

University of Groningen

## Kac-Moody Symmetries and Gauged Supergravity

Nutma, Teake

**IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.**

*Document Version*

Publisher's PDF, also known as Version of record

*Publication date:*

2010

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Nutma, T. A. (2010). Kac-Moody Symmetries and Gauged Supergravity Groningen: s.n.

**Copyright**

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

**Take-down policy**

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

*Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.*



rijksuniversiteit  
groningen

# Kac-Moody symmetries and gauged supergravity

Proefschrift

ter verkrijging van het doctoraat in de  
Wiskunde en Natuurwetenschappen  
aan de Rijksuniversiteit Groningen  
op gezag van de  
Rector Magnificus, dr. F. Zwarts,  
in het openbaar te verdedigen op  
maandag 13 september 2010  
om 11.00 uur

door

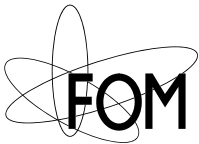
**Teake Aant Nutma**

geboren op 1 juni 1981  
te Heerenveen

Promotor: Prof. dr. E.A. Bergshoeff

Beoordelingscommissie: Prof. dr. A. Kleinschmidt  
Prof. dr. H. Samtleben  
Prof. dr. M. Trigiante





The work described in this thesis was performed at the Centre for Theoretical Physics of the University of Groningen. This work is part of the research programme of the Foundation for Fundamental Research on Matter (FOM), which is part of the Netherlands Organisation for Scientific Research (NWO).

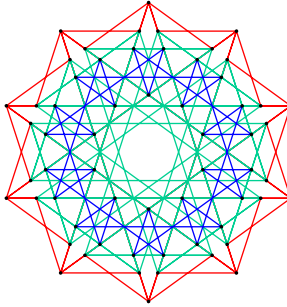
On the cover: the root system of the hyperbolic Kac-Moody algebra  $E_{10}$  up to height 124 projected onto the Coxeter plane of a regular  $E_8$  subalgebra.

Printed by GVO drukkers & vormgevers B.V. | Ponsen & Looijen

ISBN 978-90-367-4473-7 (printed version)

ISBN 978-90-367-4474-4 (electronic version)

Copyright © 2010 Teake A. Nutma



# Contents

<b>1</b>	<b>Introduction</b>	<b>9</b>
1.1	Physical background . . . . .	9
1.2	From symmetry to groups . . . . .	11
1.3	From groups to algebras . . . . .	13
1.4	Infinite Lie algebras . . . . .	14
<b>2</b>	<b>Lie algebras</b>	<b>17</b>
2.1	Lie algebras . . . . .	17
2.1.1	Basic definitions . . . . .	17
2.1.2	The Cartan matrix . . . . .	19
2.1.3	Roots . . . . .	23
2.1.4	Weights . . . . .	29
2.1.5	The Weyl vectors and the highest root . . . . .	32
2.1.6	The Weyl group . . . . .	33
2.1.7	A bound on root norms . . . . .	35
2.2	Representations . . . . .	36
2.2.1	Integrable lowest weight representations . . . . .	38
2.3	Multiplicities . . . . .	42
2.4	Real forms . . . . .	45
2.5	Cosets and non-linear sigma models . . . . .	48

<b>3</b>	<b>Visualizations</b>	<b>53</b>
3.1	Hasse diagrams . . . . .	53
3.1.1	Visualizing the Serre construction . . . . .	58
3.2	Coxeter projections . . . . .	59
3.2.1	The Coxeter plane . . . . .	59
3.2.2	Projections to subalgebras . . . . .	63
<b>4</b>	<b>Kac-Moody algebras</b>	<b>65</b>
4.1	Affine algebras . . . . .	65
4.1.1	Affine algebras as extensions of finite algebras . . . . .	71
4.2	Over- and very-extended algebras . . . . .	72
4.3	Level decomposition . . . . .	76
4.4	Non-linear realizations . . . . .	82
<b>5</b>	<b>Supergravity</b>	<b>85</b>
5.1	Maximal supergravity . . . . .	86
5.2	Half-maximal supergravity . . . . .	90
5.3	Gaugings . . . . .	92
5.3.1	The embedding tensor . . . . .	92
5.3.2	The tensor hierarchy . . . . .	94
<b>6</b>	<b>The comparison</b>	<b>97</b>
6.1	Kinematics . . . . .	97
6.1.1	Maximal supergravity . . . . .	98
6.1.2	Half-maximal supergravity . . . . .	101
6.2	Fundamental $p$ -forms and type- $p$ deformations . . . . .	105
6.3	Dynamics . . . . .	106
6.3.1	$E_{10}$ or $E_{11}$ ? . . . . .	106
6.3.2	The $E_{10}/K(E_{10})$ coset model . . . . .	108
6.3.3	Gauged supergravity in three dimensions . . . . .	111
6.3.4	The correspondence . . . . .	113
<b>7</b>	<b>Conclusions</b>	<b>117</b>
<b>A</b>	<b>Simplie</b>	<b>121</b>
A.1	User manual . . . . .	123
A.2	List of papers . . . . .	125
<b>B</b>	<b>Hasse diagrams and Coxeter projections</b>	<b>129</b>

---

<b>C Decomposition tables</b>	<b>143</b>
C.1 $E_{11}$ decompositions . . . . .	143
C.2 $D_8^{+++}$ decompositions . . . . .	152
<b>Index</b>	<b>161</b>
<b>Publications</b>	<b>165</b>
<b>Bibliography</b>	<b>167</b>
<b>Nederlandse samenvatting</b>	<b>175</b>
<b>Dankwoord</b>	<b>179</b>



