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*Published in:*
Journal of High Energy Physics

*DOI:*
10.1088/1126-6708/2003/11/022

Published by Institute of Physics Publishing for SISSA/ISAS
Received: *October 29, 2003*
Accepted: *November 11, 2003*

# Potential and mass-matrix in gauged $N = 4$ supergravity

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Abstract: We discuss the potential and mass-matrix of gauged $N = 4$ matter coupled supergravity for the case of six matter multiplets, extending previous work by considering the dependence on all scalars. We consider all semi-simple gauge groups and analyse the potential and its first and second derivatives in the origin of the scalar manifold. Although we find in a number of cases an extremum with a positive cosmological constant, these are not stable under fluctuations of all scalar fields.

Keywords: Extended Supersymmetry, Supergravity Models, Cosmology of Theories beyond the SM.

## Contents



## 1. Introduction

Recent astronomical observations [1, 2] have led to the conclusion that our universe is presently in a state of accelerating expansion, and that its cosmological properties are best described by assuming the presence of a large amount of dark matter of unknown origin, as well as an even larger amount of dark energy, which one considers as due to a cosmological constant. This situation has renewed interest in fundamental theories with scalar potentials, as these may have nonzero extremal values, thus presenting a possibility to explain the cosmological constant. Motivated by string and M theory, the search for the appropriate fundamental theory concentrates on the supergravity theories which arise as low energy limit of compactified string theories. A major problem in this respect is that the sign of the cosmological constant should be positive to explain the expansion, a property which is hard to reconcile with theories which have their origin in string theory ([3, 4, 5], for a recent discussion see [6]).

In this paper we take the point of view that one should first obtain a solution of a four-dimensional supergravity theory with the appropriate properties (de Sitter, stability), and consider the connection with string theory as a second step. We will concentrate on the properties of gauged $N = 4$ supergravity theories coupled to additional matter multiplets. It is well-known that these theories do allow extrema with a positive cosmological constant [7]. In a previous paper [8] a situation where the scalar manifold is truncated to four scalar fields was discussed, and it was shown that a positive, stable extremum of the potential is possible when the scalar fluctuations are limited to the four directions which survive the truncation. In the present paper we will extend the scope of these investigations to include more general gauge groups, and also consider the fluctuations in all scalars present in the model.

A further motivation for this investigation arises from [9], where a stable solution in de Sitter space was constructed in gauged $N = 2$ supergravity. There is no obvious connection of this solution with string theory, but the authors indicated a possible relation with $N = 4$. Since $N = 4$ is a step closer to the maximal ten and eleven dimensional supergravity theories, the connection with string theory might be easier to obtain once the $N = 2$ case is raised to $N = 4$. In section 4 we will discuss the $N = 2$ aspect of our work in more detail.

We leave for later work the question of how these theories arise from the higher dimensional low-energy limit of string theory. It is well known that ungauged $N = 4$ supergravity theories in $d = 4$ can be obtained by toroidal compactification from $d = 10$ supergravity [10]. Gauged $N = 4$ supergravity obtained from Scherk-Schwarz compactifications [11] has been considered in the past [12, 13, 14], and also more recently in combination with flux compactification [15]–[20]. Nevertheless, to our knowledge the SU$(1, 1)$ duality angles [21], which play an essential role in obtaining extrema with a positive cosmological constant, have not been given a higher-dimensional origin.

Throughout this paper we will use the notation of and results from [8]. We will discuss the value of the potential and its derivatives, as well as the definition of the bosonic mass-matrices, in section 2. Nine semi-simple gauge groups satisfy the conditions that (i) the potential has an extremum for all scalar fields, (ii) the value of the potential in the extremum is positive. These groups are introduced in section 2.2 and their properties are discussed in detail in the different subsections of section 3.

To complete this Introduction we will review some basic properties of the parametrisation of the scalar sector of matter coupled $N = 4$ supergravity [22, 21]. We consider gauged $N = 4$ supergravity coupled to $n$ vector multiplets. The scalar fields of the theory are[1] ${Z_a}^R$ (real) and $\phi_\alpha$ (complex), satisfying the constraints

$$\phi^\alpha \phi_\alpha = 1 \,, \tag{1.1}$$

[1]The indices $\alpha, \beta, \dots$ take on values 1 and 2, indices $R, S \dots$ the values $1, \dots, 6 + n$, and the indices $a, b, \dots$ the values $1, \dots, 6$. The metric $\eta_{RS}$ can be chosen as diag$(-1, -1, -1, -1, -1, -1, +1, \dots, +1)$, with $n$ positive entries. In comparison to [21] we have replaced the complex scalars ${\phi_{ij}}^R$ by real scalars ${Z_a}^R$: ${\phi_{ij}}^R = \frac{1}{2} {Z_a}^R (G^a)_{ij}$, where the $G^a$ are six matrices which ensure that ${Z_a}^R$ transforms as a vector under SO(6).

$$\eta_{RS} Z_a{}^R Z_b{}^S = -\delta_{ab} \,. \tag{1.2}$$

The scalars $\phi_\alpha$ ($\phi^1 = (\phi_1)^*, \phi^2 = -(\phi_2)^*$) transform under global $\mathrm{SU}(1,1)$ and local $\mathrm{U}(1)$, the $Z_a{}^R$ transform under local $\mathrm{SO}(6) \times \mathrm{SO}(n)$, and under global $\mathrm{SO}(6,n)$. The constraints and the local symmetry restrict the scalars to the cosets $\mathrm{SU}(1,1)/\,\mathrm{U}(1)$ (two physical scalars) and $\mathrm{SO}(6,n)/\,\mathrm{SO}(6) \times \mathrm{SO}(n)$ ($6n$ physical scalars).

There is a certain freedom in coupling the vector multiplets: for each multiplet, labeled by $R$, we can introduce an $\mathrm{SU}(1,1)$ element, of which only a single angle $\alpha_R$ turns out to be important. These angles $\alpha_R$ can be reinterpreted as a modification of the $\mathrm{SU}(1,1)$ scalars coupling to the multiplet $R$ in the form

$$\phi^1_{(R)} = e^{i\alpha_R}\phi^1 \,, \quad \phi^2_{(R)} = e^{-i\alpha_R}\phi^2 \,, \quad \Phi_{(R)} = e^{i\alpha_R}\phi^1 + e^{-i\alpha_R}\phi^2 \,. \tag{1.3}$$

The gauge group has to be a subgroup of $\mathrm{SO}(6,n)$. For a semi-simple gauge group the $\alpha_R$ (called $\mathrm{SU}(1,1)$ angles in this paper) have to be the same for all $R$ belonging to the same factor of the gauge group. This gauging breaks the global $\mathrm{SO}(6,n)$ symmetry of the ungauged theory. In the remainder of this paper we will limit ourselves to $n = 6$. The reason for the choice $n = 6$ is, besides its relative simplicity, that this case follows by toroidal compactification from $d = 10$ $N = 1$ supergravity without additional matter.

## 2. The potential, its derivatives, and the mass-matrix

### 2.1 The potential

The scalar potential $V(\phi, Z)$ can be written in the form

$$V = \sum_{i,j} \left(R^{(ij)}(\phi)\, V_{ij}(Z) + I^{(ij)}\, W_{ij}(Z)\right). \tag{2.1}$$

The indices $i, j, \ldots$ label the different factors in the gauge group $G$, which we will take to be semi-simple. $R^{(ij)}$ and $I^{(ij)}$ contain the $\mathrm{SU}(1,1)$ scalars and depend on the gauge coupling constants and the $\mathrm{SU}(1,1)$ angles, $V_{ij}$ and $W_{ij}$ contain the structure constants, depend on the matter fields, and are symmetric resp. antisymmetric in the indices $i, j$.

The extremum of the potential in the $\phi$ direction has been determined in [8]. For completeness we briefly review this analysis in appendix A. The conclusion is that in the extremum in the $\mathrm{SU}(1,1)$ scalars the potential takes on the form

$$V_0 = \mathrm{sgn} C_- \sqrt{\Delta} - T_- \,, \tag{2.2}$$

where (see [8])

$$C_- = \sum_{ij} g_i g_j \cos(\alpha_i - \alpha_j) V_{ij} \,, \tag{2.3}$$

$$T_- = \sum_{ij} a_{ij} W_{ij} \,, \tag{2.4}$$

$$\Delta = 2 \sum_{ij} \sum_{kl} V_{ij} V_{kl} a_{ik} a_{jl} \,, \tag{2.5}$$

| Group | $-$ | $+$ | c | nc | Group | $-$ | $+$ | c | nc |
|---|---|---|---|---|---|---|---|---|---|
| $\mathrm{SO}(3)_-$ | 3 | 0 | 3 | 0 | $\mathrm{SO}(3)_+$ | 0 | 3 | 3 | 0 |
| $\mathrm{SO}(2,1)_-$ | 1 | 2 | 1 | 2 | $\mathrm{SO}(2,1)_+$ | 2 | 1 | 1 | 2 |
| $\mathrm{SO}(3,1)_-$ | $3_c$ | $3_{nc}$ | 3 | 3 | $\mathrm{SO}(3,1)_+$ | $3_{nc}$ | $3_c$ | 3 | 3 |
| $\mathrm{SL}(3,\mathbb{R})_-$ | 3 | 5 | 3 | 5 | $\mathrm{SL}(3,\mathbb{R})_+$ | 5 | 3 | 3 | 5 |
| $\mathrm{SU}(2,1)_-$ | $4_c$ | $4_{nc}$ | 4 | 4 | $\mathrm{SU}(2,1)_+$ | $4_{nc}$ | $4_c$ | 4 | 4 |

**Table 1:** List of allowed simple groups. The first two columns indicate how the group is embedded in SO(6, 6) with respect to the signs in the metric $\eta_{RS}$, the column $c$ and $nc$ indicate the number of compact and noncompact generators, respectively. The structure constants of these groups are presented in appendix B.

$$R^{(ij)} = \frac{\mathrm{sgn} C_-}{\sqrt{\Delta}} \sum_{kl} V_{kl}(2a_{ik}a_{jl} - a_{ij}a_{kl})\,, \tag{2.6}$$

$$I^{(ij)} = -a_{ij}\,. \tag{2.7}$$

and

$$a_{ij} \equiv g_i g_j \sin(\alpha_i - \alpha_j)\,. \tag{2.8}$$

The condition for this extremum is that $\Delta > 0$, which implies that at least two of the SU(1, 1) angles must be different. This in turn implies that the gauge group must contain at least two simple subgroups.

Let us now make a list of possible simple subgroups of $G$, and discuss their embedding in SO(6, 6). The metric $\eta_{RS}$ is the invariant metric of the global symmetry group SO(6, 6), which acts on the fields in the fundamental representation. The gauge group $G$ acts in the adjoint representation, and has 12 or less generators (for $n = 6$). So the adjoint representation of $G$ must fit into the fundamental representation of SO(6, 6). The generators $T_R$ of the simple subgroups of $G$ in the fundamental representation are chosen in such a way that $g_{RS} \equiv \mathrm{tr}\, T_R T_S = \pm 2\delta_{RS}$, with positive entries for the compact, and negative entries for the noncompact directions. The embedding of $G$ in SO(6, 6) is such that the metric $g_{RS}$ coincides, up to an overall factor $\pm 2$, with $\eta_{RS}$. The factor 2 we absorb in the coupling constant of the corresponding gauge group (see appendix B for further properties of these groups). With these restrictions one can easily list all allowed simple factors $G_i$ of $G$. These are presented in table 1. Note that groups of dimension 10 and higher are excluded since they do not leave enough room for a second nonabelian subgroup.

