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Supersymmetric Yang-Mills theory at order α'^3

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Abstract: We construct the order α'^3 terms in the supersymmetric Yang-Mills action in ten dimensions for an arbitrary gauge group. The result can be expressed in terms of the structure constants of the Yang-Mills group, and is therefore independent of abelian factors. The α'^3 invariant obtained here is independent of the α'^2 invariant, and we argue that additional superinvariants will occur at all odd orders of α'.

Keywords: Superstrings and Heterotic Strings, D-branes, Supersymmetric Effective Theories.

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1. Introduction

The abelian Born-Infeld action provides us with an effective theory, which reproduces to all orders in α' the tree level scattering amplitudes of massless modes of open strings that end on a single D-brane, with the assumption that the fields vary slowly [1]. As was recently shown in [2], this assumption implies that gravitational effects are large. Small derivatives imply that the fields stay large over a vast region, and an estimate of the total energy and the corresponding volume indicates that under gravitational forces such a system would collapse to a black hole. To avoid this, fields have to fall off over a short distance, making derivatives large. Physically it is hard to make sense of the Born-Infeld action in string theory, where gravitational forces are implied by the presence of closed strings.

When n D-branes coincide, the gauge group is enhanced to $\mathrm{U}(n)$ [3], making the task of writing an effective action much more complicated. Now there is an additional, practical, argument for including derivative terms. A constant field strength is not a gauge-invariant concept, and one has to take into account that $[\mathcal{D},\mathcal{D}]F = [F,F]$. So, if we were to neglect derivatives of fields, we would also have to neglect commutators of field strengths, which amounts to going back to the abelian situation.

In the abelian case the complete supersymmetric action for slowly varying fields is known [4]. According to the argument of [2], this does not mean that derivative terms are necessarily small. Contributions to the abelian action involving derivatives have been obtained in [5, 6] from string amplitude calculations. Under supersymmetry these terms should form an invariant, which is independent of the Born-Infeld action. In the nonabelian case these invariants are no longer independent because of $[\mathcal{D},\mathcal{D}]F = [F,F]$. A straightforward approach to the Yang-Mills case is then to compute tree-level string scattering amplitudes and calculate the corresponding effective action. This method has been applied to the string four-point function (see [7] and references therein) and has yielded complete results for orders α'^2 [8], and partial results for order α'^3 and α'^4 [2]. For instance, at order α'^3 terms of the form $(\mathcal{D}F)^2F^2$ and $F\mathcal{D}F\chi\gamma\mathcal{D}\mathcal{D}\chi$ (plus terms that are quartic in χ, which we will not deal with in this paper) have been computed in this way. This leaves the F^5 and $F^3\chi\gamma\mathcal{D}\chi$ terms to be determined.

Another approach consists in calculating the deformations allowed by supersymmetry of the $d = 10$ super Yang-Mills theory. In [9, 10] this idea is put to the test up to order α'^2. One finds that α' terms can be eliminated via field redefinitions, and the α'^2 terms match string theory predictions. In the calculation of [9] a significant simplification is reached by the assumption that only symmetric traces of the Yang-Mills generators appear. A superspace calculation by [10] yields a result to all orders in the fermions, where all Yang-Mills indices enter symmetrically. At α'^3 a symmetric single trace is not possible, since the symmetric trace of F^5 vanishes. However, the string theoretical calculations performed in [11] show that terms of the form F^5 and $(\mathcal{D}F)^2F^2$ are needed.

Recently, two calculations of the bosonic α'^3 terms have been performed. In [12] the one-loop five-point amplitude is calculated in $N = 4$ super-Yang-Mills theory in four dimensions. This leads to an effective α'^3 action that reproduces this amplitude, and, assuming that supersymmetry uniquely determines such an action, this $N = 4$, $d = 4$ result should then correspond (although not uniquely) to the ten-dimensional effective action. In [13] deformations of the $d = 10$ Yang-Mills theory that preserve a BPS solution to the equations of motion are studied. This method also yields an effective action at α'^3, now directly in $d = 10$. Although [12] and [13] find the same $(\mathcal{D}F)^2F^2$ terms, they disagree on the F^5 contributions.

We obtain in this paper the α'^3 terms in the effective action, including the terms bilinear in the fermions, by imposing supersymmetry to order α'^3. The result agrees with [2, 11, 12, 13] for the bosonic terms with derivatives, and with [13] for the bosonic F^5 terms. The group structure of the action and transformation rules that we obtain can be expressed completely in terms of the structure constants. This implies that the result vanishes in the abelian case, and also that it is trivially invariant under nonlinear supersymmetry transformations, which act only on a U(1) factor in the gauge group.

This paper is organized as follows. In section 2 we explain our calculational method, showing, as an example, that no effective action at order α' is needed. The result at order α'^3 is presented in section 3. In section 4 we construct the α'-expansion of the string four-point function, and discuss consequences of this expansion for the effective action at higher orders in α'. In particular, we will argue that al all odd orders in α' a new, independent, superinvariant begins. We present our conclusions in section 5.

2. Constructing the order α'^3 action

First we review the $d = 10$, $N = 1$ supersymmetric Yang-Mills theory in order to set the stage for our calculations. The lagrangian is given by:[1]

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{g^2}\,\mathrm{Tr}\left\{-\frac{1}{4}F_{ab}F_{ab} + \frac{1}{2}\bar{\chi}\not{\mathcal{D}}\chi\right\}. \tag{2.1}$$

g is the Yang-Mills coupling constant; it has mass dimension -3. The gauge field A_a and the derivatives $\mathcal{D}$ and ∂ have dimension $+1$, the gaugino χ dimension $+3/2$. From now on we will drop the factor of $1/g^2$ for notational clarity, the dimension of the remaining lagrangian then equals $+4$.

Variation of this action gives $\delta\mathcal{L}_{\mathrm{YM}} = -\,\mathrm{Tr}\,\{(\mathcal{D}_a F_{ab} - \bar{\chi}\gamma_b\chi)\delta A_b + \delta\bar{\chi}\not{\mathcal{D}}\chi\}$, from which one obtains the equations of motion:

$$0 = \mathcal{D}_a F^A_{ab} - \frac{1}{2} f^{ABC}\bar{\chi}^B\gamma_b\chi^C\,, \tag{2.2}$$

$$0 = \not{\mathcal{D}}\chi^A\,. \tag{2.3}$$

$\mathcal{L}_{\mathrm{YM}}$ is invariant under the following supersymmetry transformations:

$$\delta_\epsilon A_a = \bar{\epsilon}\gamma_a\chi\,, \tag{2.4}$$

$$\delta_\epsilon\chi = \frac{1}{2}F_{ab}\gamma_{ab}\,\epsilon\,, \tag{2.5}$$

where ϵ is a constant Majorana-Weyl spinor of dimension $+1/2$. As is well known, the supersymmetry algebra only closes on-shell and involves a field-dependent gauge transformation:

$$\begin{aligned}
[\delta_{\epsilon_1},\delta_{\epsilon_2}]A_a &= 2\bar{\epsilon}_1\not{\partial}\epsilon_2 A_a - \mathcal{D}_a\left(2\bar{\epsilon}_1\not{A}\epsilon_2\right),\\
[\delta_{\epsilon_1},\delta_{\epsilon_2}]\chi &= 2\bar{\epsilon}_1\not{\mathcal{D}}\epsilon_2\chi - \left(\frac{7}{8}\bar{\epsilon}_1\gamma_a\epsilon_2\gamma_a - \frac{1}{5!16}\bar{\epsilon}_1\gamma_{a_1\cdots a_5}\epsilon_2\gamma_{a_1\cdots a_5}\right)\not{\mathcal{D}}\chi\,.
\end{aligned} \tag{2.6}$$

Before moving on to the actual α'^3 corrections to (2.1) we will first discuss our method.

