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Published in:
Nuclear Physics B

DOI:
10.1016/S0920-5632(00)91638-4

ELSEVIER

Nuclear Physics B (Proc. Suppl.) 83–84 (2000) 250–252

NUCLEAR PHYSICS B
PROCEEDINGS SUPPLEMENTS
www.elsevier.nl/locate/npe

On $K \to \pi\pi$ Decays in Quenched and Unquenched Chiral Perturbation Theory

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We calculate the logarithmic corrections to the matrix elements for $K^+ \to \pi^+$ and $K \to$ vacuum (which are used on the lattice to determine $K \to \pi\pi$ amplitudes), in one-loop quenched and unquenched Chiral Perturbation Theory. We find that these corrections can be large. We also discuss, and present some results for, the direct determination of $K \to \pi\pi$ amplitudes. In particular, we address effects from choosing $m_s = m_d$ and vanishing external spatial momenta, finite volume and quenching. In the quenched octet case, we find *enhanced* finite-volume contributions which may make numerical estimates of this matrix element unreliable for *large* volumes.

1. Introduction

Chiral Perturbation Theory (ChPT) helps us understand several systematic errors which afflict lattice computations of $K \to \pi\pi$ decay amplitudes, and thus plays an important role in assessing the reliability of such computations. In particular, ChPT can be used to gain insight into the size of finite-volume and quenching effects, as well as the modifications induced by an unphysical choice of kinematics and/or the values of light quark masses. We consider two approaches to these amplitudes. The first is the direct computation of such amplitudes with $m_s = m_d$ and external mesons at rest [1]. Three key questions can be studied in ChPT: 1) how much do these unphysical choices affect the size of the chiral logarithms (with and without quenching)?; 2) are there quenched chiral logarithms [2]?; 3) are there *enhanced* finite-volume corrections? Here we mainly answer the last two questions, while a more detailed analysis will be given elsewhere [3]. Second, we calculate the $K \to \pi$ and $K \to 0$ (K to vacuum) matrix elements at one loop in ChPT, unquenched and quenched. The motivation comes from the possibility of performing an indirect determination of $K \to \pi\pi$ amplitudes through the computation of reduced matrix elements such as $K \to \pi$ and $K \to 0$, which is simpler on the lattice [4].

2. The Unphysical $K^0 \to \pi^+\pi^-$ amplitude

The Euclidean effective Lagrangian for $\Delta S = 1$ hadronic weak transitions can, at leading order in ChPT, be written as [4,5] (notation of [4]):

$$\mathcal{L}_{\Delta S=1} = -\alpha^{27} T^{ij}_{kl} (\Sigma\partial_\mu\Sigma^\dagger)^k{}_i (\Sigma\partial_\mu\Sigma^\dagger)^l{}_j \quad (1)$$
$$-\alpha_1^8 \mathrm{tr}[\Lambda(\partial_\mu\Sigma)(\partial_\mu\Sigma^\dagger)] + \alpha_2^8 \frac{8v}{f^2} \mathrm{tr}[\Lambda(\Sigma M + M^\dagger\Sigma^\dagger)] ,$$

where the first term transforms as $(27_L, 1_R)$ under SU(3)×SU(3) and the last two terms as $(8_L, 1_R)$.

The term with coupling α_2^8 is known as the "weak mass term," and mediates the $K \to 0$ transition at tree level. Its odd-parity part, which in principle can also contribute to the octet $K \to \pi\pi$ amplitude, is proportional to $m_s - m_d$. For $m_s \neq m_d$ the weak mass term is a total derivative [4,6], and therefore does not contribute to any physical matrix element. Whether this term contributes to the octet $K \to \pi\pi$ matrix element for unphysical external momenta and $m_s = m_d$ is a more subtle question. What actually is computed on the lattice is the Euclidean correlation function $C(t_1, t_2) = \langle 0|\pi^+(t_2)\pi^-(t_2) O_8(t_1) \overline{K}^0(0)|0\rangle$. Any contribution generated by the insertion of the weak mass term to the Euclidean correlation function at fixed times is proportional to $m_s - m_d$ in the limit $m_s \to m_d$ and therefore zero

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PII S0920-5632(00)00243-7

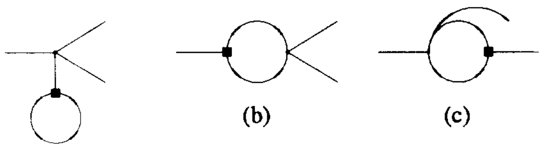

Figure 1. Some $K \to \pi\pi$ diagrams in ChPT. The box is a weak vertex, the dot a strong vertex.

at $m_s = m_d$; there are no subtleties with propagator poles in Euclidean space from tree-level tadpole diagrams [3]. This is also true for tadpole contributions as in diagram (a) of Fig. 1 with the insertion of an octet or a 27-plet weak operator. Such contributions are absent for $m_s = m_d$. We note that, choosing quark masses such that $m_K = 2m_\pi$, as proposed in [7], the contribution from α_2^8 vanishes for the same reason as for the physical $K \to \pi\pi$ amplitude, but that, in general, there are contributions from Fig. 1(a).