The starting point for the remaining analysis is then (2.1). The ingredients are

$$V_{ij} = \left( -\frac{1}{12} Z^{RU} Z^{SV} Z^{TW} + \frac{1}{4} Z^{RU} Z^{SV} (\eta^{TW} + Z^{TW}) \right) f^{(i)}{}_{RST} f^{(j)}{}_{UVW}\,, \tag{2.9}$$

$$W_{ij} = \frac{1}{36} \epsilon^{abcdef} Z_a{}^R Z_b{}^S Z_c{}^T Z_d{}^U Z_e{}^V Z_f{}^W \, f^{(i)}{}_{RST} f^{(j)}{}_{UVW}\,, \tag{2.10}$$

where $Z^{RS} = Z_a{}^R Z_a{}^S$. It is important to remember that the $Z_a{}^R$ are not the independent scalars, due to the constraint (1.2). A useful parametrisation is given in terms of $6 \times 6$ matrices $G$ (symmetric) and $B$ (antisymmetric). We split the indices $R, S, \ldots$ of $\eta_{RS}$ in

$A, B, \ldots = 1, \ldots, 6$, $(\eta_{AB} = -\delta_{AB})$ and $I, J, \ldots = 7, \ldots, 12$, $(\eta_{IJ} = +\delta_{IJ})$. The scalar constraint (1.2) then reads

$$XX^T - YY^T = \mathbb{1}_6 \,, \tag{2.11}$$

where $X_a{}^A = Z_a{}^A$, $Y_a{}^{I-6} = Z_a{}^I$, which is solved by

$$X = \frac{1}{2}\left(G + G^{-1} + BG^{-1} - G^{-1}B - BG^{-1}B\right), \tag{2.12}$$

$$Y = \frac{1}{2}\left(G - G^{-1} - BG^{-1} - G^{-1}B - BG^{-1}B\right). \tag{2.13}$$

In [8] we limited ourselves to the case where

$$G = \begin{pmatrix} a\mathbb{1}_3 & 0 \\ 0 & a\mathbb{1}_3 \end{pmatrix} \quad (a > 0)\,, \qquad B = \begin{pmatrix} 0 & b\mathbb{1}_3 \\ -b\mathbb{1}3 & 0 \end{pmatrix}, \tag{2.14}$$

in this paper we will use the complete $G$ and $B$.

We will analyse the potential at the point $Z_0$, the origin of the scalar manifold, given by $G = \mathbb{1}$, $B = 0$. We will see in section 2.2 that for many gauge groups the origin corresponds to an extremum of the potential in all directions. At $Z = Z_0$ we have

$$Z_{0\,a}{}^R = \begin{pmatrix} \mathbb{1}_6 & 0 \end{pmatrix}, \quad Z_0{}^{RS} = \begin{pmatrix} \mathbb{1}_6 & 0 \\ 0 & 0 \end{pmatrix}, \quad (\eta + Z)_0{}^{RS} = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1}_6 \end{pmatrix}. \tag{2.15}$$

Consider $V_{ij}$, as given in (2.9). The first term contains a product of three $Z_0{}^{RS}$, which are diagonal and only non-vanishing if all of $RSTUVW$ are in the range $1 \ldots 6$. The second term containing $\eta + Z$ is also diagonal with the two indices in the range $7 \ldots 12$. Therefore in $V_{ij}$ the indices of the structure constants of $G_i$ and $G_j$ are contracted, implying that they belong to the same factor in the product of groups. Therefore $V_{ij} = 0$ for $i \neq j$.

$W_{ij}$ can only be nonzero if there are subgroups $G_i$ and $G_j$ of $G$, $i \neq j$, such that both have three generators in the range $1 \ldots 6$ with structure constants $f_{ABC}$. These $G_i$ must therefore be SO(3) or SU(2) subgroups of $G_i$ and $G_j$.

Therefore we have:

$$\begin{aligned}
V_{0\,ii} &= -\frac{1}{12} f^{(i)}{}_{ABC} f^{(i)}{}_{ABC} + \frac{1}{4} f^{(i)}{}_{ABI} f^{(i)}{}_{ABI}\,, \\
&= -\frac{1}{2} \qquad \text{for } \mathrm{SO}(3)_-,\ \mathrm{SO}(3,1)_-,\ \mathrm{SL}(3,\mathbb{R})_-\,, \\
&\quad\ -2 \qquad \text{for } \mathrm{SU}(2,1)_-\,, \\
&\quad\ \frac{1}{2} \qquad \text{for } \mathrm{SO}(2,1)_+\,, \\
&\quad\ \frac{3}{2} \qquad \text{for } \mathrm{SO}(3,1)_+\,, \\
&\quad\ \frac{15}{2} \qquad \text{for } \mathrm{SL}(3,\mathbb{R})_+\,, \\
&\quad\ 6 \qquad \text{for } \mathrm{SU}(2,1)_+\,, \\
&\quad\ 0 \qquad \text{for } \mathrm{SO}(2,1)_-,\ \mathrm{SO}(3)_+\,, \\
V_{0\,ij} &= 0 \qquad i \neq j\,,
\end{aligned}$$

$$
\begin{aligned}
W_{0\,ij} &= 1 \qquad \text{for subgroups } \mathrm{SO}(3)_-,\ \mathrm{SO}(3,1)_-,\ \mathrm{SL}(3,\mathbb{R})_-\,, \\
&= 0 \qquad \text{otherwise}\,.
\end{aligned} \tag{2.16}
$$

Since $V_{0\,ij} = 0$ for $i \neq j$ we can simplify (2.2). We find

$$
C_{0-} = \sum_i g_i^2 V_{0\,ii}\,, \tag{2.17}
$$

$$
T_{0-} = \sum_{ij} a_{ij} W_{0\,ij}\,, \tag{2.18}
$$

$$
\Delta_0 = 2 \sum_{ij} V_{0\,ii} V_{0\,jj} a_{ij}^2\,, \tag{2.19}
$$

$$
R^{(0\,ii)} = \frac{2\,\mathrm{sgn} C_{0-}}{\sqrt{\Delta_0}} \sum_j V_{0\,jj} a_{ij}^2\,. \tag{2.20}
$$

The off-diagonal $R^{ij}$ can also be nonzero, but they do not appear in the potential (or its first and second derivatives, as we shall see) in $Z_0$ because $V_{0\,ij}$ is diagonal.

Further restrictions come from the value of $V_{0\,ii}$. To have $\Delta > 0$, we must have at least two subgroups $G_i$ for which $V_{ii} \neq 0$. Note for instance that the groups $\mathrm{SO}(3)_+$ and $\mathrm{SO}(2,1)_-$ have zero $V_{0\,ii}$, and do not contribute to $\Delta$ in $Z_0$. More restrictions will come from the requirement that $Z_0$ corresponds to an extremum of the potential.

There are two ways to make $V_0$ positive. One is to have $C_- > 0$, to which groups with positive $V_{0\,ii}$, such as $\mathrm{SO}(2,1)_+$ or $\mathrm{SO}(3,1)_+$, contribute. A further positive contribution can come from $T_-$, if $W_{ij}$ is nonzero and the $\mathrm{SU}(1,1)$ angles are appropriately chosen. This can be done for groups with $\mathrm{SO}(3)_-^2$ as a subgroup.

### 2.2 The first derivatives of the potential

The unconstrained, independent scalar fields in the 12 vector multiplets are 21 components of the symmetric matrix $G$, and 15 components of the antisymmetric matrix $B$. It is convenient to introduce a single matrix $P = (G + B)$ for the 36 independent scalars. In appendix C we give the derivatives of $X$ and $Y$ in $Z_0$ with respect to these parameters. The first derivatives of $V$ and $W$ with respect to $P$ are:

$$
\frac{\partial V_{ij}}{\partial P} = \frac{1}{2}\left(\frac{\partial Z_f{}^R}{\partial P} Z_f{}^U + Z_f{}^R \frac{\partial Z_f{}^U}{\partial P}\right) Z^{SV} (\eta + Z)^{TW} f^{(i)}{}_{RST} f^{(j)}{}_{UVW}\,, \tag{2.21}
$$

$$
\begin{aligned}
\frac{\partial W_{ij}}{\partial P} = \frac{1}{12}\epsilon^{abcdef} \Bigg(&\frac{\partial Z_a{}^R}{\partial P} Z_b{}^S Z_c{}^T Z_d{}^U Z_e{}^V Z_f{}^W + \\
&+ Z_a{}^R Z_b{}^S Z_c{}^T \frac{\partial Z_d{}^U}{\partial P} Z_e{}^V Z_f{}^W\Bigg) f^{(i)}{}_{RST} f^{(j)}{}_{UVW}\,.
\end{aligned} \tag{2.22}
$$

Now we evaluate this in $Z_0$, using the fact that the derivative of $X$ with respect to $P$ vanishes in $Z_0$. The derivatives of $V$ and $W$ in $Z_0$ are then:

$$
\left.\frac{\partial V_{ii}}{\partial P}\right|_0 = \left.\frac{\partial Z_A{}^I}{\partial P}\right|_0 f^{(i)}{}_{IBJ} f^{(i)}{}_{ABJ}\,, \quad \left.\frac{\partial V_{ij}}{\partial P}\right|_0 = 0 \text{ for } i \neq j\,, \tag{2.23}
$$

| Groups | Properties |
|---|---|
| $\mathrm{SO}(3)_-^2 \otimes \mathrm{SO}(3)_+^2$ | |
| $\mathrm{SO}(3)_- \otimes \mathrm{SO}(2,1)_+ \otimes \mathrm{SO}(2,1)_- \otimes \mathrm{SO}(3)_+$ | $\Delta_0 < 0$, no extremum |
| $\mathrm{SO}(3)_- \otimes \mathrm{SO}(2,1)_-^3$ | $\Delta_0 = 0$ |
| $\mathrm{SO}(2,1)_+^3 \otimes \mathrm{SO}(3)_+$ | |
| $\mathrm{SO}(2,1)_+^2 \otimes \mathrm{SO}(2,1)_-^2$ | |
| $\mathrm{SO}(3,1)_+ \otimes \mathrm{SO}(2,1)_+ \otimes \mathrm{SO}(2,1)_-$ | |
| $\mathrm{SO}(3,1)_+ \otimes \mathrm{SO}(3,1)_+$ | |
| $\mathrm{SO}(3,1)_+ \otimes \mathrm{SO}(3)_- \otimes \mathrm{SO}(3)_+$ | $\Delta_0 < 0$, no extremum |
| $\mathrm{SO}(3,1)_- \otimes \mathrm{SO}(2,1)_+ \otimes \mathrm{SO}(2,1)_-$ | $\Delta_0 < 0$, no extremum |
| $\mathrm{SO}(3,1)_- \otimes \mathrm{SO}(3,1)_-$ | |
| $\mathrm{SO}(3,1)_- \otimes \mathrm{SO}(3,1)_+$ | $\Delta_0 < 0$, no extremum |
| $\mathrm{SO}(3,1)_- \otimes \mathrm{SO}(3)_- \otimes \mathrm{SO}(3)_+$ | |
| $\mathrm{SL}(3,\mathbb{R})_+ \otimes \mathrm{SO}(3)_+$ | $\Delta_0 = 0$ |
| $\mathrm{SL}(3,\mathbb{R})_- \otimes \mathrm{SO}(3)_-$ | |
| $\mathrm{SL}(3,\mathbb{R})_+ \otimes \mathrm{SO}(2,1)_-$ | $\Delta_0 = 0$ |
| $\mathrm{SL}(3,\mathbb{R})_- \otimes \mathrm{SO}(2,1)_+$ | $\Delta_0 < 0$, no extremum |
| $\mathrm{SU}(2,1)_+ \otimes \mathrm{SO}(2,1)_+$ | |
| $\mathrm{SU}(2,1)_+ \otimes \mathrm{SO}(2,1)_-$ | $\Delta_0 = 0$ |
| $\mathrm{SU}(2,1)_- \otimes \mathrm{SO}(2,1)_+$ | $\Delta_0 < 0$, no extremum |
| $\mathrm{SU}(2,1)_- \otimes \mathrm{SO}(2,1)_-$ | $\Delta_0 = 0$ |

**Table 2:** List of possible gauge groups $G$. Nine groups have an extremum with respect to the matter scalars in $Z_0$ with positive $\Delta_0$.

$$\left.\frac{\partial W_{ij}}{\partial P}\right|_0 = \frac{1}{12}\epsilon^{ABCDEF}\left.\frac{\partial Z_A{}^I}{\partial P}\right|_0 f^{(i)}{}_{IBC}f^{(j)}{}_{DEF} - (i \leftrightarrow j)\,. \tag{2.24}$$

In the derivatives of $V_{ij}$ there are always contractions between the different groups, implying that the derivatives vanish for $i \neq j$. For the derivative of $V_{ii}$ to be nonzero we see that $G_i$ must have two indices $AB$ and two indices $IJ$. This is not the case for the groups in table 1.

