{\underline{M_{p+1}}} \dots \partial_{M_1} Z^{\underline{M_1}} B_{\underline{M_1} \dots \underline{M_{p+1}}} \\ &- \frac{i}{2mn(p+1)} \Gamma^{\alpha\beta}_{c_{p+1}} \Big[E^{c_{p+1}}_{M_{p+1}} \dots E^{c_1}_{M_1} H_{\alpha\beta c_1 \dots c_p} \\ &+ \mathrm{cyclic}(M_1 \dots M_{p+1}) \Big]. \end{aligned} \tag{3.4}$$

The grading factor is given by

$$\epsilon_{p+1}(M, \underline{M}) = \sum_{n=1}^{p} (M_1 + \dots M_n)(M_{n+1} + \underline{M_{n+1}}), \tag{3.5}$$

and the pull-back of H by

$$H_{A_1 \dots A_{p+2}} = (-1)^{\epsilon_{p+2}(A,\underline{A})} E^{\underline{A_{p+2}}}_{A_{p+2}} \dots E^{\underline{A_1}}_{A_1} H_{\underline{A_1} \dots \underline{A_{p+2}}}. \tag{3.6}$$

The field equation for $P^{\alpha}_{\underline{a}}$ is

$$E^{\underline{a}}_{\alpha} = 0. \tag{3.7}$$

The integrability condition for this equation yields the analog of the twistor constraint (2.13) for super p-branes. It can be obtained from (2.20), (3.1), (3.2) and takes the form

$$\left(E_{\alpha} \Gamma^{\underline{a}} E_{\beta} \right) = \Gamma^{a}_{\alpha\beta} E^{\underline{a}}_{a}. \tag{3.8}$$

Recall that $\Gamma^{a}_{\alpha\beta} = \Gamma^{a}_{\alpha'\beta'} \eta_{rs}$. We shall use (3.7) and (3.8) repeatedly in the following calculations.

The field equation for $P^{M_1 \cdots M_{p+1}}$ is

$$\tilde{H}_{M_1 \dots M_{p+2}} = \partial_{M_1} \tilde{B}_{M_2 \dots M_{p+2}} + \mathrm{cyclic}(M_1 \dots M_{p+2}) = 0. \tag{3.10}$$

Given $\tilde{B}$ as in (3.4), it is nontrivial to show (3.10). To this end, we first refer to the tangent space components of (3.10) which read

$$\begin{aligned}\tilde{H}_{A_1\dots A_{p+2}} = H_{A_1\dots A_{p+2}} - \Bigg(& \frac{i}{2mn}\Gamma^{\alpha\beta}_{c_1} T_{A_1A_2}{}^{[c_1}\delta^{c_2}_{A_3}\dots\delta^{c_{p+1}]}_{A_{p+2}} H_{\alpha\beta c_2\dots c_{p+1}} \\ & + \frac{i(-1)^{p+1}}{2mn(p+1)}\Gamma^{\alpha\beta}_{c_1}\delta^{c_1}_{A_1}\dots\delta^{c_{p+1}}_{A_{p+1}} D_{A_{p+2}} H_{\alpha\beta c_2\dots c_{p+1}} \\ & + \text{cyclic}(A_1\dots A_{p+2})\Bigg) \\ = 0. \end{aligned} \tag{3.11}$$

Using (3.1), (3.2) and (3.7) we find that all the projections of $\tilde{H}$ are identically vanishing except $\tilde{H}_{\alpha\beta c_1\dots c_p}$ and $\tilde{H}_{\alpha c_1\dots c_{p+1}}$. The vanishing of the former gives the equation

$$H_{\alpha\beta c_1\dots c_p} = \frac{1}{mn(p+1)}\Gamma^{c_{p+1}}_{\alpha\beta}\Big[\Gamma^{\gamma\delta}_{c_{p+1}} H_{\gamma\delta c_1\dots c_p} + \text{cyclic}(c_1\dots c_{p+1})\Big]. \tag{3.12}$$

