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The M5-brane and non-commutative open strings

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Abstract

The M-theory origin of non-commutative open-string theory is examined by investigating the M-theory 5-brane at near critical field strength. In particular, it is argued that the open-membrane metric provides the appropriate moduli when calculating the duality relations between M and II non-commutative theories.

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The natural generalization of the spatial non-commutative geometry that occurs on D-branes [1] is to include field strengths with non-zero electric components that induce spatio-temporal non-commutativity on the D-brane. This was examined in [2, 3]. The somewhat remarkable result was that by examining the D-brane in a decoupling limit near critical electric field strength[1] one naturally constructed a unitary decoupled spatio-temporally non-commutative open-string theory (NCOS).

The crucial property of the NCOS limit described in [2, 3] is to keep fixed both the open-string two-point function and the effective open-string coupling constant

$$\langle X^A X^B \rangle = 2\pi\alpha' G^{AB} + \Theta^{AB} = 2\pi\alpha' \left(\frac{1}{g + 2\pi\alpha'\mathcal{F}} \right)^{AB}, \tag{1}$$

$$G_0 = g_s \frac{\det^{1/2}(g + 2\pi\alpha'\mathcal{F})}{\det^{1/2}(g)}, \tag{2}$$

where G^{AB} is the symmetric part and Θ^{AB} is the antisymmetric part of the two-point function. In this limit the leading divergent parts of $\mathcal{F}$ cancel the contribution from $g_{\mu\nu}/\alpha'$, leaving the two-point function and coupling governed by the finite subleading terms[2]. As discussed in [2], this NCOS limit introduces a fixed metric G^{AB}_{OS} and a new effective length scale α'_{eff} defined as follows:

$$\alpha' G^{AB} = \alpha'_{\mathrm{eff}} G^{AB}_{\mathrm{OS}}. \tag{3}$$

[1] The concept of a critical field only works for the electric case as it relies crucially on the Lorentz signature.
[2] Here and in the rest of this paper we will ignore factors of 2π, etc.

In this paper we will consider a decoupling limit in the M5-brane analogous to the non-commutative open-string limit and examine its properties through an open-membrane probe. Even though the analogue of the string two-point function is not available for the open membrane, it was conjectured in [4] that the decoupled 5-brane theory should be formulated in terms of a so-called *open-membrane metric* with properties analogous to those of the open-string metric. Thus we demand that the six-dimensional proper lengths measured by the open-membrane metric are fixed in units of the 11-dimensional Planck length ℓ_p. This defines a finite effective length scale of the decoupled 5-brane theory that we shall denote ℓ_g. This is suggestive of an open-membrane theory underlying the decoupled spatio-temporal non-commutative M5-brane [6].

Evidence for the decoupled non-commutative 5-brane theory can be obtained by comparing various reductions of the 5-brane limit to NCOS and NCYM limits in string theory. Conversely, this provides a direct interpretation of the strong-coupling behaviour of the decoupled NCOS and NCYM theories.

The 5-brane may be effectively described by a six-dimensional self-dual 2-form field theory (that is neglecting the superpartners in the (2, 0) supermultiplet). The adapted field strength is

$$\mathcal{H} = \mathrm{d}b + C, \tag{4}$$

where C is the pull-back to the 5-brane of the 3-form potential in 11-dimensional supergravity and b is the 2-form potential on the 5-brane worldvolume. The self-duality condition provides a nonlinear algebraic constraint involving the components of the field strength and the induced metric $g_{\mu\nu}$ on the brane as follows [5]:

$$\frac{\sqrt{-\det g}}{6}\epsilon_{\mu\nu\rho\sigma\lambda\tau}\mathcal{H}^{\sigma\lambda\tau} = \frac{1+K}{2}(G^{-1})_\mu{}^\lambda\mathcal{H}_{\nu\rho\lambda}, \tag{5}$$

where $\epsilon^{012345} = 1$ and the scalar K and the tensor $G_{\mu\nu}$ are given by

$$K = \sqrt{1 + \tfrac{1}{24}\ell_p^6\mathcal{H}^2}, \qquad G_{\mu\nu} = \frac{1+K}{2K}\left(g_{\mu\nu} + \tfrac{1}{4}\ell_p^6\mathcal{H}^2_{\mu\nu}\right), \tag{6}$$

where ℓ_p is the 11-dimensional Planck scale.