The derivative of $W_{ij}$ is nonzero only if one of the groups $G_j$ or $G_i$ has an SO(3) subgroup on the indices $DEF$, and the other has an $\mathrm{SO}(2,1)$ subgroup for which the $IBC$ structure constants are nonzero. Groups $G$ with an $\mathrm{SO}(3)_- \otimes \mathrm{SO}(2,1)_+$ subgroup are therefore excluded.

If we require that $Z_0$ is an extremum of the potential, and that $\Delta > 0$ in the extremum is possible for a suitable choice of the parameters, the number of allowed groups becomes sufficiently small to make a complete analysis possible. The list of allowed groups is given in table 2.

To illustrate table 2, let's consider the group $\mathrm{SO}(3,1)$. The commutation relations for the $\mathrm{SO}(3,1)$ Lie algebra can be found in appendix B. The generators of the rotation subgroup are denoted by $T$, the boosts by $K$. To embed the adjoint of $\mathrm{SO}(3,1)$ in $\mathrm{SO}(6,6)$ we have two choices: either the three $T$ correspond to the negative, and the boosts $K$ to



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the positive entries of $\eta_{RS}$ ($\mathrm{SO}(3,1)_-$), or the other way round ($\mathrm{SO}(3,1)_+$). If we choose $\mathrm{SO}(3)_- \otimes \mathrm{SO}(3,1)_-$, the nonzero contributions to $V_{0\,ii}$ (2.16) come from $\mathrm{SO}(3)_-$ and from the rotation subgroup of $\mathrm{SO}(3,1)_-$, both contribute $-\frac{1}{2}$, and $\Delta_0$ (2.19) is positive. The structure constants $f_{ABI}$ vanish, and do not contribute to $V_0$ in this case. For the same reason, the first derivatives of $V$ and $W$ (2.23), (2.24) vanish. So this case is interesting, and will appear as a subgroup of the groups considered in section 3.6 and 3.7. On the other hand, if we choose $\mathrm{SO}(3)_- \otimes \mathrm{SO}(3,1)_+$ then the boosts are in the $ABC$ range of the indices, and structure constants $f_{ABI}$ are nonzero. The potential $V_{0\,ii}$ gets contributions from $\mathrm{SO}(3)_-$, and now the second term in (2.16) will contribute positively. This will make $\Delta_0$ negative. Also the first derivative of $W$ will be nonzero, so this group cannot be used for our purposes.

### 2.3 The second derivatives of the potential

The full potential is given in (2.1), and its second derivatives are:

$$\frac{\partial^2 V}{\partial \phi^2} = \sum_{ij} \frac{\partial^2 R^{(ij)}}{\partial \phi^2} V_{ij}\,, \tag{2.25}$$

$$\frac{\partial^2 V}{\partial \phi \partial P} = \sum_{ij} \frac{\partial R^{(ij)}}{\partial \phi} \frac{\partial V_{ij}}{\partial P}\,, \tag{2.26}$$

$$\frac{\partial^2 V}{\partial P^2} = \sum_{ij} R^{(ij)} \frac{\partial^2 V_{ij}}{\partial P^2} + I^{(ij)} \frac{\partial^2 W_{ij}}{\partial P^2}\,. \tag{2.27}$$

The second derivatives (2.25) were studied in [8]. The sign of (2.25) depends on the sign of $C_-$. For positive (negative) $C_-$ the extremum in the $\mathrm{SU}(1,1)$ scalars is a minimum (maximum). The mixed second derivatives vanish if either the derivatives with respect to the $\mathrm{SU}(1,1)$ scalars $\phi$ or with respect to the matter scalars vanishes. In this section we will evaluate the second derivatives (2.27) with respect to the matter scalars in $Z_0$.

We therefore calculate the second derivatives of $V_{ij}$ and $W_{ij}$ with respect to the independent scalars $P$. They are:

$$\begin{aligned}
\frac{\partial^2 V_{ij}}{\partial P_1 \partial P_2} = \frac{1}{2}\Bigg\{ &\left( \frac{\partial^2 Z_f{}^R}{\partial P_1 \partial P_2} Z_f{}^U + Z_f{}^R \frac{\partial^2 Z_f{}^U}{\partial P_1 \partial P_2} \right) Z^{SV} (\eta + Z)^{TW} + \\
&+ \left( \frac{\partial Z_f{}^R}{\partial P_1} \frac{\partial Z_f{}^U}{\partial P_2} + \frac{\partial Z_f{}^R}{\partial P_2} \frac{\partial Z_f{}^U}{\partial P_1} \right) Z^{SV} (\eta + Z)^{TW} + \\
&+ \left( \frac{\partial Z_f{}^R}{\partial P_1} Z_f{}^U + Z_f{}^R \frac{\partial Z_f{}^U}{\partial P_1} \right) \left( \frac{\partial Z_g{}^S}{\partial P_2} Z_g{}^V + Z_g{}^S \frac{\partial Z_g{}^V}{\partial P_2} \right) (\eta + Z)^{TW} + \\
&+ \left( \frac{\partial Z_f{}^R}{\partial P_1} Z_f{}^U + Z_f{}^R \frac{\partial Z_f{}^U}{\partial P_1} \right) Z^{SV} \left( \frac{\partial Z_g{}^T}{\partial P_2} Z_g{}^W + Z_g{}^T \frac{\partial Z_g{}^W}{\partial P_2} \right) \Bigg\} \times \\
&\times f^{(i)}{}_{RST} f^{(j)}{}_{UVW}\,,
\end{aligned} \tag{2.28}$$

$$\begin{aligned}
\frac{\partial^2 W_{ij}}{\partial P_1 \partial P_2} = \frac{1}{12}\, \epsilon^{a_1 \ldots a_6} \Bigg\{ &\frac{\partial^2 Z_{a_1}{}^R}{\partial P_1 \partial P_2} Z_{a_2}{}^S Z_{a_3}{}^T Z_{a_4}{}^U Z_{a_5}{}^V Z_{a_6}{}^W + \\
&+ 2 \frac{\partial Z_{a_1}{}^R}{\partial P_1} \frac{\partial Z_{a_2}{}^S}{\partial P_2} Z_{a_3}{}^T Z_{a_4}{}^U Z_{a_5}{}^V Z_{a_6}{}^W +
\end{aligned}$$

$$+\,3\frac{\partial Z_{a_1}{}^R}{\partial P_1} Z_{a_2}{}^S Z_{a_3}{}^T \frac{\partial Z_{a_4}{}^U}{\partial P_2} Z_{a_5}{}^V Z_{a_6}{}^W - \\ -\,(RST \leftrightarrow UVW)\Big\} f^{(i)}{}_{RST} f^{(j)}{}_{UVW}\,. \tag{2.29}$$

We now evaluate both expressions for $Z = Z_0$. From (2.28) it is clear that in $Z = Z_0$ we will get contractions between the two structure constants, so that they must belong to the same subgroup. Therefore only the second derivatives of $V_{ii}$ are nonzero in $Z_0$. We find:

$$\begin{aligned}
\frac{\partial^2 V_{ii}}{\partial P_1 \partial P_2}\bigg|_0 &= \bigg\{ \frac{\partial^2 Z_A{}^R}{\partial P_1 \partial P_2}\bigg|_0 f^{(i)}{}_{RBJ} f^{(i)}{}_{ABJ} + \frac{\partial Z_f{}^I}{\partial P_1}\bigg|_0 \frac{\partial Z_f{}^J}{\partial P_2}\bigg|_0 f^{(i)}{}_{IBK} f^{(i)}{}_{JBK} + \\
&\quad + \left( \frac{\partial Z_A{}^I}{\partial P_1}\bigg|_0 \frac{\partial Z_B{}^J}{\partial P_2}\bigg|_0 f^{(i)}{}_{IJK} f^{(i)}{}_{ABK} + \frac{\partial Z_A{}^I}{\partial P_1}\bigg|_0 \frac{\partial Z_B{}^J}{\partial P_2}\bigg|_0 f^{(i)}{}_{AJK} f^{(i)}{}_{IBK} \right) \\
&\quad + \left( \frac{\partial Z_A{}^I}{\partial P_1}\bigg|_0 \frac{\partial Z_B{}^J}{\partial P_2}\bigg|_0 f^{(i)}{}_{ICJ} f^{(i)}{}_{ACB} + \frac{\partial Z_A{}^I}{\partial P_1}\bigg|_0 \frac{\partial Z_B{}^J}{\partial P_2}\bigg|_0 f^{(i)}{}_{ICB} f^{(i)}{}_{ACJ} \right\}, \\
\frac{\partial^2 W_{ij}}{\partial P_1 \partial P_2}\bigg|_0 &= \frac{1}{12}\epsilon^{ABCDEF} \left( \frac{\partial^2 Z_A{}^R}{\partial P_1 \partial P_2}\bigg|_0 f^{(i)}{}_{RBC} f^{(j)}{}_{DEF} + 2\,\frac{\partial Z_A{}^I}{\partial P_1}\bigg|_0 \frac{\partial Z_B{}^J}{\partial P_2}\bigg|_0 f^{(i)}{}_{IJC} f^{(j)}{}_{DEF} + \right. \\
&\quad \left. + \,3\,\frac{\partial Z_A{}^I}{\partial P_1}\bigg|_0 \frac{\partial Z_D{}^J}{\partial P_2}\bigg|_0 f^{(i)}{}_{IBC} f^{(j)}{}_{JEF} - (i \leftrightarrow j) \right).
\end{aligned} \tag{2.30}$$

The second derivatives of $Z$ ($X$ and $Y$) are given in appendix C.

