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Stabilizing switching control of power converters:
the lossy line and nonlinear case

Marius Zainea, Arjan van der Schaft and Jean Buisson

Abstract— This paper proposes a switching control approach for the set-point stabilization of power converters connected via a lossy transmission line to a resistive load. The approach employs a Lyapunov function that is directly based on the energy functions of the power converter and of the transmission line described by the telegraph equations. The method allows a certain freedom in the choice of the stabilizing switching control law, and in a simple example a comparison is made between a maximum descent strategy and a minimum commutation strategy. Finally it is shown how the method can be extended to the case of power converters with nonlinear energy-storing elements.

I. INTRODUCTION

Power converters (Boost, Buck, Čuk, multilevel converters) are widespread industrial devices. They are used in many applications such as variable speed DC motor drives, computer power supply, cell phone and cameras. When they are operating in normal conditions, these circuits have been designed in such a way that the commutation of the switches does not produce discontinuities. In this case they can be modelled by switching systems without jumps. For this class of systems, multiple approaches for control have been developed, mainly based either on continuous time approaches (i.e. sliding mode [1], passivity based control [2], stabilizing control [3],[4]...), or on discretization approaches (i.e. model predictive control [5], supervisory control [6],...).

The goal of this paper is to show how the switching stabilizing control scheme [7], designed for the situation where the energy-storing elements are linear, can be extended, first, to the case where the power converter is connected to the resistive load via a lossy transmission line, and, second, to the case where the energy-storing elements are nonlinear. The main advantages of the method proposed in [7] are that it uses a simple Lyapunov function deduced from energy considerations and that the control variable is directly boolean.

The difficulty of the first problem resides in the fact that the transmission line model is a distributed parameter model described by PDEs (the telegraph equations), to which the original switching stabilizing control method of [7] cannot be directly applied. To solve this problem, the power converter part and the line and the load part are analyzed separately, where for each part a candidate Lyapunov function is proposed based on the same energy considerations as in the

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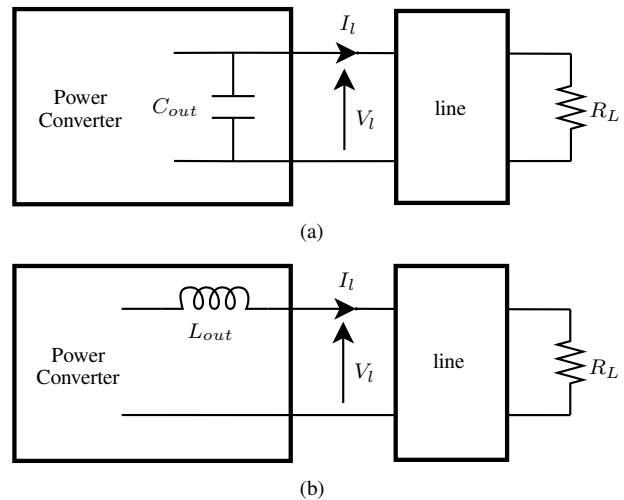
Fig. 1. The two most common cases when connecting the load to the power converter through a transmission line

original method. Then it can be shown that the sum of the two candidate Lyapunov functions constitutes a Lyapunov function for the entire system. Finally, it will be shown how in the case of a power converter with nonlinear energy-storing elements a candidate Lyapunov function can be obtained.

Section II introduces the models used for the power converter as well as for the line and load subsystem. Section III starts by recalling, for the linear case, how the stabilizing control can be applied when the power converter and the load are directly connected to each other. Then it is shown how this method can be extended to the power converter – line – load system. An illustrative example is discussed at the end of the section, where two control strategies are analyzed. In Section IV the construction of the candidate Lyapunov function in the nonlinear case is presented, while the conclusions of the paper are in Section V.

II. MODELS OF THE SYSTEMS WITH SWITCHING POWER CONVERTERS

When connecting a resistive load to a power converter the most common cases are those presented in figure 1. The situation depicted by figure 1(a) corresponds, for example, to the popular "boost", "buck" and "buckboost" converters, while the situation depicted by figure 1(b) corresponds, for example, to the multicellular converter.

A. The Power Converter Model

In order to derive models for physical systems, different energy based approaches, such as circuit theory, bond graphs [8], Euler Lagrange, Hamiltonian approach [9] can be used. For switching systems, extensions have been proposed in [10] for the Hamiltonian approach or in [11], [12] among many other references for the bond graph approach. These approaches consider the switch as an ideal element: the voltage drop is zero when the switch is *on* and the current is zero if the switch is *off*. Moreover, the system is considered to operate in normal conditions, i.e. storage elements are independent for all configurations of the switches.