Consider a general lagrangian $\mathcal{L}_0[\phi]$ that possesses a symmetry, with infinitesimal transformations $\delta_0\phi$. If $\mathcal{L} = \mathcal{L}_0 + \lambda\mathcal{L}_1$, where λ is some expansion parameter, then the variation of $\mathcal{L}_1$ due to $\delta_0\phi$ generically yields terms that, to preserve the symmetry, should be cancelled by an λ variation of ϕ in $\mathcal{L}_0$. Cancellation occurs if and only if the variation of $\mathcal{L}_1$ is proportional to the order λ^0 equations of motion. The lagrangian $\mathcal{L}$ one obtains in this way is uniquely defined up to total derivatives and field redefinitions. A field redefinition $\phi \to \phi + \lambda\delta\phi$ gives rise to order λ terms of the form

$$\lambda\delta\phi\,\frac{\delta\mathcal{L}_0}{\delta\phi_i}\,, \tag{2.7}$$

i.e., is proportional to the order λ^0 equations of motion. Therefore, any term in $\mathcal{L}_1$ of the form (2.7) can be eliminated by a field redefinition. We will choose our α'^3 action such that no explicit terms of the form (2.7) appear.

[1] Our conventions for the γ-matrices follow [14]. For the gauge fields, they are presented in Appendix A. We will always write spacetime indices as lower indices.

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Let us illustrate how this works by considering order α'^1. From this point on we discard any terms of higher than quadratic order in the fermions. Since α' has mass dimension -2 we can write down terms that have dimension $+6$. These terms must be Lorentz and gauge invariant. We also take only terms with a single trace over the generators T^A since we want to make contact with a string theory tree level effective action. Possible terms are:

$$\begin{array}{lll}
(1) & \mathrm{Tr}\ T^A T^B T^C & F_{ab}{}^A F_{bc}{}^B F_{ca}{}^C\,, \\
(2) & \mathrm{Tr}\ T^A T^B T^C & \mathcal{D}_a F_{ab}{}^A\,\bar{\chi}^B \gamma_b \chi^C\,, \\
(3) & \mathrm{Tr}\ T^A T^B T^C & \mathcal{D}_a F_{bc}{}^A\,\bar{\chi}^B \gamma_{abc} \chi^C\,, \\
(4) & \mathrm{Tr}\ T^A [T^B, T^C] & F_{ab}{}^A\,\bar{\chi}^B \gamma_a \mathcal{D}_b \chi^C\,, \\
(5) & \mathrm{Tr}\ T^A \{T^B, T^C\} & F_{ab}{}^A\,\bar{\chi}^B \gamma_a \mathcal{D}_b \chi^C\,, \\
(6) & \mathrm{Tr}\ T^A [T^B, T^C] & F_{ab}{}^A\,\bar{\chi}^B \gamma_{ab} \not{\mathcal{D}} \chi^C\,, \\
(7) & \mathrm{Tr}\ T^A \{T^B, T^C\} & F_{ab}{}^A\,\bar{\chi}^B \gamma_{ab} \not{\mathcal{D}} \chi^C\,.
\end{array}$$

In choosing these terms we put no restriction on the group structure other than the cyclic property of the trace. We do not need to take terms with more than one derivative: it is not difficult to convince oneself that such terms always contain $[\mathcal{D}, \mathcal{D}]$ and/or lowest order equations of motion. We see that (3) vanishes due to the Bianchi identity. Furthermore, (2), (6) and (7) are proportional to the order α'^0 field equations, so we do not allow them in the lagrangian. Since $\mathcal{L}$ is only defined up to total derivatives we also consider all of these:

$$\begin{aligned}
\mathrm{Tr}\ T^A T^B T^C \partial_a \left(F_{ab}{}^A\,\bar{\chi}^B \gamma_b \chi^C\right) &= (2) - (4), \\
\mathrm{Tr}\ T^A T^B T^C \partial_a \left(F_{bc}{}^A\,\bar{\chi}^B \gamma_{abc} \chi^C\right) &= (3) + (7) - 2 \times (5)\,.
\end{aligned}$$

So we see that we also need not include (4) and (5) since they can be rewritten as a total derivative and terms that can be cancelled by a field redefinition. This analysis leaves only the term (1).

We now show that the remaining term $f^{ABC} F_{ab}{}^A F_{bc}{}^B F_{ca}{}^C$ is not allowed by supersymmetry. Varying this term with the transformation rule (2.4) gives:

$$6 f^{ABC} F_{ac}{}^A F_{cb}{}^B\,\bar{\epsilon} \gamma_a \mathcal{D}_b \chi^C\,.$$

We adopt the rule that any derivative on χ in a variation is partially integrated to act on the bosonic fields—except in the situation where this derivative takes on the form of the order α'^0 equation of motion $\not{\mathcal{D}}\chi^A$. This rule leads to

$$3 f^{ABC} \mathcal{D}_a F_{bc}{}^A F_{bc}{}^B \bar{\epsilon} \gamma_a \chi^C + 6 f^{ABC} \mathcal{D}_a F_{ab}{}^A F_{bc}{}^B \bar{\epsilon} \gamma_c \chi^C\,. \tag{2.8}$$

The second term in (2.8) contains the A_a equation of motion, and can therefore be cancelled by an order α' transformation, while the first term cannot. Therefore term (1) does not allow supersymmetrization; the only terms allowed by supersymmetry at order α' can be eliminated by a field redefinition.

In the present case we can see by inspection that the first term in (2.8) cannot be rewritten as a total derivative plus terms containing equations of motion. In a more complicated situation one would parametrize all possible total derivatives, which lead to the same structures as those in (2.8) to verify this fact.

So our method comes down to the following: first we write down an action involving all possible terms that are independent up to partial integrations. To this we add all possible total derivatives, and use these to reduce the starting point to a minimal number of terms. This results in the Ansatz for the effective action, in which each term gets an arbitrary coefficient to be determined later on. We then vary the Ansatz with the lowest order variations of A and χ.

To this variation we add all possible total derivatives, which lead to contributions having the same structure as the variations. These also have arbitrary coefficients. All terms proportional to lowest order equations of motion of A and χ are saved for later use in determining the new transformation rules. After eliminating all remaining derivatives on χ by partial integrations, the rest has to vanish, and this gives rise to linear equations between the unknown coefficients. Note that the fact that all variations are ultimately written without derivatives on χ implies that the total derivatives that we add to the variation must give rise to a lowest order fermion equation of motion—otherwise the partial integration away from χ just reproduces the original total derivative, and the term does not influence the calculation. An important part of the calculation is to rewrite the remaining terms such that the minimal number of independent structures is left. This is done by using Bianchi identities for $\mathcal{D}F$, $\mathcal{D}\mathcal{D}F$, etc., and by ordering the field strengths. Each independent structure gives rise to an equation between the coefficients. If these equations have non-trivial solutions then these correspond to supersymmetric actions.