### 2.4 Masses of scalar and vector fields

The mass-matrix should be normalised in relation to the kinetic terms. The kinetic term of the matter scalar fields $Z$ is independent of the gauging - the gauge fields occur only in the covariantisations. The kinetic term of the $Z_a{}^R$ reads (ignoring the gauge field contributions)

$$-\frac{1}{2}\eta_{RS}\partial_\mu Z_a{}^R \partial^\mu Z_a{}^S - \frac{1}{8}\eta_{RS}\eta_{TU} Z_a{}^R \overleftrightarrow{\partial_\mu} Z_b{}^S\, Z_a{}^T \overleftrightarrow{\partial^\mu} Z_b{}^U\,. \tag{2.31}$$

This should be evaluated in $Z = Z_0$ and expressed in terms of $P_{ab}$, giving

$$-\frac{1}{2}\partial_\mu P_{ab} \partial^\mu P_{ab}\,, \tag{2.32}$$

the standard normalization for scalar fields. Together with the contribution from the potential we therefore get

$$-\frac{1}{2}\partial_\mu P_{ab}\partial^\mu P_{ab} - V_0 - \frac{1}{2}(P_{ab} - \delta_{ab})(P_{cd} - \delta_{cd})\frac{\partial^2 V}{\partial P_{ab}\partial P_{cd}}\bigg|_0 . \tag{2.33}$$

On shifting the scalar fields in the kinetic term by the constant $\delta_{ab}$ we see that the matrix of second derivatives we have calculated in the previous section is precisely the mass-matrix.

The mass-matrix for the SU(1,1) scalars was given is given in A.10 in appendix A. The kinetic and mass terms for these scalars are:

$$-\frac{1}{2}(\partial_\mu x' \partial^\mu x' + \partial_\mu y' \partial^\mu y') - \frac{1}{2}\mathrm{sgn}C_- \sqrt{\Delta_0}(x'^2 + y'^2)\,. \tag{2.34}$$

In the case $C_- < 0$ we have two tachyons. The relation between $\phi_{1,2}$ and $x$ and $y$ is explained in appendix A.

The vector masses follow from the coupling of the vectors to the scalars $Z$. We have in the covariant derivative:

$$\mathcal{D}_\mu Z_a{}^R = \partial_\mu Z_a{}^R - V_{\mu\, ab} Z_b{}^R - A_\mu{}^S g f_{ST}{}^R Z_a{}^T \,. \tag{2.35}$$

After elimination of $V$ the scalar kinetic term becomes:

$$-\frac{1}{2}\eta_{RS} D_\mu Z_a{}^R D^\mu Z_a{}^S - \frac{1}{2}\eta_{RS}\eta_{TU} Z_a{}^R Z_a{}^T D_\mu Z_b{}^S D^\mu Z_b{}^U \,, \tag{2.36}$$

where $D$ contains the gauge field $A$ only. Now substitute $Z = Z_0$ and isolate the $A^2$ terms. The result is, after writing out the indices $R, S$ in terms of $A, B$ and $I, J$:

$$-\frac{1}{2} g_i^2 A_\mu{}^A A^{\mu\, B} f^{(i)}{}_{ACI} f^{(i)}{}_{BCI} - g_i^2 A_\mu{}^A A^{\mu\, K} f^{(i)}{}_{ACI} f^{(i)}{}_{KCI} - \frac{1}{2} g_i^2 A_\mu{}^K A^{\mu\, L} f^{(i)}{}_{KCI} f^{(i)}{}_{LCI} \,. \tag{2.37}$$

We see that there are vector masses only for noncompact groups. That is to be expected, since these are the noncompact generators do not leave $Z_0$ invariant. The second term vanishes for all groups considered. The first term get contributions from gauge groups with an $\mathrm{SO}(2,1)_+$ subgroups, the last one from $\mathrm{SO}(2,1)_-$ subgroups. The masses are proportional to the corresponding $g_i^2$, and independent of the $\mathrm{SU}(1,1)$ angles.

## 3. Semi-simple gauge groups

Table 2 left us with nine allowed groups. In this section we will discuss these nine cases separately. In table 3 we give a list of the allowed groups, with their basic properties. The sign of $C_-$ is important, because it determines the behaviour of the $\mathrm{SU}(1,1)$ scalars: if $C_- < 0$, the $\mathrm{SU}(1,1)$ scalars are at a maximum, if $C_- > 0$ at a minimum. Clearly a stable minimum in all 38 scalar directions requires $C_- > 0$. We now discuss the nine groups in the order of table 3.

| Groups | $C_-$ | $V_0$ |
|---|---|---|
| $\mathrm{SO}(2,1)^3_+ \otimes \mathrm{SO}(3)_+$ | $\frac{1}{2}(g_1^2 + g_2^2 + g_3^2)$ | $\sqrt{a_{12}^2 + a_{13}^2 + a_{23}^2}$ |
| $\mathrm{SO}(2,1)^2_+ \otimes \mathrm{SO}(2,1)^2_-$ | $\frac{1}{2}(g_1^2 + g_2^2)$ | $\lvert a_{12}\rvert$ |
| $\mathrm{SO}(3,1)_+ \otimes \mathrm{SO}(2,1)_+ \otimes \mathrm{SO}(2,1)_-$ | $\frac{1}{2}(3g_1^2 + g_2^2)$ | $\sqrt{3}\lvert a_{12}\rvert$ |
| $\mathrm{SO}(3,1)_+ \otimes \mathrm{SO}(3,1)_+$ | $\frac{3}{2}(g_1^2 + g_2^2)$ | $3\lvert a_{12}\rvert$ |
| $\mathrm{SO}(3)^2_- \otimes \mathrm{SO}(3)^2_+$ | $-\frac{1}{2}(g_1^2 + g_1^2)$ | $-\lvert a_{12}\rvert - 2a_{12}$ |
| $\mathrm{SO}(3)_- \otimes \mathrm{SO}(3,1)_- \otimes \mathrm{SO}(3)_+$ | $-\frac{1}{2}(g_1^2 + g_1^2)$ | $-\lvert a_{12}\rvert - 2a_{12}$ |
| $\mathrm{SO}(3,1)_- \otimes \mathrm{SO}(3,1)_-$ | $-\frac{1}{2}(g_1^2 + g_2^2)$ | $-\lvert a_{12}\rvert - 2a_{12}$ |
| $\mathrm{SL}(3,\mathbb{R})_- \otimes \mathrm{SO}(3)_-$ | $-\frac{1}{2}(g_1^2 + g_2^2)$ | $-\lvert a_{12}\rvert - 2a_{12}$ |
| $\mathrm{SU}(2,1)_+ \otimes \mathrm{SO}(2,1)_+$ | $6g_1^2 + \frac{1}{2}g_2^2$ | $2\sqrt{3}\lvert a_{12}\rvert$ |

**Table 3:** List of possible gauge groups $G$. $V_0$ is the value of the potential in $Z = Z_0$. $C_- > 0$ ($< 0$) implies that the $\mathrm{SU}(1,1)$ scalars are at a minimum (maximum).

In the following sections 3.1–3.9 we will discuss the nine gauge groups which have $\Delta_0 > 0$ and an extremum for the matter scalars in the origin of the scalar manifold. We present for each case the mass-matrix for the matter scalars. The masses of the SU(1,1) scalars are always given by $\mathrm{sgn} C_- \sqrt{\Delta_0}$, indicating that they are always both tachyonic or both positive.

For the first case, the group $\mathrm{SO}(2,1)_+^3 \otimes \mathrm{SO}(3)_+$ presented in section 3.1, we will give full details of the analysis. For the other cases the procedure should then be clear, and we will limit ourselves to the presentation of the results.

### 3.1 $\mathrm{SO}(2,1)_+^3 \otimes \mathrm{SO}(3)_+$

In this case the groups have to be arranged as follows:

$$R, S, \ldots = \overbrace{1\ 2\ 7}^{i=1}\ \overbrace{3\ 4\ 8}^{i=2}\ \overbrace{5\ 6\ 9}^{i=3}\ \overbrace{10\ 11\ 12}^{i=4}\ . \tag{3.1}$$

Then all $V_{0\,ii} = 1/2$ for $i = 1,2,3$, while $V_{044} = 0$, and we have:

$$C_{0-} = \frac{1}{2}(g_1^2 + g_2^2 + g_3^2)\,, \tag{3.2}$$
$$T_{0-} = 0\,, \tag{3.3}$$
$$\Delta_0 = a_{12}^2 + a_{13}^2 + a_{23}^2\,, \tag{3.4}$$
$$R^{(0\,ii)} = \frac{1}{\sqrt{\Delta_0}}\left(a_{i1}^2 + a_{i2}^2 + a_{i3}^2\right). \tag{3.5}$$

giving

$$V_0 = \sqrt{\Delta_0}\,, \tag{3.6}$$
$$\left.\frac{\partial V}{\partial P_{ab}}\right|_0 = 0\,, \tag{3.7}$$
$$\left.\frac{\partial^2 V}{\partial P_{ab}\partial P_{cd}}\right|_0 = \frac{1}{\sqrt{\Delta_0}}\sum_{i=1}^{3}\left(a_{i1}^2 + a_{i2}^2 + a_{i3}^2\right)\left.\frac{\partial^2 V_{ii}}{\partial P_{ab}\partial P_{cd}}\right|_0 - \sum_{i<j} 2a_{ij}\left.\frac{\partial^2 W_{ij}}{\partial P_{ab}\partial P_{cd}}\right|_0 . \tag{3.8}$$

Notice that the group $\mathrm{SO}(3)_+$ does not contribute to the second derivatives, structure constants with indices $IJK$ appear in (2.30) in the third term, but in combination with a structure constant $f_{ABK}$, which $\mathrm{SO}(3)_+$ does not have. The non-vanishing second derivative matrices required for calculating the mass-matrix are:

$$\left.\frac{\partial^2 V_{11}}{\partial P_{ab}\partial P_{cd}}\right|_0 = (\delta_{ac} - \delta_{a1}\delta_{c1})(\delta_{b1}\delta_{d1} + \delta_{b2}\delta_{d2})\,,$$
$$\left.\frac{\partial^2 V_{22}}{\partial P_{ab}\partial P_{cd}}\right|_0 = (\delta_{ac} - \delta_{a2}\delta_{c2})(\delta_{b3}\delta_{d3} + \delta_{b4}\delta_{d4})\,,$$
$$\left.\frac{\partial^2 V_{33}}{\partial P_{ab}\partial P_{cd}}\right|_0 = (\delta_{ac} - \delta_{a3}\delta_{c3})(\delta_{b5}\delta_{d5} + \delta_{b6}\delta_{d6})\,,$$
$$\left.\frac{\partial^2 W_{12}}{\partial P_{ab}\partial P_{cd}}\right|_0 = (\delta_{a1}\delta_{c2} - \delta_{a2}\delta_{c1})(\delta_{b5}\delta_{d6} - \delta_{b6}\delta_{d5})\,,$$



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$$\left.\frac{\partial^2 W_{13}}{\partial P_{ab}\partial P_{cd}}\right|_0 = (\delta_{a1}\delta_{c3} - \delta_{a3}\delta_{c1})(\delta_{b3}\delta_{d4} - \delta_{b4}\delta_{d3}) \,,$$
$$\left.\frac{\partial^2 W_{23}}{\partial P_{ab}\partial P_{cd}}\right|_0 = (\delta_{a2}\delta_{c3} - \delta_{a3}\delta_{c2})(\delta_{b1}\delta_{d2} - \delta_{b2}\delta_{d1}) \,. \tag{3.9}$$

