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The supersymmetric effective action of the heterotic string in ten dimensions

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We construct the supersymmetric completion of quartic $R + R^4$-actions in the ten-dimensional effective action of the heterotic string. Two invariants, of which the bosonic parts are known from one-loop string amplitude calculations, are obtained. One of these invariants can be generalized to an $R + F^2 + F^4$-invariant for supersymmetric Yang–Mills theory coupled to supergravity. Supersymmetry requires the presence of $B \wedge R \wedge R \wedge R \wedge R$-terms, ($B \wedge F \wedge F \wedge F \wedge F$ for Yang–Mills) which correspond to counterterms in the Green–Schwarz anomaly cancellation. Within the context of our calculation the $\zeta(3)R^4$-term from the tree-level string effective action does not allow supersymmetrization.

1. Introduction

In recent years much work has been devoted to the study of the low-energy effective action of string theory. In the limit of low energy, string theory can be approximated by ordinary field theory, in which string effects should appear as higher derivative interaction terms. This effective action provides a useful tool to investigate the impact of string theory on particle physics.

In this context, the heterotic string [1] is of particular interest. Its zero slope limit (the limit in which the inverse string tension, α', goes to zero) is given by ten-dimensional supergravity coupled to Yang–Mills [2,3]. Corrections to this zero slope limit, proportional to α', are required in $d = 10$, $N = 1$ supergravity to achieve the cancellation of anomalies [4]. These corrections involve the introduction of the Lorentz Chern–Simons term, on the same footing as the Yang–Mills Chern–Simons term required by supersymmetry in the Einstein–Yang–Mills supergravity theory [3].

One method of investigating the implications of string theory for particle physics involves the compactification of the effective field theory from ten to four dimensions [5]. The inclusion of the Lorentz Chern–Simons term makes it possible to obtain in this way phenomenologically interesting models in four dimensions [6].

Supersymmetry in four dimensions, a remnant of the space–time supersymmetry of the heterotic string, is a common feature of most of these models.

Much is known about the bosonic contributions to the ten-dimensional string effective action, $\mathscr{L}_{\mathrm{eff}}$. In this paper we investigate the supersymmetric completion of $\mathscr{L}_{\mathrm{eff}}$. We may characterize the different contributions to $\mathscr{L}_{\mathrm{eff}}$ by the power of the Riemann tensor in the R^n-terms which they contain:

$$\mathscr{L}_{\mathrm{eff}} = \sum_n \mathscr{L}_{R^n}. \tag{1.1}$$

The main issue in this paper is the supersymmetrization of the R^4-terms in $\mathscr{L}_{\mathrm{eff}}$. Partial results about this work were presented in ref. [7].

Before discussing our results it is useful to present schematically what is known about the bosonic part of $\mathscr{L}_{\mathrm{eff}}$. We use the results obtained by string amplitude methods. Here one calculates string S-matrix elements for scattering of massless particles, and then reconstructs a field theoretical action which reproduces these amplitudes. There are contributions from the tree-level (classical) string theory, from one-loop string effects, etc. This action is expressed in terms of the physical fields of $d = 10$, $N = 1$ supergravity. The bosonic fields are the tenbein field $e_\mu{}^a$, an antisymmetric tensor gauge field $B_{\mu\nu}$ (with field strength $H_{\mu\nu\lambda}$), the dilaton field ϕ, and the Yang–Mills gauge field A_μ (the fermions are introduced in sect. 2, where we present some basic properties of ten-dimensional supergravity). The presence of the dilaton in this action is limited by global scale invariance [8]. Our fields (except the dilaton) are scale invariant, while ϕ transforms as $\phi \to \phi\xi$, ξ being the parameter of scale transformations. Scale invariance implies that ϕ occurs only in the combination $\phi^{-1}\partial\phi$, or as an overall multiplicative factor in the lagrangian.

From the tree-level string calculation [9–13] one obtains $\mathscr{L}_R$:

$$\mathscr{L}_R \sim \frac{1}{\kappa^2}\phi^{-3}\left[R + H^2 + \left(\phi^{-1}\partial\phi\right)^2\right], \tag{1.2}$$

where κ is the ten-dimensional gravitational coupling constant, of dimension $[\mathrm{mass}]^{-4}$. Also from the string tree-level one obtains a quadratic action *

$$\mathscr{L}_{R^2} \sim \phi^{-3}\left(\frac{\alpha'}{\kappa^2}R^2 + \beta\ \mathrm{tr}\ F^2\right) \tag{1.3}$$

* Here $\beta = 1/(g_{10})^2$, g_{10} the Yang–Mills coupling constant. The dimension of α' is $[\mathrm{mass}]^{-2}$, of β $[\mathrm{mass}]^6$. The number of string loops is counted by the dimensionless coupling g^2, which satisfies, for the heterotic string, the relation $g = 2\kappa(2\alpha')^{-2}$. β is fixed by $\beta = \alpha'/(2\kappa^2)$ [1].

and a quartic action *

$$\mathscr{L}_{R^4} \sim \alpha' \kappa^2 \phi^{-3} \left(\frac{\alpha'}{\kappa^2} R^2 + \beta \text{ tr } F^2 \right)^2 + \frac{\alpha'^3}{\kappa^2} \phi^{-3} \zeta(3) X, \tag{1.4}$$

where X is the term [14,11]

$$X = t^{\mu_1 \cdots \mu_8} t^{\nu_1 \cdots \nu_8} R_{\mu_1 \mu_2 \nu_1 \nu_2} R_{\mu_3 \mu_4 \nu_3 \nu_4} R_{\mu_5 \mu_6 \nu_5 \nu_6} R_{\mu_7 \mu_8 \nu_7 \nu_8}. \tag{1.5}$$

The tensor t is discussed in sect. 3. The transcendental coefficient $\zeta(3)$ makes it impossible to relate the two contributions in $\mathscr{L}_{R^4}$ by supersymmetry.

At the one-loop level [15–17] $\mathscr{L}_{\text{eff}}$ obtains corrections to the quartic action:

$$\mathscr{L}_{R^4} \sim \alpha' \kappa^2 g^2 \left[\left(\frac{\alpha'}{\kappa^2} R^2 + \beta \text{ tr } F^2 \right)^2 + \beta^2 \text{ tr } F^4 \right] + \frac{\alpha'^3 g^2}{\kappa^2} X. \tag{1.6}$$

Note the absence of the factor ϕ^{-3} in the one-loop contributions. In fact, each string loop will give a factor $\phi^3 g^2$. This can be understood in terms of a background field sigma-model calculation from the coupling of the dilaton to the Euler character of the world sheet [18–20].

Besides the above terms due to four-point scattering amplitudes there are also contributions from one-loop five-point amplitudes [21,22]. These are of the form

$$\mathscr{L}_{R^4} \sim \varepsilon^{\mu_1 \cdots \mu_{10}} B_{\mu_1 \mu_2} \text{ tr } F_{\mu_3 \mu_4} \cdots F_{\mu_9 \mu_{10}}, \tag{1.7}$$

while similar terms with F replaced by R also appear.

Other information about the quartic action comes from the counterterms in the $d = 10$ action which are required for anomaly cancellations [4]. We would expect these terms to be part of the string effective action. Indeed, terms of the form (1.7) are among the counterterms of ref. [4]. It is then of interest to see, whether or not they are linked by supersymmetry to some of the terms already present in (1.4) and (1.6).

Let us now discuss the supersymmetrization of the effective action. The action $\mathscr{L}_R$ corresponds to the supersymmetric Einstein action of $d = 10$, $N = 1$ supergravity [2]. The inclusion of the term β tr F^2 leads to the supersymmetric action of ref. [3]. The field strength H then has to be extended with the Yang-Mills Chern–Simons term. The introduction of the Lorentz Chern–Simons term requires, by supersymmetry, the presence of the R^2-action. The supersymmetrization of the R^2-action has been achieved by the Noether method [23–25] and by superspace methods ** [27,28]. In ref. [25] an explicit supersymmetric action for the Lorentz

* The absence of the cubic action $\mathscr{L}_{R^3}$ is understood from the vanishing of three-point string scattering amplitudes.

** For a recent review of superspace methods in connection with the Lorentz Chern–Simons terms, see ref. [26].

Chern–Simons term, including terms quartic in R, was presented. In the absence of Yang–Mills couplings it is of the schematic form:

$$\mathscr{L}_{\mathrm{LCS}} = \mathscr{L}_R + \phi^{-3}\alpha R^2 + \phi^{-3}\alpha^3 R^4 + \dots . \tag{1.8}$$

Each term has the same power of ϕ, and, consistent with string amplitude results, the $n = 3$ contribution is absent. Supersymmetry holds only iteratively in α, so that the supersymmetry transformation rules of a generic field V are

$$\delta V = \sum_{n=0} \alpha^n \delta_n V. \tag{1.9}$$

Here $\delta_0 V$ are the transformation rules corresponding to the action $\mathscr{L}_R$. This can easily be generalized to the case where Yang–Mills couplings are present. Again schematically, one should make everywhere the replacement $\alpha R^2 \to \alpha R^2 + \beta$ tr F^2. On identifying the *a priori* independent coupling α with α'/κ^2 one then obtains exactly the terms in the tree-level string amplitude result (1.3), (1.4), except for the $\zeta(3)X$-term.

In this paper we address the problem of supersymmetrizing terms quartic in the Riemann tensor. These include the remaining tree-level term $\zeta(3)\phi^{-3}X$ and the one-loop contributions (1.6). Since the supersymmetrization of the R^2-terms in $\mathscr{L}_{\mathrm{eff}}$ is complete, this supersymmetric R^4-action should be of the form

$$\mathscr{L} = \mathscr{L}_R + \gamma R^4 + \dots, \tag{1.10}$$

with modifications to the supersymmetry transformation rules of refs. [2,3] proportional to γ. Here γ is an additional parameter, of dimension $[\text{mass}]^2$, *a priori* independent of α and β. Relations between α, β and γ will be required if quartic contributions to $\mathscr{L}$ and the string effective action $\mathscr{L}_{\mathrm{eff}}$ are to be identified, or if the cancellation of anomalies is imposed. Supersymmetry by itself will not relate α, β and γ.

An obvious problem is already evident from the schematic form of the action given above. There are two contributions proportional to X, one with and one without the dilaton-dependent factor. One would expect that supersymmetry gives a unique value for the power of ϕ which appears in front of X. The same problem arises for the terms with $\alpha R^2 + \beta$ tr F^2. In that case one should realize however that in the tree-level quartic action this term is determined largely by the presence of R^2 in (1.8), so that the tree-level and one-loop contributions to $(\alpha R^2 + \beta$ tr $F^2)^2$ do not appear on the same footing.

A second indication that factors of ϕ are important can be seen from (1.7). This term is invariant under gauge transformations of the B-field only if the factor ϕ^{-3}

is absent. Therefore the presence of the parity-violating terms (1.7) requires the absence of the factor ϕ^{-3}.

As we shall show in this paper the supersymmetrization of *any* action of the form (1.10) requires ϵBR^4-terms, and therefore the absence of ϕ^{-3}. Thus we achieve the supersymmetrization of the one-loop contributions (1.6), but not that of the $\zeta(3)$-term in (1.3).

Some results about the supersymmetrization of R^4-actions have been obtained in superspace [29–31]. However, the supersymmetrization of X (1.5) in refs [29,30] depends on an off-shell formulation of $d=10$, $N=1$ supergravity, which has not yet been proven to exist. Also, it has not been worked out whether the proposed superspace invariant for X represents the tree-level contribution (1.4) or the one-loop term in (1.6). On the basis of our work we would have to conclude that this can only be the one-loop term. Since other R^4-terms besides X appear in $\mathscr{L}_{\text{eff}}$, we prefer to search systematically for the most general supersymmetric invariant with the generic structure (1.10).

In this paper we use the component field Noether method. One starts with an ansatz for the supersymmetric action that one wants to construct. The ansatz should contain all possible terms, each with an unknown coefficient. Invariance under supersymmetry is then used to determine these coefficients. This method has the disadvantage of being algebraically complex. The ansatz contains many terms, so working out the variations involves a large amount of work. However, this tedious task can and has all been done by a computer program for algebraic manipulations. Then the explicit nature of this method turns into an advantage. The resulting invariant can be compared in detail with the results from string amplitude calculations. Also, the explicit form of the modified transformation rules is obtained. The transformation rules of the fermions play a crucial role in the study of compactification to four dimensions [5].

The full calculation will be done for the gravitational sector only, i.e., without the Yang–Mills coupling. We shall see that our results can be generalized to the case were the Yang–Mills multiplet is present as well.

This paper is organized as follows. In sect. 2 we present some basic material on $d=10$, $N=1$ supergravity. We also briefly discuss results about the supersymmetric R^2-action. In sect. 3 we construct the ansatz (given explicitly in Appendix A) for the supersymmetric R^4-action. Of course, for practical reasons we have to limit ourselves to certain sectors of the complete action (for instance, we never include four-fermion terms). These limitations are also discussed in sect. 3. In sect. 4 we give a schematic overview of the calculation, and consider in some detail a particular sub-calculation which leads us to conclude that terms such as (1.7) must be present in the final result. The full result, and its generalization to the Yang–Mills case, is then presented in sect. 5 and Appendix B. Sect. 6 compares our results with the string amplitude calculations and discusses the relation with other work.

2. $N=1$ supergravity in $d=10$ and the R^2-action

The basic multiplet of $N=1$ supergravity in ten space–time dimensions consists of the tenbein field $e_\mu{}^a$, the dilation field ϕ, an antisymmetric tensor gauge field $B_{\mu\nu}$ and the Majorana–Weyl fermions ψ_μ (gravitino) and λ (dilatino) [2]. This multiplet transforms under local supersymmetry as follows *:

$$\delta e_\mu{}^a = \tfrac{1}{2}\bar{\epsilon}\Gamma^a\psi_\mu, \tag{2.1}$$

$$\delta\psi_\mu = \left(\partial_\mu - \tfrac{1}{4}\Omega_{\mu+}^{ab}\Gamma_{ab}\right)\epsilon + \epsilon(\text{fermi})^2, \tag{2.2}$$

$$\delta B_{\mu\nu} = \tfrac{1}{2}\sqrt{2}\ \bar{\epsilon}\Gamma_{[\mu}\psi_{\nu]}, \tag{2.3}$$

$$\delta\lambda = -\tfrac{3}{8}\sqrt{2}\ \Gamma^\mu\epsilon\phi^{-1}D_\mu\phi + \tfrac{1}{8}\Gamma^{abc}\epsilon\hat{H}_{abc} + \epsilon(\text{fermi})^2, \tag{2.4}$$

$$\phi^{-1}\delta\phi = -\tfrac{1}{3}\sqrt{2}\ \bar{\epsilon}\lambda. \tag{2.5}$$

The derivatives $\mathscr{D}_\mu$ are Lorentz covariant, supercovariant derivatives are denoted by D_μ. In the variation of the gravitino field we encounter a torsionful spin-connection defined by

$$\Omega_{\mu\pm}^{ab} \equiv \omega_\mu{}^{ab}(e,\psi) \pm \tfrac{3}{2}\sqrt{2}\ \hat{H}_\mu{}^{ab}. \tag{2.6}$$

Here, $\omega_\mu{}^{ab}(e,\psi)$ is the usual spin connection with ψ-torsion, i.e., the solution of $D_{[\mu}(\omega)e_{\nu]}{}^a = 0$. The additional torsion is determined by the supercovariant field strength of the B – field, $H_\mu{}^{ab}$, given by

$$\hat{H}_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]} - \tfrac{1}{4}\bar{\psi}_{[\mu}\Gamma_\nu\psi_{\rho]}, \tag{2.7}$$

which is invariant under gauge transformations

$$\delta B_{\mu\nu} = \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu. \tag{2.8}$$

Under local supersymmetry, $\omega_\mu{}^{ab}$ and $\hat{H}$ transform as

$$\delta\omega_\mu{}^{ab}(e,\psi) = \tfrac{1}{4}\bar{\epsilon}\Gamma_\mu\psi^{ab} + \tfrac{1}{2}\bar{\epsilon}\Gamma^{[a}\psi_\mu{}^{b]} + \tfrac{3}{4}\sqrt{2}\ \bar{\epsilon}\Gamma_c\psi_\mu\hat{H}^{abc}, \tag{2.9}$$

$$\delta\hat{H}_{abc} = -\tfrac{1}{4}\sqrt{2}\ \bar{\epsilon}\Gamma_{[a}\psi_{bc]}. \tag{2.10}$$

* Note that $\Gamma^{a_1\cdots a_n} \equiv \Gamma^{[a_1}\Gamma^{a_2}\ldots\Gamma^{a_n]}$. Throughout this paper we use the conventions of ref. [25]. In our calculations we will never consider terms quartic in fermions in the action, and, consequently, we may ignore terms quadratic in fermions in the transformation rules.

