Elliptic delsarte surfaces

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Summary

Elliptic curves can be represented as plane curves given by an equation of degree three. Moreover, this plane curve contains a special point called zero, with the property that the tangent line to the curve at zero intersects the curve only at zero. On such curves we can define a group law by stating that three points on a straight line add up to zero. We call the resulting group the Mordell-Weil group.

An elliptic surface is a surface with a map to a curve, such that the inverse image of almost all points of this curve is an elliptic curve. Every elliptic surface corresponds uniquely to an elliptic curve over a function field. This allows us to talk about the Mordell-Weil group of an elliptic surface. We can use both the theory of (elliptic) curves and surfaces to study these objects.

In this thesis elliptic Delsarte surfaces are studied. These are elliptic surfaces that are given by an equation having the form of the sum of four monomials. In the first chapter the general theory of elliptic surfaces is described.

In the second chapter an algorithm by Shioda is described. With this algorithm one can calculate the so-colled Lefschetz number of a Delsarte surface. For an elliptic Delsarte surface this number can be used to compute the minimal number of generators of the Mordell-Weil group of this surface. We will call this the Mordell-Weil rank of the elliptic surface.

In the third chapter we consider, which Delsarte surfaces are elliptic. To answer this question we make use of Newton polygons. Using these polygons one can determine the genus of a curve. This gives a classification of elliptic Delsarte surfaces. Using this we describe eleven families of elliptic surfaces, such that every elliptic Delsarte surface corresponds with a surface from one of these families.

In the fourth chapter we use Shioda's algorithm to compute the maximal Mordell-Weil rank for each of these eleven families of elliptic surfaces. In doing this we construct an elliptic surface for which this maximal rank is attained. From this we conclude that the maximal rank of an elliptic Delsarte surface is 68.

In the fifth chapter we pick from each of the eleven families of elliptic Delsarte surfaces, a surface with maximal Mordell-Weil rank. For ten of the chosen surface we calculate explicitly a maximal set of linearly independent elements of the Mordell-Weil group. In one case we did not succeed in finding the required number of elements of the Mordell-Weil group.

In the sixth chapter we compare our results with a paper of Fastenberg. Several families she describes correspond to families of elliptic Delsarte surfaces. In this chapter we make this explicit.

In the final chapter we try to generalise our results to positive characteristic.

Several proofs seem not to work in this case. Due to this we are not able to prove that we can classify all elliptic Delsarte surfaces using a finite number of Newton polygons. We also present a new example illustrating that in positive characteristic the Mordell-Weil rank is unbounded.