The resulting eigenvalues of $\left.\frac{\partial^2 V}{\partial P_{ab}\partial P_{cd}}\right|_0$ are then

$$\begin{aligned}
&0 \ (6\ \times), \quad \frac{1}{\sqrt{\Delta_0}}(a_{12}^2 + a_{13}^2)\ (6\ \times), \quad \frac{1}{\sqrt{\Delta_0}}(a_{13}^2 + a_{23}^2)\ (6\ \times), \quad \frac{1}{\sqrt{\Delta_0}}(a_{12}^2 + a_{23}^2)\ (6\ \times)\,, \\
&\frac{1}{\sqrt{\Delta_0}}(a_{12}^2 + a_{13}^2) \pm 2a_{23}\ (2\ \times)\,, \quad \frac{1}{\sqrt{\Delta_0}}(a_{13}^2 + a_{23}^2) \pm 2a_{12}\ (2\ \times)\,, \\
&\frac{1}{\sqrt{\Delta_0}}(a_{12}^2 + a_{23}^2) \pm 2a_{13}\ (2\ \times)\,.
\end{aligned} \tag{3.10}$$

One can obtain these eigenvalues as follows. The second derivatives of $V_{ii}$ are diagonal in the parameters $P$, in the sense that always $a = c,\ b = d$. The list of elements of $P$ giving a nonzero second derivative of $V_{ii}$ is:[2]

$$\begin{aligned}
V_{11}: &\ \frac{1}{\sqrt{\Delta_0}}(a_{12}^2 + a_{13}^2)\,[\,21,\ 22,\ 31,\ 32,\ 41,\ 42,\ 51,\ 52,\ 61,\ 62\,]\,, \\
V_{22}: &\ \frac{1}{\sqrt{\Delta_0}}(a_{12}^2 + a_{23}^2)\,[\,13,\ 14,\ 33,\ 34,\ 43,\ 44,\ 53,\ 54,\ 63,\ 64\,]\,, \\
V_{33}: &\ \frac{1}{\sqrt{\Delta_0}}(a_{13}^2 + a_{23}^2)\,[\,15,\ 16,\ 25,\ 26,\ 45,\ 46,\ 55,\ 56,\ 65,\ 66\,]\,.
\end{aligned}$$

On the other hand, $W_{ij}$ has a non-vanishing second derivative for the following pairs of elements of $P$:

$$\begin{aligned}
W_{12}: &\ -2a_{12}[\,(15,\ 26),\ (16,\ 25),\ (25,\ 16),\ (26,\ 15)\,]\,, \\
W_{13}: &\ -2a_{13}[\,(13,\ 34),\ (14,\ 33),\ (33,\ 14),\ (34,\ 13)\,]\,, \\
W_{23}: &\ -2a_{23}[\,(21,\ 32),\ (22,\ 31),\ (31,\ 22),\ (32,\ 21)\,]\,.
\end{aligned}$$

The six eigenvalues $(a_{13}^2 + a_{23}^2)/\sqrt{\Delta_0}$ are associated with the six diagonal elements of the second derivative matrix coming from $V_{11}$: 41, 42, 51, 52, 61, 62 are the corresponding elements of $P$. The six eigenvalues $(a_{12}^2 + a_{23}^2)/\sqrt{\Delta_0}$ and $(a_{12}^2 + a_{13}^2)/\sqrt{\Delta_0}$ arise in a similar way from the derivatives of $V_{22}$ and $V_{33}$, respectively. The remaining diagonal contributions from $V_{ii}$ combine with the corresponding derivatives of $W_{ij}$: $V_{11}$ with $W_{23}$, etc. These elements of $P$ give rise to $2 \times 2$ submatrices in the matrix of second derivatives, with the eigenvalues as indicated. The zero eigenvalues of the second derivative matrix are associated with the elements 11, 12, 23, 24, 35, 36 of $P$ which do not occur anywhere in the second derivatives.

The six zero eigenvalues indicate that the solution $Z = Z_0$ breaks the gauge symmetry. In each of the $SO(2,1)_+$ groups the $SO(2,1)$ symmetry is broken to $U(1)$. The six massless scalars give masses to the gauge vectors, as we have seen in section 2.4.

[2] We use the notation $ab$ to indicate the second derivative with respect to $P_{ab}$ (diagonal elements of the $36 \times 36$ second derivative matrix), and $(ab, cd)$ for the second derivative with respect to $P_{ab}$ and $P_{cd}$ (off-diagonal elements of the second derivative matrix).

The extremum we have obtained in this case is not stable. Assume that all 36 eigenvalues are nonnegative. If $(a_{12}^2 + a_{13}^2)/\sqrt{\Delta_0} \pm 2a_{23}$ has to be positive then necessarily

$$|a_{23}| < \sqrt{\Delta_0}(\sqrt{2} - 1)\,. \tag{3.11}$$

This has to be true then for $a_{13}$ and $a_{23}$ as well, implying

$$\Delta_0 < 3\Delta_0(\sqrt{2} - 1)^2 < \Delta_0\,. \tag{3.12}$$

Therefore our assumption that all eigenvalues are positive must be false. In fact, there are two, four or six negative eigenvalues. To have six negative eigenvalues choose $a_{12} = a_{23} = a_{13} = \sqrt{\Delta_0/3}$.

### 3.2 $\mathrm{SO}(2,1)_+^2 \otimes \mathrm{SO}(2,1)_-^2$

The embedding of the subgroups is as follows:

$$R, S, \ldots = \overbrace{1\ 2\ 7}^{i=1}\ \overbrace{3\ 4\ 8}^{i=2}\ \overbrace{5\ 9\ 10}^{i=3}\ \overbrace{6\ 11\ 12}^{i=4}\,. \tag{3.13}$$

Although the $V_{0\,ii}$ vanish for each $\mathrm{SO}(2,1)_-$ factor, the second derivatives don't. We now have:

$$\begin{aligned} C_{0-} &= \frac{1}{2}(g_1^2 + g_2^2)\,, \\ T_{0-} &= 0\,, && (3.14) \\ \Delta_0 &= a_{12}^2\,, && (3.15) \\ R^{(0\,ii)} &= \frac{1}{|a_{12}|}\,(a_{i1}^2 + a_{i2}^2)\,, && (3.16) \\ V_0 &= |a_{12}|\,. && (3.17) \end{aligned}$$

Now we find the following eigenvalues:

$$\begin{aligned} &0\ (8\ \times)\,, \quad |a_{12}|\ (4\ \times)\,, \quad |a_{12}| + R^{(0\,33)}\ (8\ \times)\,, \quad |a_{12}| + R^{(0\,44)}\ (8\ \times)\,, \\ &R^{(0\,33)}\ (2\ \times)\,, \quad R^{(0\,44)}\ (2\ \times)\,, \quad 2a_{12}\ (2\ \times)\,, \quad -2a_{12}\ (2\ \times)\,. \end{aligned} \tag{3.18}$$

$Z = Z_0$ breaks the gauge symmetry to $\mathrm{U}(1)^4$, so the eight zero eigenvalues correspond to the Goldstone bosons that produce the masses of the gauge fields.

There are two negative eigenvalues, proportional to $a_{12}$. It does not help to set $\alpha_1 = \alpha_2$ to eliminate them, since this would make $\Delta = 0$ and invalidate the analysis of the $\mathrm{SU}(1,1)$ scalars.

### 3.3 $\mathrm{SO}(3,1)_+ \otimes \mathrm{SO}(2,1)_+ \otimes \mathrm{SO}(2,1)_-$

In this case the groups are arranged as follows:

$$R, S, \ldots = \overbrace{1\ 2\ 3\ 10\ 11\ 12}^{i=1}\ \overbrace{4\ 5\ 7}^{i=2}\ \overbrace{6\ 8\ 9}^{i=3}\,. \tag{3.19}$$

The rotation subgroup of the SO(3,1) subgroup is embedded on the indices $10\dots12$, the boosts on the indices $1\dots3$. Here we have:

$$\begin{aligned}
C_{0-} &= \frac{1}{2}(3g_1^2+g_2^2)\,, && (3.20)\\
T_{0-} &= 0\,, && (3.21)\\
\Delta_0 &= 3a_{12}^2\,, && (3.22)\\
R^{(0\,ii)} &= \frac{1}{\sqrt{3}|a_{12}|}\left(3a_{i1}^2+a_{i2}^2\right), && (3.23)\\
V_0 &= \sqrt{3}|a_{12}|\,. && (3.24)
\end{aligned}$$

The complete list of eigenvalues in this case is:

$$\begin{aligned}
&0\ (7\,\times)\,,\quad \frac{2|a_{12}|}{\sqrt{3}}\ (5\,\times)\,,\quad \sqrt{3}|a_{12}|\ (6\,\times)\,,\\
&\frac{|a_{12}|(1+\sqrt{13})}{\sqrt{3}}\ (3\,\times)\,,\quad \frac{|a_{12}|(1-\sqrt{13})}{\sqrt{3}}\ (3\,\times)\,,\\
&\frac{|a_{12}|(1+\sqrt{37})}{\sqrt{3}}\ (1\,\times)\,,\quad \frac{|a_{12}|(1-\sqrt{37})}{\sqrt{3}}\ (1\,\times)\,,\\
&\frac{1}{\sqrt{3}|a_{12}|}(3a_{12}^2+3a_{13}^2+a_{23}^2)\ (4\,\times)\,,\quad \frac{1}{\sqrt{3}|a_{12}|}(2a_{12}^2+3a_{13}^2+a_{23}^2)\ (6\,\times)\,. \qquad (3.25)
\end{aligned}$$

Altogether then we find that this group gives rise to four negative eigenvalues of the mass-matrix. The zero eigenvalues can again correspond to the Goldstone bosons of the broken noncompact gauge symmetries. The negative eigenvalues are proportional to $|a_{12}|$, which however we cannot set to zero because then also $\Delta_0 = 0$.

### 3.4 $\mathrm{SO}(3,1)_+ \otimes \mathrm{SO}(3,1)_+$

In this case the groups have to be arranged as follows:

$$R,S,\ldots = \overbrace{1\;2\;3\;10\;11\;12}^{i=1}\;\overbrace{4\;5\;6\;7\;8\;9}^{i=2}\,. \qquad (3.26)$$

In this case the rotation subgroup of the two SO(3,1) subgroups is embedded on the indices $7\dots12$, the boosts on the indices $1\dots6$. Here $V_{0\,11}$ and $V_{0\,22}$ each are $\frac{3}{2}$, $W_{12}=0$. and we have:

$$\begin{aligned}
C_{0-} &= \frac{3}{2}\,(g_1^2+g_2^2)\,, && (3.27)\\
T_{0-} &= 0\,, && (3.28)\\
\Delta_0 &= 9a_{12}^2\,, && (3.29)\\
R^{(0\,ii)} &= \frac{1}{|a_{12}|}\left(a_{i1}^2+a_{i2}^2\right), && (3.30)\\
V_0 &= 3|a_{12}|\,. && (3.31)
\end{aligned}$$

The eigenvalues are:

$$0\ (15\,\times)\,,\quad 2|a_{12}|\ (10\,\times)\,,\quad 4|a_{12}|\ (9\,\times)\,,\quad 8|a_{12}|\ (1\,\times)\,,\quad -4|a_{12}|\ (1\,\times)\,. \qquad (3.32)$$

In this case there is a single negative eigenvalue. There are now more zero eigenvalues than the number required by the breaking of gauge invariance.