While the unphysical choice of kinematics and quark masses modifies the size of chiral logarithms and finite-volume corrections, quenching also causes new "quenched-artifact" contributions, due to the presence of the double pole in the singlet propagator. These are of two types: quenched chiral logarithms (QχL) and *enhanced* finite-volume corrections (discovered in quenched $\pi\pi$ scattering [8]). In principle, both artifacts occur in the quenched octet $K \to \pi\pi$ amplitude.

Only diagrams of type (b), (c) in Fig. 1 with the weak operator α_1^8 give rise to QχL in the unphysical $K^0 \to \pi^+\pi^-$ amplitude at $m_s = m_d$. However, we find that *no* QχL is present at one loop, due to a cancellation between contributions from type-(b) and type-(c) diagrams.

In finite volume, only the "rescattering diagram" of type (b) gives rise to power-like finite-volume corrections. It was shown in the case of $\pi\pi$ scattering [8] how, in the quenched approximation in a similar diagram, the presence of a double-pole singlet propagator gives rise to enhanced (infrared divergent!) finite-volume corrections. The same happens for the octet $K^0 \to \pi^+\pi^-$ amplitude.

We have calculated the chiral logs and power-like finite-volume corrections for $C(t_1, t_2)_{\rm octet}$ to one loop. In the unquenched case we find

$$\begin{aligned} C(t_1,t_2) &= \frac{8i\alpha_1^8}{f^3}\frac{M^2L^3}{(2M)^3}e^{-2M(t_2-t_1)-Mt_1} \\ &\left[1-\mu(M)+\frac{7}{6}\frac{1}{f^2L^3}(t_2-t_1)\right. \\ &\left.-\frac{M^2}{(4\pi f)^2}\left(\frac{41.597}{ML}+\frac{62}{3}\frac{\pi^2}{(ML)^3}\right)\right], \end{aligned} \tag{2}$$

where M is the degenerate meson mass, f is the pion decay constant in the chiral limit (normalized such that its value is 132 MeV at the physical pion mass), and $\mu(M) = (M^2/(16\pi^2 f^2))\log(M^2/\Lambda^2)$ is the chiral logarithm. In the quenched case we obtain

$$\begin{aligned} C(t_1,t_2) &= \frac{8i\alpha_1^8}{f^3}\frac{M^2L^3}{(2M)^3}e^{-2M(t_2-t_1)-Mt_1} \\ &\left[1+\delta\left(-\frac{\pi^2}{M^2L^3}(t_2-t_1)-\frac{2\pi^2}{ML^3}(t_2-t_1)^2\right.\right. \\ &\left.+\frac{3\pi^2}{4(ML)^3}+\frac{2.2284}{ML}-0.41877ML\right)+2\alpha\mu(M) \\ &+\frac{\alpha}{3}\frac{M^2}{(4\pi f_\pi)^2}\left(\frac{5\pi^2}{2(ML)^3}-\frac{14\pi^2}{M^2L^3}(t_2-t_1)\right. \\ &\left.\left.+\frac{4\pi^2}{ML^3}(t_2-t_1)^2+\frac{31.198}{ML}+0.83754ML\right)\right], \end{aligned} \tag{3}$$

where $\delta = m_0^2/(24\pi^2 f^2)$ contains the singlet mass m_0 and α is another singlet parameter renormalizing its kinetic term [2].

We have ignored $O(p^4)$ contact terms, exponentially suppressed finite-volume effects, and contributions from excited states. The term linear in $t_2 - t_1$ can be related to finite-volume energy shifts of the two-particle internal states of type-(b) diagrams (at least in the unquenched case). The term linear in ML inside the square brackets of Eq. (3) is the enhanced finite-volume contribution, which is a quenched artifact, as is the term quadratic in $t_2 - t_1$.