Here, ψ_{ab} denotes the gravitino curvature

$$\psi_{\mu\nu} = 2\mathscr{D}_{[\mu}(\Omega_+)\psi_{\nu]} + (\text{fermi})^3. \tag{2.11}$$

The transformations (2.9) and (2.10) can be combined to yield

$$\delta\Omega_{\mu -}{}^{ab} = \tfrac{1}{2}\bar{\epsilon}\Gamma_\mu\psi^{ab}. \tag{2.12}$$

The gravitino curvature ψ^{ab} itself has the following variation:

$$\delta\psi^{ab} = -\tfrac{1}{4}\Gamma^{\mu\nu}\epsilon R_{m\nu}{}^{ab}(\Omega_-) + \epsilon(\text{fermi})^2, \tag{2.13}$$

where $R_{\mu\nu}{}^{ab}(\Omega_-)$ denotes the Riemann curvature tensor with spin connection Ω_-.
The ten-dimensional action which is invariant under the transformations (2.1)–(2.5) is given by

$$\begin{aligned}\mathscr{L}_R = e\phi^{-3}\Big[& -\tfrac{1}{2}R(\omega(e)) - \tfrac{3}{4}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + \tfrac{9}{2}\big(\phi^{-1}\partial_\mu\phi\big)^2 \\ & -\tfrac{1}{2}\bar{\psi}_\mu\Gamma^{\mu\nu\rho}\mathscr{D}_\nu(\omega(e))\psi_\rho + 2\sqrt{2}\,\bar{\lambda}\Gamma^{\mu\nu}\mathscr{D}_\mu(\omega(e))\psi_\nu \\ & +4\bar{\lambda}\not{\mathscr{D}}(\omega(e))\lambda + 3\sqrt{2}\,\bar{\psi}_\mu\Gamma^\nu\Gamma^\mu\lambda\big(\phi^{-1}\partial_\nu\phi\big) - \tfrac{3}{2}\bar{\psi}_\mu\Gamma^\mu\psi_\nu\big(\phi^{-1}\partial^\nu\phi\big) \\ & +\tfrac{1}{16}\sqrt{2}\,H^{\rho\sigma\tau}\Big(\bar{\psi}_\mu\Gamma^{[\mu}\Gamma_{\rho\sigma\tau}\Gamma^{\nu]}\psi_\nu + 4\sqrt{2}\,\bar{\psi}^\mu\Gamma_{\mu\rho\sigma\tau}\lambda - 8\bar{\lambda}\Gamma_{\rho\sigma\tau}\lambda\Big)\Big] \\ & +(\text{fermi})^4. \end{aligned}\tag{2.14}$$

The equations of motion which follow from (2.14) will play an important role in this paper. These equations, for the fields ϕ, $e_\mu{}^a$, $\bar{\lambda}$, $\bar{\psi}_\mu$ and $B_{\mu\nu}$, respectively, read

$$\Phi = e\phi^{-3}\Big[\tfrac{3}{2}R(\omega) - 9\mathscr{D}_a(\omega)(\phi^{-1}\partial^a\phi) + \tfrac{27}{2}\big(\phi^{-1}\partial_\mu\phi\big)^2 + \tfrac{9}{4}H_{\mu\nu\lambda}H^{\mu\nu\lambda}\Big], \tag{2.15}$$

$$\mathscr{E}_a^\mu = e\phi^{-3}e^\mu{}_b e^\nu{}_a\Big[R_\nu{}^b(\omega) - 3\mathscr{D}_\nu(\omega)(\phi^{-1}\partial^b\phi) + \tfrac{9}{2}H_{\nu\lambda\rho}H^{b\lambda\rho}\Big] - \tfrac{1}{3}e^\mu{}_a\Phi, \tag{2.16}$$

$$\Lambda = e\phi^{-3}\big[8\not{\mathscr{D}}(\omega)\lambda + \sqrt{2}\,\Gamma^{\mu\nu}\psi_{\mu\nu} - 12(\phi^{-1}\not{\mathscr{D}}\phi)\lambda - \sqrt{2}\,\Gamma^{abc}\lambda H_{abc}\big], \tag{2.17}$$

$$\Psi_\mu = e\phi^{-3}\big(\Gamma^\nu\psi_{\mu\nu} + 2\sqrt{2}\,\mathscr{D}_\mu(\Omega_+)\lambda\big) - \tfrac{1}{4}\sqrt{2}\,\Gamma_\mu\Lambda, \tag{2.18}$$

$$\mathscr{B}^{\mu\nu} = \tfrac{3}{2}\partial_\lambda\big(e\phi^{-3}H^{\lambda\mu\nu}\big). \tag{2.19}$$

In this paper we frequently use identities which are implied by the fermionic equations of motion and the Bianchi identity

$$\mathscr{D}_{[\mu}(\omega)\psi_{\nu\lambda]} = -\tfrac{1}{4}\Gamma^{ab}\psi_{[\mu}R_{\nu\lambda]ab}(\omega) \qquad (2.20)$$

for the gravitino curvature. First of all, we use (2.18) to solve for the single Γ-contraction of the gravitino curvature

$$\Gamma^{b}\psi_{ab} = e^{-1}\phi^{3}\left(\Psi_{a} + \tfrac{1}{4}\sqrt{2}\ \Gamma_{a}\Lambda\right) - 2\sqrt{2}\ \mathscr{D}_{a}\lambda. \qquad (2.21)$$

From (2.21) we obtain by contracting with a further Γ-matrix

$$\Gamma^{ab}\psi_{ab} = e^{-1}\phi^{3}\left(2\Gamma^{a}\Psi_{a} + \tfrac{9}{2}\sqrt{2}\ \Lambda\right). \qquad (2.22)$$

Combining (2.18) and (2.20) one derives two additional identities involving the derivative of the gravitino field equation:

$$\begin{aligned}\not{\mathscr{D}}\psi_{ab} = {} & -2\mathscr{D}_{[a}\left(e^{-1}\phi^{3}\Psi_{b]}\right) + \tfrac{1}{2}\sqrt{2}\ \Gamma_{[a}\mathscr{D}_{b]}\left(e^{-1}\phi^{3}\Lambda\right)\\ & -\tfrac{1}{4}\Gamma^{c}\Gamma^{ef}\psi_{c}R_{abef} - \tfrac{1}{2}\sqrt{2}\ \Gamma^{cd}\lambda R_{abcd} + \Gamma^{c}\psi_{[a}R_{b]c},\end{aligned} \qquad (2.23)$$

$$\begin{aligned}\mathscr{D}^{b}\psi_{ab} = {} & \not{\mathscr{D}}\left(e^{-1}\phi^{3}\Psi_{a}\right) - \tfrac{1}{4}\sqrt{2}\left[\Gamma_{a}\not{\mathscr{D}}\left(e^{-1}\phi^{3}\Lambda\right) - \mathscr{D}_{a}\left(e^{-1}\phi^{3}\Lambda\right)\right]\\ & -\tfrac{1}{4}\Gamma^{ef}\psi^{b}R_{abef} + \tfrac{1}{2}R_{ab}\left(\Gamma^{bc}\psi_{c} - \psi^{b}\right) + \tfrac{1}{4}R\psi_{a},\end{aligned} \qquad (2.24)$$

while (2.17) and (2.22) give

$$\not{\mathscr{D}}\lambda = -e^{-1}\phi^{3}\left(\tfrac{1}{4}\sqrt{2}\ \Gamma^{a}\Psi_{a} + \Lambda\right). \qquad (2.25)$$

In the identities (2.20)–(2.25) we have not written contributions of H and $\phi^{-1}\partial\phi$. In the next section we will discuss why these are neglected in our calculations.

In $d=10$, $N=1$ supergravity the only matter multiplet is the Yang–Mills multiplet, which consists of the gauge field A_{μ}, and a Majorana–Weyl spinor χ, both in the adjoint representation of an arbitrary gauge group. The transformation rules are

$$\delta A_{\mu} = \tfrac{1}{2}\bar{\epsilon}\Gamma_{\mu}\chi, \qquad (2.26)$$

$$\delta\chi = -\tfrac{1}{4}\Gamma^{\mu\nu}F_{\mu\nu}(A)\epsilon + \epsilon\ (\text{fermi})^{2}. \qquad (2.27)$$

The coupling of the Yang–Mills multiplet (A_{μ}, χ) to ten-dimensional supergravity [2,3] leads to a supersymmetric action of the form $\mathscr{L}_{R} + \mathscr{L}_{F^{2}}$. This requires the inclusion of the Yang–Mills Chern–Simons term in the field strength (2.7) of the B-field [3], and a corresponding modification of the B transformation rule.

The cancellation of anomalies requires a further modification of H by the corresponding Lorentz Chern–Simons term [4]. However, this mechanism breaks the local supersymmetry. The fact that the transformation rules (2.12) and (2.13) of $\Omega_{\mu-}^{ab}$ and the gravitino curvature ψ^{ab} have the same structure as those of the Yang–Mills multiplet (A_μ, χ) (2.26), (2.27) simplifies the restoration of supersymmetry [24]. By replacing in the action $R+\beta$ tr F^2, and in the corresponding transformation rules A_μ by $\Omega_{\mu-}^{ab}$, χ by ψ^{ab}, $F_{\mu\nu}(A)$ by the corresponding curvature $R_{\mu\nu}{}^{ab}(\Omega_-)$, and the coupling β by an *a priori* independent coupling α, the tr F^2 Yang–Mills action can be immediately extended to a supersymmetric action of the form $R+\beta$ tr $F^2+\alpha R^2$. This requires a modification, proportional to α, of the supersymmetry transformation rule of the B-field. Since $(\Omega_{\mu-}^{ab}, \psi^{ab})$ depend on B (see relations (2.6) and (2.11)), the transformation rules of $\Omega_{\mu-}^{ab}$ and ψ^{ab} obtain order α terms, besides the order β terms already present due to the Yang–Mills coupling. This breaks the invariance of the $R+\beta$ tr $F^2+\alpha R^2$-action by terms which are of order α^2 and $\alpha\beta$. The best one can hope for in this explicit supersymmetrization of the Lorentz Chern–Simons term is an iterative invariance in the couplings α and β.

The iterative procedure outlined above was worked out for the cubic α^2R^3, $\alpha\beta R$ tr F^2, and for the quartic α^3R^4, $\alpha^2\beta R^2$ tr F^2, $\alpha\beta^2(\text{tr } F^2)^2$-contributions to the supersymmetric effective action. Bosonic cubic terms in the supersymmetric action are not required. Contributions from the variation of the quadratic and cubic action play a crucial role in the cancellations which lead to the final form of the quartic action [25]. Thus the quartic action obtained in ref. [25] is directly linked to the inclusion of the Lorentz Chern–Simons form, and *a priori* unrelated to the quartic actions which we will construct in this paper, which do not include quadratic or cubic contributions.

3. R^4-invariants and the ansatz

The supersymmetrization of R^4-action starts with the construction of an ansatz, which should contain all terms that might be linked to the R^4-terms by supersymmetry.

In order to make the supersymmetrization feasible one has to put restrictions on the terms which are included in the ansatz, and, correspondingly, on the contributions to its supersymmetry variation. In this section we will discuss the structure of our ansatz and the restrictions we have imposed.

As we have already mentioned in sect. 2, we will not consider terms in the action which are quartic in fermions. Hence, in the R^4-action only purely bosonic terms and terms quadratic in fermions will appear. Correspondingly, in the supersymmetry transformations of the bosonic fields only terms linear in fermions, in the transformations of the fermionic fields only the bosonic contributions have to be considered:

$$\delta(\text{boson}) = \bar{\epsilon}(\text{fermion}),$$

$$\delta(\text{fermion}) = (\text{boson})\epsilon.$$

In the R^4-action we do not write terms which contain the equations of motion of the R-action (2.15)–(2.19). Such contributions can always be eliminated by a suitable redefinition of the corresponding field. As was outlined in refs. [11,13], the results obtained from scattering amplitude calculations are insensitive to such redefinitions of the fields. Thus, we do not have to include terms in the ansatz containing a Ricci tensor or a curvature scalar. The same applies to terms containing a contracted derivative of the Riemann tensor, since

$$\mathscr{D}_\mu(\omega) R_{\lambda\rho}{}^{\mu a}(\omega) = 2\mathscr{D}_{[\lambda}(\omega) R_{\rho]}{}^{a}(\omega). \tag{3.1}$$

Similarly, fermionic terms containing the left-hand-side of (2.21)–(2.25) can be left out.

The presence of the fields ϕ and $B_{\mu\nu}$ in $d = 10$ supergravity complicates our calculations considerably. The occurrence of $B_{\mu\nu}$ itself is of course restricted by the requirement of gauge invariance (see (2.8)), but many contributions containing the field strength H are possible. One may attempt to restrict the contributions of H by requiring that H only occurs as torsion (2.6), as seems to be indicated by string amplitude calculations. However, we prefer not to bias our calculations by introducing such input. Similarly, the appearance of ϕ can be restricted by requiring global scale invariance, but $\phi^{-1}\phi$ may appear anywhere.

We compromise by including in the action only terms independent of or linear in H and $\phi^{-1}\partial\phi$. In the variation of the action we should then consider only those terms in which H and $\phi^{-1}\partial\phi$ are absent. From (2.4) we see that this implies for instance that we never have to vary the field λ, and that consequently there is no need to include λ^2-terms in the action. Furthermore, we can restrict the terms containing H and $\phi^{-1}\partial\phi$ to be purely bosonic.

In the ansatz we use the spin connection with ψ-torsion, i.e., $\omega_\mu{}^{ab}(e, \psi)$, as the argument of the Riemann tensor, and parametrize the terms linear in H separately. In the H-dependent terms in the ansatz we use the supercovariant field strength $\hat{H}$, given in (2.7).

Note that with the above restrictions, it is no longer guaranteed that our method will yield a useful result. It may well be, for instance, that the cancellation of variations containing H is required to fix the coefficients of the terms linear in H in the action uniquely. As the next sections will show, a large part of the supersymmetric action is determined, even though we do not consider the cancellation of all possible variations. More fundamentally, we must admit that our method does not strictly prove the existence of a supersymmetric invariant, since the procedure may still fail for variations which we do not consider. The results, and their relation with string amplitude calculations, give us confidence that our procedure could in principle be continued to the end without essential obstructions.

The purpose of our present work is the supersymmetrization of R^4-actions, with in view the application to the effective action of heterotic string theory. As discussed in the introduction, the bosonic part corresponding to tree-level and one-loop contributions to string amplitudes are known. There, the following actions quartic in the Riemann tensor arise:

$$X = t^{\mu\nu\lambda\rho\sigma\tau\alpha\beta} t_{abcdefgh} R_{\mu\nu}{}^{ab} R_{\lambda\rho}{}^{cd} R_{\sigma\tau}{}^{ef} R_{\alpha\beta}{}^{gh}, \tag{3.2}$$

$$Y_1 = t^{\mu\nu\lambda\rho\sigma\tau\alpha\beta} R_{\mu\nu}{}^{ab} R_{\lambda\rho}{}^{ab} R_{\sigma\tau}{}^{cd} R_{\alpha\beta}{}^{cd}, \tag{3.3}$$

$$Y_2 = t^{\mu\nu\lambda\rho\sigma\tau\alpha\beta} R_{\mu\nu}{}^{ab} R_{\lambda\rho}{}^{bc} R_{\sigma\tau}{}^{cd} R_{\alpha\beta}{}^{da}, \tag{3.4}$$

$$Z = R_{[ab}{}^{ab} R_{cd}{}^{cd} R_{ef}{}^{ef} R_{gh]}{}^{gh}. \tag{3.5}$$

The tensor t has the following structure when acting on commuting, antisymmetric tensors * M_i, $i = 1, \dots, 4$:

$$\begin{aligned} &t_{abcdefgh} M_1^{ab} M_2^{cd} M_3^{ef} M_4^{gh} \\ &\quad = -2(\operatorname{tr} M_1 M_2 \operatorname{tr} M_3 M_4 + \operatorname{tr} M_2 M_3 \operatorname{tr} M_4 M_1 + \operatorname{tr} M_1 M_3 \operatorname{tr} M_2 M_4) \\ &\qquad + 8(\operatorname{tr} M_1 M_2 M_3 M_4 + \operatorname{tr} M_1 M_3 M_2 M_4 + \operatorname{tr} M_1 M_3 M_4 M_2). \end{aligned} \tag{3.6}$$

The action (3.2) was obtained from a calculation of the two-loop β-function in a supersymmetric nonlinear sigma-model [14] and independently in string amplitude calculations [11]. This action appears in the tree-level string effective action with a characteristic coefficient $\zeta(3)$.

The action Y_1, which has the structure $t^{\cdots}(\operatorname{tr} R^2)^2$, was also found in tree-level string amplitude calculations ** [13]. Note that Y_2 has a different trace structure $t^{\cdots}(\operatorname{tr} R^4)$. Finally, (3.5) is invariant under linearized supersymmetry transformations, since by the Bianchi identity of the Riemann tensor,

$$\mathscr{D}_{[\mu}(\omega) R_{\nu\rho]}{}^{ab}(\omega) = 0, \tag{3.7}$$

* In string amplitude considerations (see e.g. ref. [13]) the indices of the t-tensor indicate the eight transverse directions in light-cone coordinates, and then t-contains an additional eight-dimensional Levi–Cività symbol. Here we extend the range of the indices to all ten values.