We observe that the expression in the square brackets is totally antisymmetric in $(c_1\dots c_{p+1})$ and therefore it must be proportional to the Levi-Civita symbol $\epsilon^{c_1\dots c_{p+1}}$. Thus we can write

$$H_{\alpha\beta c_1\dots c_p} = \epsilon_{c_1\dots c_{p+1}}\Gamma^{c_{p+1}}_{\alpha\beta} Q, \tag{3.13}$$

for some Q. Introducing the notation

$$H_{\alpha\beta c_1\dots c_p} := \epsilon_{c_1\dots c_p a} H^a_{\alpha\beta}, \tag{3.14}$$

we can write (3.13) as

$$H^a_{\alpha\beta} = \Gamma^a_{\alpha\beta} Q. \tag{3.15}$$

From the definition of the pull-back of H and using the constraints (3.2) we have

$$H_{\alpha\beta c_1\dots c_p} = i\xi^{-1} E^{\underline{c}_1}_{c_1}\dots E^{\underline{c}_p}_{c_p}\big(E_\alpha \eta \Gamma_{\underline{c}_1\dots\underline{c}_p} E_\beta\big). \tag{3.16}$$

Using this equation, we now have to show that (3.15) is satisfied. Our strategy is to replace one of the $E^{\underline{c}}_a$ factors in (3.16), by using the following identity which follows from (3.8):

$$E^{\underline{a}}_a = \frac{1}{mn}\Gamma^{\alpha\beta}_a\big(E_\alpha \Gamma^{\underline{a}} E_\beta\big), \tag{3.17}$$

and then making use of the super p-brane Dirac matric identity

$$\Gamma^{\underline{c}}_{(\underline{\alpha\beta}}(\eta\Gamma_{\underline{c}\,\underline{c}_1\dots\underline{c}_{p-1}})_{\underline{\gamma\delta})} = 0. \tag{3.18}$$

In this fashion, after a little bit of algebra, from (3.16) we obtain

$$p(mn+4)H^a_{\alpha\beta} = \left(H^c_{\gamma\delta}\Gamma_c^{\gamma\delta}\Gamma^a_{\alpha\beta} - H^a_{\gamma\delta}\Gamma_c^{\gamma\delta}\Gamma^c_{\alpha\beta}\right) + 2H^c_{\alpha\gamma}\left(\Gamma_c{}^a\right)^\gamma{}_\beta + 2H^c_{\beta\gamma}\left(\Gamma_c{}^a\right)^\gamma{}_\alpha. \tag{3.19}$$

We now decompose $H^a_{\alpha\beta}$ as follows:

$$H^a_{\alpha\beta} = \Gamma^a_{\alpha\beta}Q + H^{ab}(\Gamma_b)_{\alpha\beta} + \hat{H}^a_{\alpha\beta}, \tag{3.20}$$

with H^{ab} traceless in ab and

$$(\Gamma^a)^{\alpha\beta}\hat{H}^b_{\alpha\beta} = 0. \tag{3.21}$$

Substituting the above parametrization of $H^a_{\alpha\beta}$ into (3.19), after some calculation, we find that Q is not determined, and that the expansion coefficient H^{ab} is equal to zero. This leaves us with the following equation for $\hat{H}^a_{\alpha\beta}$:

$$p(mn+4)\hat{H}^a_{\alpha\beta} = 2\hat{H}^c_{\alpha\gamma}\left(\Gamma_c{}^a\right)^\gamma{}_\beta + 2\hat{H}^c_{\beta\gamma}\left(\Gamma_c{}^a\right)^\gamma{}_\alpha. \tag{3.22}$$

We now rewrite the 2-gamma matrix in the second term on the RHS of (3.22) as $(\Gamma_c{}^a)^\gamma{}_\alpha = (\Gamma_c{}^a)_\alpha{}^\gamma$ (see Table 1 for the symmetry of gamma matrices). Next we write the 2-gamma matrices in Eq. (3.22) as products of 1-gamma matrices, and multiplying this equation with $\Gamma_a^{\beta\eta}$ we obtain

$$[p(mn+2)+2]\left(\hat{H}^a_{\alpha\beta}\Gamma_a^{\beta\eta}\right) = 2\left(\hat{H}^c_{\beta\gamma}(\Gamma_c)_{\alpha\delta}\right)\left((\Gamma^a)^{\delta\gamma}\Gamma_a^{\beta\eta}\right). \tag{3.23}$$