3. $K^+ \to \pi^+$ and $K \to 0$ matrix elements

Ref. [4] proposed an indirect determination of the $K \to \pi\pi$ amplitudes with $\Delta I = 1/2$ and $3/2$

by computing on the lattice the reduced amplitudes $K^+ \to \pi^+$ and $K \to 0$. The ratio of the $\Delta I = 1/2$ and $3/2$ $K \to \pi\pi$ amplitudes can then be determined at tree level in ChPT through

$$\frac{[K^0\to\pi^+\pi^-]_{\frac{1}{2}}}{[K^0\to\pi^+\pi^-]_{\frac{3}{2}}} = \frac{[K^+\to\pi^+]_{\frac{1}{2}} - b[K^0\to 0]_{\frac{1}{2}}}{[K^+\to\pi^+]_{\frac{3}{2}}}, \quad (4)$$

where $b = iM^2/f(m_K^2 - m_\pi^2)$, M is the degenerate mass used to compute $K^+ \to \pi^+$, and m_K, m_π are the nondegenerate masses used to compute $K \to 0$. The question arises how one-loop corrections modify Eq. (4). This problem was already addressed in [6] in the unquenched case, however, what is calculated there is the full pseudoscalar two-point function, and not the amplitude $K \to \pi$.

Here, we present the chiral logs for $K^+ \to \pi^+$ and $K \to 0$. For $K^+ \to \pi^+$, with degenerate masses, we find for $\Delta I = 1/2$, unquenched,

$$\frac{[K^+\to\pi^+]}{4M^2/f^2} = \alpha_1^8\left(1 - \frac{1}{3}\mu(M)\right) - \alpha_2^8\left(1 + \frac{4}{3}\mu(M)\right) - \alpha^{27}(1 - 12\mu(M)), \quad (5)$$

while in the quenched case we obtain

$$\frac{[K^+\to\pi^+]}{4M^2/f^2} = \alpha_1^8\left(1 - 2\delta\log\frac{M^2}{\Lambda^2} + 4\alpha\mu(M)\right) - \alpha_2^8\left(1 + \frac{4}{3}\alpha\mu(M)\right) - \alpha^{27}(1 - 6\mu(M)). \quad (6)$$

The $\Delta I = 3/2$ amplitudes are obtained from this by setting $\alpha_1^8 = \alpha_2^8 = 0$. Note that the contribution from ordinary chiral logarithms is substantially reduced by quenching. One should keep in mind that the values of the α's are in principle different in the quenched and unquenched theories. The one-loop $K \to 0$ amplitude to leading order in $m_K^2 - m_\pi^2$ is (with M^2 some average of m_K^2 and m_π^2)), unquenched,

$$\frac{[K\to 0]f}{4(m_K^2 - m_\pi^2)} = i\alpha_2^8\left(1 - \frac{13}{3}\mu(M)\right) + i\alpha_1^8\frac{10}{3}\mu(M),$$

while the quenched amplitude is

$$\frac{[K\to 0]f}{4(m_K^2 - m_\pi^2)} = i\alpha_2^8\left(1 - \frac{4}{3}\alpha\mu(M)\right) + i\alpha_1^8\left(2\delta\log\frac{M^2}{\Lambda^2} - 4\alpha\mu(M)\right).$$

Again, the chiral logarithms are potentially large, and reduced by quenching. One can now in principle extract unquenched

$$\frac{\alpha_1^8}{\alpha^{27}} = \frac{[K^+\to\pi^+]_8(1-3\mu) - b[K\to 0](1+\frac{8}{3}\mu)}{-[K^+\to\pi^+]_{27}(1+12\mu)},$$

and quenched

$$\frac{\alpha_1^8}{\alpha^{27}} = \frac{[K^+\to\pi^+]_8 - b[K\to 0](1+\frac{8}{3}\alpha\mu)}{-[K^+\to\pi^+]_{27}(1+6\mu)},$$

where $\mu \equiv \mu(M) = (M^2/(16\pi^2 f^2))\log(M^2/\Lambda^2)$. It is clear that one-loop corrections are potentially large in the determination of weak-Lagrangian parameters from lattice computations. (For $M = 400$ MeV, $\Lambda = m_\rho$ and $f = 132$ MeV, $\mu(M) = -0.076$, and *e.g.* $1 + 6\mu = 0.54$.)

Obviously, in order to go beyond these "leading-log" estimates, it is necessary to consider also the contributions of $O(p^4)$ LECs to $K^+ \to \pi^+$ and $K \to 0$. Then, it is worth looking for ratios less sensitive to one-loop effects, if such exist (considering also other channels like $K \to \eta$ and/or varying momenta or masses), and also, whether (combinations of) the $O(p^4)$ LECs that appear in $K \to \pi\pi$ amplitudes can be extracted from amplitudes with less external legs.

We thank Claude Bernard and Steve Sharpe for very useful discussions. MG is supported by the US Dept. of Energy, and EP by the Minist. de Educacion y Cultura (Spain).

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