** For comparison to tree-level string amplitude results we will use the very detailed result given in ref. [13].

the variation of Z is a total derivative for any variation of ω. If Z is reduced to eight dimensions it becomes a total derivative. This implies that it does not play a role in light-cone gauge string amplitude calculations. Therefore one has no *a priori* knowledge from string amplitude or sigma-model calculations about its effects in a ten-dimensional supersymmetric invariant. The fact that in ten dimensions one should allow the inclusion of a Z-action was emphasized in refs. [32–34].

In the supersymmetrization of R^4-actions we look for invariants of the form

$$\mathscr{L} = R + \gamma R^4 + \mathrm{O}(\gamma^2), \tag{3.8}$$

where R is the pure $d = 10$, $N = 1$ supergravity action (2.14). Supersymmetry may hold iteratively in γ, so that the supergravity fields will need modifications of the supersymmetry transformation rules of $\mathrm{O}(\gamma)$ in order to achieve invariance of the action (3.8) to $\mathrm{O}(\gamma)$.

Our ansatz in the search for the supersymmetric completion of R^4-actions is written in the form

$$\mathscr{L}_{\mathrm{tot}} = \gamma e \phi^{y} \sum_i \mathscr{L}_i . \tag{3.9}$$

The sum is over the different structures that may occur in the action. We consider 15 different sectors, which are presented in Appendix A, four involving purely bosonic terms ($\mathscr{L}_1$–$\mathscr{L}_4$), four sectors involving the gravitino field ψ_μ and its curvature $\psi_{(2)}$ ($\mathscr{L}_5$–$\mathscr{L}_8$) and seven sectors containing the dilatino field λ ($\mathscr{L}_9$–$\mathscr{L}_{15}$). We give a few comments on our ansatz.

We include an arbitrary power of the dilaton in front of the action (the R-action also has such a structure, with ϕ^{-3}). Note that by supersymmetry the power of ϕ has to be independent of the index i labelling the different sectors, since the supersymmetry transformation rules (2.1)–(2.5) contain no explicit powers of ϕ.

The sector $\mathscr{L}_1$ (A.1) contains all possible contractions of four Riemann tensors. Therefore, the actions (3.2)–(3.5) can be written as linear combinations of the terms given in (A.1). Using pair exchange and cyclic identities for the Riemann tensor, and neglecting terms containing the Ricci tensor or curvature scalar, one finds *

$$\begin{aligned}
X &= 12(A_1 - 16A_2 + 2A_3 - 32A_5 + 16A_6 + 32A_7),\\
Y_1 &= -2A_1 + 16A_2 - 4A_3 + 8A_4,\\
Y_2 &= -4A_2 + 2A_4 - 16A_5 + 8A_6 + 16A_7,\\
Z &= \frac{1}{7\times 5!}(A_1 - 16A_2 + 2A_3 + 16A_4 - 32A_5 + 16A_6 - 32A_7).
\end{aligned} \tag{3.10}$$

* A capital letter denotes the term in the ansatz without the corresponding parameter (always given in lower case) (see Appendix A).

Note that the actions X, Y_1 and Y_2 are related by

$$X + 6Y_1 - 24Y_2 = 0. \tag{3.11}$$

The sector $\mathscr{L}_1$ is the only one for which the variation of ϕ in front of the action has to be evaluated.

The $\mathscr{L}_3$-sector consists of the following two terms:

$$K_1 = ie^{-1}\varepsilon^{\mu_1 \dots \mu_{10}} B_{\mu_1\mu_2} R_{\mu_3\mu_4}{}^{ab} R_{\mu_5\mu_6}{}^{ab} R_{\mu_7\mu_8}{}^{cd} R_{\mu_9\mu_{10}}{}^{cd}, \tag{3.12}$$

$$K_2 = ie^{-1}\varepsilon^{\mu_1 \dots \mu_{10}} B_{\mu_1\mu_2} R_{\mu_3\mu_4}{}^{ab} R_{\mu_5\mu_6}{}^{ac} R_{\mu_7\mu_8}{}^{bd} R_{\mu_9\mu_{10}}{}^{cd}. \tag{3.13}$$

Both terms are clearly invariant under gauge transformations (2.8) of the B-field because of the Bianchi identity (3.7). Note that this gauge invariance requires the absence of the dilaton field in (3.12) and (3.13), i.e. $y = 0$ in (3.9). One-loop string amplitude calculations reveal that these K-terms must be part of the effective string action [21,22].

The sectors $\mathscr{L}_{5-8}$ parametrize terms of type $\bar{\psi}_{(2)}\Gamma\psi_{(2)}R\mathscr{D}R$, $\bar{\psi}_{(2)}\Gamma\mathscr{D}\psi_{(2)}R^2$, $\bar{\psi}\Gamma\psi_{(2)}R^3$ and $\bar{\psi}\Gamma\psi R^2\mathscr{D}R$ respectively. As we noted above, in constructing these sectors we do not allow terms with any contractions of the form (2.21)–(2.24). Note that a partial integration and the use of the Bianchi identity (2.20) may relate terms of these sectors. Therefore, in order to find a minimal set of independent terms for the ansatz only those terms are taken into account which are not related by any of these operations.

Similar arguments apply for the λ-sectors $\mathscr{L}_{9-15}$. There we do not write terms which are related to the equation of motions Λ (2.17) or Ψ_μ (2.18).

4. The calculation

In this section we will discuss some of the technical aspects concerning the calculation we have outlined in the previous section.

TABLE 1
The schematic form of the supersymmetry transformation rules considered in this paper. The symbol ψ represents the gravitino, $\psi_{(2)}$ the gravitino curvature.

#	Transformation
(1)	$\delta\psi = \mathscr{D}(\omega)\epsilon$
(2)	$\delta H = \bar{\epsilon}\psi_{(2)}, \quad \delta B = \bar{\epsilon}\psi$
(3)	$\delta\omega = \bar{\epsilon}\psi_{(2)}$
(4)	$\delta\psi_{(2)} = \epsilon R$
(5)	$\delta e = \bar{\epsilon}\psi$
(6)	$\delta\phi = \bar{\epsilon}\lambda$
(7)	$\delta(\phi^{-1}\partial\phi) = \bar{\epsilon}\mathscr{D}(\omega)\lambda$

TABLE 2
The different structures in the variation of the action. The third column indicates identities used to rewrite various contributions. The last column shows how these contributions are cancelled. A $\delta\psi$ or a $\delta\lambda$-entry indicates a modification of the transformation rules of the corresponding fermion.

#	Variation	Identity	Cancelled by
(A)	$\bar{\epsilon}\psi_{(2)}R^2\mathscr{D}R$	(2.21), (2.22)	$\delta\psi$, $\delta\lambda$
(B)	$\bar{\epsilon}\mathscr{D}\psi_{(2)}R^3$	(2.20), (2.23)–(2.24)	(I), (J), $\delta\psi$, $\delta\lambda$
(C)	$\bar{\epsilon}\psi R(\mathscr{D}R)^2$	–	–
(D)	$\bar{\epsilon}\psi R^2\mathscr{D}\mathscr{D}R$	(4.4)	(I)
(E)	$\bar{\epsilon}\mathscr{D}\lambda R^2\mathscr{D}R$	(2.25)	$\delta\lambda$, $\delta\psi$
(F)	$\bar{\epsilon}\mathscr{D}\mathscr{D}\lambda R^3$	(2.25), (4.3)	(J), $\delta\lambda$
(G)	$\bar{\epsilon}\lambda R(\mathscr{D}R)^2$	–	–
(H)	$\bar{\epsilon}\lambda R^2\mathscr{D}\mathscr{D}R$	(4.4)	(J)
(I)	$\bar{\epsilon}\psi R^4$	–	–
(J)	$\bar{\epsilon}\lambda R^4$	–	–

In table 1 we present a schematic form of the supersymmetry transformations relevant for our purposes. Their precise form is given in sect. 2. Note that due to the restrictions we have imposed we may refrain from considering various other contributions such as $\delta\omega = \bar{\epsilon}\psi H$.

Table 2 shows the generic structure of the variations of the action that emerge when applying the transformations (1)–(7) to the ansatz. In calculating the variation of the ansatz we always integrate away from the supersymmetry parameter ϵ by performing a partial integration. The variation is then simplified by working out products of Γ-matrices, etc., and brought to a standard form. The result then has to vanish, which determines the unknown coefficients.

In many cases however, contributions to a variation do not have to cancel against each other. If a variation is proportional to one of the equations of motion (2.15)–(2.19) it can be cancelled by changing the transformation rule of the corresponding field with a contribution of $O(\gamma)$. Consider for example a variation which is of the form

$$\delta\mathscr{L}_{\text{tot}} = \gamma\bar{\epsilon}O^{\mu}\Psi_{\mu}, \tag{4.1}$$

where O^{μ} is a field-dependent object which may contain Γ-matrices. Since Ψ_{μ} is the gravitino equation of motion of the action $\mathscr{L}_R$, a variation $\delta_{\gamma}\overline{\psi}_{\mu}$ of the gravitino in $\mathscr{L}_R$ with parameter $-\gamma\bar{\epsilon}O_{\mu}$ will give

$$\delta_{\gamma}\mathscr{L}_R = -\gamma\bar{\epsilon}O^{\mu}\Psi_{\mu}. \tag{4.2}$$

This new transformation rule of the gravitino cancels (4.1) in the variation of $\mathscr{L}_R + \mathscr{L}_{\text{tot}}$. The new transformation applied to $\mathscr{L}_{\text{tot}}$ gives a contribution proportional to γ^2, which we need not consider in this stage of our procedure.

TABLE 3

All contributions to the variations considered in table 2. The numbers in the table correspond to the supersymmetry transformations given in table 1. The $\mathscr{L}_i$-entries denote the different sectors of the ansatz, given in the appendix.

$\mathscr{L}_i$	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(J)
$\mathscr{L}_1$	(3)	–	–	–	–	–	–	–	(5)	(6)
$\mathscr{L}_2$	(2)	–	–	–	–	–	–	–	–	–
$\mathscr{L}_3$	–	–	–	–	–	–	–	–	(2)	–
$\mathscr{L}_4$	–	–	–	–	(7)	–	–	–	–	–
$\mathscr{L}_5$	(4)	–	–	–	–	–	–	–	–	–
$\mathscr{L}_6$	(4)	(4)	–	–	–	–	–	–	–	–
$\mathscr{L}_7$	(1)	(1)	–	–	–	–	–	–	(4)	–
$\mathscr{L}_8$	(1)	–	(1)	(1)	–	–	–	–	–	–
$\mathscr{L}_9$	–	–	–	–	–	–	–	–	–	(4)
$\mathscr{L}_{10}$	–	–	–	–	(1)	–	(1)	(1)	–	–
$\mathscr{L}_{11}$	–	–	–	–	(1)	(1)	–	–	–	–
$\mathscr{L}_{12}$	–	–	–	–	–	(4)	–	–	–	–
$\mathscr{L}_{13}$	–	–	–	–	(4)	–	–	–	–	–
$\mathscr{L}_{14}$	–	–	–	–	–	–	–	(4)	–	–
$\mathscr{L}_{15}$	–	–	–	–	–	–	(4)	–	–	–

If a variation of $\mathscr{L}_{\text{tot}}$ can be rewritten using an identity such as (2.20) then its contribution is shifted to another part of the calculation. Besides (2.20) one also has the useful relations

$$\mathscr{D}_{[\mu}\mathscr{D}_{\nu]}\lambda = -\tfrac{1}{8}\Gamma^{ab}\lambda R_{\mu\nu ab}, \tag{4.3}$$

$$\mathscr{D}_{[\mu}\mathscr{D}_{\nu]}R_{abcd} = R_{\mu\nu[a}{}^{f}R_{b]fcd} - R_{\mu\nu[\underline{c}}{}^{f}R_{abf\underline{d}]}. \tag{4.4}$$

In some cases, using identities such as (2.21)–(2.24), a contribution can be rewritten in terms of equations of motion and additional terms which contribute to other variations. This mechanism is indicated in the third and fourth columns of table 2. In the fourth column we have not indicated explicitly cancellation through modifications to the transformation rule of the tenbein. Ricci tensors occur, either directly or through (3.1), in all the variations (A)–(J).

The basic tactic is then to shift as much as possible of a particular variation to equations of motion and/or the variations (I) and (J) of table 2, by using the identities mentioned in the third column of the table. Everything which cannot be shifted, which is true in particular for all contributions to the variations (I) and (J), has to cancel and is used to fix coefficients. Table 3 indicates how the different sectors of the ansatz contribute, through the supersymmetry transformations of table 1, to the variations of table 2.

As an example consider variations of type (B), i.e. $\bar{\epsilon}\mathscr{D}\psi_{(2)}R^3$. From table 3 we see that these variations are generated by the sectors $\mathscr{L}_6$ and $\mathscr{L}_7$. From $\mathscr{L}_6$, the

$\bar{\psi}_{(2)}\Gamma\mathscr{D}\psi_{(2)}R^2$-terms, we obtain (B) by varying the first gravitino curvature, which is the transformation numbered (4) in table 1. From $\mathscr{L}_7$, the Noether terms $\bar{\psi}\Gamma\psi_{(2)}R^3$, we find this variation by varying the gravitino, and taking, after the partial integration away from ϵ, the contribution containing $\mathscr{D}\psi_{(2)}$. This is transformation (1). On simplifying these variations we isolate those contributions which can be written as a Bianchi identity (2.20), or which take on the form (2.23)–(2.24). This gives variations of type (I) and (J) ((J) only in the case (2.23) is used) and equations of motion. Note that in the variations (B) we will not encounter the left-hand-side of (2.21)–(2.22). Such contractions between $\psi_{(2)}$ and Γ-matrices are absent in the ansatz, as explained in sect. 3. Contributions containing (2.21)–(2.22) would therefore have to come from the variation of the $\mathscr{L}_6$-terms, but it is easy to see that the products of Γ-matrices in these variations do not involve the indices of the gravitino curvature.

Of course also the bosonic equations of motion, and the Bianchi identity (3.7) are used in the same way. However, the use of these does not generate a remainder.

An important role in the calculation is played by the two K-terms (A.4). If they are part of the action, the power of the dilaton in front of the action (3.9) will have to vanish. We find that indeed the presence of the K-terms is unavoidable. Interestingly, this result can be seen relatively easily, since only a few terms in the ansatz interact with the K-terms. As an example, which also illustrates explicitly our procedure *, we will work out the contribution of the K_1-term.

In the variation of K_1 we only have to consider the transformation of the field B, (2.3). The ϵ-tensor and the Γ-matrix are combined to give

$$\delta K_1 = -\tfrac{1}{2}\sqrt{2}\; R_{mn}{}^{ab}R_{pqab}R_{rs}{}^{cd}R_{tucd}\bar{\epsilon}\Gamma^{mnpqrstuv}\psi_v . \tag{4.5}$$

The only term in the ansatz which gives rise to a similar variation is M_{106} in (A.8). In M_{106} we have to vary the gravitino and the gravitino curvature. After a partial integration, and upon using the Bianchi identity (3.7) we find

$$\begin{aligned}\delta M_{106} = &-R_{mn}{}^{ab}R_{pqab}R_{rs}{}^{cd}\bar{\epsilon}\Gamma^{mnpqrst}\mathscr{D}_t\psi_{cd}\\ &+\tfrac{1}{4}R_{mn}{}^{ab}R_{pqab}R_{rs}{}^{cd}R_{tucd}\bar{\epsilon}\Gamma^{tu}\Gamma^{mnpqr\,sv}\psi_v .\end{aligned} \tag{4.6}$$

In the first term we extract Γ^t from the Γ-matrix, using

$$\Gamma^{mnpqr\,st} = \Gamma^{mnpqr\,s}\Gamma^t - 6\Gamma^{[mnpqr}\delta^{s]t} .$$

* Except for the fact that the algebraic manipulations in the following calculation were of course performed by our computer program!