The relation between the tensor $G_{\mu\nu}$ and the open-membrane metric for the 5-brane in analogy with the open-string metric that occurs on D-branes was discussed in [4]. It should be noted that the overall conformal scale of the open-membrane metric was not determined. In this paper such an overall scale will play a role. We therefore define the open-membrane metric as follows:

$$\hat{G}_{\mu\nu} \equiv \phi(x)\left(g_{\mu\nu} + \tfrac{1}{4}\ell_p^6\mathcal{H}^2_{\mu\nu}\right), \qquad x = \tfrac{1}{6}\ell_p^6\mathcal{H}^2. \tag{7}$$

Below we shall determine the asymptotic behaviour of $\phi(x)$ for large x from the requirements of the decoupling limit. We now proceed with the definition of the decoupling limit. The properties that we demand for the decoupling limit we wish to take are as follows.

(a) The Planck length $\ell_p \to 0$, so that the gravitational interactions can be decoupled.
(b) The proper six-dimensional lengths $\mathrm{d}s^2\,(\hat{G})$ of the open-membrane metric are fixed in 11-dimensional Planck units in the limit, i.e. $\ell_p^{-2}\,\mathrm{d}s^2\,(\hat{G})$ is fixed, so that the limit describes a genuine six-dimensional theory with a finite length scale ℓ_g.
(c) The electric components contain a divergent piece and a constant piece, in analogy with the limit discussed in [2] for open strings.

The first condition we satisfy by scaling $\ell_p \sim \epsilon^{1/3}$ ($\epsilon \to 0$). In order to satisfy the second and third condition we impose that $\mathcal{H}\ell_p^3$ diverges in the appropriate way (such that they solve the 5-brane equations of motion). We are therefore led to consider the following limit:

$$\begin{aligned} &g_{\alpha\beta} \sim \epsilon^0 \qquad g_{ab} \sim \epsilon^1 \qquad \ell_p \sim \epsilon^{1/3} \\ &\epsilon \to 0, \qquad \alpha, \beta = 0, 1, 2 \qquad a, b = 3, 4, 5 \end{aligned} \tag{8}$$

with the components of $\mathcal{H}$ behaving as follows:

$$\begin{aligned} &\mathcal{H}_{012} \sim \epsilon^{-1} + \epsilon^0, \\ &\mathcal{H}_{345} \sim \epsilon^0. \end{aligned} \tag{9}$$

In order to satisfy requirement (b) we demand in analogy with (3)

$$\ell_p^2\, (\hat{G}^{-1})^{\mu\nu} \equiv \ell_g^2\, G_{\mathrm{OM}}^{\mu\nu} \text{ is fixed.} \tag{10}$$

This allows us to fix the conformal factor as follows:

$$\phi(x) \sim x^{-2/3} \qquad \text{as} \quad x \to \infty. \tag{11}$$

This defines a non-commutative M5-brane length scale ℓ_g, a fixed metric $G_{\mathrm{OM}}^{\mu\nu}$. We now show that the decoupling limit (8) on the M-theory 5-brane reduces to the NCOS limit on the D4-brane. This provides an interpretation of the spatio-temporal non-commutative 5-brane as the strong-coupling dual of the NCOS on the D4-brane.

In order to show this we wrap the 5-brane, for finite ϵ, on a circle of fixed radius R in the direction x^2 and identify $x^2 = X^{11} \sim X^{11} + R$ and $\mathcal{F}_{AB} = R\mathcal{H}_{AB2}$ for $A, B = 0, 1, 3, 4, 5$. Clearly, this means that only $\mathcal{F}_{01}$ is non-zero on the D4-brane. We also use the following standard relations between M-theory and IIA string theory parameters:

$$g_s = \left(\frac{R}{\ell_p}\right)^{3/2}, \qquad \alpha' = \frac{\ell_p^3}{R}. \tag{12}$$

The scaling of the metric components in $D = 11$ induces the same scaling for the ten-dimensional metric components and together with the requirement of fixed radius R we find the following limit on the D4-brane (we reset our conventions such that $\alpha, \beta = 0, 1$):

$$\begin{aligned} &g_{\alpha\beta} \sim \epsilon^0, \qquad g_{ab} \sim \epsilon^1, \qquad \mathcal{F}_{01} \sim \epsilon^{-1} + \epsilon^0, \\ &\alpha' \sim \epsilon^1, \qquad g_s \sim \epsilon^{-1/2}, \qquad \epsilon \to 0. \end{aligned} \tag{13}$$