Contracting this equation with δ^α_η we find

$$\hat{H}^a_{\alpha\beta}\Gamma_a^{\beta\alpha} = 0. \tag{3.24}$$

It is convenient now to distinguish between different values of (p, m, n) (see table). We first consider the cases with $p = 2$, i.e. $(p, m, n) = (2, 2, 8)$, $(2, 2, 4)$, $(2, 2, 2)$ or $(2, 2, 1)$. At this point it is useful to write out explicitly $\Gamma^a_{\alpha\beta} = \Gamma^a_{\alpha'\beta'}\delta_{rs}$ ($\alpha' = 1, 2$, $r = 1, \dots, n$) where $\Gamma^a_{\alpha'\beta'}$ are the two by two Pauli matrices. Multiplying (3.23) with $\delta^{\alpha'}_{\eta'}$ we find a stronger version of (3.24) to hold, namely: $\hat{H}^a_{\alpha'r,\beta's}\Gamma_a^{\beta'\alpha'} = 0$. Using this equation and using the fact that the Pauli matrices satisfy the relation

$$\Gamma^a_{\alpha'\beta'}(\Gamma_a)_{\gamma'\delta'} = \left(C_{\beta'\gamma'}C_{\alpha'\delta'} + C_{\alpha'\gamma'}C_{\beta'\delta'}\right), \tag{3.25}$$

it is not too difficult to show that $\hat{H}^a_{\alpha\beta} = 0$. From the decomposition (3.20) then immediately follows the desired equation (3.15).

We next consider the cases $(p, m, n) = (5, 4, 2)$ and $(3, 4, 1)$ where the following gamma matrix identity holds:

$$(\Gamma^a)^{\delta(\gamma}\Gamma_a^{\beta\eta)} = 0. \tag{3.26}$$

This identity is related to the construction of superstrings in $d = 6$ and $d = 4$ target space dimensions, respectively. Using this identity, the fact that $\hat{H}_{\beta\gamma} = \hat{H}_{\gamma\beta}$ and Eq. (3.21), it is then not too difficult to show that again $\hat{H}^a_{\alpha\beta} = 0$, and thus (3.15) is indeed satisfied.

This leaves us with the cases $(p, m, n) = (4, 4, 2)$ and $(3, 4, 2)$. For these case we can neither find a Fierz identity of the form (3.25) nor can we in the (4, 4, 2) case apply the gamma matrix identity (3.26) since there are no superstrings in $d = 5$ target space dimensions. So far, we have not been able to proof the identity (3.15) for these cases by other means. This completes our discussion of the identity (3.15).

To complete the proof of (3.11) there remains to be shown that $\tilde{H}_{\alpha c_1 \dots c_{p+1}}$ vanishes. From (3.11) we obtain

$$H_{\gamma c_1 \dots c_{p+1}} = \frac{i}{2mn(p+1)} \Gamma_{c_1}^{\alpha\beta} D_\gamma H_{\alpha\beta c_2 \dots c_{p+1}} + \text{cyclic}(c_1 \dots c_{p+1}). \tag{3.27}$$

Introducing the notation

$$H_{\alpha c_1 \dots c_{p+1}} := \epsilon_{c_1 \dots c_{p+1}} Q_\alpha, \tag{3.28}$$

and using (3.12), we can write (3.27) as follows:

$$Q_\alpha = \tfrac{1}{2} i(-1)^{p+1} D_\alpha Q. \tag{3.29}$$

To prove this equation, we consider the Bianchi identity $D_{(\alpha} H_{\beta\gamma) c_1 \dots c_p} + 2i\Gamma^c_{(\alpha\beta} H_{\gamma) c c_1 \dots c_p} = 0$. Using the notations (3.13) and (3.28), this can be written as

$$D_{(\alpha} H^a_{\beta\gamma)} + 3i(-1)^{p+1} \Gamma^a_{(\alpha\beta} Q_{\gamma)} = 0. \tag{3.30}$$