Thus we obtain

$$-R_{mn}{}^{ab}R_{pqab}R_{rs}{}^{cd}\bar{\epsilon}\Gamma^{mnpqr\,s}\not{\mathcal{D}}\psi_{cd},$$

and other terms, which will never contribute to a variation with a nine-index Γ-matrix. Now we use (2.23) to obtain terms proportional to equations of motion, as well as

$$+\tfrac{1}{4}R_{mn}{}^{ab}R_{pqab}R_{rs}{}^{cd}R_{tucd}\bar{\epsilon}\Gamma^{mnpqr\,s}\Gamma^{v}\Gamma^{tu}\psi_{v}. \tag{4.7}$$

We now work out all the products of Γ-matrices in the second term in (4.6) and (4.7), and finally obtain the following contribution with a $\Gamma^{(9)}$:

$$\delta M_{106}=\tfrac{1}{2}R_{mn}{}^{ab}R_{pqab}R_{rs}{}^{cd}R_{tucd}\bar{\epsilon}\Gamma^{mnpqr\,stuv}\psi_{v}. \tag{4.8}$$

The contributions (4.5) and (4.8) must cancel, since none of the other terms in the ansatz produces such a variation. Therefore

$$k_1=\tfrac{1}{2}\sqrt{2}\ m_{106}. \tag{4.9}$$

To find out whether or not a term of type K_1 is present we therefore have to know the value of m_{106}. This coefficient is determined by considering the following two variations:

$$R_{mn}{}^{ab}R_{pqab}R_{stcd}R^{stcd}\bar{\epsilon}\Gamma^{mnpqr}\psi_{r}, \tag{4.10}$$

$$R_{mnab}R^{mnab}R_{stcd}R^{stcd}\bar{\epsilon}\Gamma^{r}\psi_{r}. \tag{4.11}$$

To (4.10) we get contributions from M_{106}, on working out the product of Γ-matrices in (4.6) and (4.7). We also get contributions from M_{30}. By a calculation similar to the one outlined above for M_{106}, using the equation of motion (2.23), we get two equal contributions from M_{30}. We then find $m_{30}=2m_{106}$. Finally we calculate the contributions to the variation (4.11). These come from the previous calculation of the variation of M_{30}, and also from the tenbein variation in A_1. The result is that $m_{30}=\frac{1}{2}a_1$. Thus we conclude that this calculation determines k_1:

$$k_1=\tfrac{1}{8}\sqrt{2}\ a_1. \tag{4.12}$$

The presence of the K_1-term is therefore linked by supersymmetry to the presence of A_1. The possibility of having $a_1=0$ will be discussed in the next section. None of the other terms in the ansatz contributes to (4.10) or (4.11). The feature which singles out these variations is the contraction between the index of the gravitino and the Γ-matrix. Such a contraction can only arise from the variation of the tenbein in the A-terms (A.1), or from terms in $\mathscr{L}_7$ (A.8), which already have such a contraction. A glance at such terms in the ansatz shows that indeed only M_{106} and M_{30} have the appropriate structure.

The K_1-term is only invariant under the gauge transformations of the B-field, if the factor dependent on the dilaton in (3.9) is absent. We expect then, given the presence of the K-term, that supersymmetry will fix $y=0$. To see this we will consider variations of type (J), $\bar{\epsilon}\lambda R^4$. There are three variations which play a determining role in fixing the value of y. These are

$$R_{mnab}R^{mnab}R_{stcd}R^{stcd}\bar{\epsilon}\lambda, \tag{4.13}$$

$$R_{mn}{}^{ab}R_{pqab}R_{stcd}R^{stcd}\bar{\epsilon}\Gamma^{mnpq}\lambda, \tag{4.14}$$

$$R_{mn}{}^{ab}R_{pqab}R_{rscd}R^{tucd}\bar{\epsilon}\Gamma^{mnpqrstu}\lambda. \tag{4.15}$$

To these variations we will get contributions from M_{106} and M_{30}. These arise from the use of (2.23) in (4.6) and in the related variation of M_{30}. Then there are contributions from (A.10), in particular from P_1 and P_{21}, obtained from the variation of the gravitino curvature $\psi_{(2)}$. Finally there is of course a contribution to (4.13) from the variation of ϕ^y in front of the A_1-term. The resulting equations for the coefficients read

$$(4.13) \to -\tfrac{1}{3}y\sqrt{2}\ a_1 - \sqrt{2}\ m_{30} + \tfrac{1}{2}p_1 = 0,$$

$$(4.14) \to +2\sqrt{2}\ m_{30} - 4\sqrt{2}\ m_{106} - p_1 + 2p_{21} = 0,$$

$$(4.15) \to +2\sqrt{2}\ m_{106} - p_{21} = 0.$$

These three equations fix p_1 and p_{21}, and set $y=0$ (unless $a_1=0$, in which case y remains arbitrary at this stage).

The above calculation shows that any solution with $a_1 \neq 0$ will require the presence of K_1, and therefore the absence of an overall dilaton-dependent factor in front of the action.

The result of the above calculation should be compared with the results presented in ref. [25]. There the same terms that we consider above appeared in the ansatz for the quartic action, and a similar calculation was done. The major difference is, however, that in ref. [25] an R^2-action, related to the supersymmetrization of the Lorentz Chern–Simons terms, is present as well. Then the cancellation of the variation of the quartic action also involves contributions which arise iteratively from the quadratic and cubic action. One may check, that these contributions (which can be found in ref. [25]) have the effect of setting $k_1=0$ and $y=-3$.

The above calculation is a small part of the complete calculation which determines all coefficients in the ansatz. But the general procedure should now be clear. The contributions to the variations are brought to a standard form, in such a way that the remaining structures are all independent. Of course one uses the identities mentioned in table 2 to express the variation in terms of independent structures. For each independent structure in the variation of the action one finds an equation between the coefficients in the ansatz. In solving the equations, free parameters may remain. Certainly one free parameter is associated with the normalization of the action. Free parameters may also indicate that the ansatz is overcomplete in the sense that a subset of the contributions to the ansatz may be dependent. This occurs, for instance, for the seven terms in (A.2), of which only four are independent because of the identities (A.3). Other free parameters indicate the presence of more than one solution to the problem of supersymmetrization. These aspects of our result will be discussed in the following section.

Table 3 shows that the calculation splits in a natural way in two almost independent parts. The variations (A)–(D) and (I) (the ψ-sector) are independent of the dilatino λ, the variations (E)–(H) and (J) (the λ-sector) do depend on λ. All these λ-dependent variations come from λ-dependent terms in the ansatz, except those due to the variation of the dilaton (see table 1). The transformation (7) in table 2 is only applied to a single sector of the ansatz, (A.5), which does not contribute to the ψ-sector. Therefore it seems that, except for the variation of the dilaton factor in front of the total action, there is no contact between the ψ-sector and the λ-sector. However, the use of (2.21)–(2.24) provides contributions which move from the ψ-sector to the λ-sector. Therefore it is essential to first work out the variations in the ψ-sector.

As we shall see, the equations resulting from the ψ-sector are very restrictive, and result in two independent solutions. The equations in the λ-sector are much less restrictive. As we discussed above, the cancellation of the (B)-variations involves only the identities (2.23)–(2.24). Using these, the (B)-variation produces $\bar{\epsilon}\lambda R^4$, (J)-terms. We expect the identities (2.21) and (2.22) to play a role in the variation (A). Since (2.21) contains a $\mathscr{D}\lambda$-contribution, the use of (2.21) in the cancellation of (A) provides a link between the ψ-sector and a variation containing $\mathscr{D}\lambda$. In table 3 we see that there are several contributions to (A). Since no contractions between a Γ-matrix and the gravitino curvature are present in the ansatz, only the variation of $\mathscr{L}_8$, the $\bar{\psi}\psi R^2\mathscr{D}R$-terms, can produce such a contraction. Therefore, all contributions containing $\mathscr{D}\lambda$ arising from the λ-sector are proportional to the parameters in $\mathscr{L}_8$. However, the equations arising from the ψ-sector require, that all these parameters vanish!

We conclude that the only link between the ψ- and λ-sector is through (B) and (J), and through the variation of ϕ^{y} in front of the action, which also gives (J). Therefore we may choose a minimal option in the λ-sector, which is to include only those λ-dependent terms in the action which contribute to (J). As we see in table 3, this is the sector $\mathscr{L}_9$, the $\bar{\lambda}\psi_{(2)}R^3$-terms. Indeed, the calculation shows that cancellation of all λ-dependent terms in the variation can be achieved by including $\mathscr{L}_9$ only.

Besides this minimal option we have also considered the inclusion of the sectors $\mathscr{L}_4$, $\mathscr{L}_{10-15}$. The variations from these terms have to cancel against each other. We have found that the resulting equations are not sufficiently restrictive to solve for all parameters in this part of the ansatz. When discussing our results, in the next section, we will restrict ourselves to the minimal option mentioned above. Of course, this does not mean that we think that the coefficients in $\mathscr{L}_4$, $\mathscr{L}_{10-15}$ are actually zero. It only means that these coefficients cannot be determined, in terms of a small number of free parameters, in the present calculation. The same remark holds for other sectors in the R^4-action which we have not considered in the construction of the ansatz (see sect. 3).

5. Results

Using the procedure discussed in the previous section, we find that supersymmetry requires that the bosonic terms must occur in the following combination:

$$\begin{aligned}\mathscr{L} = {} & a_1 A_1 + (-16a_1 + b)A_2 + 2a_1 A_3 + (12a_1 - 2b)A_4 \\ & + (-32a_1 + 4b)A_5 + (16a_1 - 2b)A_6 + (-16a_1 + 2b)A_7 \\ & + b_1 B_1 + b_2 B_2 + b_3 B_3 + \left(-\tfrac{1}{2}b_2 - b_3 + 6\sqrt{2}\, b\right)B_4 \\ & + 2b_2 B_5 + \left(b_1 + \tfrac{1}{4}b_2 + \tfrac{1}{2}b_3 + 3\sqrt{2}\, b\right)B_6 - \left(b_2 - 2b_3 - 12\sqrt{2}\, b\right)B_7 \\ & + \tfrac{1}{8}\sqrt{2}\, a_1 K_1 + \tfrac{1}{2}\sqrt{2}\left(-a_1 + \tfrac{1}{8}b\right)K_2, \end{aligned} \tag{5.1}$$

where $b = \frac{1}{24}\sqrt{2}(b_2 + 2b_3 + 2b_4)$. The coefficients b_{1-4} remain free parameters after solving the equations. Three of these are redundant because of the three identities (A.3), which imply that B_{1-7} are not independent. We can therefore take arbitrary values for b_{1-3}, without changing the action. Thus b and a_1 are the only true free parameters remaining in the action, which can therefore be written as a linear combination of two independent invariants.

Expressed in terms of X, Y_1, Y_2 and Z the R^4-contribution in (5.1) reads

$$\begin{aligned}\mathscr{L} = {} & cX + \tfrac{1}{8}7!\left(a_1 - \tfrac{1}{8}b\right)Z \\ & + \left[6c - \left(\tfrac{1}{8}a_1 + \tfrac{3}{64}b\right)\right]Y_1 + \left[-24c + \tfrac{1}{2}\left(a_1 - \tfrac{1}{8}b\right)\right]Y_2. \end{aligned} \tag{5.2}$$

Here the coefficient c is arbitrary and reflects the dependence of X, Y_1 and Y_2 discussed in sect. 3.

In (5.1) we remark that for any nontrivial choice of a_1 and b at least one of the K-terms is present. Our conclusion from sect. 4, that the exponent y in the factor ϕ^y must vanish, is therefore valid for arbitrary a_1 and b. Thus $a_1 = 0$ plays no special role in this respect.

We will now discuss the two independent solutions contained in (5.1). The first one is associated with $b = 0$, the second with $b = 8a_1$. The most convenient way to express these two solutions in terms of X, Z, Y_1 and Y_2 is to take $c = \frac{1}{48}(a_1 - \frac{1}{8}b)$ in (5.2). The parameter a_1 is then a normalization factor, which we set equal to one.

The complete action corresponding to the choice $b = 0$ (with $b_{1-3} = 0$) is displayed in Appendix B (B.1). The bosonic part of this invariant reads

$$\begin{aligned} I_1 = e\big(&R_{abef}R_{abef}R_{cdgh}R_{cdgh} - 16R_{acef}R_{bcef}R_{adgh}R_{bdgh} \\ &+ 2R_{abef}R_{cdef}R_{abgh}R_{cdgh} \\ &+ 12R_{abef}R_{cdef}R_{acgh}R_{bdgh} - 32R_{abef}R_{cdef}R_{agch}R_{bgdh} \\ &+ 16R_{aebf}R_{cedf}R_{agbh}R_{cgdh} - 16R_{aebf}R_{cedf}R_{agch}R_{bgdh}\big) \\ &+ \tfrac{1}{8}i\sqrt{2}\ \varepsilon^{\mu_1\cdots\mu_{10}}B_{\mu_1\mu_2}R_{\mu_3\mu_4}{}^{ab}R_{\mu_5\mu_6}{}^{ab}R_{\mu_7\mu_8}{}^{cd}R_{\mu_9\mu_{10}}{}^{cd} \\ &- \tfrac{1}{2}i\sqrt{2}\ \varepsilon^{\mu_1\cdots\mu_{10}}B_{\mu_1\mu_2}R_{\mu_3\mu_4}{}^{ab}R_{\mu_5\mu_6}{}^{ac}R_{\mu_7\mu_8}{}^{bd}R_{\mu_9\mu_{10}}{}^{cd}. \end{aligned} \tag{5.3}$$

The R^4-terms in (5.3) correspond to the combination $\frac{1}{48}[X + (6 \times 7!)Z]$.

Note that this solution has no terms linear in H. In ref. [13] it was found that in the string effective action the Riemann tensor should depend on the modified spin-connection Ω_- (see (2.6)). However, when X and Z are written in terms of the modified spin-connection Ω_-, and one then expands in H, terms linear in H cancel. Thus the effect of torsion appears only in the terms at least quadratic in H, which we do not consider here.

The complete action corresponding to the choice $b = 8a_1$ (with $b_1 = b_2 = 0$, $b_3 = -48\sqrt{2}\ a_1$) is presented in (B.2). The bosonic part of this invariant is given by

$$\begin{aligned} I_2 = e\Big[&R_{abef}R_{abef}R_{cdgh}R_{cdgh} - 8R_{acef}R_{bcef}R_{adgh}R_{bdgh} \\ &+ 2R_{abef}R_{cdef}R_{abgh}R_{cdgh} - 4R_{abef}R_{cdef}R_{acgh}R_{bdgh} \\ &+ 96\sqrt{2}\ \hat{H}^{abc}\big(-\tfrac{1}{2}R_{abem}R_{ghfm}\mathscr{D}_c R_{efgh} + R_{abem}R_{ghfm}\mathscr{D}_e R_{cfgh}\big)\Big] \\ &+ \tfrac{1}{8}i\sqrt{2}\ \varepsilon^{\mu_1\cdots\mu_{10}}B_{\mu_1\mu_2}R_{\mu_3\mu_4}{}^{ab}R_{\mu_5\mu_6}{}^{ab}R_{\mu_7\mu_8}{}^{cd}R_{\mu_9\mu_{10}}{}^{cd}. \end{aligned} \tag{5.4}$$

The R^4-terms in (5.4) are $-\frac{1}{2}Y_1$. The presence of K_1 implies that there is no factor ϕ^y in front of I_2.

Using pair exchange for the Riemann tensor, all R^4-terms in (5.4) can be rewritten in terms of

$$V_{\mu\nu\lambda\rho} \equiv R_{\mu\nu}{}^{ab}(\omega) R_{\lambda\rho}{}^{ab}(\omega) \tag{5.5}$$

and its contractions. Note that $V_{\mu\nu\lambda\rho}$ is the Lorentz analogue of the Yang–Mills invariant tr $F_{\mu\nu}F_{\lambda\rho}$. Because the connection Ω_- transforms under supersymmetry as a Yang–Mills gauge field (compare (2.12) and (2.26)), this analogy only holds if the spin-connection in V is Ω_-. This suggests that the action should be rewritten in terms of the torsionful connection Ω_-. Indeed, the two terms linear in H in (5.4) are precisely what is needed to introduce H-torsion, with the coefficient as in (2.6), in the R^4-terms.

The fermionic contributions to both I_1 and I_2 can be found in Appendix B. One surprise (for us) in this fermionic sector is that all terms of the type $\bar{\psi}\Gamma\psi R^2\mathscr{D}R$ have a vanishing coefficient. Note that implicitly such terms appear in the action in (A.1) in the ψ^2-torsion in ω, and in (A.2) in the supercovariantization in $\hat{H}$. Another way of presenting our result about $\mathscr{L}_8$ is to say that all such terms can be absorbed into ψ^2-torsion in ω and in supercovariantizations.

Both the actions I_1 and I_2 contain terms dependent on the field λ. In sect. 4 we discussed our procedure with respect to the λ-sector. Because of the vanishing of $\mathscr{L}_8$, it is possible to include only $\mathscr{L}_9$ in the λ-sector, the so-called minimal option. All the coefficients p_i are then determined.