In the case of the α'^3 modification to the Yang-Mills action the number of terms at intermediate stages of the calculation reaches 10^4. Therefore, the required algebraic manipulations, such as obtaining the variation of the Ansatz, working out products of γ-matrices, partial integrations, the use of Bianchi identities, are all done by computer.

3. SYM at order α'^3

We saw that at order α' there are no non-trivial modifications to the supersymmetric action (2.1). At order α'^2 there are non-trivial corrections to the super Yang-Mills lagrangian and supersymmetry transformation rules [9, 10]. However, in the iterative procedure these terms cannot contribute to the order α'^3 variations, precisely because there are no order α' terms in the transformation rules. This means that at α'^3 the analysis follows the outline given in the previous section.

However, there is one complication. At order α'^3 we have to go through a two-step procedure, since in the Ansatz we have not only terms with five fields, i.e., F^5 and the corresponding terms involving fermions, but also terms with four fields, such as $(\mathcal{D}F)^2F^2$ with fermionic partners. In this case the analysis, both in determining the Ansatz and in cancelling the variation, has to start at the higher-derivative terms. The reason is that the higher-derivative terms produce terms with less derivatives because of $[\mathcal{D},\mathcal{D}]F = [F,F]$ and $[\mathcal{D},\mathcal{D}]\chi = [F,\chi]$. It is easily seen that all terms with four derivatives and two F's, and their fermionic partners, can be eliminated by field redefinitions.

The leading terms in this analysis are therefore the higher-derivative terms $(\mathcal{D}F)^2F^2$ and partners. As we mentioned before, the Ansatz is not unique. We found that the bosonic part of the Ansatz must contain 13 terms (in agreement with [13]), and one may choose for instance to have only $(\mathcal{D}F)^2F^2$ terms and no F^5 terms [15]. However, for the terms involving fermions the partners of F^5 cannot all be eliminated. We have chosen for the bosonic part of our Ansatz the 13 terms in the starting point of [13]. Our Ansatz then contains 13 bosonic terms, and 110 terms involving fermions: $7+18$ terms of the form $(\mathcal{D}F)^2F^2$ and fermionic partners, and $6+92$ of type F^5 with partners.

After simplifying the resulting variations there remain 128 linear equations from the sector with four fields, and 320 equations from the sector with five fields. These equations must be solved for the 123 coefficients from the Ansatz and the 182 coefficients that parametrize total derivatives having the same structure as the variations (see section 2).

The result is that there is *one* unique deformation of $d=10$, $N=1$ supersymmetric Yang-Mills theory at order α'^3, up to a single multiplicative constant, which according to string theory equals $\zeta(3)/2$. In one particular parametrization, the result is:

$$
\begin{aligned}
\mathcal{L}_3 &= \\
&= f^{XYZ}f^{VWZ}\Big[2\,F_{ab}{}^X F_{cd}{}^W \mathcal{D}_e F_{bc}{}^V \mathcal{D}_e F_{ad}{}^Y - 2\,F_{ab}{}^X F_{ac}{}^W \mathcal{D}_d F_{be}{}^V \mathcal{D}_d F_{ce}{}^Y + \\
&\qquad + F_{ab}{}^X F_{cd}{}^W \mathcal{D}_e F_{ab}{}^V \mathcal{D}_e F_{cd}{}^Y - 4\,F_{ab}{}^W \mathcal{D}_c F_{bd}{}^Y \bar\chi^X \gamma_a\,\mathcal{D}_d \mathcal{D}_c \chi^V - \\
&\qquad - 4\,F_{ab}{}^W \mathcal{D}_c F_{bd}{}^Y \bar\chi^X \gamma_d\,\mathcal{D}_a \mathcal{D}_c \chi^V + 2\,F_{ab}{}^W \mathcal{D}_c F_{de}{}^Y \bar\chi^X \gamma_{ade}\,\mathcal{D}_b \mathcal{D}_c \chi^V + \\
&\qquad + 2\,F_{ab}{}^W \mathcal{D}_c F_{de}{}^Y \bar\chi^x \gamma_{abd}\,\mathcal{D}_e \mathcal{D}_c \chi^V\Big] + \\
&\quad + f^{XYZ}f^{UVW}f^{TUX}\Big[4F_{ab}{}^Y F_{cd}{}^Z F_{ac}{}^V F_{be}{}^W F_{de}{}^T + 2F_{ab}{}^Y F_{cd}{}^Z F_{ab}{}^V F_{ce}{}^W F_{de}{}^T - \\
&\qquad - 11F_{ab}{}^Y F_{cd}{}^Z F_{cd}{}^V \bar\chi^T \gamma_a \mathcal{D}_b \chi^W + 22F_{ab}{}^Y F_{cd}{}^Z F_{ac}{}^V \bar\chi^T \gamma_b \mathcal{D}_d \chi^W + \\
&\qquad + 18F_{ab}{}^Y F_{cd}{}^V F_{ac}{}^W \bar\chi^T \gamma_b \mathcal{D}_d \chi^Z + 12F_{ab}{}^T F_{cd}{}^Y F_{ac}{}^V \bar\chi^Z \gamma_b \mathcal{D}_d \chi^W + \\
&\qquad + 28F_{ab}{}^T F_{cd}{}^Y F_{ac}{}^V \bar\chi^W \gamma_b \mathcal{D}_d \chi^Z - 24F_{ab}{}^Y F_{cd}{}^V F_{ac}{}^T \bar\chi^W \gamma_b \mathcal{D}_d \chi^Z + \\
&\qquad + 8F_{ab}{}^T F_{cd}{}^Y F_{ac}{}^Z \bar\chi^V \gamma_b \mathcal{D}_d \chi^W - 12F_{ab}{}^T F_{ac}{}^Y \mathcal{D}_b F_{cd}{}^V \bar\chi^Z \gamma_d \bar\chi^W - \\
&\qquad - 8F_{ab}{}^Y F_{ac}{}^T \mathcal{D}_b F_{cd}{}^V \bar\chi^Z \gamma_d \bar\chi^W + 22\,F_{ab}{}^V F_{ac}{}^Y \mathcal{D}_b F_{cd}{}^T \bar\chi^Z \gamma_d \bar\chi^W - \\
&\qquad - 4F_{ab}{}^Y F_{cd}{}^T \mathcal{D}_e F_{ac}{}^V \bar\chi^Z \gamma_{bde} \bar\chi^W + 4F_{ab}{}^Y F_{ac}{}^T \mathcal{D}_c F_{de}{}^V \bar\chi^Z \gamma_{bde} \bar\chi^W + \\
&\qquad + 4F_{ab}{}^T F_{cd}{}^Y F_{ce}{}^V \bar\chi^Z \gamma_{abd} \mathcal{D}_e \chi^W - 8F_{ab}{}^Y F_{cd}{}^T F_{ce}{}^V \bar\chi^Z \gamma_{abd} \mathcal{D}_e \chi^W + \\
&\qquad + 6F_{ab}{}^V F_{cd}{}^Y F_{ce}{}^W \bar\chi^Z \gamma_{abd} \mathcal{D}_e \chi^T + 5\,F_{ab}{}^V F_{cd}{}^W F_{ce}{}^Y \bar\chi^Z \gamma_{abd} \mathcal{D}_e \chi^T + \\
&\qquad + 6F_{ab}{}^Y F_{ac}{}^T F_{de}{}^V \bar\chi^Z \gamma_{bcd} \mathcal{D}_e \chi^W - 2F_{ab}{}^Y F_{ac}{}^T F_{de}{}^Z \bar\chi^V \gamma_{bcd} \mathcal{D}_e \chi^W + \\
&\qquad + 4F_{ab}{}^Y F_{ac}{}^V F_{de}{}^Z \bar\chi^W \gamma_{bcd} \mathcal{D}_e \chi^T + 4F_{ab}{}^T F_{cd}{}^V F_{ce}{}^Y \bar\chi^Z \gamma_{abd} \mathcal{D}_e \chi^W - \\
&\qquad - 4F_{ab}{}^Y F_{cd}{}^V F_{ce}{}^W \bar\chi^Z \gamma_{abd} \mathcal{D}_e \chi^T + \frac{1}{2} F_{ab}{}^Y F_{cd}{}^T F_{ef}{}^V \bar\chi^Z \gamma_{abcde} \mathcal{D}_f \chi^W + \\
&\qquad + \frac{1}{2}\,F_{ab}{}^Y F_{cd}{}^T F_{ef}{}^Z \bar\chi^V \gamma_{abcde}\,\mathcal{D}_f \chi^W\Big]\,.
\end{aligned}
\tag{3.1}
$$