Substituting (3.15) into this equation, we obtain the equation we wanted to prove, namely (3.29). This completes the proof of Eq. (3.11). As a consequence, the action (3.3) has the additional symmetry

$$\delta P^{M_1 \dots M_{p+1}} = \partial_N \Sigma^{N M_1 \dots M_{p+1}}, \tag{3.31}$$

where the parameter is completely graded antisymmetric.

Now we turn to the equation of motion for the Lagrange multiplier $Q^{M_1 \dots M_p}$ given by

$$\partial_{M_1} P^{M_1 \dots M_{p+1}} = 0. \tag{3.32}$$

In analogy with (2.30), using the gauge invariance (3.31), the solution of the above equation can be put into the form

$$P^{M_1 \dots M_{p+1}} + T\epsilon^{m_1 \dots m_{p+1}} \delta^{M_1}_{m_1} \dots \delta^{M_{p+1}}_{m_{p+1}} \theta^{mn}. \tag{3.33}$$

Substituting this into the action (3.3), we obtain (with $T = 1$)

$$S = \int d^{p+1}\sigma \; d^{mn}\theta \; P^\alpha_{\underline{a}} E^{\underline{a}}_\alpha + \tfrac{1}{2} i(p+1)! \int d^{p+1}\sigma (\det E^a_m) Q\big|_{\theta=0}$$
$$+ \int d^{p+1}\sigma \; \epsilon^{m_1 \dots m_{p+1}} \partial_{m_{p+1}} Z^{\underline{M_{p+1}}} \dots \partial_{m_1} Z^{\underline{M_1}} B_{\underline{M_1} \dots \underline{M_{p+1}}}\big|_{\theta=0}. \tag{3.34}$$

The last term coincides with the Wess–Zumino term of the usual super p-brane action, but to show that the second term is the Nambu–Goto term requires quite a bit of further work. To this end, it is convenient to introduce the following matrix:

$$\Gamma = \frac{\xi}{\sqrt{-g}\,(p+1)!}\epsilon^{c_1\cdots c_{p+1}} E_{c_1}^{\underline{a_1}}\cdots E_{c_{p+1}}^{\underline{a_{p+1}}}\Gamma_{\underline{a_1\cdots a_{p+1}}}, \tag{3.35}$$

where $g = \det\, g_{ab}$, with the definition

$$g_{ab} = E_a^{\underline{c}} E_b^{\underline{c}}. \tag{3.36}$$

Using (3.36) one can easily show that $\Gamma^2 = 1$. Next, we define the matrix

$$\tau_a = E_a^{\underline{a}}\Gamma_{\underline{a}}, \tag{3.37}$$

which satisfies

$$\{\tau_a, \tau_b\} = 2g_{ab}, \quad [\tau_a, \Gamma] = 0, \quad \tau_{c_1\ldots c_p} = -\xi^{-1}\epsilon_{c_1\ldots c_{p+1}}\Gamma\tau^{c_{p+1}}\sqrt{-g}\,. \tag{3.38}$$

Using (3.37) and (3.16) we can write (3.13) as

$$\left(E_\alpha\eta\tau_{c_1\ldots c_p}E_\beta\right) = -i\xi\epsilon_{c_1\ldots c_p a}\Gamma^a_{\alpha\beta}Q. \tag{3.39}$$