In the calculations leading to I_1 and I_2 we use the identities (2.23) and (2.24). The terms in the variation in which we encounter the left-hand-side of (2.3) and (2.24) are for I_1:

$$\begin{aligned}
&\left(R_{abcd}R_{ajef}R_{bkgh} - \tfrac{1}{4}R_{abcd}R_{abef}R_{jkgh}\right)\bar{\epsilon}\Gamma_{cdefgh}\not{\mathscr{D}}\psi_{jk} \\
&\quad + \left(2R_{abde}R_{abci}R_{fgcj} + 12R_{acde}R_{afbi}R_{bgcj}\right)\bar{\epsilon}\Gamma_{defg}\not{\mathscr{D}}\psi_{ij} \\
&\quad + \left(-8R_{acbd}R_{aebh}R_{cfdi} + 4R_{abcd}R_{abce}R_{dfhi} + 4R_{bcad}R_{efah}R_{bcdi}\right. \\
&\quad + 20R_{bcae}R_{adfh}R_{bcdi} + 2R_{adef}R_{bcah}R_{bcdi} \\
&\quad - 16R_{abde}R_{cfah}R_{bcdi} + 24R_{abde}R_{afch}R_{bcdi} - \tfrac{1}{2}R_{abcd}R_{abcd}R_{efhi} \\
&\quad \left. + 2R_{abce}R_{abdf}R_{cdhi} - R_{abcd}R_{abef}R_{cdhi} + 8R_{acbe}R_{adbf}R_{cdhi}\right)\bar{\epsilon}\Gamma_{ef}\not{\mathscr{D}}\psi_{hi} \\
&\quad - 2R_{abde}R_{acfg}R_{bhci}\bar{\epsilon}\Gamma_{defgh}\mathscr{D}_j\psi_{ij} + \left(8R_{abce}R_{abdf}R_{cdgh} - 4R_{acbe}R_{adbf}R_{cdgh}\right.
\end{aligned}$$

$$-8R_{abef}R_{acdg}R_{bcdh}+8R_{adef}R_{bcag}R_{bcdh})\bar{\epsilon}\Gamma_{efg}\mathscr{D}_i\psi_{hi}$$

$$+(32R_{acbe}R_{adbf}R_{cdeg}-20R_{abcd}R_{abef}R_{cdeg}+20R_{abcd}R_{abce}R_{dfeg})\bar{\epsilon}\Gamma_f\mathscr{D}_h\psi_{gh}. \tag{5.6}$$

Using the identities (2.23) and (2.24) this can be expressed as derivatives of the equations of motion Ψ_μ and Λ of ψ_μ and λ, and terms proportional to ψR and λR, which contribute to other variations. These last terms have been taken into account in the calculation. The equations of motion always occur in the combination $\Psi_\mu+\frac{1}{4}\sqrt{2}\ \Gamma_\mu\Lambda$. The required additional variations of ψ_μ and of λ, $\delta_\gamma\psi_\mu$ and $\delta_\gamma\lambda$, are given in (B.2). Of course, the combination of the two equations of motion implies a relation between $\delta_\gamma\psi_\mu$ and $\delta_\gamma\lambda$. The fact that the only changes in the λ transformation rules occur in this particular combination with $\delta_\gamma\psi_\mu$ is a consequence of the fact that we need only $\mathscr{L}_9$ in the λ-sector. The variation of $\mathscr{L}_9$ never gives rise to additional λ equations of motion.

For the invariant I_2 (5.4) the remaining fermionic equations of motion arise from

$$-\tfrac{1}{4}R_{cdab}R_{efab}R_{ghjk}\bar{\epsilon}\Gamma_{cdefgh}\not{\mathscr{D}}\psi_{jk}+\big(2R_{ceab}R_{dfab}R_{cdhi}-\tfrac{1}{2}R_{cdab}R_{cdab}R_{efhi}$$

$$-R_{cdab}R_{efab}R_{cdhi}+4R_{cdab}R_{ceab}R_{dfhi}\big)\bar{\epsilon}\Gamma_{ef}\not{\mathscr{D}}\psi_{hi}. \tag{5.7}$$

The corresponding modifications to the transformation rules of ψ_μ and λ are given in (B.6). The remaining variations containing bosonic equations of motion, which imply additional transformation rules for the bosonic fields, will not be presented explicitly. The new transformation rules of the bosonic fields are not immediately relevant for the compactification procedure.

Let us now come back in more detail to the analogy between these R^4-actions and quartic Yang–Mills invariants. We already mentioned above that the Riemann tensors in the bosonic part of I_2 can be expressed in terms of $V_{\mu\nu\lambda\rho}$ (5.5), if we use the torsionful spin-connection Ω_-. This requires the use of pair exchange for the Riemann tensor, which gives rise to additional $\bar{\psi}\Gamma\psi_{(2)}R^3$ Noether terms, since

$$R_{ab}{}^{cd}(\omega)=R^{cd}{}_{ab}(\omega)$$

$$-\tfrac{1}{2}\bar{\psi}^{[c}\Gamma^{d]}\psi_{ab}-\bar{\psi}^{[c}\Gamma_{[a}\psi^{d]}{}_{b]}+\tfrac{1}{2}\bar{\psi}_{[a}\Gamma_{b]}\psi^{cd}+\bar{\psi}_{[a}\Gamma^{[c}\psi_{b]}{}^{d]}. \tag{5.8}$$

The additional fermionic terms due to pair exchange give contributions to the action which make it possible to write I_2 in terms of V and

$$W_{\mu\nu}\equiv R_{\mu\nu}{}^{ab}(\omega)\psi_{ab}. \tag{5.9}$$

W is also the Lorentz form of a Yang–Mills invariant: tr $F_{\mu\nu}\chi$.

All contributions to (5.4) can be generalized to the $d=10$ Yang–Mills multiplet, if we replace in the action

$$V_{\mu\nu\lambda\rho} \to \mathrm{tr}\ F_{\mu\nu}(A) F_{\lambda\rho}(A),$$
$$W_{\mu\nu} \to \mathrm{tr}\ F_{\mu\nu}(A)\chi, \tag{5.10}$$

where A_μ and χ are the fields of the $d=10$ Yang–Mills multiplet. The resulting quartic Yang–Mills action will then be invariant under the transformations (2.26), (2.27), if the Yang–Mills analogue of the terms (5.7) allows the same treatment as in the case of the R^4-action. Writing (5.7) in terms of Yang–Mills fields we obtain

$$-\tfrac{1}{4}\mathrm{tr}\ F_{cd}F_{ef}\bar{\epsilon}\Gamma_{cdefgh}\ \mathrm{tr}\ F_{gh}\mathscr{D}\!\!\!/\,\chi + (2\mathrm{tr}\ F_{ce}F_{df} - \mathrm{tr}\ F_{cd}F_{ef})\bar{\epsilon}\Gamma_{ef}\ \mathrm{tr}\ F_{cd}\mathscr{D}\!\!\!/\,\chi$$
$$-\tfrac{1}{2}\mathrm{tr}\ F_{cd}F_{cd}\bar{\epsilon}\Gamma_{ef}\ \mathrm{tr}\ F_{ef}\mathscr{D}\!\!\!/\,\chi + 4\ \mathrm{tr}\ F_{cd}F_{ce}\bar{\epsilon}\Gamma_{ef}\ \mathrm{tr}\ F_{df}\mathscr{D}\!\!\!/\,\chi. \tag{5.11}$$

Now, the relevant terms in the χ equation of motion which follows from the *quadratic* Yang–Mills action read *

$$\mathscr{X} = e\phi^{-3}\left(\mathscr{D}\!\!\!/\,(\omega, A)\chi + \tfrac{1}{4}\Gamma^c\Gamma^{ab}\psi_c F_{ab} + \tfrac{1}{2}\sqrt{2}\ \Gamma^{ab}F_{ab}\lambda\right), \tag{5.12}$$

so that the identity corresponding to (2.23) is

$$\mathscr{D}\!\!\!/\,(\omega, A)\chi = e^{-1}\phi^3\mathscr{X} - \tfrac{1}{4}\Gamma^c\Gamma^{ab}\psi_c F_{ab} - \tfrac{1}{2}\sqrt{2}\ \Gamma^{ab}F_{ab}\lambda. \tag{5.13}$$

So indeed we can express $\mathscr{D}\!\!\!/\,\chi$ in terms of $\mathscr{X}$, and ψR and λR-terms. Note that these last terms take on exactly the same form as the ψR and λR contributions in (2.23). This is of course essential for the invariance of the quartic Yang–Mills action, since after the use of the identity (5.13) the rest of the calculation should proceed in the same fashion as in the R^4-case.

$\mathscr{X}$ is the fermionic equation of motion of the F^2-action. Therefore, the $\mathscr{X}$-contributions in (5.11) can only be cancelled by changing the χ transformation rule if we include the supersymmetric F^2-action. In this way we obtain an action

$$\mathscr{L} = R + \beta\ \mathrm{tr}\ F^2 + \gamma(\mathrm{tr}\ F^2)^2, \tag{5.14}$$

and supersymmetry will require new transformation rules of χ and A_μ of order γ/β. As a byproduct of our analysis of R^4-actions we therefore find also the

* We use here the form of the Yang–Mills supergravity action given in ref. [25].

following Yang–Mills invariant (with $W_{\mu\nu} = \mathrm{tr}\ F_{\mu\nu}\chi$):

$$
\begin{aligned}
\mathscr{L}_{\mathrm{YM}} = {} & \mathscr{L}_R + \mathscr{L}_{F^2} + \gamma e\Big(-\tfrac{1}{2} t^{\mu_1\cdots\mu_8}\ \mathrm{tr}\ F_{\mu_1\mu_2} F_{\mu_3\mu_4}\ \mathrm{tr}\ F_{\mu_5\mu_6} F_{\mu_7\mu_8} \\
& + \tfrac{1}{8} i e^{-1}\sqrt{2}\ \varepsilon^{\mu_1\cdots\mu_{10}} B_{\mu_1\mu_2}\ \mathrm{tr}\ F_{\mu_3\mu_4} F_{\mu_5\mu_6}\ \mathrm{tr}\ F_{\mu_7\mu_8} F_{\mu_9\mu_{10}} \\
& + 4\overline{W}^{\mu\nu}\Gamma^\lambda\ \mathrm{tr}\ \chi\mathscr{D}_\lambda F_{\mu\nu} - 2\ \mathrm{tr}\ F^{\mu\nu}\mathscr{D}_\mu F^{\lambda\rho}\ \mathrm{tr}\ \Gamma_{\nu\lambda\rho} X - 4\overline{W}^{\mu\nu}\Gamma^{\lambda\rho}{}_\nu\ \mathrm{tr}\ \chi\mathscr{D}_\mu F_{\lambda\rho} \\
& - 8\ \mathrm{tr}\ F^{\mu\lambda}F^\nu{}_\lambda\ \mathrm{tr}\ \bar{\chi}\Gamma_\mu\mathscr{D}_\nu\chi - 16\overline{W}^{\mu\nu}\Gamma_\nu\ \mathrm{tr}(\mathscr{D}^\lambda\chi)F_{\mu\lambda}\Big) \\
& + \gamma e\sqrt{2}\Big[\Big(\mathrm{tr}\ F^{\mu\nu}F_{\mu\nu}\bar{\lambda}\Gamma_{\lambda\rho} - 8\ \mathrm{tr}\ F^{\mu\nu}F_{\mu\lambda}\bar{\lambda}\Gamma_{\nu\rho} - 4\ \mathrm{tr}\ F^\mu{}_\lambda F_{\nu\rho}\bar{\lambda}\Gamma_{\mu\nu} \\
& + 2\ \mathrm{tr}\ F^{\mu\nu}F_{\lambda\rho}\bar{\lambda}\Gamma_{\mu\nu}\Big)W^{\lambda\rho} + \tfrac{1}{2}\ \mathrm{tr}\ F^{\mu\nu}F^{\sigma\tau}\bar{\lambda}\Gamma_{\mu\nu\sigma\tau\lambda\rho}W^{\lambda\rho}\Big] \\
& + \text{Noether terms.}
\end{aligned}
\tag{5.15}
$$

The complete invariant is presented in (B.7), the $O(\gamma)$ transformation rules of χ in (B.8). In the above we have not considered the bosonic equations of motion nor the new transformation rules of A_μ. We have checked that indeed the bosonic counterpart of (5.7) also allows the generalization to an arbitrary Yang–Mills group.

In the abelian case (5.15) reduces to the quartic contribution to the Born–Infeld action [35] coupled to supergravity, and agrees in the flat limit with the globally supersymmetric Born–Infeld action presented in ref. [36]. In the Yang–Mills case the structure of (5.15) differs in the flat limit from the result of ref. [36], since in ref. [36] only the symmetric Yang–Mills trace (i.e., $t \cdots \mathrm{tr}\ F^4$) is considered.

The invariant I_2 corresponds to one particular choice of the coefficients a_1 and b in (5.1). One may wonder, whether other choices also lead to actions which have a Yang–Mills generalization. There are, for an arbitrary Yang–Mills group, eight independent $\mathrm{tr}\ F^4$ invariants. These are given by

$$
\begin{aligned}
\mathrm{YM}_1 &= F_{\mu\nu}{}^I F^{\mu\nu\ J} F_{\lambda\rho}{}^K F^{\lambda\rho\ L}, \\
\mathrm{YM}_2 &= F_{\mu\nu}{}^I F^\nu{}_\lambda{}^J F^{\mu\rho\ K} F_{\rho\lambda}{}^L, \\
\mathrm{YM}_3 &= F_{\mu\nu}{}^I F^\nu{}_\lambda{}^J F_{\lambda\rho}{}^K F_\rho^{\ \mu\ L}, \\
\mathrm{YM}_4 &= F_{\mu\nu}{}^I F_{\lambda\rho}{}^J F^{\mu\nu K} F^{\lambda\rho\ L},
\end{aligned}
$$

multiplied by either $\mathrm{tr}\ T_I T_J\ \mathrm{tr}\ T_K T_L$, giving $\mathrm{YM}_i(1)$, or $\mathrm{tr}\ T_I t_J T_K T_L$, giving $\mathrm{YM}_i(2)$. Here T_I are the Yang–Mills generators in the fundamental representation. These eight possibilities give the following R^4-actions if we work them out for the SO(9,1) Lorentz group *:

$$\begin{aligned} &\mathrm{YM}_1(1) \to A_1, \quad \mathrm{YM}_1(2) \to A_2, \\ &\mathrm{YM}_2(1) \to A_2, \quad \mathrm{YM}_2(2) \to \tfrac{1}{4}A_4 - A_5 + A_7, \\ &\mathrm{YM}_3(1) \to A_2, \quad \mathrm{YM}_3(2) \to A_6, \\ &\mathrm{YM}_4(1) \to A_3, \quad \mathrm{YM}_4(2) \to A_4. \end{aligned} \tag{5.16}$$

Note that Z (3.10) has the wrong combination of A_5 and A_7 to be the Lorentz case of a general Yang–Mills invariant. The only way to avoid having Z in our solution (5.1) is to choose $b = 8a_1$, which leads to I_2. Thus I_{YM} is the only Yang–Mills invariant which we can reconstruct from our result. This implies that a supersymmetric action of the type $t^{\cdots}$ tr F^4, which would correspond to the generalization of Y_2, does not exist f or arbitrary Yang–Mills groups.

The action (5.15) can be generalized in the following way. We may choose a semi-simple gauge group of of the form $\mathrm{G} \times \mathrm{SO}(9,1)$. Then we can identify the gauge field of SO(9, 1) with Ω_-, and the corresponding field strength with the Riemann tensor. The invariant (5.14) then takes on the form

$$\mathscr{L} = R + \beta \operatorname{tr} F^2 + \gamma\left(R^2 + \operatorname{tr} F^2\right)^2. \tag{5.17}$$

Note that an R^2-term is not required for invariance. In the absence of quadratic terms invariance holds up to (5.11) for $\mathrm{G} \times \mathrm{SO}(9, 1)$. For the contributions containing $\mathscr{D}\!\!\!/\chi$, where χ is the partner of the G gauge field, we use (5.13). This requires the presence of the standard F^2-action. For the contributions containing $\mathscr{D}\!\!\!/\psi_{ab}$ we use (2.23), which contains an equation of motion of the R-action. Therefore no R^2-action is needed to cancel particular variations.

6. Discussion

In this paper, we have found that two supersymmetric invariants of the type $R + \gamma R^4$ exist. As a byproduct, we have also obtained the leading terms of a locally supersymmetric tr $F^2 + \gamma(\operatorname{tr} F^2)^2$-invariant.

Let us now compare our results to the effective action obtained by other methods. The tree-level string amplitude contributions to $\mathscr{L}_{\mathrm{eff}}$ contain the action $\mathscr{L}_R$, (2.14), with the Yang–Mills contribution $\mathscr{L}_{F^2}$. The field strength H of the antisymmetric tensor gauge field $B_{\mu\nu}$ is modified with Yang–Mills and Lorentz Chern–Simons terms. As discussed in sect. 2, supersymmetry requires the presence

* In this calculation we use pair exchange and the cyclic identity for the Riemann tensor.

of $\mathscr{L}_{R^2}$-terms, and quartic contributions of the form $(R^2 + \text{tr}\, F^2)^2$. In these quadratic and quartic actions the Riemann tensor depends on Ω_-, and the couplings to the dilaton are limited to the same overall factor ϕ^{-3} which is also present in $\mathscr{L}_R$. As we discussed in sect. 4, this action does not contain a term K_1 (3.12), so that the overall factor ϕ^{-3} does not interfere with the $B_{\mu\nu}$ gauge transformations. The result of supersymmetrizing the Lorentz Chern–Simons terms [25] agrees (up to field redefinitions) with the determination of the bosonic part by a string amplitude calculation [13].

In ref. [13] a different basis is used for the independent fields. The dilaton is denoted by the field D, with the correspondence $\phi = \exp(\frac{2}{3}\sqrt{2}\, D)$, ϕ being our scalar field. The tenbein in ref. [13] differs by a factor $\phi^{-3/8}$ from our tenbein. With this rescaling, we find indeed that the modified Riemann tensor $\bar{R}$ in ref. [13], which contains $e^D H$ and $\mathscr{D}\mathscr{D}D$-contributions, becomes equal to $R_{\mu\nu}{}^{ab}(\Omega_-)$.