All authors [2, 11, 12, 13] agree on the the bosonic terms $(\mathcal{D}F)^2F^2$. Our bosonic terms F^5 agree with [13], but are given here in a different parametrization. The higher derivative terms with fermions agree with [2].

Note that the group structure is completely specified in terms of structure constants. This was not assumed at the start of our calculation. In fact, the Ansatz was given in terms of traces of four and five generators, for which only the cyclic property was used. In [2] it is shown that all terms with four fields can be written in terms of structure constants. We now find that all terms with five fields allow such a formulation as well.

The implication of this is that if the group contains a U(1) factor, the corresponding U(1) fields, which are certainly present at order α'^0 and α'^2, do not occur in the α'^3 action. It also implies that the action (3.1) is trivially invariant under the nonlinear supersymmetry present at order α'^0 and α'^2. The nonlinear transformation acts at order α'^0 only on χ (at order α'^2 there are modifications [10]) as

$$\delta\chi^A = \eta^A \,, \tag{3.2}$$

where η is a constant spinor, satisfying $f^{ABC}\eta^C = 0$. This implies that η commutes with all group generators, and must therefore be in a U(1) factor. The invariance of (3.1) under (3.2) is then obvious.

The required α'^3 modifications to the transformation rules for the Yang-Mills vector and fermions are presented in Appendix B. We only show supersymmetry transformations that may modify the supersymmetry algebra with additional field dependent gauge transformations. That leaves many supersymmetry transformations that are proportional to the lowest order equations of motion. Those will modify the on-shell terms in the algebra, but play no role in the closure. Since we do not consider quartic fermions in the action we cannot say anything about terms bilinear in χ in the transformation rules, nor about closure of the algebra on χ. On A we have checked that the algebra closes, and obtain the following new gauge transformations in addition to those of order α'^0 (2.6) and α'^2 [10]:

$$\begin{aligned}
[\delta_{\epsilon_1},\delta_{\epsilon_2}]A_a{}^Z = {} & 2\bar\epsilon_1\not{\partial}\epsilon_2 A_a{}^Z - \mathcal{D}_a\left(2\bar\epsilon_1\gamma_b\epsilon_2 A_b{}^Z\right) + \\
& + f^{XYZ} f^{VWX}\mathcal{D}_a\big(-16\mathcal{D}_bF_{cd}{}^V F_{be}{}^Y F_{cd}{}^W\,\bar\epsilon_1\gamma_e\epsilon_2 + \\
& \qquad + S8\mathcal{D}_bF_{cd}{}^V F_{be}{}^W F_{cd}{}^Y\,\bar\epsilon_1\gamma_e\epsilon_2 - \\
& \qquad - 16\mathcal{D}_bF_{cd}{}^V F_{be}{}^W F_{ce}{}^Y\,\bar\epsilon_1\gamma_d\epsilon_2 - \\
& \qquad - 2\mathcal{D}_bF_{cd}{}^V F_{ef}{}^Y F_{bg}{}^W\,\bar\epsilon_1\gamma_{cdefg}\epsilon_2\big)\,.
\end{aligned} \tag{3.3}$$

4. String theory and higher orders in α'

In [2] the relation between the tree-level open string four-point function and the effective action was explored to order α'^4. In this section we will discuss the relation between this four-point function and supersymmetric invariants in the effective action, also at higher orders in α'. The string theory four-point function takes on the following form:

$$A_4 = -8ig^2\,K(1,2,3,4)\left(T_1^{ABCD}G(s,u) + T_2^{ABCD}G(s,t) + T_3^{ABCD}G(t,u)\right)\,, \tag{4.1}$$

where

$$\begin{aligned}
T_1^{ABCD} &= \mathrm{Tr}\, T^A T^B T^C T^D + \mathrm{Tr}\, T^D T^C T^B T^A\,,\\
T_2^{ABCD} &= \mathrm{Tr}\, T^A T^B T^D T^C + \mathrm{Tr}\, T^C T^D T^B T^A\,,\\
T_3^{ABCD} &= \mathrm{Tr}\, T^A T^C T^B T^D + \mathrm{Tr}\, T^D T^B T^C T^A\,,
\end{aligned} \tag{4.2}$$

g is the Yang-Mills coupling constant and s, t and u are the standard Mandelstam variables satisfying $s+t+u=0$. K contains the polarization and wave-functions of the external lines, where the different permutations have to be taken into account. The last factor in A_4 can always be written as a sum of terms that are proportional to $T_1+T_2+T_3$ (which is the symmetric trace) and $T_i - T_j$ (which can be written in terms of structure constants only):

$$\begin{aligned}
&\frac{1}{3}(T_1-T_2)(G(s,u)+G(t,u)-2G(s,t)) +\\
&+\frac{1}{3}(T_1-T_3)(G(s,u)+G(s,t)-2G(t,u)) +\\
&+\frac{1}{3}(T_1+T_3+T_3)(G(s,u)+G(t,u)+G(s,t))\,.
\end{aligned} \tag{4.3}$$

The Veneziano amplitude G contains the α' dependence:

$$G(s,t) = \frac{1}{st}\frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)}{\Gamma(1-\alpha'(s+t))}\,, \tag{4.4}$$

and can be expanded in orders of α'.

A_4 has to be reproduced by the effective action. At order α'^0 the standard Yang-Mills action gives, from the $({A_a}^A)^4$ vertex and from a reducible diagram involving three-point vertices, the correct four-point function. At higher orders in α' it is always the irreducible four-point vertex $\alpha'^p \mathcal{D}^{2p-4} F^4$, where the derivatives have to be distributed in agreement with the kinematic factors in A_4, which yields the string four-point function. Therefore we can read off from the string four-point function what the coefficients of the terms in the effective action will be.