We now derive an identity for Q. Multiplying the cyclic identity (3.18) by $\epsilon^{c_1\cdots c_{p-1}ab}\Gamma_a^{\alpha\beta}\Gamma_b^{\gamma\delta}E_{c_1}^{\underline{c_1}}\cdots E_{c_{p-1}}^{\underline{c_{p-1}}}(E_\alpha\Gamma\eta)^{\underline{\alpha}}E_\beta^{\underline{\beta}}E_\gamma^{\underline{\gamma}}E_\delta^{\underline{\delta}}$, and then using Eqs. (3.8), (3.35), (3.37), (3.38) and (3.39), we find that

$$Q^2 = \det\, g + \frac{2(-1)^{(p+1)}\xi\sqrt{-g}\;\epsilon^{c_1\cdots c_{p-1}ab}}{(mn)^2(p+1)!}$$
$$\times\left(E_\alpha\eta\Gamma\Gamma^{\underline{a}}E_\delta\right)\Gamma_a^{\alpha\beta}\Gamma_b^{\gamma\delta}\left(E_\gamma\eta\Gamma^{\underline{a}}{}_{c_1\ldots c_{p-1}}E_\beta\right), \tag{3.40}$$

where $\Gamma^{\underline{a}}{}_{c_1\ldots} = \Gamma^{\underline{a}}{}_{\underline{b}\ldots}E_{c_1\ldots}^{\underline{b}}$.

The last term in this equation can be shown to vanish by Fierz rearranging the expression $\epsilon^{c_1\cdots c_{p-1}ab}\Gamma_a^{\alpha\beta}\Gamma_b^{\gamma\delta}$, and then using the following identities (in fact, we need only the trace of these identities in their symmetrized indices):

$$\begin{aligned}
&\Gamma_{(b}^{\alpha'\beta'}\left(E_{\alpha' r}\eta\Gamma^{\underline{c_1}}{}_{a)}E_{\beta' s}\right) = 0 && (p=2;\ n\neq 1, 2),\\
&\left(\Gamma_5\Gamma_{c(b}\right)^{\alpha\beta}\left(E_\alpha\eta\Gamma^{\underline{c_1}}{}_{a)c_2}E_\beta\right) = 0 && (p=3),\\
&\Gamma^{\alpha\beta}_{b_1\ldots(b_r}\left(E_\alpha\eta\Gamma^{\underline{c_1}}{}_{a)c_2\ldots c_{p-1}}E_\beta\right) = 0 && (p=3,\ r=1,2;\ p=5,\ r=1).
\end{aligned} \tag{3.41}$$

The above identities can be derived by multiplying the cyclic identity (3.18) with $\Gamma_a^{\alpha'\beta'}\Gamma_b^{\gamma'\delta'}E_{\alpha' r}^{\underline{\alpha}}E_{\beta' s}^{\underline{\beta}}E_{\gamma q}^{\underline{\gamma}}E_{\delta' q}^{\underline{\delta}}$ or with $\Gamma_a^{\alpha\beta}(\Gamma_5\Gamma_{bc})^{\gamma\delta}E_{c_2}^{\underline{c_2}}E_\alpha^{\underline{\alpha}}E_\beta^{\underline{\beta}}E_\gamma^{\underline{\gamma}}E_\delta^{\underline{\delta}}$, or with $\Gamma_a^{\alpha\beta}\Gamma^{\gamma\delta}_{b_1\cdots b_r}E_{c_2}^{\underline{c_2}}\cdots E_{c_{p-1}}^{\underline{c_{p-1}}}E_\alpha^{\underline{\alpha}}E_\beta^{\underline{\beta}}E_\gamma^{\underline{\gamma}}E_\delta^{\underline{\delta}}$, respectively, and then using Eq. (3.8).

With the last term vanishing in (3.40), it follows that †

$$Q = (\det\, g)^{1/2}. \tag{3.42}$$

† We are grateful to M. Tonin for explaining to us the derivation of this identity for the supermembrane.