Among the tree-level terms obtained in ref. [13] is also the contribution $\zeta(3)X$, with X given in (3.10). After the rescaling mentioned above, this term also obtains the overall factor ϕ^{-3}. Therefore we must conclude from our analysis, that this term does not have a supersymmetric completion. As we have seen, the supersymmetrization of X requires the presence of both K_1 and K_2 (3.12), (3.13), which because of B gauge invariance conflicts with the presence of the ϕ^{-3}-factor *. Therefore we still do not understand the properties of $\zeta(3)X$ in relation to supersymmetry in ten dimensions.

At the one-loop level string amplitudes reveal again the presence of the X-term, as well as further $(R^2 + \text{tr}\, F^2)^2$-terms [16,17]. However, the one-loop contributions to $\mathscr{L}_{\text{eff}}$ have *no* overall dilaton factor. One also finds a contribution proportional to tr F^4. For $E_8 \times E_8$ this term can be rewritten in the form $(\text{tr}\, F^2)^2$, but this is not possible for SO(32).

Comparing now to our results in sect. 5, we see that we can indeed supersymmetrize the one-loop contributions to the effective action, except for tr F^4, which remains a problem in case the gauge group is SO(32). In sect. 5 we showed that the supersymmetrization of $(R^2 + \text{tr}\, F^2)^2$ requires an F^2-contribution to the action, but no R^2-terms. This implies that the R^2-contributions to the effective action are completely determined by the supersymmetrization of the Lorentz Chern–Simons terms, or, in string amplitude terminology, by the tree-level contributions.

The counterterms required for the cancellation of anomalies for the gauge group $E_8 \times E_8$ are, schematically, [4]

$$\mathscr{L}_{\text{counter}} \sim \varepsilon^{\mu_1 \cdots \mu_{10}} B_{\mu_1\mu_2} \times \left[\text{tr}\, R^4 + \tfrac{1}{4}(\text{tr}\, R^2)^2 + (\text{tr}\, R^2)(\text{tr}\, F^2) + (\text{tr}\, F^2)^2\right]_{\mu_3 \cdots \mu_{10}}.$$

* The terms $\phi^{-3}K_i$ are gauge invariant under modified B gauge transformations: $\delta B_{\mu\nu} = 2[\partial_{[\mu}\Lambda_{\nu]} + 3(\phi^{-1}\partial_{[\mu}\phi)\Lambda_{\nu]}]$. However, the conflict is now shifted to the H-dependence in $\mathscr{L}_R$. The field strength H has to be modified to be invariant under the new B gauge transformations. This breaks the supersymmetry of $\mathscr{L}_R$. These modified gauge transformations are discussed in ref. [37].

All these counterterms can be seen as part of the supersymmetric actions presented in sect. 5. Note in particular, that we also obtain the relative coefficient $\frac{1}{4}$ between the two R^4-terms. Thus we find that these counterterms are indeed linked by supersymmetry to the known bosonic one-loop contributions to the quartic effective string action. The other counterterms presented in ref. [4], which contain products of Chern–Simons forms, belong in our terminology to actions R^n with $n > 4$.

In a recent paper by Duff and Lu [38] it was argued that the coupling of the heterotic five-brane [39] σ-model to background supergravity fields implies the existence of quartic terms in the Riemann tensor and Yang–Mills field strength. However, these are obtained in the version of $N = 1$, $d = 10$ supergravity with a six-index antisymmetric gauge field, which is related to our $B_{\mu\nu}$ by a duality transformation. Let us therefore consider the effect of a duality transformation on the quartic action we obtain in this paper.

For this duality transformation we focus again on the $B \wedge R \wedge R \wedge R \wedge R$-terms. They are related to Chern–Simons forms. The usual Lorentz Chern–Simons term ω_3 appears as a modification to the field strength H of the gauge field B, schematically, this reads: $H \sim \partial B + \mathrm{tr}(\omega \wedge \partial\omega + \omega \wedge \omega \wedge \omega)$, along with the Yang–Mills Chern–Simons term [3]. In the dual version of $d = 10$ supergravity with a six-index gauge field Chern–Simons terms are absent, but are replaced by an interaction term of the form $A_{(6)} \wedge R \wedge R$ in the action.

By a similar duality transformation, the terms $B \wedge R \wedge R \wedge R \wedge R$ will give rise to the Chern–Simons forms ω_7,

$$H_{(7)} \sim \partial A_{(6)} + \mathrm{tr}(\,\omega \wedge \partial\omega \wedge \partial\omega \wedge \partial\omega\,) + \dots,$$

in the seven-index field strength of $A_{(6)}$ in the six-index version of $d = 10$ supergravity. Such terms are indeed required in the anomaly cancellations in the six-index version [40].

In this paper we have supersymmetrized the one-loop, quartic terms which appear in the bosonic string effective action. We do not find a supersymmetric completion for the $\zeta(3)\phi^{-3}X$-term, which is part of the tree-level effective action. This failure may be due to the fact that we limited ourselves to the use of the physical fields of $d = 10$, $N = 1$ supergravity. Failure of the Noether method may of course indicate the necessity of introducing additional fields. These could correspond to massive fields, perhaps related to auxiliary fields of the $d = 10$, $N = 1$ supergravity multiplet, which become propagating fields in the higher derivative actions we have considered.

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Appendix A

This appendix is devoted to the presentation of the various sectors we constructed for the ansatz. We write the ansatz as the sum (3.9)

$$\mathscr{L}_{\text{tot}} = \gamma e \phi^{y} \sum_{i} \mathscr{L}_{i}.$$

The first purely bosonic sector is formed by seven terms of the form R^4 and therefore contains all possible independent contractions of four Riemann tensors:

$$\begin{aligned}
\mathscr{L}_1 = & +a_1 R_{abcd} R_{abcd} R_{efgh} R_{efgh} + a_2 R_{abcd} R_{abce} R_{dfgh} R_{efgh} \\
& + a_3 R_{abcd} R_{abef} R_{cdgh} R_{efgh} + a_4 R_{abce} R_{abdf} R_{cdgh} R_{efgh} \\
& + a_5 R_{abce} R_{abdg} R_{cfdh} R_{efgh} + a_6 R_{acbd} R_{aebg} R_{cfdh} R_{efgh} \\
& + a_7 R_{acbe} R_{adbg} R_{cfdh} R_{efgh}.
\end{aligned} \tag{A.1}$$

The second sector in our ansatz consists of seven terms of type $HR^2 \mathscr{D} R$. Its explicit form is

$$\begin{aligned}
\mathscr{L}_2 = & +b_1 H_{aef} R_{adbc} R_{ghbc} \mathscr{D}_d R_{efgh} - b_2 H_{aef} R_{abcg} R_{bdch} \mathscr{D}_d R_{efgh} \\
& + b_3 H_{abd} R_{abce} R_{ghcf} \mathscr{D}_d R_{efgh} + b_4 H_{abe} R_{abcd} R_{ghcf} \mathscr{D}_d R_{efgh} \\
& + b_5 H_{abe} R_{adcg} R_{bfch} \mathscr{D}_d R_{efgh} + b_6 H_{abc} R_{abef} R_{cdgh} \mathscr{D}_d R_{efgh} \\
& + b_7 H_{abe} R_{adcf} R_{bcgh} \mathscr{D}_d R_{efgh}.
\end{aligned} \tag{A.2}$$

It is important to realize that the seven terms in this sector are overcomplete. This is due to the fact that the Bianchi identity for the H-field implies the following relations among the different terms:

$$\begin{aligned}
& 0 = D_{[a} \hat{H}_{bcd]} R_{abef} R_{cdgh} R_{efgh} = B_1 + B_6, \\
& 0 = D_{[a} \hat{H}_{bcd]} R_{abef} R_{cegh} R_{dfgh} = \tfrac{1}{4} B_1 - \tfrac{1}{2} B_3 + \tfrac{1}{2} B_4 + B_7, \\
& 0 = D_{[a} \hat{H}_{bcd]} R_{abef} R_{cgeh} R_{dgfh} = \tfrac{1}{2} B_2 - \tfrac{1}{4} B_3 + B_5.
\end{aligned} \tag{A.3}$$

The latter results are obtained by performing a partial integration. Note that these identities are valid modulo terms of the form $\bar{\psi} \Gamma \psi_{(2)} R^3$. This is related to the fact that the Bianchi identity of the H-field involves a supercovariant derivative. The eqs. (A.3) imply that three of the coefficients b_i can be chosen arbitrarily.

The third purely bosonic sector consists of two terms of type BR^4:

$$\mathscr{L}_3 = +k_1 ie^{-1}\varepsilon_{abcdefijkl}B_{ab}R_{cdgh}R_{efgh}R_{ijmn}R_{klmn}$$
$$+k_2 ie^{-1}\varepsilon_{abcdefhikl}B_{ab}R_{cdgj}R_{efgm}R_{hijn}R_{klmn}. \qquad \text{(A.4)}$$

There are four terms of the structure $(\phi^{-1}\partial\phi)R^2\mathscr{D}R$:

$$\mathscr{L}_4 = +\phi^{-1}\partial_a\phi\big(c_1 R_{bcde}R_{fgde}\mathscr{D}_a R_{bcfg} + c_2 R_{bcde}R_{cfeg}\mathscr{D}_a R_{bfdg}$$
$$-c_3 R_{abcd}R_{defg}\mathscr{D}_b R_{cefg} - c_4 R_{abcd}R_{defg}\mathscr{D}_c R_{befg}\big). \qquad \text{(A.5)}$$

This completes the list of the purely bosonic sectors.

We considered 17 terms of the type $\bar\psi_{(2)}\Gamma\psi_{(2)}R\mathscr{D}R$:

$$\mathscr{L}_5 = +\big(d_1 R_{bcae}\mathscr{D}_a R_{bcdf} + d_2 R_{bcad}\mathscr{D}_a R_{bcef} + d_3 R_{abcd}\mathscr{D}_e R_{abcf}\big)\bar\psi_{dg}\Gamma_e\psi_{fg}$$
$$+\big(d_5 R_{cdab}\mathscr{D}_e R_{fgab} + d_6 R_{ceab}\mathscr{D}_d R_{fgab} + d_7 R_{cfab}\mathscr{D}_d R_{egab}$$
$$+d_8 R_{acbf}\mathscr{D}_e R_{adbg} + d_9 R_{acbf}\mathscr{D}_d R_{aebg} + d_{10} R_{aebf}\mathscr{D}_d R_{acbg}\big)\bar\psi_{cd}\Gamma_e\psi_{fg}$$
$$+d_{21} R_{bcaf}\mathscr{D}_a R_{bcde}\bar\psi_{gh}\Gamma_{def}\psi_{gh}$$
$$+\big(d_{22} R_{deab}\mathscr{D}_c R_{fgab} + d_{23} R_{dgab}\mathscr{D}_c R_{efab} + d_{24} R_{acbd}\mathscr{D}_f R_{aebg}\big)\bar\psi_{ch}\Gamma_{def}\psi_{gh}$$
$$+\big(-d_{25} R_{efac}\mathscr{D}_a R_{bdgh} - d_{26} R_{ghaf}\mathscr{D}_a R_{bcde} - d_{27} R_{chaf}\mathscr{D}_a R_{bgde}$$
$$-d_{28} R_{efac}\mathscr{D}_b R_{adgh} + d_{29} R_{cfah}\mathscr{D}_b R_{agde}\big)\bar\psi_{bc}\Gamma_{def}\psi_{gh}. \qquad \text{(A.6)}$$

Next, there are six terms with the structure $\bar\psi_{(2)}\Gamma\mathscr{D}\psi_{(2)}R^2$:

$$\mathscr{L}_6 = +f_1 R_{adbc}R_{aebc}\bar\psi_{fg}\Gamma_d\mathscr{D}_e\psi_{fg} + f_2 R_{acbd}R_{aebf}\bar\psi_{cg}\Gamma_d\mathscr{D}_e\psi_{fg}$$
$$+f_3 R_{bcad}R_{fgae}\bar\psi_{bc}\Gamma_d\mathscr{D}_e\psi_{fg} + f_4 R_{bcaf}R_{deag}\bar\psi_{bc}\Gamma_d\mathscr{D}_e\psi_{fg}$$
$$-f_5 R_{cdaf}R_{egab}\bar\psi_{bh}\Gamma_{cde}\mathscr{D}_f\psi_{gh} + f_6 R_{cdab}R_{efgh}\bar\psi_{ab}\Gamma_{cde}\mathscr{D}_f\psi_{gh}. \qquad \text{(A.7)}$$

For the Noether sector, the terms of type $\bar{\psi}\Gamma\psi_{(2)}R^3$, we constructed 92 independent terms:

$$\begin{aligned}
\mathscr{L}_7 = {} & +\left(m_1 R_{afbg} R_{acde} R_{bcde} + m_2 R_{afbc} R_{agde} R_{bcde} + m_3 R_{bfad} R_{cgae} R_{bcde}\right)\bar{\psi}_h \Gamma_f \psi_{gh} \\
& +\left(m_4 R_{efgh} R_{abcd} R_{abcd} + m_5 R_{efag} R_{bhcd} R_{abcd} + m_6 R_{egaf} R_{bhcd} R_{abcd}\right. \\
& + m_7 R_{ghae} R_{bfcd} R_{abcd} + m_8 R_{ghaf} R_{becd} R_{abcd} + m_9 R_{efab} R_{ghcd} R_{abcd} \\
& + m_{10} R_{egab} R_{fhcd} R_{abcd} + m_{11} R_{efab} R_{agcd} R_{bhcd} + m_{12} R_{egab} R_{afcd} R_{bhcd} \\
& + m_{13} R_{fgab} R_{aecd} R_{bhcd} + m_{14} R_{ghab} R_{aecd} R_{bfcd} + m_{15} R_{efac} R_{bgad} R_{bhcd} \\
& + m_{16} R_{egac} R_{bfad} R_{bhcd} + m_{17} R_{fgac} R_{bead} R_{bhcd} + m_{18} R_{ghac} R_{bead} R_{bfcd} \\
& + m_{19} R_{aecg} R_{bfdh} R_{abcd} + m_{20} R_{aecg} R_{bfad} R_{bhcd} + m_{21} R_{afcg} R_{bead} R_{bhcd} \\
& \left. + m_{22} R_{aebg} R_{afcd} R_{bhcd} + m_{23} R_{afbg} R_{aecd} R_{bhcd}\right)\bar{\psi}_e \Gamma_f \psi_{gh} \\
& +\left(m_{30} R_{efhi} R_{abcd} R_{abcd} + m_{31} R_{efah} R_{bicd} R_{abcd} + m_{32} R_{hiae} R_{bfcd} R_{abcd}\right. \\
& + m_{34} R_{efab} R_{hicd} R_{abcd} + m_{35} R_{ehab} R_{ficd} R_{abcd} + m_{37} R_{efab} R_{ahcd} R_{bicd} \\
& + m_{38} R_{ehab} R_{afcd} R_{bicd} + m_{39} R_{hiab} R_{aecd} R_{bfcd} + m_{42} R_{efac} R_{bhad} R_{bicd} \\
& + m_{43} R_{ehac} R_{bfad} R_{bicd} + m_{44} R_{hiac} R_{bead} R_{bfcd} + m_{47} R_{aech} R_{bfdi} R_{abcd} \\
& \left. + m_{48} R_{aech} R_{bfad} R_{bicd} + m_{49} R_{aebh} R_{ficd} R_{abcd}\right)\bar{\psi}_g \Gamma_{efg} \psi_{hi} \\
& +\left(m_{33} R_{efah} R_{bgcd} R_{abcd} + m_{36} R_{efab} R_{ghcd} R_{abcd} + m_{40} R_{efab} R_{agcd} R_{bhcd}\right. \\
& \left. + m_{41} R_{ehab} R_{afcd} R_{bgcd} + m_{45} R_{efac} R_{bgad} R_{bhcd} + m_{46} R_{ehac} R_{bfad} R_{bgcd}\right)\bar{\psi}_i \Gamma_{efg} \psi_{hi} \\
& \quad +\left(m_{501} R_{dhef} R_{agbc} R_{aibc} + m_{502} R_{efhi} R_{adbc} R_{agbc} + m_{51} R_{efad} R_{hibc} R_{agbc}\right. \\
& \quad + m_{52} R_{efad} R_{ghbc} R_{aibc} + m_{53} R_{efah} R_{dgbc} R_{aibc} + m_{54} R_{efah} R_{gibc} R_{adbc} \\
& \quad + m_{55} R_{efah} R_{dibc} R_{agbc} + m_{56} R_{deah} R_{fgbc} R_{aibc} - m_{57} R_{deah} R_{fibc} R_{agbc} \\
& \quad + m_{58} R_{ehad} R_{fgbc} R_{aibc} - m_{59} R_{ehad} R_{fibc} R_{agbc} + m_{60} R_{hiae} R_{dfbc} R_{agbc} \\
& \quad + m_{61} R_{hiae} R_{fgbc} R_{adbc} + m_{62} R_{hiad} R_{efbc} R_{agbc} + m_{63} R_{efbd} R_{agch} R_{aibc} \\
& \quad + m_{64} R_{efbh} R_{adcg} R_{aibc} + m_{65} R_{efbh} R_{agci} R_{adbc} + m_{66} R_{efbh} R_{adci} R_{agbc}
\end{aligned}$$