Using the Taylor expansion for $\log\Gamma(1+z)$,

$$\log\Gamma(1+z) = -\gamma z + \sum_{m=2}^{\infty}(-1)^m \zeta(m)\frac{z^m}{m}\,, \tag{4.5}$$

we obtain the following expression for G:

$$G(s,t) = \frac{1}{st}\exp\left\{\sum_{m=2}^{\infty}\alpha'^m \frac{\zeta(m)}{m}(s^m + t^m - (s+t)^m)\right\}. \tag{4.6}$$

$\zeta(n)$ is the Riemann zeta-function, γ the Euler-Mascheroni constant. The expansion of the exponential gives the required result in orders of α', of which the first few terms read:

$$\begin{aligned}G(s,t) =& +\alpha'^0\,\frac{1}{st} - \alpha'^2\,\frac{1}{6}\,\pi^2 - \alpha'^3\,(s+t)\zeta(3) - \alpha'^4\,\frac{1}{360}\,\pi^4\left(4s^2+st+4t^2\right) + \\ &+\alpha'^5\left(\frac{1}{6}\,\pi^2 st(s+t)\zeta(3) - (s+t)(s^2+st+t^2)\zeta(5)\right) - \\ &-\alpha'^6\left(\frac{1}{15120}\,\pi^6(16s^4+12s^3t+23s^2t^2+12st^3+16t^4) - \frac{1}{2}\,st(s+t)^2\zeta(3)^2\right) + \\ &+\alpha'^7\left(\frac{1}{360}\,\pi^4 st(s+t)(4s^2+st+4t^2)\zeta(3) + \frac{1}{6}\,\pi^2 st(s^2+st+t^2)\zeta(5)\right. \\ &\qquad\left. - \ (s^2+st+t^2)^2\zeta(7)\right) + \cdots\,. \end{aligned} \tag{4.7}$$

In this way we understand that the series at even p involving only powers of π and no ζ-functions corresponds to the supersymmetric invariant that starts at order α'^2. Similarly, the series of terms with $\zeta(3)^k$ at order $p = 3k,\ k = 1, 2, \ldots$ is the invariant that starts at order $p = 3$. We see now that necessarily a new invariant starts at every odd power of α'. For instance, the term with $\zeta(5)$ at order $p = 5$ can only be part of the $p = 3$ invariant if there were a relation with rational coefficients between $\pi^2\zeta(3)$ and $\zeta(5)$. To our knowledge, no such relation between the $\zeta(2n + 1)$ for different n exist, and new invariants therefore appear at all odd orders of α'.

The leading term with $\alpha'^n\zeta(n)$ is proportional to $(s^n + t^n - (s + t)^n)/st$. For n odd this is of the form

$$(s+t)P(s,t), \quad \text{with } P(s,t) = -\frac{s^n+t^n+u^n}{stu}\,. \tag{4.8}$$

Now in (4.3) the symmetric trace is proportional to $G(s, t) + G(s, u) + G(t, u)$, which for the leading term with $\alpha'^n\zeta(n)$, n odd, gives a factor:

$$(s+t)P(s,t) + (s+u)P(s,u) + (t+u)P(t,u) = 2(s+t+u)P(s,t) = 0\,. \tag{4.9}$$

Therefore, all new invariants starting at α'^n for n odd can be expressed in terms of structure constants only, and thus vanish in the abelian limit.

The conclusion must be that supersymmetry by itself cannot be sufficient to determine the open string effective action. The effective action is a sum of an infinite number of superinvariants, of which the relative coefficients can be determined from string theory, but not from supersymmetry alone. Our argument does not exclude the possiblity that additional invariants, which do not contribute to the four-point function, appear in the effective action.

In the abelian case A_4 simplifies to

$$A_4 = -8ig^2\,K(1,2,3,4)\,(G(s,u) + G(s,t) + G(t,u))\,. \tag{4.10}$$

The expansion in α' now reads

$$G(s,u) + G(s,t) + G(t,u) = -\,\alpha'^2\frac{1}{2}\,\pi^2 - \alpha'^4\frac{1}{24}\,\pi^4(s^2+st+t^2) + \alpha'^5\,\frac{1}{2}\pi^2 st(s+t)\zeta(3) -$$

$$
\begin{aligned}
&-\alpha'^6\frac{1}{240}\pi^6(s^2+st+t^2)^2+\\
&+\alpha'^7\frac{1}{48}\pi^2 st(s^3+2s^2t+2st^2+t^3)(2\pi^2\zeta(3)+24\zeta(5))+\\
&+\cdots\,,
\end{aligned}
\tag{4.11}
$$

where we have used $s+t+u=0$. Of course there is now no order α'^0 term, also the term at order α'^3 vanishes. However, at order α'^4 there is a four-point function, which in the effective action must be represented by a term $\alpha'^4\partial^4F^4$. Such terms can indeed be found in the analysis of [5, 6]. Since the terms at order α'^4 without derivatives on F belong to the Born-Infeld superinvariant, these higher derivatives must be invariant by themselves.

The expansion (4.11) shows terms proportional to $\pi^2\zeta(2k+1)$ at odd orders α'^{2k+3}. These also appear in the expansion of $G(s,t)$ that we presented for the nonabelian case (4.7). There it would be tempting to interpret these terms as an "interference" between the α'^2 invariant and the α'^{2k+1} invariant proportional to $\zeta(2k+1)$. In that case they would be required to cancel the α'^2 variation of the $\zeta(2k+1)$-invariant and the α'^{2k+1} variation of the α'^2 invariant. However, if that interpretation were correct, these terms should vanish in the abelian case, because the α'^{2k+1} invariant does. A closer look at (4.3) shows that in the nonabelian case these terms contain only the symmetric trace contribution, and not the terms $T_i - T_j$, proportional to structure constants. Therefore, they correspond to independent invariants in the nonabelian case, which survive the abelian limit.

5. Discussion

In this paper we have obtained the contribution to the open superstring effective action at order α'^3, with the exception of terms quartic in the fermions. We assume that the nonabelian structure is given by a single trace of group generators, in agreement with what one would expect from tree level string theory. The result is then unique, up to total derivatives and field redefinitions. In the sectors that allow comparison with previous work we agree with [2, 13]. We disagree with the result of [12], which is an effective action in four dimensions. That does not imply that the action of [12] is not supersymmetric - it may well be that more invariants can be found in four than in ten dimensions.

The traces over group generators turn out to give products of structure constants only. It was known that the nonabelian result should vanish in the abelian limit, but that is a much weaker statement than structure constants only. It implies that fields in U(1) factors of the gauge group are absent from this part of the effective action, and that therefore the nonlinear supersymmetry is trivial.