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### 3.5 $SO(3)^2_- \otimes SO(3)^2_+$

In the case of four SO(3) groups these are arranged over the index values $R, S = 1, \ldots 12$ as follows:

$$R, S, \ldots = \overbrace{1\ 2\ 3}^{i=1}\ \overbrace{4\ 5\ 6}^{i=2}\ \overbrace{7\ 8\ 9}^{i=3}\ \overbrace{10\ 11\ 12}^{i=4}\,. \tag{3.33}$$

This simplifies considerably the expressions derived in the section 2 for the potential, and its first and second derivatives in the point $Z_0$. The only terms which contribute are those for which the indices on the structure constants are either $ABC$ (in the range $1 \ldots 6$) or $IJK$ (in the range $7 \ldots 12$). We find

$$C_{0-} = -\frac{1}{2}(g_1^2 + g_2^2)\,, \tag{3.34}$$
$$T_{0-} = 2a_{12}\,, \tag{3.35}$$
$$\Delta_0 = a_{12}^2\,, \tag{3.36}$$
$$R^{(0\,ii)} = \frac{1}{|a_{12}|}\sum_i (a_{i1}^2 + a_{i2}^2)\,, \tag{3.37}$$
$$V_0 = -|a_{12}| - 2a_{12}\,. \tag{3.38}$$

The eigenvalues of the mass-matrix are

$$-2a_{12}\ (36\ \times)\,. \tag{3.39}$$

If the SU(1, 1) angles are chosen such that $V_0$ is positive (de Sitter) then the eigenvalues of the mass-matrix are also positive for all 36 scalars.

In the present case $C_- < 0$ in $Z_0$, which implies that for the SU(1, 1) scalars we have a maximum. So in this example there are two tachyons in the SU(1, 1) sector. In [8] we showed in the truncated model with two scalars that $V_{11}$ and $V_{22}$, and therefore $C_-$, change sign on a circle around $Z_0$. This turns the maximum for the SU(1, 1) scalars into a minimum. It will be interesting to see if this phenomenon also holds when all 36 matter scalars are taken into account.

### 3.6 $SO(3,1)_- \otimes SO(3)_- \otimes SO(3)_+$

In this case the groups have to be arranged as follows:

$$R, S, \ldots = \overbrace{1\ 2\ 3\ 10\ 11\ 12}^{i=1}\ \overbrace{4\ 5\ 6}^{i=2}\ \overbrace{7\ 8\ 9}^{i=3}\,. \tag{3.40}$$

In this case the rotation subgroup of the SO(3, 1) subgroup is embedded on the indices $1 \ldots 3$, the boosts on the indices $10 \ldots 12$. Here we have:

$$C_{0-} = -\frac{1}{2}(g_1^2 + g_2^2)\,, \tag{3.41}$$
$$T_{0-} = 2a_{12}\,, \tag{3.42}$$
$$\Delta_0 = a_{12}^2\,, \tag{3.43}$$
$$R^{(0\,ii)} = \frac{1}{|a_{12}|}\left(a_{i1}^2 + a_{i2}^2\right)\,, \tag{3.44}$$

$$V_0 = -|a_{12}| - 2a_{12}\,. \tag{3.45}$$

We have the following eigenvalues:

$$\begin{aligned} &0\ (3\ \times)\,, \quad -2a_{12}\ (18\ \times)\,, \quad 2|a_{12}| - 2a_{12}\ (9\ \times)\,, \\ &2|a_{12}| - 4a_{12}\ (5\ \times)\,, \quad 2|a_{12}| + 2a_{12}\ (1\ \times)\,. \end{aligned} \tag{3.46}$$

For $a_{12} < 0$ (de Sitter) there are no negative eigenvalues, and four zero eigenvalues. However, since $\mathrm{sgn} C_- < 0$, the SU(1,1) scalars produce the instability.

### 3.7 $\mathrm{SO}(3,1)_- \otimes \mathrm{SO}(3,1)_-$

In this case the groups have to be arranged as follows:

$$R, S, \ldots = \overbrace{1\ 2\ 3\ 10\ 11\ 12}^{i=1}\ \overbrace{4\ 5\ 6\ 7\ 8\ 9}^{i=2}\,. \tag{3.47}$$

In this case the rotation subgroup of the two SO(3,1) subgroups is embedded on the indices $1\ldots6$, the boosts on the indices $7\ldots12$. Here $V_{0\,11}$ and $V_{0\,22}$ each are $-\frac{1}{2}$, $W_{12} = 1$, and we have:

$$C_{0-} = -\frac{1}{2}(g_1^2 + g_2^2)\,, \tag{3.48}$$

$$T_{0-} = 2a_{12}\,, \tag{3.49}$$

$$\Delta_0 = a_{12}^2\,, \tag{3.50}$$

$$R^{(0\,ii)} = \frac{1}{|a_{12}|}\left(a_{i1}^2 + a_{i2}^2\right), \tag{3.51}$$

$$V_0 = -|a_{12}| - 2a_{12}\,. \tag{3.52}$$

To make $V_0$ positive we have to choose $a_{12} < 0$. The eigenvalues of the mass-matrix are:

$$\begin{aligned} &0\ (6\ \times)\,, \quad 2|a_{12}| - 2a_{12}\ (18\ \times)\,, \\ &2|a_{12}| - 4a_{12}\ (10\ \times)\,, \quad 2|a_{12}| + 2a_{12}\ (2\ \times)\,. \end{aligned} \tag{3.53}$$

For $a_{12} < 0$ there are no negative eigenvalues, and eight zero eigenvalues. However, since $C_- < 0$, the SU(1,1) scalars produce the instability.

### 3.8 $\mathrm{SL}(3,\mathbb{R})_- \otimes \mathrm{SO}(3)_-$

In this case the groups are arranged as follows:

$$\overbrace{1\ 2\ 3\ 4\ 5\ 7\ 8\ 9}^{i=1}\ \overbrace{4\ 5\ 6}^{i=2}\,. \tag{3.54}$$

The rotation subgroup of $\mathrm{SL}(3,\mathbb{R})$ is placed on the indices $7,\ldots 9$, the noncompact generators on $1,\ldots 5$. In this case we have

$$C_{0-} = -\frac{1}{2}(g_1^2 + g_2^2)\,, \tag{3.55}$$

$$T_{0-} = 2a_{12}\,, \tag{3.56}$$

$$\Delta_0 = a_{12}^2\,, \tag{3.57}$$
$$R^{(0\,ii)} = \frac{1}{|a_{12}|}\left(a_{i1}^2 + a_{i2}^2\right), \tag{3.58}$$
$$V_0 = -|a_{12}| - 2a_{12}\,. \tag{3.59}$$

The eigenvalues are:

$$0\ (5\ \times)\,,\quad -2a_{12}\ (6\ \times)\,,\quad 6|a_{12}|+4a_{12}\ (3\ \times)\,,\quad 6|a_{12}|-2a_{12}\ (15\ \times)\,,$$
$$6|a_{12}|-6a_{12}\ (7\ \times)\,, \tag{3.60}$$

In the de Sitter case ($a_{12} < 0$) there are no negative eigenvalues, but of course the $\mathrm{SU}(1,1)$ scalars are tachyons because $C_- < 0$. The five zero eigenvalues are associated to the Goldstone fields which give mass to the gauge fields corresponding to the non–compact gauge generators.

In the AdS case ($a_{12} > 0$) the potential in the extremum is $V_0 = -3|a_{12}|$, in that case there are seven additional zero eigenvalues.

### 3.9 $\mathrm{SU}(2,1)_+ \otimes \mathrm{SO}(2,1)_+$

The gauge generators are arranged as follows:

$$\overbrace{1\ 2\ 3\ 4\ 7\ 8\ 9\ 10}^{i=1}\ \overbrace{5\ 6\ 11}^{i=2}\ . \tag{3.61}$$

The compact generators of $\mathrm{SU}(2,1)$ are placed on the indices $7,\dots 10$, the noncompact ones on $1,\dots 4$. In this case we have:

$$C_{0-} = 6g_1^2 + \frac{1}{2}g_2^2\,, \tag{3.62}$$
$$T_{0-} = 0\,, \tag{3.63}$$
$$\Delta_0 = 12a_{12}^2\,, \tag{3.64}$$
$$R^{(0\,ii)} = \frac{1}{\sqrt{3}|a_{12}|}\left(6a_{i1}^2 + \frac{1}{2}\,a_{i2}^2\right), \tag{3.65}$$
$$V_0 = 2\sqrt{3}\,|a_{12}| > 0\,. \tag{3.66}$$

Again we find a de Sitter vacuum. The mass of the $\mathrm{SU}(1,1)$ scalars is $2\sqrt{3}|a_{12}|$ while in the matter sector the mass eigenvalues are:

$$0\ (6\ \times)\,,\quad \sqrt{3}|a_{12}|\ (12\ \times)\,,\quad 2\sqrt{3}|a_{12}|\ (10\ \times)\,,\quad \sqrt{3}|a_{12}|(1\pm 2\sqrt{2}\ (4\ \times)\,. \tag{3.67}$$

The spectrum contains four tachyonic modes. The six zero modes correspond to the Goldstone bosons of the broken noncompact generators.

## 4. Conclusions

In this paper we have searched in gauged $N = 4$ supergravity with semi-simple gauge groups for examples that give a positive cosmological constant and a non-negative mass-matrix for all scalar fields. In all examples considered the scalar potential does allow a positive extremum, but we have always found tachyons. In our search we limited ourselves to six vector multiplets, and therefore 36 matter scalars.

We found that there are two classes of gauge groups in the nine that were considered. Five have a positive extremum for all values of the parameters in the problem (coupling constants, SU(1,1) angles), have tachyonic modes in the matter sector, and positive $m^2$ for the SU(1,1) scalars. In four cases we find that the sign of potential in the extremum depends on the choice of parameters, that the matter scalars all have positive $m^2$ (if the parameters are chosen such that the extremum occurs for positive potential), and the two SU(1,1) scalars are the tachyons. These last four are precisely all the cases with an $\mathrm{SO}(3)^2_-$ subgroup. This distinction is not yet understood.

Certain features of the mass spectrum are clear. We always find the appropriate number of massless modes to provide for the Goldstone bosons of the broken gauge symmetries. In a number of cases we have more zero modes than Goldstone bosons. This might be an indication that such models can be embedded in a gauged supergravity with $N > 4$, and that these extra zero modes are related to Goldstone bosons that occur in the larger supergravity theory.

Another feature of gauged supergravity theories, remarked in [24], is the fact that the mass spectrum of gauged supergravity is often such that $3m^2/V_0$ is an integer. This is an interesting observation, since it makes such models unsuitable as candidates for slow-roll inflationary scenarios. We find the same property for the ratio of mass and potential in cases where the gauge group is a product of two simple groups, with one exception in the $\mathrm{SU}(2,1)_+ \otimes \mathrm{SO}(2,1)_+$ gauging in section 3.9, where this ratio contains a factor $\sqrt{2}$. For groups that have three simple factors and positive cosmological constant, the ratio becomes parameter dependent for some of the masses. However, also in these cases one does not have enough freedom to tune the parameters such that all tachyon masses become small.

As far as supersymmetry breaking is concerned the analysis of [21] can be applied. For the groups in sections 3.1–3.4 and 3.9 we always have $V_0 > 0$, and supersymmetry is completely broken. In sections 3.5–3.8 $V_0$ depends on the sign of $a_{12}$, and one finds that for $V_0 > 0$ supersymmetry is completely broken, for $V_0 \leq 0$ $N = 4$ supersymmetry is preserved. The supersymmetry variations of the fermions in these last three cases are proportional to $g_1\Phi_{(1)} - ig_2\Phi_{(2)}$, for which

$$|g_1\Phi_{(1)} - ig_2\Phi_{(2)}|^2 = R^{(0\,11)} + R^{(0\,22)} + 2I^{(0\,12)} = 2\left(|a_{12}| - a_{12}\right). \tag{4.1}$$

This vanishes for $a_{12} > 0$, leading to unbroken supersymmetry in AdS spacetime. In these cases the potential in the AdS extremum is $V_0 = -3|a_{12}|$. This value follows also from the integrability condition arising from the supersymmetry variation of the gravitinos.