$$+m_{67}R_{debh}R_{aicf}R_{agbc}+m_{68}R_{ehbd}R_{aicf}R_{agbc}+m_{69}R_{hibe}R_{adcf}R_{agbc}$$

$$-m_{70}R_{deab}R_{fgac}R_{hibc}-m_{71}R_{dhab}R_{efac}R_{gibc}-m_{72}R_{efab}R_{hiac}R_{bdcg}$$

$$-m_{73}R_{ehab}R_{fiac}R_{bdcg}-m_{74}R_{efab}R_{ghac}R_{bdci}-m_{75}R_{efab}R_{dhac}R_{bgci}$$

$$-m_{76}R_{deab}R_{fhac}R_{bgci}+m_{77}R_{efab}R_{cdah}R_{bgci}+m_{78}R_{ehab}R_{cdaf}R_{bgci}$$

$$+m_{79}R_{adbe}R_{cfah}R_{bgci})\bar{\psi}_d\Gamma_{efg}\psi_{hi}$$

$$+\left(m_{90}R_{deai}R_{fgbc}R_{ahbc}-m_{91}R_{deab}R_{fgac}R_{bhci}\right)\bar{\psi}_j\Gamma_{defgh}\psi_{ij}$$

$$+\left(m_{92}R_{deai}R_{fgbc}R_{ajbc}+m_{93}R_{ijad}R_{efbc}R_{agbc}+m_{94}R_{deai}R_{fjbc}R_{agbc}\right.$$

$$\left.+m_{95}R_{debi}R_{afcj}R_{agbc}-m_{96}R_{deab}R_{fiac}R_{bgcj}\right)\bar{\psi}_h\Gamma_{defgh}\psi_{ij}$$

$$+\left(m_{97}R_{deci}R_{fgab}R_{hjab}+m_{98}R_{cdij}R_{efab}R_{ghab}+m_{981}R_{deij}R_{cfab}R_{ghab}\right.$$

$$+m_{99}R_{deai}R_{fgbj}R_{chab}+m_{100}R_{deac}R_{ijbf}R_{ghab}+m_{101}R_{diac}R_{efbj}R_{ghab}$$

$$+m_{102}R_{cdai}R_{efbj}R_{ghab}+m_{103}R_{deac}R_{fgbi}R_{hjab}$$

$$\left.+m_{104}R_{deac}R_{fgbi}R_{ahbj}\right)\bar{\psi}_c\Gamma_{defgh}\psi_{ij}$$

$$+\left(m_{105}R_{cdaj}R_{efbk}R_{ghab}+m_{106}R_{cdjk}R_{efab}R_{ghab}\right)\bar{\psi}_i\Gamma_{cdefghi}\psi_{jk}. \tag{A.8}$$

Finally, in the cancellation mechanism we also included

$$\mathscr{L}_8\text{: Terms of type } \bar{\psi}\Gamma^{(1)}\psi R^2\mathscr{D}R \text{ and } \bar{\psi}\Gamma^{(5)}\psi R^2\mathscr{D}R. \tag{A.9}$$

Altogether there are 70 terms of this type. In our solutions we find that all these terms have to vanish. We will therefore not write them explicitly.

In principle there are 19 additional sectors to be included in the ansatz. Roughly speaking, these have fewer fields and more derivatives. These sectors consist of the following structures:

$$(\bar{\psi}\psi)R\mathscr{D}^3R;\quad (\bar{\psi}\psi)\mathscr{D}R\mathscr{D}^2R;\quad (\bar{\psi}\psi_{(2)})R\mathscr{D}^2R;\quad (\bar{\psi}\psi_{(2)})(\mathscr{D}R)^2;$$

$$(\bar{\psi}\psi)\mathscr{D}^5R;\quad (\bar{\psi}\psi_{(2)})\mathscr{D}^4R;\quad (\bar{\psi}_{(2)}\psi_{(2)})\mathscr{D}^3R;\quad (\bar{\psi}_{(2)}\mathscr{D}\psi_{(2)})\mathscr{D}^2R;$$

$$(\mathscr{D}\bar{\psi}_{(2)}\mathscr{D}\psi_{(2)})\mathscr{D}R;\quad (\bar{\psi}_{(2)}\mathscr{D}^2\psi_{(2)})\mathscr{D}R;\quad (\mathscr{D}\bar{\psi}_{(2)}\mathscr{D}^2\psi_{(2)})R;\quad (\bar{\psi}_{(2)}\mathscr{D}^3\psi_{(2)})R;$$

$$(\bar{\psi}\mathscr{D}\psi_{(2)})R\mathscr{D}R;\quad (\bar{\psi}\mathscr{D}^2\psi_{(2)})R^2;\quad (\bar{\psi}\mathscr{D}\psi_{(2)})\mathscr{D}^3R;\quad (\bar{\psi}\mathscr{D}^2\psi_{(2)})\mathscr{D}^2R;$$

$$(\bar{\psi}\mathscr{D}^3\psi_{(2)})\mathscr{D}R;\quad (\bar{\psi}\psi_{(2)})\mathscr{D}^4R;\quad \hat{H}\mathscr{D}R\mathscr{D}^2R.$$

They participate in the cancellation mechanism through the use of the relation (4.4). We have constructed all possible terms of this type (for a few of the above structures there are actually no contributions), and found that the equations require that the corresponding coefficients vanish. Therefore these terms have no effect on our solutions, and we refrain from presenting their explicit parametrization.

This completes the discussion of all sectors of the ansatz which contain the gravitino field and the gravitino curvature.

There are six sectors which contain the dilatino field λ. We constructed 21 independent terms of the structure $\bar{\lambda}\Gamma\psi_{(2)}R^3$:

$$\begin{aligned}\mathscr{L}_9 = \big(&p_1 R_{efgh}R_{abcd}R_{abcd} + p_2 R_{efag}R_{bhcd}R_{abcd} + p_3 R_{ghae}R_{bfcd}R_{abcd}\\ &+p_4 R_{efab}R_{ghcd}R_{abcd} + p_5 R_{egab}R_{fhcd}R_{abcd} + p_6 R_{efab}R_{agcd}R_{bhcd}\\ &+p_7 R_{egab}R_{afcd}R_{bhcd} + p_8 R_{ghab}R_{aecd}R_{bfcd} + p_9 R_{efac}R_{bgad}R_{bhcd}\\ &+p_{10} R_{egac}R_{bfad}R_{bhcd} + p_{11} R_{ghac}R_{bead}R_{bfcd} + p_{12} R_{aecg}R_{bfdh}R_{abcd}\\ &+p_{13} R_{aecg}R_{bfad}R_{bhcd} + p_{14} R_{aebg}R_{afcd}R_{bhcd}\big)\bar{\lambda}\Gamma_{ef}\psi_{gh}\\ &+\big(p_{15} R_{deah}R_{fgbc}R_{aibc} + p_{16} R_{hiad}R_{efbc}R_{agbc} + p_{17} R_{deah}R_{fibc}R_{agbc}\\ &+p_{18} R_{debh}R_{afci}R_{agbc} - p_{19} R_{deab}R_{fhac}R_{bgci}\big)\bar{\lambda}\Gamma_{defg}\psi_{hi}\\ &+\big(p_{20} R_{cdai}R_{efbj}R_{ghab} + p_{21} R_{cdij}R_{efab}R_{ghab}\big)\bar{\lambda}\Gamma_{cdefgh}\psi_{ij}. \end{aligned} \tag{A.10}$$

Besides the sector $\mathscr{L}_9$ there are the following λ-dependent contributions:

$$\mathscr{L}_{10} \sim \bar{\psi}\lambda R^2 \mathscr{D}R, \tag{A.11}$$

$$\mathscr{L}_{11} \sim \bar{\psi}\mathscr{D}\lambda R^3, \tag{A.12}$$

$$\mathscr{L}_{12} \sim \bar{\psi}_{(2)}\mathscr{D}^2\lambda R^2, \tag{A.13}$$

$$\mathscr{L}_{13} \sim \bar{\psi}_{(2)}\mathscr{D}\lambda R\mathscr{D}R, \tag{A.14}$$

$$\mathscr{L}_{14} \sim \bar{\psi}_{(2)}\lambda R\mathscr{D}^2 R, \tag{A.15}$$

$$\mathscr{L}_{15} \sim \bar{\psi}_{(2)}\lambda \mathscr{D}R\mathscr{D}R. \tag{A.16}$$

As we explained in sect. 4, these additional sectors may be included, but are not actually required to achieve the cancellation of the variations we consider. Since we choose in sect. 5 for the minimal option of the including only (A.10) in the presentation our results, we shall not give the parametrization of $\mathscr{L}_{10-15}$ explicitly. For the same reason, we do not display λ-dependent terms containing more derivatives, which might participate through the use of (4.3).

Appendix B

This appendix is devoted to the presentation of the two solutions we have found. If we choose in (5.1) $b=0$, $b_1=b_2=b_3=0$ and $a_1=1$ we obtain

$$
\begin{aligned}
e^{-1}I_1 = {} & +R_{abcd}R_{abcd}R_{efgh}R_{efgh} - 16R_{abcd}R_{abce}R_{dfgh}R_{efgh} \\
& + 2R_{abcd}R_{abef}R_{cdgh}R_{efgh} + 12R_{abce}R_{abdf}R_{cdgh}R_{efgh} \\
& - 32R_{abce}R_{abdg}R_{cfdh}R_{efgh} + 16R_{acbd}R_{aebg}R_{cfdh}R_{efgh} \\
& - 16R_{acbe}R_{adbg}R_{cfdh}R_{efgh} \\
& \quad + \tfrac{1}{8}i\sqrt{2}\ e^{-1}\varepsilon_{abcdefijkl}B_{ab}R_{cdgh}R_{efgh}R_{ijmn}R_{klmn} \\
& \quad - \tfrac{1}{2}i\sqrt{2}\ e^{-1}\varepsilon_{abcdefhikl}B_{ab}R_{cdgj}R_{efgm}R_{hijn}R_{klmn} \\
& + \left(8R_{bcad}\mathscr{D}_aR_{bcef} - 8R_{abcd}\mathscr{D}_eR_{abcf}\right)\bar{\psi}_{dg}\Gamma_e\psi_{fg} \\
& + \left(-4R_{ceab}\mathscr{D}_dR_{fgab} - 8R_{cfab}\mathscr{D}_dR_{egab}\right. \\
& \left. + 16R_{acbf}\mathscr{D}_dR_{aebg} - 16R_{aebf}\mathscr{D}_dR_{acbg}\right)\bar{\psi}_{cd}\Gamma_e\psi_{fg} \\
& + 4R_{deab}\mathscr{D}_cR_{fgab}\bar{\psi}_{ch}\Gamma_{def}\psi_{gh} \\
& + \left(8R_{efac}\mathscr{D}_aR_{bdgh} + 4R_{chaf}\mathscr{D}_aR_{bgde} - 4R_{efac}\mathscr{D}_bR_{adgh}\right)\bar{\psi}_{bc}\Gamma_{def}\psi_{gh} \\
& + 16R_{acbd}R_{aebf}\bar{\psi}_{cg}\Gamma_d\mathscr{D}_e\psi_{fg} + 32R_{bcaf}R_{deag}\bar{\psi}_{bc}\Gamma_d\mathscr{D}_e\psi_{fg} \\
& + 8R_{cdaf}R_{egab}\bar{\psi}_{bh}\Gamma_{cde}\mathscr{D}_f\psi_{gh} + \left(-20R_{afbg}R_{acde}R_{bcde}\right. \\
& \left. + 20R_{afbc}R_{agde}R_{bcde} - 32R_{bfad}R_{cgae}R_{bcde}\right)\bar{\psi}_h\Gamma_f\psi_{gh} \\
& + \left(-R_{efgh}R_{abcd}R_{abcd} + 8R_{efag}R_{bhcd}R_{abcd} + 4R_{ghae}R_{bfcd}R_{abcd}\right. \\
& - 4R_{ghaf}R_{becd}R_{abcd} - 2R_{efab}R_{ghcd}R_{abcd} + 2R_{egab}R_{fhcd}R_{abcd}
\end{aligned}
$$

$$+4R_{efab}R_{agcd}R_{bhcd}-28R_{egab}R_{afcd}R_{bhcd}+20R_{fgab}R_{aecd}R_{bhcd}$$

$$+8R_{ghab}R_{aecd}R_{bfcd}+24R_{efac}R_{bgad}R_{bhcd}+48R_{egac}R_{bfad}R_{bhcd}$$

$$+16R_{fgac}R_{bead}R_{bhcd}-16R_{ghac}R_{bead}R_{bfcd}-24R_{aecg}R_{bfdh}R_{abcd}$$

$$+8R_{aecg}R_{bfad}R_{bhcd}+16R_{aebg}R_{afcd}R_{bhcd}-4R_{afbg}R_{aecd}R_{bhcd}\big)\bar{\psi}_e\Gamma_f\psi_{gh}$$

$$+\big(\tfrac{1}{2}R_{efhi}R_{abcd}R_{abcd}-4R_{efah}R_{bicd}R_{abcd}-4R_{hiae}R_{bfcd}R_{abcd}$$

$$+R_{efab}R_{hicd}R_{abcd}-2R_{efab}R_{ahcd}R_{bicd}+20R_{ehab}R_{afcd}R_{bicd}$$

$$-2R_{hiab}R_{aecd}R_{bfcd}-16R_{ehac}R_{bfad}R_{bicd}-8R_{hiac}R_{bead}R_{bfcd}$$

$$+8R_{aech}R_{bfdi}R_{abcd}-8R_{aech}R_{bfad}R_{bicd}\big)\bar{\psi}_g\Gamma_{efg}\psi_{hi}$$

$$+\big(-8R_{efab}R_{agcd}R_{bhcd}-8R_{ehab}R_{afcd}R_{bgcd}$$

$$+8R_{efac}R_{bgad}R_{bhcd}+4R_{ehac}R_{bfad}R_{bgcd}\big)\bar{\psi}_i\Gamma_{efg}\psi_{hi}$$

$$+\big(-4R_{dhef}R_{agbc}R_{aibc}-2R_{efhi}R_{adbc}R_{agbc}-2R_{efad}R_{hibc}R_{agbc}$$

$$+2R_{efah}R_{dibc}R_{agbc}-4R_{ehad}R_{fgbc}R_{aibc}-2R_{hiae}R_{fgbc}R_{adbc}$$

$$+8R_{efbd}R_{agch}R_{aibc}-4R_{efbh}R_{adcg}R_{aibc}+8R_{efbh}R_{agci}R_{adbc}$$

$$+4R_{efbh}R_{adci}R_{agbc}+16R_{debh}R_{aicf}R_{agbc}+16R_{ehbd}R_{aicf}R_{agbc}$$

$$+8R_{hibe}R_{adcf}R_{agbc}-8R_{deab}R_{fgac}R_{hibc}-8R_{dhab}R_{efac}R_{gibc}$$

$$-4R_{efab}R_{hiac}R_{bdcg}+8R_{ehab}R_{fiac}R_{bdcg}-12R_{efab}R_{ghac}R_{bdci}$$

$$+4R_{efab}R_{dhac}R_{bgci}-16R_{deab}R_{fhac}R_{bgci}+8R_{ehab}R_{cdaf}R_{bgci}$$

$$-8R_{adbe}R_{cfah}R_{bgci}\big)\bar{\psi}_d\Gamma_{efg}\psi_{hi}$$

$$+2R_{deab}R_{fgac}R_{bhci}\bar{\psi}_j\Gamma_{defgh}\psi_{ij}$$

$$+\big(2R_{deai}R_{fgbc}R_{ajbc}-12R_{deab}R_{fiac}R_{bgcj}\big)\bar{\psi}_h\Gamma_{defgh}\psi_{ij}$$

$$+\big(-2R_{deci}R_{fgab}R_{hjab}-\tfrac{1}{2}R_{cdij}R_{efab}R_{ghab}+2R_{deac}R_{ijbf}R_{ghab}$$

$$-4R_{diac}R_{efbj}R_{ghab}+4R_{deac}R_{fgbi}R_{hjab}-2R_{deac}R_{fgbi}R_{ahbj}\big)\bar{\psi}_c\Gamma_{defgh}\psi_{ij}$$

$$+\left(-R_{cdaj}R_{efbk}R_{ghab}+\tfrac{1}{4}R_{cdjk}R_{efab}R_{ghab}\right)\bar{\psi}_i\Gamma_{cdefghi}\psi_{jk}$$

$$+\sqrt{2}\Big[\big(R_{efgh}R_{abcd}R_{abcd}-8R_{efag}R_{bhch}R_{abcd}-8R_{ghae}R_{bfcd}R_{abcd}$$

$$+2R_{efab}R_{ghcd}R_{abcd}-4R_{efab}R_{agcd}R_{bhcd}+40R_{egab}R_{afcd}R_{bhcd}$$

$$-4R_{ghab}R_{aecd}R_{bfcd}-32R_{egac}R_{bfad}R_{bhcd}+16R_{ghac}R_{bead}R_{bfcd}$$

$$+16R_{aecg}R_{bfdh}R_{abcd}-16R_{aecg}R_{bfad}R_{bhcd}-32R_{aebg}R_{afcd}R_{bhcd}\big)\bar{\lambda}\Gamma_{ef}\psi_{gh}$$

$$+\left(-4R_{deah}R_{fgbc}R_{aibc}-24R_{deab}R_{fhac}R_{bgci}\right)\bar{\lambda}\Gamma_{defg}\psi_{hi}$$

$$+\left(-2R_{cdai}R_{efbj}R_{ghab}+\tfrac{1}{2}R_{cdij}R_{efab}R_{ghab}\right)\bar{\lambda}\Gamma_{cdefgh}\psi_{ij}\Big]. \qquad \text{(B.1)}$$