Although our procedure works for an arbitrary gauge group at order α'^3, we do not expect this to hold at higher orders. Continuing the iteration to order α'^4 would give two kinds of contributions. In the first place there are terms that come from the variation of the order α'^4 Ansatz with the α'^0 transformation rules. If we still assume the Ansatz to be proportional to a single trace, these terms are proportional to $\mathrm{Tr}(T^AT^BT^CT^DT^ET^F)$. In the second place there are contributions from the variation of the α'^2 action with the α'^2 transformation rules. Such terms are proportional to a product of two traces,

$\mathrm{Tr}(T^A T^B T^C T^G)\,\mathrm{Tr}(T^D T^E T^F T^G)$. These different terms can only communicate with each other if the generators T satisfy requirements which are analogous to the unitarity conditions on Chan-Paton factors. We therefore expect that at higher orders supersymmetry requires the generators to be in the fundamental representation of $\mathrm{U}(n)$, $\mathrm{SO}(n)$ or $\mathrm{USp}(n)$.

We have argued that the ${\alpha'}^3$ invariant is just the first of an infinite number of invariants appearing at all odd orders ${\alpha'}^{2p+1}$ in the effective action, with a coefficient proportional to $\zeta(2p+1)$. This sheds new light on efforts [16] to obtain the effective action through κ-symmetry, a method, which was extremely successfull in the abelian situation. κ-symmetry with parameters in the adjoint representation of the gauge group turns out not to work [8], κ-symmetry that only transforms fields in the $\mathrm{U}(1)$ direction of the group will not see the ${\alpha'}^3$ effective action we have just obtained. The most likely scenario, if κ-symmetry works at all in the nonabelian context, is that it gives the part of the action generated by the ${\alpha'}^2$ terms, i.e., the terms that are not proportional to ζ-functions.

In [15] the result of [13] was tested by calculating the spectrum of the deformed Yang-Mills theory in a constant magnetic background. By T-duality the constant magnetic field corresponds to D-branes at angles, and in this context string theory allows an alternative calculation of the spectrum [17]. This test uses configurations of Yang-Mills fields in the Cartan subalgebra of the gauge group. It would be interesting to find a true nonabelian generalization of the method of [17], also including fermions.

Terms with derivatives in the field strength F are inevitably present in the nonabelian effective action, and also in the abelian case there is no reason to assume that such terms are small in general. In section 4 we have discussed such terms in the context of the open string four-point function. From the plethora of supersymmetric invariants that are indicated by the four-point function, it is clear that the construction of the complete open string effective action, in both the abelian and the nonabelian cases, requires perhaps additional symmetries beyond supersymmetry, but certainly new insights. One may conclude that the real surprise in this field is still the apparent simplicity of the abelian Born-Infeld action, which disappears completely as soon as one deviates from the context of slowly varying abelian fields.

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A. Conventions

We consider a compact gauge group G and parametrize elements g that are connected to the identity by $g = \exp \Lambda \cdot T$. The generators T^A satisfy the orthonormality condition $\mathrm{Tr}\ T^A T^B = -\delta^{AB}$ and the algebra

$$\left[T^A, T^B\right] = f^{ABC} T^C\,, \tag{A.1}$$

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where the f^{ABC} are completely antisymmetric real structure constants. No further restrictions are imposed on the generators. We freely raise and lower indices on the structure constants. All fields in this paper transform in the adjoint representation, $(T^A_{\rm adj})^{BC} = -f^{ABC}$. For such fields we use the notation $\Phi \equiv \Phi \cdot T = \Phi^A T^A$, where T^A can be any representation. Since under a gauge transformation $\Phi \to g\Phi g^{-1}$, we can form gauge invariant objects by tracing, e.g. ${\rm T}r\ \Phi_1 \cdots \Phi_n$.

The infinitesimal gauge transformations of the nonabelian Yang-Mills multiplet (A_a, χ) are

$$\delta A_a = -\mathcal{D}_a \Lambda\,, \tag{A.2}$$

$$\delta F_{ab} = [\Lambda, F_{ab}]\,, \tag{A.3}$$

$$\delta\chi = [\Lambda, \chi]\,. \tag{A.4}$$

The covariant derivative and the field strength are defined by

$$\mathcal{D}_a \Phi = \partial_a \Phi + [A_a, \Phi]\,, \tag{A.5}$$

$$[\mathcal{D}_a, \mathcal{D}_b]\Phi = [F_{ab}, \Phi]\,, \tag{A.6}$$

so that

$$F_{ab} = \partial_a A_b - \partial_b A_a + [A_a, A_b]\,. \tag{A.7}$$

F satisfies a Bianchi identity: $\mathcal{D}_{[a} F_{bc]} \equiv 0\,.$

B. Transformation rules

We now present the supersymmetry transformation rules that leave the action (3.1) invariant. The transformation rules of the fermions are:

$$\begin{aligned}
\delta_3 \chi^Z = f^{XYZ} f^{VWX} \Big[& -4\,\mathcal{D}_a F_{bc}{}^Y \mathcal{D}_b F_{ad}{}^V F_{cd}{}^W + 2\,\mathcal{D}_a F_{bc}{}^Y \mathcal{D}_a F_{bd}{}^V F_{cd}{}^W + \\
& + 4\,\mathcal{D}_a F_{bc}{}^Y \mathcal{D}_a F_{bd}{}^V F_{ce}{}^W \gamma_{de} - 6\,\mathcal{D}_a F_{bc}{}^V \mathcal{D}_a F_{bd}{}^Y F_{ce}{}^W \gamma_{de} - \\
& - 2\,\mathcal{D}_a F_{bc}{}^Y \mathcal{D}_b F_{ad}{}^V F_{ce}{}^W \gamma_{de} - 2\,\mathcal{D}_a F_{bc}{}^Y \mathcal{D}_b F_{de}{}^V F_{ad}{}^W \gamma_{ce} + \\
& + 2\,\mathcal{D}_a F_{bc}{}^Y \mathcal{D}_d F_{be}{}^V F_{ad}{}^W \gamma_{ce} + 2\,\mathcal{D}_a F_{bc}{}^Y \mathcal{D}_a F_{de}{}^V F_{bd}{}^W \gamma_{ce} - \\
& - 3\,\mathcal{D}_a F_{bc}{}^Y \mathcal{D}_a F_{de}{}^V F_{bc}{}^W \gamma_{de} + \frac{3}{2}\,\mathcal{D}_a F_{bc}{}^V \mathcal{D}_a F_{de}{}^Y F_{bc}{}^W \gamma_{de} + \\
& + \frac{3}{2}\,\mathcal{D}_a F_{bc}{}^Y \mathcal{D}_a F_{bc}{}^V F_{de}{}^W \gamma_{de} - \mathcal{D}_a \mathcal{D}_b F_{cd}{}^Y F_{be}{}^V F_{cd}{}^W \gamma_{ae} - \\
& - 4\,\mathcal{D}_a \mathcal{D}_b F_{cd}{}^V F_{ae}{}^Y F_{be}{}^W \gamma_{cd} + 3\,\mathcal{D}_a \mathcal{D}_b F_{cd}{}^V F_{ae}{}^W F_{be}{}^Y \gamma_{cd} - \\
& - 3\,\mathcal{D}_a F_{bc}{}^Y \mathcal{D}_a F_{de}{}^V F_{df}{}^W \gamma_{bcef} - \mathcal{D}_a F_{bc}{}^V \mathcal{D}_a F_{de}{}^Y F_{df}{}^W \gamma_{bcef} + \\
& + 3\,\mathcal{D}_a F_{bc}{}^Y \mathcal{D}_a F_{bd}{}^V F_{ef}{}^W \gamma_{cdef} - \mathcal{D}_a F_{bc}{}^Y \mathcal{D}_d F_{ef}{}^V F_{ad}{}^W \gamma_{bcef} + \\
& + \frac{1}{4}\,\mathcal{D}_a F_{bc}{}^Y \mathcal{D}_a F_{de}{}^V F_{fg}{}^W \gamma_{bcdefg} \Big] \epsilon + \\
+ f^{WXY} f^{TUV} f^{TWZ} \Big[& 7\,F_{ab}{}^X F_{ac}{}^U F_{de}{}^V F_{de}{}^Y \gamma_{bc} - 2\,F_{ab}{}^X F_{cd}{}^U F_{ae}{}^V F_{ce}{}^Y \gamma_{bd} -
\end{aligned}$$