The relation of our $N = 4$ work with the $N = 2$ results of [9] remains intriguing. There are three cases presented in [9]. The one which seems most directly related to $N = 4$ supergravity has five vector multiplets, gauging, with the graviphoton, a six-dimensional $\mathrm{SO}(2,1)\otimes\mathrm{SO}(3)$ group. In addition, the model has two hyper–multiplets, giving a total of 18 scalar fields. The scalar manifold is

$$\left[\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(2,4)}{\mathrm{SO}(2)\times\mathrm{SO}(4)}\right] \times \left[\frac{\mathrm{SO}(4,2)}{\mathrm{SO}(4)\times\mathrm{SO}(2)}\right], \tag{4.2}$$

where the last factor corresponds to the hyper–multiplets. The two $\mathrm{SU}(1,1)$ scalars play a similar role as in $N = 4$ and allow the introduction of $\mathrm{SU}(1,1)$ mixing angles in the coupling to the vectors. The gauge group is embedded in both $\mathrm{SO}(4,2)$ groups, and it was shown in [9] how to obtain the manifold (4.2) from a truncation of

$$\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(6,6)}{\mathrm{SO}(6)\times\mathrm{SO}(6)}\,. \tag{4.3}$$

In the $N = 4$ case the gauge group would be $[\mathrm{SO}(2,1)\times\mathrm{SO}(3)]^2$, the $N = 2$ group being the diagonal subgroup. The only way to embed this group in $N = 4$ is as $\mathrm{SO}(3)_- \times \mathrm{SO}(2,1)_+ \times \mathrm{SO}(2,1)_- \times \mathrm{SO}(3)_+$, which we have in table 2 in section 2.2 but which does not have an extremum in the matter and $\mathrm{SU}(1,1)$ scalars. We may restrict our analysis to points in the moduli space preserving the diagonal compact subgroup $\mathrm{SO}(2)_D \times \mathrm{SO}(3)_D$ of the gauge group (in this analysis we set the $g_1 = g_4,\ g_2 = g_3,\ \alpha_1 = \alpha_4,\ \alpha_2 = \alpha_3$) . To this end it suffices to study the behavior of the scalar potential as a function of the $\mathrm{SO}(2)_D \times \mathrm{SO}(3)_D$ singlets only (indeed, the scalar potential being invariant in particular under $\mathrm{SO}(2)_D\times\mathrm{SO}(3)_D$, its dependence on scalar fields which are not singlets with respect to this group will be at least quadratic, and therefore in order to analyse the critical points of the potential exhibiting $\mathrm{SO}(2)_D \times \mathrm{SO}(3)_D$ symmetry, we may set these fields to zero). These singlets in the matter sector are four. This can be seen by first considering the action of the compact subgroup $\mathrm{SO}(3)_- \times \mathrm{SO}(2)_+ \times \mathrm{SO}(2)_- \times \mathrm{SO}(3)_+$ under which the matter scalar fields transform as follows:

$$(\mathbf{3}+\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1}) + (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{3}+\mathbf{1}) + (\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1}) + (\mathbf{3}+\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{3}+\mathbf{1})\,. \tag{4.4}$$

The scalars in $(\mathbf{3}+\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})$ and $(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{3}+\mathbf{1})$ correspond to the 16 scalars in the $\mathrm{SO}(2,4)/\,\mathrm{SO}(2)\times\mathrm{SO}(4)$ and $\mathrm{SO}(4,2)/\,\mathrm{SO}(4)\times\mathrm{SO}(2)$ cosets, those in $(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})$ and $(\mathbf{3}+\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{3}+\mathbf{1})$ will be projected out by the $N = 4 \to N = 2$ truncation. Branching the above representations with respect to $\mathrm{SO}(2)_D \times \mathrm{SO}(3)_D$ we obtain two singlets $\varphi_{1,2}$ from $(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})$, one singlet $\xi$ from $(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{3})$ and finally the $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$ singlet $\xi_0$. A non vanishing value for $\xi$ would therefore break $\mathrm{SO}(3)_- \times \mathrm{SO}(3)_+$ to $\mathrm{SO}(3)_D$. Although the potential does not have critical points in the $\mathrm{SU}(1,1)$ and $\varphi_{1,2}$, $\xi_0$, $\xi$ scalars, a numerical analysis shows that it can be extremized with respect to all the scalars except $\xi$, for large enough values of $|\xi|$ (at the origin of the matter sector the potential does not have an extremum with respect to the $\mathrm{SU}(1,1)$ scalars). In other words there is an infinite range of values for $\xi$ for which all the other scalars can be fixed so that the potential has a run–away behavior in $\xi$ field only. In these points the potential is positive. It seems therefore that in lifting the $N = 2$ model with vector and hyper–multiplets to $N = 4$ new scalar fields emerge which destabilize the $N = 2$ de Sitter vacuum.

The program presented in this paper can be extended in many directions. One possibility could be to consider contractions ($CSO$ groups) of the gauge groups studied here. Also the present analysis can be generalized to include Peccei–Quinn symmetries. These symmetries naturally appear in Scherk–Schwarz reductions [11, 15, 17], therefore such an investigation could also elucidate the relation between $N = 4$ supergravity with nonzero $\mathrm{SU}(1,1)$ angles and string theory. Gaugings related to Scherk–Schwarz reductions from



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$D = 5$ could also provide new ways for obtaining some of the $N = 2$ models with stable de Sitter vacua [9] as effective realizations of a larger gauged $N = 4$ theory (although a definite statement about this possibility would require an analysis that we postpone to future work). As an example we could consider the $N = 4$ supergravity coupled to six matter multiplets and gauge a group of the form $G_{S-S} \times \mathrm{SO}(2,1) \times \mathrm{SO}(3)$ where $G_{S-S}$ is a non–semi-simple gauge group á la Scherk–Schwarz. It was shown indeed that, for a certain choice of the gauge parameters, the effect of $G_{S-S}$ alone amounts to a partial supersymmetry breaking from $N = 4$ to $N = 2$, in which the final effective supergravity is coupled to five vector multiplets and no hyper–multiplet. This is the ungauged version of one of the models considered in [9]. Therefore if we gauge in the $N = 4$ theory the group $G_{S-S} \times \mathrm{SO}(2,1) \times \mathrm{SO}(3)$ and introduce $\mathrm{SU}(1,1)$ angles for each simple factor, we would be left at the level of the $N = 2$ effective model with a surviving $\mathrm{SO}(2,1) \times \mathrm{SO}(3)$ gauge group. However, in order to recover in this framework one of the models without hyper–multiplets constructed in [9] a crucial ingredient would be the presence of the Fayet–Iliopoulos term corresponding to the $\mathrm{SO}(3)$ factor, whose $N = 4$ origin is still unclear.

There are (at least) two aspects of this program which remain to be elucidated. The first is the existence or non-existence of stable de Sitter vacua in $N = 4$ supergravity, the second is the relation of gauged $N = 4$ supergravity with $\mathrm{SU}(1,1)$ angles with ten and/or eleven dimensions. If such a relation could be established, the no-go theorem of [3, 4, 5] would come into play, and one would know that to solve the first problem would require flux reduction or hyperbolic reduction and/or other ways around the no-go theorem [6].

## Acknowledgments

MT would like to thank S. Ferrara for useful remarks. The work of DBW is part of the research programme of the “Stichting voor Fundamenteel Onderzoek van de Materie” (FOM). This work is supported in part by the European Commission RTN programme HPRN-CT-2000-00131, in which MdR and DBW are associated to the University of Utrecht. The work of MT is supported by a European Community Marie Curie Fellowship under contract HPRN-CT-2001-01276. SP thanks the ICTP in Trieste and the Centre for Theoretical Physics in Groningen for their hospitality.

## A. $\mathrm{SU}(1,1)$ scalars and angles

We parametrise the scalars of the $\mathrm{SU}(1,1)/\mathrm{U}(1)$ coset in a suitable $\mathrm{U}(1)$ gauge as

$$\phi_1 = \frac{1}{\sqrt{1-r^2}}\,, \qquad \phi_2 = \frac{r e^{i\varphi}}{\sqrt{1-r^2}}\,. \tag{A.1}$$

The scalars $r$ and $\varphi$ then appear in the potential (2.1) through

$$\begin{aligned} R^{(ij)} &= \frac{g_i g_j}{2}(\Phi_i^* \Phi_j + \Phi_j^* \Phi_i) \\ &= g_i g_j \left( \cos(\alpha_i - \alpha_j) \frac{1+r^2}{1-r^2} - \frac{2r}{1-r^2} \cos(\alpha_i + \alpha_j + \varphi) \right), \end{aligned} \tag{A.2}$$

$$
\begin{aligned}
I^{(ij)} &= \frac{g_i g_j}{2i}(\Phi_i^*\Phi_j - \Phi_j^*\Phi_i) \\
&= -g_i g_j \sin(\alpha_i - \alpha_j)\,.
\end{aligned}
\tag{A.3}
$$

Introducing

$$
C_\pm = \sum_{ij} g_i g_j \cos(\alpha_i \pm \alpha_j) V_{ij}\,, \qquad S_+ = \sum_{ij} g_i g_j \sin(\alpha_i + \alpha_j) V_{ij}\,, \tag{A.4}
$$

$$
T_- = \sum_{ij} g_i g_j \sin(\alpha_i - \alpha_j) W_{ij}\,, \tag{A.5}
$$

we rewrite the potential as

$$
V = C_- \frac{1+r^2}{1-r^2} - \frac{2r}{1-r^2}\left(C_+\cos\varphi - S_+ \sin\varphi\right) - T_-\,. \tag{A.6}
$$

This extremum in $r$ and $\varphi$ takes on the form

$$
\begin{aligned}
\cos\varphi_0 &= \frac{s_1 C_+}{\sqrt{C_+^2 + S_+^2}}\,, \qquad \sin\varphi_0 = -\frac{s_1 S_+}{\sqrt{C_+^2+S_+^2}}\,, \\
r_0 &= \frac{1}{\sqrt{C_+^2+S_+^2}}\left(s_1 C_- + s_2\sqrt{\Delta}\right)\,, \qquad \Delta \equiv C_-^2 - C_+^2 - S_+^2\,,
\end{aligned}
\tag{A.7}
$$

where $s_1$ and $s_2$ are signs. These are determined by requiring $r_0 \geq 0$ and $r_0 < 1$, this gives $s_1 = \mathrm{sgn} C_-$ and $s_2 = -1$. Substitution of $r_0$ and $\varphi_0$ in $V$ leads to

$$
V_0 = \mathrm{sgn} C_- \sqrt{\Delta} - T_-\,. \tag{A.8}
$$

In the case that all SU$(1,1)$ angles $\alpha_i$ vanish, $S_+ = T_- = 0$ and $C_- = C_+$, and one finds $r_0 = 1$ and $\Delta = 0$. This is a singular point of the parametrisation, which we will exclude. It is generalisation of the Freedman-Schwarz potential [23] to the case of general matter coupling.

