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THE ROLE OF FINAL STATE INTERACTIONS IN ε'/ε

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The Standard Model prediction for ε'/ε is updated, taking into account the chiral loop corrections induced by final state interactions. The resulting value, $\varepsilon'/\varepsilon = (17\pm 6)\times 10^{-4}$, is in good agreement with present measurements.

1. Introduction

The CP–violating ratio ε'/ε constitutes a fundamental test for our understanding of flavour–changing phenomena. The present experimental world average,[1] $\mathrm{Re}\,(\varepsilon'/\varepsilon) = (19.3 \pm 2.4)\cdot 10^{-4}$, provides clear evidence for a non-zero value and, therefore, the existence of direct CP violation.

The theoretical prediction has been rather controversial since different groups, using different models or approximations, have obtained different results.[2–9] In terms of the $K \to \pi\pi$ isospin amplitudes, $\mathcal{A}_I = A_I\, e^{i\delta_I}$ $(I = 0, 2)$,

$$\frac{\varepsilon'}{\varepsilon} = e^{i\Phi}\,\frac{\omega}{\sqrt{2}|\varepsilon|}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right], \qquad \Phi \approx \delta_2 - \delta_0 + \frac{\pi}{4} \approx 0\,, \tag{1}$$

where $\omega = \mathrm{Re}A_2/\mathrm{Re}A_0 \approx 1/22$. The CP–conserving amplitudes $\mathrm{Re}A_I$, their ratio ω and ε are usually set to their experimentally determined values. A theoretical calculation is then only needed for the quantities $\mathrm{Im}A_I$.

Since $M_W \gg M_K$, there are large short–distance logarithmic contributions which can be summed up using the Operator Product Expansion and the renormalization group.[10,11] To predict the physical amplitudes one also needs to compute long–distance hadronic matrix elements of light four–quark operators Q_i. They are usually parameterized in terms of the so-called bag parameters B_i, which measure them in units of their vacuum insertion approximation values.

To a very good approximation, the Standard Model prediction for ε'/ε can be written (up to global factors) as[5]

$$\frac{\varepsilon'}{\varepsilon} \sim \left[B_6^{(1/2)}(1-\Omega_{IB}) - 0.4\,B_8^{(3/2)}\right], \qquad \Omega_{IB} = \frac{1}{\omega}\frac{(\mathrm{Im}A_2)_{IB}}{\mathrm{Im}A_0}\,. \tag{2}$$

Thus, only two operators are numerically relevant: the QCD penguin operator Q_6 governs $\mathrm{Im}A_0$ $(\Delta I = 1/2)$, while $\mathrm{Im}A_2$ $(\Delta I = 3/2)$ is dominated by the electroweak penguin operator Q_8. The parameter Ω_{IB} takes into account isospin breaking corrections; the value $\Omega_{IB} = 0.25$ was usually adopted in all calculations.[12] Together with $B_i \sim 1$, this produces a numerical cancellation leading to values of $\varepsilon'/\varepsilon \sim 7 \times 10^{-4}$. This number has been slightly increased by a recent Chiral Perturbation Theory (χPT) calculation at $O(p^4)$ which finds $\Omega_{IB} = 0.16 \pm 0.03$.[13]

2. Chiral Loop Corrections

Chiral symmetry determines the low-energy hadronic realization of the operators Q_i, through a perturbative expansion in powers of momenta and quark masses. The corresponding chiral couplings can be calculated in the large-N_C limit of QCD. The usual input values $B_8^{(3/2)} \approx B_6^{(1/2)} = 1$ correspond to the lowest-order approximation in both the $1/N_C$ and χPT expansions.

The lowest-order calculation does not provide any strong phases δ_I. Those phases originate in the final rescattering of the two pions and, therefore, are generated by higher-order chiral loops. Analyticity and unitarity require the presence of a corresponding dispersive effect in the moduli of the isospin amplitudes. Since the S-wave strong phases are quite large, specially in the isospin-zero case, one should expect large unitarity corrections.

The one-loop analyses of $K \to 2\pi$ show in fact that pion loop diagrams provide an important enhancement of the $\mathcal{A}_0$ amplitude.[14] This chiral loop correction destroys the accidental numerical cancellation in eq. (2), generating a sizeable enhancement of the ε'/ε prediction.[2] The large one-loop correction to $\mathcal{A}_0$ has its origin in the strong final state interaction (FSI) of the two pions in S-wave, which generates large infrared logarithms involving the light pion mass.[3] Using analyticity and unitarity constraints, these logarithms can be exponentiated to all orders in the chiral expansion.[2,3] For the CP-conserving amplitudes, the result can be written as

$$\mathcal{A}_I = \left(M_K^2 - M_\pi^2\right) a_I(M_K^2) = \left(M_K^2 - M_\pi^2\right) \Omega_I(M_K^2, s_0)\, a_I(s_0)\,, \tag{3}$$

where $a_I(s)$ denote reduced off-shell amplitudes with $s \equiv (p_{\pi_1} + p_{\pi_2})^2$ and

$$\Omega_I(s, s_0) \equiv e^{i\delta_I(s)}\, \Re_I(s, s_0) = \exp\left\{\frac{(s - s_0)}{\pi} \int \frac{dz}{(z - s_0)} \frac{\delta_I(z)}{(z - s - i\epsilon)}\right\} \tag{4}$$

provides an evolution of $a_I(s)$ from an arbitrary low-energy point s_0 to $s = M_K^2$. The physical amplitude $a_I(M_K^2)$ is of course independent of s_0.

Taking the chiral prediction for $\delta_I(z)$ and expanding the exponential to first order, one just reproduces the one-loop χPT result. Eq. (4) allows us to get a much more accurate prediction, by taking s_0 low enough that the χPT corrections to $a_I(s_0)$ are small and exponentiating the large logarithms with the Omnès factor $\Omega_I(M_K^2, s_0)$. Moreover, using the experimental phase-shifts in the dispersive integral one achieves an all-order resummation of FSI effects. The numerical accuracy of this exponentiation has been successfully tested through an analysis of the scalar pion form factor,[3] which has identical FSI than $\mathcal{A}_0$.

3. Numerical Predictions

At $s_0 = 0$, the chiral corrections are rather small. To a very good approximation,[4] we can just multiply the tree–level χPT result for $a_I(0)$ with the experimentally determined Omnès exponentials:[3]

$$\Re_0 \equiv \Re_0(M_K^2, 0) = 1.55 \pm 0.10\,, \qquad \Re_2 \equiv \Re_2(M_K^2, 0) = 0.92 \pm 0.03\,. \tag{5}$$

Thus, $B_6^{(1/2)} \approx \Re_0 \times B_6^{(1/2)}\Big|_{N_C\to\infty} = 1.55$, $B_8^{(3/2)} \approx \Re_2 \times B_8^{(3/2)}\Big|_{N_C\to\infty} \approx 0.92$ and $\Omega_{IB} \approx 0.16 \times \Re_2/\Re_0 = 0.09$. This agrees with the result $\Omega_{IB} = 0.08 \pm 0.05$, obtained recently with an explicit chiral loop calculation.[15]

The large FSI correction to the $I = 0$ amplitude gets reinforced by the mild suppression of the $I = 2$ contributions. The net effect is a large enhancement of ε'/ε by a factor 2.4, pushing the predicted central value from[5,6] 7×10^{-4} to[3] 17×10^{-4}. A more careful analysis, taking into account all hadronic and quark–mixing inputs gives the Standard Model prediction:[4]

$$\varepsilon'/\varepsilon = (17 \pm 6) \times 10^{-4}\,, \tag{6}$$

which compares well with the present experimental world average.

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