As a result we find that length scales on the D4-brane, as measured by the open-string metric G^{AB}, are kept fixed in the limit. This also holds for the non-commutativity parameters Θ^{AB} appearing in the two-point function (1) and the open-string coupling G_{O} given by (2).

$$G_{\mathrm{O}} = \left(\frac{R}{\ell_g}\right)^{3/2}. \tag{14}$$

Therefore, the NCOS limit on the D4-brane has an effective open-string scale α'_{eff} and non-commutativity parameter θ given by

$$\alpha'_{\mathrm{eff}} \equiv \theta = \frac{\ell_g^3}{R}, \qquad G_{\mathrm{OM}}^{AB} = G_{\mathrm{OS}}^{AB}. \tag{15}$$

Thus we find the following relations between open-string moduli and M-theory open-membrane moduli:

$$R = G_O\sqrt{\alpha'_{\mathrm{eff}}} \tag{16}$$

$$l_g = G_O^{1/3}\sqrt{\alpha'_{\mathrm{eff}}}. \tag{17}$$

These are formally equivalent to the standard relations between the moduli of M-theory and IIA superstring theory provided that we give ℓ_g a six-dimensional interpretation analogous to that of the 11-dimensional Planck scale ℓ_p in M-theory. This suggests that the NCOS theory on the D4-brane generates an extra dimension when we increase the open-string coupling and in the limit $R \to \infty$ we end up with a non-commutative (in all directions!) six-dimensional theory governed by the scale ℓ_g. Note that $\alpha'_{\text{eff}} = \theta$ implies that a field theory limit taking $\alpha'_{\text{eff}} \to 0$ will at the same time also result in vanishing spatio-temporal non-commutativity.

Next we carry out the double-dimensional reduction of the M-theory 5-brane on a fixed two-torus in order to compare with the limits given in [2] for the D3-brane directly reduced on a circle. We drop all Kaluza–Klein modes and identify the wrapped 5-brane with the directly reduced D3-brane in nine dimensions. This gives the relations between M-theory 5-brane and IIB 3-brane quantities [7–9].

The natural fixed moduli for the non-commutative M-5-brane are l_g, the complex structure of the torus, τ_{OM} and the area of the torus, A_{OM} as measured by the open-membrane metric. We then recover the following relations between M-theory and the S-dual descriptions of IIB for the non-commutative open-string/membrane moduli. For the non-commutative open string,

$$
\begin{aligned}
G_O &= \tau_{OM} \\
r_B &= A_{OM}^{-3/4} \tau_{OM}^{1/4} l_g^{3/2}.
\end{aligned}
\tag{18}
$$

For the S-dual, non-commutative field theory,

$$
\begin{aligned}
g_{YM}^2 &= \frac{1}{\tau_{OM}} \\
m &= A_{OM}^{-3/4} \tau_{OM}^{-1/4} l_g^{1/2}.
\end{aligned}
\tag{19}
$$

The duality transformation is now obtained by a modular transformation on the torus as seen by the open-membrane metric so that the two theories are related by

$$
\tau_{OM} \to \frac{1}{\tau_{OM}}
\tag{20}
$$

with the appropriate interpretation of duality-related quantities.

We have argued for the existence of a decoupled non-commutative theory on the 5-brane defined by the limit (8) by showing its relation to various well defined limits of IIA and IIB string theory. The NCOS on the D4-brane with non-commutativity parameter $\theta = \alpha'_{\text{eff}}$ has a dual description in the limit of strong coupling $G_O \gg 1$ as a non-commutative 5-brane with fundamental length $\ell_g = G_O^{1/3} \sqrt{\alpha'_{\text{eff}}}$ reduced on a circle of radius $R = G_O \sqrt{\alpha'_{\text{eff}}}$. The S-duality of the NCOS and NCYM theories on a directly reduced D3-brane follows from the modular invariance of the non-commutative 5-brane wrapped on a torus. The couplings on the D3-brane are identified with the complex structure of the torus in the open-membrane metric.

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