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Comment on the pion pole part of the light-by-light contribution to the muon $g - 2$

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Abstract

We comment on the recent calculations of the pion pole part of the light-by-light contribution to the muon anomalous magnetic moment and we point out where the analysis in our previous work was mistaken. © 2002 Published by Elsevier Science B.V.

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In the recent paper [1], the pion pole part of the light-by-light contribution to the muon anomalous magnetic moment was reevaluated and analytical expressions for a large class of $\pi^0\gamma^*\gamma^*$ form factors fulfilling OPE and large-N_c QCD constraints were obtained. In [2] the leading double logarithmic dependence in the case of point-like pion–photon–photon couplings was derived analytically. This result has been confirmed in [3]. The absolute value of the contribution was found to be in good agreement with the evaluations done previously [4–8], but opposite in sign.

We have gone back to our old programs and notes in order to find the source of the discrepancy. The pion pole contribution, as well as all the others, was calculated twice independently inside our collaboration and afterwards the results were compared. Both analyses agreed and had the correct sign analytically, but unfortunately both analyses made the same sign mistake in the overall normalization factor in putting it into the integration program. Since both computations agreed and also agreed with previously known results we saw no reason to doubt our results for the pion pole. More studies of variations in the

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form factor [8] did also not indicate that there was a major problem with the size of the form-factor. The same sign problem also affects the axial-vector contribution, but not the others we discussed. The scalar exchange contribution in our approach is linked by chiral symmetry constraints to the quark-loop low-energy contribution and had the correct sign.

We have also checked that the analytical formulas for the amplitudes in [5] are all correct with the exception of an overall minus sign missing in Eq. (2.7), that was an errata in the manuscript and not the origin of the sign mistake in the pion pole contribution.

The often quoted argument that there was no matching with the short-distance quark-loop contribution was tested in our work, in particular, in Section 7.1 of [5]. There, it was found that the short-distance quark loop matched perfectly well numerically when added together with another contribution that our treatment of the low-energy part required. For us, there was thus no reason to doubt that we had a reasonable matching with the expected short-distance behaviour even with the pion-pole and the quark-loop contributions having a different sign.

One should also not forget that, while the pion and the other pseudoscalar pole contributions are the dominant ones, there are other contributions at the 20 to 30 percent level that also need to be estimated. In the works [5,6] they have been evaluated and were found to cancel to a large extent. We intend to reevaluate all contributions and come back to them in a later more extended note. Our number corrected for the sign mistakes becomes

$$\begin{aligned} a_\mu^{\mathrm{LbL}} &= (2.1 \pm 0.3) \times 10^{-10}[\text{quark-loop}] + (-0.68 \pm 0.2) \times 10^{-10}[\text{scalar}] \\ &\quad + (8.5 \pm 1.3) \times 10^{-10}[\text{pseudoscalar}] + (0.25 \pm 0.1) \times 10^{-10}[\text{axial-vector}] \\ &\quad + (-1.9 \pm 1.3) \times 10^{-10}[\pi K\text{-loop}] \\ &= (8.3 \pm 3.2) \times 10^{-10}. \end{aligned} \tag{1}$$

We have also checked that using the pointlike coupling our cut-off dependence numerically agrees with the one calculated analytically in [2].

A similar note [9] has appeared by the authors of the other full evaluation [6].

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