Substitution of (3.42) into the action (3.35) yields the following simple result which is a natural generalization of the massive superparticle case:

$$S = \int d^{p+1}\sigma \, d^{mn}\theta \, P^{\alpha}_{\underline{a}} E^{\underline{a}}_{\alpha} + \tfrac{1}{2}(p+1)! \int d^{p+1}\sigma \left(-\det E^{\underline{a}}_{m} E^{\underline{a}}_{n}\right)^{1/2}\Big|_{\theta=0}$$
$$+ \int d^{p+1}\sigma \, \epsilon^{m_1 \dots m_{p+1}} \partial_{m_{p+1}} Z^{\underline{M}_{p+1}} \dots \partial_{m_1} Z^{\underline{M}_1} B_{M_1 \dots M_{p+1}}\Big|_{\theta=0}, \tag{3.43}$$

where we have used the constraint (3.7) in manipulations similar to (2.34).

In summary, our main result for super p-branes is the action (3.3) together with the definitions (2.10) and (3.4) and the constraints (3.1) and (3.2). Elimination of the Lagrange multiplier $P^{M_1 \cdots M_{p+1}}$ yields the result (3.43). Below we shall comment on various aspects of these results and we shall discuss a number of open problems.

4. Conclusions

We have found a twistor-like formulation of a class of super p-brane theories in which κ-symmetry is replaced by worldvolume local supersymmetry. The form of the action (3.45) essentially coincides with the Nambu–Goto form of the usual super p-brane action. The difference is due to the Lagrange multiplier term. It is not altogether clear whether the equations of motions are equivalent to those which follow from the usual super p-brane action [21]. For this to happen, one must show that there is a sufficiently powerful gauge symmetry of the action which makes it possible to gauge away the Lagrange multiplier. We have shown that for the massive superparticle such a gauge symmetry indeed exists (see Eq. (2.37)). The existence of this gauge symmetry relies on the Dirac matrix identity (2.32). It remains to be seen whether a similar gauge symmetry exists for other values of p. We expect that the p-brane Dirac matrix identity (3.18) will play an essential role in proving the existence of such a symmetry.

One of the essential ingredients of the twistor-like transform is the existence of a closed super $(p+1)$-form on the worldvolume superspace which is constructed out of the pull-backs of a super $(p+1)$-form and its curvature in target superspace. We have shown that this closed $(p+1)$-form exists for the cases (p, m, n) $= (2, 2, 8)$, $(5, 4, 2)$, $(2, 2, 4)$, $(3, 4, 1)$, $(2, 2, 2)$ and $(2, 2, 1)$. The $p = 2$ cases were already considered in ref. [16]. We believe that the existence of this closed $(p+1)$-form should have some interesting geometric interpretation, independent of the role it plays in the twistor-like transform. For instance, it seems that it is related to the light-like integrability principle [24,13]. We also note an interesting relation between our work and that of refs. [25,26]. In both cases the tension parameter is supposed to emerge as an integration constant of the equations of motion. The p-form gauge potential occurring in ref. [26] seems to be closely related to the p-form gauge potential $Q_{M_1 \dots M_p}$ occurring in our work. We hope that a more precise understanding of all these connections may lead to a better understanding of the theories in question.

There are a number of open problems which deserve further investigation. To name a few, what is the precise relation between our action and the usual one [21] at the quantum level? What are the physical degrees of freedom described by this action? Are the symmetries of the action anomaly-free? Can the quantization problems of the usual κ-symmetric action be avoided by the new action? Is the theory finite?

Another open problem of considerable interest is how to couple the Yang–Mills sector to the theory (such theories are usually referred to as heterotic p-brane theories, because of their similarity to the heterotic string theory). It is tempting to think that since in the twistor-like formulation the local worldvolume supersymmetry is manifest in a superspace formalism, one may simply use the body of knowledge available on superspace formulation of matter/Yang–Mills systems coupled to supergravity. However, there is an unusual property of the twistor-like formulations, namely, the local supersymmetry does not seem to require kinetic terms for the supergravity multiplet. On the other hand, in a supergravity plus matter/Yang–Mills system, typically one encounters these kinetic terms. Thus, one may look for different than usual local supersymmetric invariants (using the usual kind of tensor calculus when available) or consider the possibility of including the supergravity kinetic terms in the spirit of ref. [28], where such terms do arise in the context of finding effective actions for heterotic p-brane solitons. We hope that the results of this paper will help in the eventual solution of this problem.

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