For the kinetic term and mass-matrix of the SU$(1,1)$ scalars we introduce:

$$
\begin{aligned}
x' &= \frac{2}{(1-r_0)^2}(r\cos\varphi - r_0\cos\varphi_0)\,, \\
y' &= \frac{2}{(1-r_0)^2}(r\sin\varphi - r_0\sin\varphi_0)\,.
\end{aligned}
\tag{A.9}
$$

In these variables we find

$$
\mathcal{L}_\phi = -\frac{1}{2}\left(\frac{1-r_0^2}{1-r^2}\right)^2\left(\partial_\mu x'\partial^\mu x' + \partial_\mu y'\partial^\mu y'\right) - V_0 - \frac{1}{2}\mathrm{sgn} C_- \sqrt{\Delta}\left(x'^2 + y'^2\right) + \dots \tag{A.10}
$$

It is clear that we have two tachyons for $\mathrm{sgn} C_- < 0$, and two positive mass scalars for $\mathrm{sgn} C_- > 0$.

It is useful to also analyse the kinetic term of the vectors, since positivity[3] of these terms might give further constraints on the SU$(1,1)$ scalars, to which the vectors couple.

[3] In [8] it was incorrectly stated that the kinetic terms of the vectors acquire the wrong sign. The discussion below should clarify this point.

In $Z = Z_0$ we have

$$\mathcal{L}_{\rm kin,A} = -\frac{1}{4} F_{\mu\nu}{}^{+A} F^{\mu\nu\,+A} \left( -\frac{\phi^1{}_{(A)} - \phi^2{}_{(A)}}{\Phi_{(A)}} + \frac{2}{\left|\Phi_{(A)}\right|^2} \right) - \\ -\frac{1}{4} F_{\mu\nu}{}^{+I} F^{\mu\nu\,+I} \left( \frac{\phi^1{}_{(I)} - \phi^2{}_{(I)}}{\Phi_{(I)}} \right) + \text{h.c.} \tag{A.11}$$

Here $\Phi_{(R)} \equiv e^{i\alpha_R}\phi^1 + e^{-i\alpha_R}\phi^2$. We find after some algebra:

$$-\frac{1}{4} F_{\mu\nu}{}^{+A} F^{\mu\nu\,+A} S^{(A)} - \frac{1}{4} F_{\mu\nu}{}^{+I} F^{\mu\nu\,+I} S^{*\,(I)} + \text{h.c.} \tag{A.12}$$

Here $S^{(R)}$ is given by

$$S^{(R)} = \frac{1 + r e^{i(\varphi + 2\alpha_R)}}{1 - r e^{i(\varphi + 2\alpha_R)}} \,. \tag{A.13}$$

The imaginary part of $S$ gives a total derivative in the kinetic term. Therefore the kinetic terms are determined by

$$Re S^{(R)} = \frac{1 - r^2}{1 + r^2 - 2r\cos(\varphi + 2\alpha_R)} \,, \tag{A.14}$$

showing that the domain of positivity is $r < 1$. In the extremum for the SU(1,1) scalars, and in the origin of the matter scalar manifold, this becomes for the i'th factor of the gauge group

$$Re S^{(i)} = \frac{g_i^2}{R^{(0\,ii)}} \,, \tag{A.15}$$

where $R^{0\,ii}$ is given in (2.20). Indeed, we find that always $R^{0\,ii} > 0$.

## B. Generators and structure constants

Our conventions for the structure constant are the following:

$$[T_R, T_S] = i f_{RS}{}^U T_U \,, \qquad f_{RST} \equiv f_{RS}{}^U \eta_{TU} \,, \tag{B.1}$$

where the structure constants $f$ are real. The structure constants $f_{RST}$ are completely antisymmetric, since $\eta RS$ is proportional to the Cartan-Killing metric $\mathrm{tr}\, T_R T_S$ of the gauge group. The factor 2 in this proportionality we absorb in a redefinition of the coupling constants $g_i$. Our choice for the generators is based on the Gell-Mann matrices (extended



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| Group | Generators (L; K) |
|---|---|
| SO(3) | $L_\alpha = \lambda_7, -\lambda_5, \lambda_2$ |
| SO(2,1) | $L_3 = \lambda_2,\ K_{1,2} = i\lambda_6, i\lambda_4,$ |
| SO(3,1) | $L_\alpha = \lambda_7, -\lambda_5, \lambda_2,\ K_\alpha = i\lambda_9, i\lambda_{10}, i\lambda_{11}$ |
| SL(3, $\mathbb{R}$) | $L_\alpha = \lambda_7, -\lambda_5, \lambda_2,\ K_\alpha = i\lambda_6, i\lambda_4, i\lambda_1,\ K_4 = i\lambda_3, K_5 = i\lambda_8$ |
| SU(2,1) | $L_\alpha = \lambda_1, \lambda_2, \lambda_3,\ L_4 = \lambda_8;\ K_1 = i\lambda_4,\ K_2 = i\lambda_5,\ K_3 = i\lambda_6,\ K_4 = i\lambda_7$ |

**Table 4:** Generators for the simple groups used in this paper. We always have $\alpha, \beta, \gamma = 1, \ldots 3$.

to $4 \times 4$ matrices to treat SO(3,1) in the same context):

$$\lambda_1 = \begin{pmatrix} 0&1&0&0\\ 1&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix}, \qquad \lambda_2 = \begin{pmatrix} 0&-i&0&0\\ i&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix}, \qquad \lambda_3 = \begin{pmatrix} 1&0&0&0\\ 0&-1&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix},$$
$$\lambda_4 = \begin{pmatrix} 0&0&1&0\\ 0&0&0&0\\ 1&0&0&0\\ 0&0&0&0 \end{pmatrix}, \qquad \lambda_5 = \begin{pmatrix} 0&0&-i&0\\ 0&0&0&0\\ i&0&0&0\\ 0&0&0&0 \end{pmatrix}, \qquad \lambda_6 = \begin{pmatrix} 0&0&0&0\\ 0&0&1&0\\ 0&1&0&0\\ 0&0&0&0 \end{pmatrix},$$
$$\lambda_7 = \begin{pmatrix} 0&0&0&0\\ 0&0&-i&0\\ 0&i&0&0\\ 0&0&0&0 \end{pmatrix}, \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&-2&0\\ 0&0&0&0 \end{pmatrix},$$
$$\lambda_9 = \begin{pmatrix} 0&0&0&1\\ 0&0&0&0\\ 0&0&0&0\\ 1&0&0&0 \end{pmatrix}, \qquad \lambda_{10} = \begin{pmatrix} 0&0&0&0\\ 0&0&0&1\\ 0&0&0&0\\ 0&1&0&0 \end{pmatrix}, \qquad \lambda_{11} = \begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{pmatrix}. \tag{B.2}$$

The groups we consider have compact generators, which in the following we will denote by $L_R$, and noncompact generators, denoted by $K_R$. We will specify the structure constants with three $L$'s, and with two $K$'s and one $L$. The choice of the generators of the simple groups used in this paper are summarized in table 4.

The structure constants for the SO(3) subgroups are in all cases

$$[L_\alpha,\ L_\beta] = i\epsilon_{\alpha\beta\gamma} L_\gamma\,, \quad (\alpha, \beta, \gamma = 1, 2, 3)\,. \tag{B.3}$$

For SO(2,1) we have

$$[L_3, K_\beta] = -i\epsilon_{3\beta\gamma} K_\gamma\,, \quad (\beta, \gamma = 1, 2)\,. \tag{B.4}$$

In the case of SO(3,1) the commutation relation involving noncompact generators are

$$[L_\alpha, K_\beta] = i\epsilon_{\alpha\beta\gamma} K_\gamma\,,\ (\alpha, \beta, \gamma = 1, 2, 3)\,. \tag{B.5}$$

For SL(3, $\mathbb{R}$) the relations between $L$ and $K$ are:

$$[L_\alpha, K_\beta] = -i\epsilon_{\alpha\beta\gamma} K_\gamma\,, \quad (\alpha \neq \beta, \quad \alpha, \beta, \gamma = 1, 2, 3)\,,$$

| | $K_1 = i\lambda_4$ | $K_2 = i\lambda_6$ | $K_3 = i\lambda_5$ | $K_4 = i\lambda_7$ |
|---|---|---|---|---|
| $L_1 = \lambda_1$ | $K_4$ | $K_3$ | $-K_2$ | $-K_1$ |
| $L_2 = \lambda_2$ | $K_2$ | $-K_1$ | $K_4$ | $-K_3$ |
| $L_3 = \lambda_3$ | $K_3$ | $-K_4$ | $-K_1$ | $K_2$ |
| $L_4 = \lambda_8$ | $\sqrt{3}K_3$ | $\sqrt{3}K_4$ | $-\sqrt{3}K_1$ | $-\sqrt{3}K_2$ |

**Table 5:** Commutation relations between compact and noncompact generators in SU(2, 1). The table reads $[L_1, K_1] = iK_4$, etc.

$$[L_1, K_1] = i(K_4 - \sqrt{3}K_5)\,, \quad [L_2, K_2] = i(K_4 + \sqrt{3}K_5)\,, \quad [L_3, K_3] = -2iK_4\,. \quad \text{(B.6)}$$

Finally, for SU(2, 1) we have

$$[L_\alpha,\ L_\beta] = 2i\epsilon_{\alpha\beta\gamma}L_\gamma\,, \quad (\alpha, \beta, \gamma = 1, 2, 3)\,, \quad [L_\alpha, L_4] = 0\,. \quad \text{(B.7)}$$

The commutation relation between the remaining generators are given in table 5. The structure constants presented for the different groups are the ${f_{RS}}^T$. For use in the calculation of second derivatives etc. the index $T$ has to be lowered by $\eta_{TU}$, which may give a sign depending on the embedding of the group in SO(6, 6). Other commutation relations, such as $[K_\alpha,\ K_\beta]$ follow from the antisymmetry of $f_{RST}$.

## C. The parameters $P$

The independent scalars are contained in $G$ and $B$. Define $G_\pm \equiv (G \pm B)$, and $P \equiv G_+$. Then $G = \frac{1}{2}(P + P^T)$, $B = \frac{1}{2}(P - P^T)$, and we find

$$\begin{aligned} \frac{\partial G_{cd}}{\partial P_{ab}} &= \frac{1}{2}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})\,, \quad \frac{\partial B_{cd}}{\partial P_{ab}} = \frac{1}{2}(\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc})\,, \\ \frac{\partial {G^{-1}}_{cd}}{\partial P_{ab}} &= -\frac{1}{2}\left({G^{-1}}_{ca}{G^{-1}}_{bd} + {G^{-1}}_{cb}G^{-1}_{ad}\right). \end{aligned} \quad \text{(C.1)}$$

In the study of the potential we need the first and second derivatives of $X$ and $Y$ with respect to $P$. We find:

$$\begin{aligned} \left.\frac{\partial X_{cd}}{\partial P_{ab}}\right|_0 &= 0\,, \\ \left.\frac{\partial Y_{cd}}{\partial P_{ab}}\right|_0 &= \delta_{ad}\delta_{bc}\,, \\ \left.\frac{\partial^2 X_{ef}}{\partial P_{ab}\partial P_{cd}}\right|_0 &= \frac{1}{2}\delta_{ac}\left(\delta_{de}\delta_{bf} + \delta_{be}\delta_{df}\right), \\ \left.\frac{\partial^2 Y_{ef}}{\partial P_{ab}\partial P_{cd}}\right|_0 &= -\frac{1}{2}(\delta_{bc}\delta_{de}\delta_{af} + \delta_{ad}\delta_{be}\delta_{cf})\,, \end{aligned} \quad \text{(C.2)}$$

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