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$$
\begin{aligned}
&- 6\, F_{ab}{}^X F_{cd}{}^U F_{ae}{}^Y F_{ce}{}^V \gamma_{bd} - 4\, F_{ab}{}^X F_{cd}{}^U F_{ef}{}^Y F_{ce}{}^V \gamma_{abdf} - \\
&- \frac{3}{2}\, F_{ab}{}^X F_{cd}{}^U F_{ef}{}^V F_{eg}{}^Y \gamma_{abcdfg} \Big] \epsilon + \\
+ f^{XYZ} f^{UVW} f^{TUX} \Big[&2\, F_{ab}{}^Y F_{ac}{}^V F_{de}{}^W F_{de}{}^T \gamma_{bc} + 2\, F_{ab}{}^V F_{cd}{}^W F_{ae}{}^Y F_{ce}{}^T \gamma_{bd} - \\
&- 8\, F_{ab}{}^V F_{cd}{}^T F_{ae}{}^Y F_{ce}{}^W \gamma_{bd} - 4\, F_{ab}{}^Y F_{cd}{}^V F_{ef}{}^T F_{ce}{}^W \gamma_{abdf} + \\
&+ F_{ab}{}^Y F_{cd}{}^V F_{ef}{}^W F_{ef}{}^T \gamma_{abcd} + \\
&+ F_{ab}{}^V F_{cd}{}^T F_{ef}{}^W F_{eg}{}^Y \gamma_{abcdfg} \Big] \epsilon \,. \qquad \text{(B.1)}
\end{aligned}
$$

The transformation rules for the vector field are:

$$
\begin{aligned}
\delta_3 A_a^Z = f^{XYZ} f^{VWX} \Big[&+ 2\, \mathcal{D}_b F_{cd}{}^Y \mathcal{D}_b F_{cd}{}^W \, \bar\epsilon \gamma_a \chi^V - \mathcal{D}_a \mathcal{D}_b F_{cd}{}^W F_{cd}{}^Y \, \bar\epsilon \gamma_b \chi^V + \\
&+ 5\, \mathcal{D}_a \mathcal{D}_b F_{cd}{}^W F_{cd}{}^V \, \bar\epsilon \gamma_b \chi^Y + 4\, \mathcal{D}_b F_{ac}{}^Y \mathcal{D}_b F_{cd}{}^W \, \bar\epsilon \gamma_d \chi^V - \\
&- \mathcal{D}_a F_{bc}{}^Y \mathcal{D}_d F_{bc}{}^W \, \bar\epsilon \gamma_d \chi^V - 5\, \mathcal{D}_a F_{bc}{}^W \mathcal{D}_d F_{bc}{}^V \, \bar\epsilon \gamma_d \chi^Y + \\
&+ 2 \mathcal{D}_b F_{ac}{}^W \mathcal{D}_b F_{de}{}^Y \, \bar\epsilon \gamma_{cde} \chi^V + \mathcal{D}_b F_{cd}{}^Y \mathcal{D}_b F_{ef}{}^W \, \bar\epsilon \gamma_{acdef} \chi^V + \\
&+ 2\, \mathcal{D}_b F_{cd}{}^Y F_{cd}{}^W \, \bar\epsilon \gamma_a \mathcal{D}_b \chi^V + 2\, \mathcal{D}_b F_{cd}{}^W F_{cd}{}^Y \, \bar\epsilon \gamma_a \mathcal{D}_b \chi^V - \\
&- 2\, \mathcal{D}_b F_{cd}{}^Y F_{ac}{}^W \, \bar\epsilon \gamma_b \mathcal{D}_d \chi^V - 2\, \mathcal{D}_b F_{cd}{}^W F_{cd}{}^Y \, \bar\epsilon \gamma_b \mathcal{D}_a \chi^V + \\
&+ 6\, \mathcal{D}_b F_{cd}{}^W F_{cd}{}^V \, \bar\epsilon \gamma_b \mathcal{D}_a \chi^Y - 2\, \mathcal{D}_b F_{ac}{}^Y F_{bd}{}^W \, \bar\epsilon \gamma_c \mathcal{D}_d \chi^V + \\
&+ 4\, \mathcal{D}_b F_{ac}{}^W F_{bd}{}^Y \, \bar\epsilon \gamma_c \mathcal{D}_d \chi^V - 8\, \mathcal{D}_a F_{bc}{}^Y F_{bd}{}^W \, \bar\epsilon \gamma_c \mathcal{D}_d \chi^V - \\
&- 2\, \mathcal{D}_a F_{bc}{}^W F_{bd}{}^V \, \bar\epsilon \gamma_c \mathcal{D}_d \chi^Y + 4\, \mathcal{D}_b F_{cd}{}^W F_{ac}{}^Y \, \bar\epsilon \gamma_d \mathcal{D}_b \chi^V - \\
&- 10\, \mathcal{D}_a F_{bc}{}^W F_{bd}{}^Y \, \bar\epsilon \gamma_d \mathcal{D}_c \chi^V + 2\, \mathcal{D}_a F_{bc}{}^W F_{bd}{}^V \, \bar\epsilon \gamma_d \mathcal{D}_c \chi^Y + \\
&+ 2\, \mathcal{D}_b F_{cd}{}^Y F_{be}{}^W \, \bar\epsilon \gamma_{acd} \mathcal{D}_e \chi^V + 2\, \mathcal{D}_b F_{cd}{}^Y F_{ce}{}^W \, \bar\epsilon \gamma_{ade} \mathcal{D}_b \chi^V + \\
&+ 2\, \mathcal{D}_b F_{ac}{}^W F_{de}{}^Y \, \bar\epsilon \gamma_{bde} \mathcal{D}_c \chi^V - 2\, \mathcal{D}_a F_{bc}{}^W F_{de}{}^Y \, \bar\epsilon \gamma_{bce} \mathcal{D}_d \chi^V + \\
&+ 2\, \mathcal{D}_a F_{bc}{}^W F_{de}{}^V \, \bar\epsilon \gamma_{bce} \mathcal{D}_d \chi^Y + \mathcal{D}_b F_{ac}{}^Y F_{de}{}^W \, \bar\epsilon \gamma_{cde} \mathcal{D}_b \chi^V - \\
&- 2\, \mathcal{D}_b F_{ac}{}^W F_{de}{}^Y \, \bar\epsilon \gamma_{cde} \mathcal{D}_b \chi^V + \mathcal{D}_b F_{cd}{}^Y F_{ae}{}^W \, \bar\epsilon \gamma_{cde} \mathcal{D}_b \chi^V - \\
&- \frac{1}{2} \mathcal{D}_b F_{cd}{}^Y F_{ef}{}^W \, \bar\epsilon \gamma_{acdef} \mathcal{D}_b \chi^V + \mathcal{D}_b F_{cd}{}^W F_{ef}{}^Y \, \bar\epsilon \gamma_{acdef} \mathcal{D}_b \chi^V + \\
&+ 10\, F_{bc}{}^Y F_{bd}{}^W \, \bar\epsilon \gamma_a \mathcal{D}_c \mathcal{D}_d \chi^V - 8\, F_{bc}{}^W F_{bd}{}^Y \, \bar\epsilon \gamma_a \mathcal{D}_c \mathcal{D}_d \chi^V - \\
&- 2\, F_{ab}{}^Y F_{cd}{}^W \, \bar\epsilon \gamma_d \mathcal{D}_b \mathcal{D}_c \chi^V + 2\, F_{ab}{}^W F_{cd}{}^Y \, \bar\epsilon \gamma_d \mathcal{D}_b \mathcal{D}_c \chi^V - \\
&- 8\, F_{bc}{}^W F_{bd}{}^Y \, \bar\epsilon \gamma_d \mathcal{D}_c \mathcal{D}_a \chi^V - 2\, F_{bc}{}^Y F_{de}{}^W \, \bar\epsilon \gamma_{cde} \mathcal{D}_a \mathcal{D}_b \chi^V - \\
&- 2\, F_{bc}{}^W F_{de}{}^V \, \bar\epsilon \gamma_{cde} \mathcal{D}_a \mathcal{D}_b \chi^Y \Big] + \\
+ f^{XYZ} f^{UVW} f^{TUX} \Big[&+ 13\, F_{ab}{}^W F_{cd}{}^Y F_{cd}{}^V \, \bar\epsilon \gamma_b \chi^T - 3\, F_{ab}{}^W F_{cd}{}^Y F_{cd}{}^T \, \bar\epsilon \gamma_b \chi^V - \\
&- 5\, F_{ab}{}^W F_{cd}{}^V F_{cd}{}^T \, \bar\epsilon \gamma_b \chi^Y - 20\, F_{bc}{}^W F_{ad}{}^Y F_{bd}{}^V \, \bar\epsilon \gamma_c \chi^T + \\
&+ 22\, F_{bc}{}^W F_{ad}{}^T F_{bd}{}^V \, \bar\epsilon \gamma_c \chi^Y - 6\, F_{bc}{}^T F_{ad}{}^Y F_{bd}{}^W \, \bar\epsilon \gamma_c \chi^V + \\
&+ 34\, F_{bc}{}^W F_{ad}{}^T F_{bd}{}^Y \, \bar\epsilon \gamma_c \chi^V + 2\, F_{bc}{}^T F_{de}{}^Y F_{bd}{}^W \, \bar\epsilon \gamma_{ace} \chi^V - \\
&- 4\, F_{bc}{}^Y F_{de}{}^W F_{bd}{}^V \, \bar\epsilon \gamma_{ace} \chi^T + 4\, F_{bc}{}^W F_{de}{}^Y F_{de}{}^V \, \bar\epsilon \gamma_{abc} \chi^T -
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{7}{2} F_{bc}{}^{W} F_{de}{}^{Y} F_{de}{}^{T}\, \bar{\epsilon}\, \gamma_{abc} \chi^{V} - F_{bc}{}^{W} F_{de}{}^{V} F_{de}{}^{T}\, \bar{\epsilon}\, \gamma_{abc} \chi^{Y} + \\
&+ 42\, F_{ab}^{W} F_{cd}{}^{Y} F_{ce}{}^{V}\, \bar{\epsilon}\, \gamma_{bde} \chi^{T} + 6\, F_{ab}^{T} F_{cd}{}^{Y} F_{ce}{}^{W}\, \bar{\epsilon}\, \gamma_{bde} \chi^{V} + \\
&+ \frac{1}{2} F_{cd}{}^{Y} F_{ef}{}^{V}\, \bar{\epsilon}\, \gamma_{bcdef} \chi^{T} + \frac{7}{2} F_{ab}{}^{W} F_{cd}{}^{Y} F_{ef}{}^{T}\, \bar{\epsilon}\, \gamma_{bcdef} \chi^{V} - \\
&- \frac{5}{2} F_{ab}{}^{W} F_{cd}{}^{V} F_{ef}{}^{T} \bar{\epsilon}\, \gamma_{bcdef} \chi^{Y} + 3 F_{bc}{}^{W} F_{de}{}^{Y} F_{ad}{}^{V} \bar{\epsilon}\, \gamma_{bce} \chi^{T} - \\
&- 6\, F_{bc}{}^{W} F_{de}{}^{V} F_{ad}{}^{T}\, \bar{\epsilon}\, \gamma_{bce} \chi^{Y} + 2\, F_{bc}{}^{W} F_{de}{}^{T} F_{ad}{}^{V}\, \bar{\epsilon}\, \gamma_{bce} \chi^{Y} + \\
&+ F_{bc}{}^{T} F_{de}{}^{Y} F_{ad}{}^{W}\, \bar{\epsilon}\, \gamma_{bce} \chi^{V} - 3\, F_{bc}{}^{W} F_{de}{}^{T} F_{ad}{}^{Y}\, \bar{\epsilon}\, \gamma_{bce} \chi^{V} - \\
&- 5 F_{bc}{}^{W} F_{de}{}^{Y} F_{df}{}^{V} \bar{\epsilon}\, \gamma_{abcef} \chi^{T} - F_{bc}{}^{T} F_{de}{}^{Y} F_{df}{}^{W} \bar{\epsilon}\, \gamma_{abcef} \chi^{V} + \\
&\left. + \frac{1}{4} F_{bc}{}^{Y} F_{de}{}^{W} F_{fg}{}^{T}\, \bar{\epsilon}\, \gamma_{abcdefg} \chi^{V} \right] + \\
+ f^{WXY} f^{TUV} f^{TWZ} &\Big[+ 8\, F_{bc}{}^{Y} F_{ad}{}^{X} F_{bd}{}^{V}\, \bar{\epsilon}\, \gamma_{c} \chi^{U} + 2\, F_{bc}{}^{Y} F_{de}{}^{V} F_{bd}{}^{X}\, \bar{\epsilon}\, \gamma_{ace} \chi^{U} - \\
&- 14\, F_{ab}{}^{Y} F_{cd}{}^{X} F_{ce}{}^{V}\, \bar{\epsilon}\, \gamma_{bde} \chi^{U} - 4\, F_{bc}{}^{Y} F_{de}{}^{X} F_{ad}{}^{V}\, \bar{\epsilon}\, \gamma_{bce} \chi^{U} + \\
&+ 7\, F_{bc}{}^{Y} F_{de}{}^{X} F_{df}{}^{V}\, \bar{\epsilon}\, \gamma_{abcef} \chi^{U} \Big] . \qquad \text{(B.2)}
\end{aligned}
$$

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