The modifications to the fermionic transformation rules follow from (5.6). The result is

$$\delta_\gamma\psi_\mu = D_b\left[\left(20R_{bcde}R_{\mu cfg}R_{defg}-20R_{bc\mu d}R_{cefg}R_{defg}-32R_{bdcf}R_{\mu ecg}R_{defg}\right)\epsilon\right]$$

$$+D_b\Big[\big(2R_{\mu bcd}R_{efcd}R_{efgh}+4R_{\mu ecd}R_{bfcd}R_{efgh}-16R_{ce\mu d}R_{bcdf}R_{efgh}$$

$$+24R_{\mu cde}R_{bcdf}R_{efgh}-24R_{\mu bcf}R_{cdeg}R_{defh}-12R_{\mu bcd}R_{cefg}R_{defh}$$

$$-8R_{\mu fcd}R_{bceg}R_{defh}+8R_{cfde}R_{\mu bcg}R_{defh}-40R_{bdcf}R_{ce\mu g}R_{defh}$$

$$+24R_{bfcd}R_{ce\mu g}R_{defh}-8R_{bfcd}R_{\mu ceg}R_{defh}$$

$$+4R_{bcde}R_{cfde}R_{\mu fgh}-16R_{cedf}R_{bcdg}R_{\mu efh}\big)\Gamma_{gh}\epsilon\Big]$$

$$+D_c\left[\left(-16R_{\mu dbf}R_{becg}R_{defh}-12R_{\mu bde}R_{bfcg}R_{defh}\right)\Gamma_{gh}\epsilon\right]$$

$$+4D_e\left(R_{\mu bcd}R_{bfcd}R_{efgh}\Gamma_{gh}\epsilon\right)+D_f\left(R_{bcde}R_{bcde}R_{\mu fgh}\Gamma_{gh}\epsilon\right)$$

$$+D_g\left[\left(-28R_{\mu fbc}R_{bcde}R_{defh}+32R_{\mu bcd}R_{becf}R_{defh}-20R_{becd}R_{bfcd}R_{\mu efh}\right)\Gamma_{gh}\epsilon\right]$$

$$+D_b\Big[\big(-2R_{\mu cde}R_{bcfg}R_{dehi}+2R_{bd\mu c}R_{cefg}R_{dehi}$$

$$+2R_{bcde}R_{\mu cfg}R_{dehi}+12R_{bcdf}R_{ce\mu g}R_{dehi}\big)\Gamma_{fghi}\epsilon\Big]$$

$$+D_c\left[\left(-8R_{\mu bdf}R_{becg}R_{dehi}+4R_{\mu dbf}R_{becg}R_{dehi}\right)\Gamma_{fghi}\epsilon\right]$$

$$+D_f\Big[\big(-8R_{\mu cbd}R_{becg}R_{dehi}+8R_{\mu dbc}R_{bceg}R_{dehi}$$

$$+4R_{bc\mu g}R_{bdeh}R_{cdei}-8R_{be\mu g}R_{cdbh}R_{cdei}\big)\Gamma_{fghi}\epsilon\Big]$$

$$+D_b\Big[\big(\tfrac{1}{2}R_{\mu bef}R_{cdgh}R_{cdij}-2R_{bcef}R_{\mu dgh}R_{cdij}\big)\Gamma_{efghij}\epsilon\Big]$$

$$-2D_e\big(R_{\mu bcf}R_{bdgh}R_{cdij}\Gamma_{efghij}\epsilon\big),$$

$$\delta_\gamma\lambda=-\tfrac{1}{4}\sqrt{2}\;\Gamma^\mu\delta_\gamma\psi_\mu. \tag{B.2}$$

Note that $\delta_\gamma\psi$ contains $R^3\mathscr{D}\epsilon$-terms. The appearance of new supersymmetry transformations containing $\mathscr{D}\epsilon$ can easily be avoided. The contributions of the equation of motion Ψ in (5.6) are, schematically, $R^3\bar{\epsilon}\mathscr{D}\Psi$, or, after a partial integration

$$-(\mathscr{D}R^3)\bar{\epsilon}\Psi-R^3(\mathscr{D}\bar{\epsilon})\Psi. \tag{B.3}$$

The first term must be cancelled by changing the transformation rule of the gravitino. The second term can also be cancelled by adding to the action:

$$R^3\bar{\psi}\Psi. \tag{B.4}$$

Of course the new term has to be varied. The variation of ψ gives $\mathscr{D}\epsilon$ and cancels the second term in (B.3) (this time we do not perform the partial integration away from ϵ!). The variation of Ψ gives a combination of bosonic equations of motion, and this can be cancelled by changing the bosonic transformation rules. If this procedure is followed, the new fermionic transformation rules are as in (B.2), but without the $\mathscr{D}\epsilon$-terms.

The second solution is obtained by taking in (5.1) $a_1=1$, $b=8$, $b_1=b_2=0$, $b_3=-48\sqrt{2}$:

$$e^{-1}I_2=+R_{abcd}R_{abcd}R_{efgh}R_{efgh}-8R_{abcd}R_{abce}R_{dfgh}R_{efgh}$$

$$+2R_{abcd}R_{abef}R_{cdgh}R_{efgh}-4R_{abce}R_{abdf}R_{cdgh}R_{efgh}$$

$$-48\sqrt{2}\;H_{abd}R_{abce}R_{ghcf}\mathscr{D}_dR_{efgh}+96\sqrt{2}\;H_{abe}R_{abcd}R_{ghcf}\mathscr{D}_dR_{efgh}$$

$$+\tfrac{1}{8}i\sqrt{2}\;e^{-1}\varepsilon_{abcdefijkl}B_{ab}R_{cdgh}R_{efgh}R_{ijmn}R_{klmn}$$

$$+4R_{cdab}\mathscr{D}_eR_{fgab}\bar{\psi}_{cd}\Gamma_e\psi_{fg}-2R_{bcaf}\mathscr{D}_aR_{bcde}\bar{\psi}_{gh}\Gamma_{def}\psi_{gh}$$

$$-4R_{ghaf}\mathscr{D}_aR_{bcde}\bar{\psi}_{bc}\Gamma_{def}\psi_{gh}$$

$$-8R_{adbc}R_{aebc}\bar{\psi}_{fg}\Gamma_d\mathscr{D}_e\psi_{fg}-16R_{bcad}R_{fgae}\bar{\psi}_{bc}\Gamma_d\mathscr{D}_e\psi_{fg}$$

$$+\left(-R_{efgh}R_{abcd}R_{abcd}+16R_{egaf}R_{bhcd}R_{abcd}+12R_{ghae}R_{bfcd}R_{abcd}\right.$$

$$-4R_{ghaf}R_{becd}R_{abcd}-2R_{efab}R_{ghcd}R_{abcd}+4R_{efab}R_{agcd}R_{bhcd}$$

$$\left.+8R_{egab}R_{afcd}R_{bhcd}+8R_{fgab}R_{aecd}R_{bhcd}+8R_{ghab}R_{aecd}R_{bfcd}\right)\bar{\psi}_e\Gamma_f\psi_{gh}$$

$$+\left(\tfrac{1}{2}R_{efhi}R_{abcd}R_{abcd}-4R_{hiae}R_{bfcd}R_{abcd}\right.$$

$$\left.+R_{efab}R_{hicd}R_{abcd}-2R_{hiab}R_{aecd}R_{bfcd}\right)\psi_g\Gamma_{efg}\psi_{hi}$$

$$+\left(-4R_{efhi}R_{adbc}R_{agbc}-4R_{hiae}R_{fgbc}R_{adbc}\right.$$

$$\left.-4R_{hiad}R_{efbc}R_{agbc}\right)\bar{\psi}_d\Gamma_{efg}\psi_{hi}$$

$$+\left(-\tfrac{1}{2}R_{cdij}R_{efab}R_{ghab}-R_{deij}R_{cfab}R_{ghab}\right)\bar{\psi}_c\Gamma_{defgh}\psi_{ij}$$

$$+\tfrac{1}{4}R_{cdjk}R_{efab}R_{ghab}\psi_i\Gamma_{cdefghi}\psi_{jk}$$

$$+\sqrt{2}\left[\left(R_{efgh}R_{abcd}R_{abcd}-8R_{ghae}R_{bfcd}R_{abcd}\right.\right.$$

$$\left.+2R_{efab}R_{ghcd}R_{abcd}-4R_{ghab}R_{aecd}R_{bfcd}\right)\bar{\lambda}\Gamma_{eg}\psi_{gh}$$

$$\left.+\tfrac{1}{2}R_{cdij}R_{efab}R_{ghab}\bar{\lambda}\Gamma_{cdefgh}\psi_{ij}\right]. \tag{B.5}$$

The modifications to the transformation rules can be calculated from (5.7). We find

$$\delta_\gamma\psi_\mu=D_b\left[\left(2R_{\mu bcd}R_{cdef}R_{efgh}\Gamma_{gh}-4R_{\mu bcf}R_{decg}R_{defh}\right)\Gamma_{gh}\epsilon\right]$$

$$-8D_e\left(R_{bcdf}R_{bcdg}R_{\mu efh}\Gamma_{gh}\epsilon\right)+D_f\left(R_{bcde}R_{bcde}R_{\mu fgh}\Gamma_{gh}\epsilon\right)$$

$$+\tfrac{1}{2}D_6\left(R_{\mu bef}R_{cdgh}R_{cdij}\Gamma_{efghij}\epsilon\right),$$

$$\delta_\gamma\lambda=-\tfrac{1}{4}\sqrt{2}\;\Gamma^\mu\delta_\gamma\psi_\mu. \tag{B.6}$$

The solution I_2 has a Yang–Mills analogon. The proper way to derive this Yang–Mills solution from I_2 consists in two steps. First, by using pair exchange (5.8), the R^4-terms must be written in such a way that the contraction over Lorentz indices corresponds to the Yang–Mills trace. Second, the spin-connection must be written with H-torsion. These steps do not require the use of the identities (A.3). We use the notation $W_{\mu\nu} = \operatorname{tr} F_{\mu\nu}\chi$. The result is

$$
\begin{aligned}
e^{-1}I_{\mathrm{YM}} = & -\tfrac{1}{2}t^{\mu_1\cdots\mu_8} \operatorname{tr} F_{\mu_1\mu_2}F_{\mu_3\mu_4} \operatorname{tr} F_{\mu_5\mu_6}F_{\mu_7\mu_8} \\
& +\tfrac{1}{8}i\sqrt{2}\; e^{-1}\varepsilon^{\mu_1\cdots\mu_{10}}B_{\mu_1\mu_2} \operatorname{tr} F_{\mu_3\mu_4}F_{\mu_5\mu_6} \operatorname{tr} F_{\mu_7\mu_8}F_{\mu_9\mu_{10}} \\
& +4\overline{W}^{\mu\nu}\Gamma^{\lambda} \operatorname{tr} \chi\mathscr{D}_{\lambda}F_{\mu\nu} - 2 \operatorname{tr} F^{\mu\nu}\mathscr{D}_{\mu}F^{\lambda\rho} \operatorname{tr} \overline{\chi}\Gamma_{\nu\lambda\rho}\chi \\
& -4\overline{W}^{\mu\nu}\Gamma^{\lambda\rho}{}_{\nu} \operatorname{tr} \chi\mathscr{D}_{\mu}F_{\lambda\rho} \\
& -8 \operatorname{tr} F^{\mu\lambda}F^{\nu}{}_{\lambda} \operatorname{tr} \overline{\chi}\Gamma_{\mu}\mathscr{D}_{\nu}\chi - 16\overline{W}^{\mu\nu}\Gamma_{\nu} \operatorname{tr}(\mathscr{D}^{\lambda}\chi)F_{\mu\lambda} \\
& +\Big(-\operatorname{tr} F^{\mu\nu}F_{\mu\nu}\overline{\psi}_{\lambda}\Gamma_{\rho} - 12 \operatorname{tr} F^{\mu\nu}F_{\mu\lambda}\overline{\psi}_{\rho}\Gamma_{\nu} + 12 \operatorname{tr} F^{\mu\nu}F_{\mu\lambda}\overline{\psi}_{\nu}\Gamma_{\rho} \\
& -2 \operatorname{tr} F^{\mu\nu}F_{\lambda\rho}\overline{\psi}_{\mu}\Gamma_{\nu} + 12 \operatorname{tr} F^{\mu}{}_{\lambda}F^{\nu}{}_{\rho}\overline{\psi}_{\mu}\Gamma_{\nu}\Big)W^{\lambda\rho} \\
& +\Big(\tfrac{1}{2} \operatorname{tr} F^{\mu\nu}F_{\mu\nu}\overline{\psi}_{\sigma}\Gamma^{\sigma}{}_{\lambda\rho} - 4 \operatorname{tr} F^{\mu\nu}F_{\mu\lambda}\overline{\psi}_{\sigma}\Gamma^{\sigma}{}_{\nu\rho} + \operatorname{tr} F^{\mu\nu}F_{\lambda\rho}\overline{\psi}_{\sigma}\Gamma^{\sigma}{}_{\mu\nu} \\
& -2 \operatorname{tr} F^{\mu}{}_{\lambda}F^{\nu}{}_{\rho}\overline{\psi}_{\sigma}\Gamma^{\sigma}{}_{\mu\nu} - 4 \operatorname{tr} F^{\mu\nu}F_{\mu}{}^{\sigma}\overline{\psi}_{\nu}\Gamma_{\sigma\lambda\rho} \\
& +4 \operatorname{tr} F^{\mu}{}_{\lambda}F^{\nu\sigma}\overline{\psi}_{\mu}\Gamma_{\nu\sigma\rho} + 4 \operatorname{tr} F^{\mu\nu}F^{\sigma}{}_{\lambda}\psi_{\rho}\Gamma_{\mu\nu\sigma}\Big)W^{\lambda\rho} \\
& +\Big(-\tfrac{1}{2} \operatorname{tr} F^{\mu\nu}F^{\sigma\tau}\overline{\psi}_{\lambda}\Gamma_{\mu\nu\sigma\tau\rho} - \operatorname{tr} F^{\mu\nu}F^{\sigma\tau}\overline{\psi}_{\mu}\Gamma_{\nu\sigma\tau\lambda\rho}\Big)W^{\lambda\rho} \\
& +\tfrac{1}{4} \operatorname{tr} F^{\mu\nu}F^{\sigma\tau}\overline{\psi}_{\xi}\Gamma^{\xi}{}_{\mu\nu\sigma\tau\lambda\rho}W^{\lambda\rho} \\
& \qquad +\sqrt{2}\Big[\Big(\operatorname{tr} F^{\mu\nu}F_{\mu\nu}\overline{\lambda}\Gamma_{\lambda\rho} - 8 \operatorname{tr} F^{\mu\nu}F_{\mu\lambda}\overline{\lambda}\Gamma_{\nu\rho} \\
& \qquad -4 \operatorname{tr} F^{\mu}{}_{\lambda}F_{\nu\rho}\overline{\lambda}\Gamma_{\mu\nu} + 2 \operatorname{tr} F^{\mu\nu}F_{\lambda\rho}\overline{\lambda}\Gamma_{\mu\nu}\Big)W^{\lambda\rho} \\
& \qquad +\tfrac{1}{2} \operatorname{tr} F^{\mu\nu}F^{\sigma\tau}\overline{\lambda}\Gamma_{\mu\nu\sigma\tau\lambda\rho}W^{\lambda\rho}\Big]. \qquad \text{(B.7)}
\end{aligned}
$$

The additional supersymmetry transformation rules of χ follow from (5.11), and read

$$
\begin{aligned}
\delta_{\gamma}\chi = & -\tfrac{1}{4}\Gamma_{cdefgh}\varepsilon F_{gh} \operatorname{tr} F_{cd}F_{ef} \\
& +\Gamma_{ef}\varepsilon F_{cd}(2 \operatorname{tr} F_{ce}F_{df} - \operatorname{tr} F_{cd}F_{ef}) \\
& -\tfrac{1}{2}\Gamma_{ef}\varepsilon F_{ef} \operatorname{tr} F_{cd}F_{cd} + 4\Gamma_{ef}\varepsilon F_{df} \operatorname{tr} F_{cd}F_{ce}. \qquad \text{(B.8)}
\end{aligned}
$$

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