Consider $\rho \in \{0, 1\}^p$ to be the boolean vector describing the configuration or mode of the system, where p is the number of switches (or pairs of physical switches). Then, all previously cited approaches lead to a model of the form (1), commonly called "port-Hamiltonian systems" (with dissipation) [9], [10], [13]. [1]

$$\begin{cases} \dot{x} = (J(\rho) - R(\rho)) z + g(\rho) u + g_l v \\ w = -g_l^T z \end{cases} \quad . \quad (1)$$

The vector $u \in \mathbb{R}^m$ corresponds to the energy sources and is supposed to be constant. The couple (v, w) is represented either by (I_l, V_l) for the case depicted by figure 1(a) or by (V_l, I_l) for the case depicted by figure 1(b). The vector $x \in \mathbb{R}^n$ is the state vector with n the number of energy-storing elements. State variables are the energy variables (flux linkages in the inductors, charges in the capacitors), while $z \in \mathbb{R}^n$ is the co-state vector. Co-state variables are the corresponding co-energy variables (currents in inductors, voltages in capacitances). In the case where the components are linear, the relation between those two vectors is given by:

$$z = Fx \quad (2)$$

where $F = F^T \succ 0$. In simple cases F is a diagonal matrix with diagonal elements being the inverse of the values of the capacitances and inductances. The quantity $\dot{x}^T z$ represents the power entering the storage elements. The energy can be expressed for the linear case as:

$$E(x) = \frac{1}{2} x^T F x \quad (3)$$

The matrix $J(\rho)$ is skew-symmetric, $J(\rho) = -J^T(\rho)$; it corresponds to a power continuous interconnection in the network model. The matrix $R(\rho)$ is nonnegative; it corresponds to the energy dissipating part of the circuit. Due to the assumption made on how the power converter and the line are connected to each other, w is a component of z and, thus, g_l does not depend on ρ. It is assumed that the

[1]In [9], [10] this was originally called a "port-controlled Hamiltonian system".

following affine dependance on ρ holds:

$$J(\rho) = J_0 + \sum_{1}^{p} \rho_i J_i, \quad (4a)$$

$$R(\rho) = R_0 + \sum_{1}^{p} \rho_i R_i, \quad (4b)$$

$$g(\rho) = g_0 + \sum_{1}^{p} \rho_i g_i, \quad (4c)$$

where ρ_i are the components of ρ. This property has been verified on many usual devices (Buck, Boost, Čuk, ...) [14], [10], and has been formally proved for multicellular serial converters [15].

B. Lossy Line and Load Model

Consider the lossy transmission line [13], where the spatial variable belongs to the interval $[0, 1]$. The energy variables associated to the line are the charge density $Q = Q(t, q)\, dq$, and the flux density $\varphi = \varphi(t, q)\, dq$. The total energy stored at time t in the transmission line is given as:

$$E_l(Q, \varphi) = \int_0^1 \frac{1}{2}\left(\frac{Q^2(t, q)}{C_l} + \frac{\varphi^2(t, q)}{L_l}\right) dq \quad (5)$$

where C_l and L_l are the uniform, and therefore constant with respect to q, distributed capacitance and the uniform distributed inductance of the line. Moreover, the voltage and the current are given by:

$$\begin{aligned} V(t, q) &= \frac{Q(t, q)}{C_l} \\ I(t, q) &= \frac{\varphi(t, q)}{L_l} \end{aligned} \quad (6)$$

satisfying the (lossy) telegraph equations:

$$\begin{aligned} \frac{\partial Q}{\partial t} &= -\frac{\partial I}{\partial q} - G_l V(q, t) \\ \frac{\partial \varphi}{\partial t} &= -\frac{\partial V}{\partial q} - R_l I(q, t), \end{aligned} \quad (7)$$

where G_l and R_l are the uniform distributed conductance and the uniform distributed resistance.

Additionally, for the system that consists of the transmission line and the resistive load, R_L, the following boundary constraints hold:

$$\begin{aligned} V(t, 0) &= V_l \\ I(t, 0) &= I_l \end{aligned} \quad (8a)$$

$$V(t, 1) = R_L I(t, 1) \quad (8b)$$

where $V(t, 0)$ and $V(t, 1)$, and, respectively, $I(t, 0)$ and $I(t, 1)$ are the voltages, respectively the currents, at the beginning and at the end of the line.

III. THE LINEAR CASE

A. The Power Converter Directly Connected to the Load

In the case where the power converter is directly connected to the load, the following additional constraint holds:

$$v = \tilde{R}_L w, \quad \text{with} \tag{9a}$$

$$\tilde{R}_L = \begin{cases} R_L, & \text{for the figure 1(b) case} \\ 1/R_L, & \text{for the figure 1(a) case} \end{cases}. \tag{9b}$$

Thus, the model expressed by (1) becomes:

$$\dot{x} = \left(J(\rho) - \tilde{R}(\rho) \right) z + g(\rho) u, \tag{10}$$

where

$$\tilde{R}(\rho) = R(\rho) + g_l \tilde{R}_L g_l^T \tag{11}$$

and $\tilde{R}(\rho)$ has the same properties as $R(\rho)$.

1) Admissible Reference: The objective is to design a switching control law such that the output of the system takes some specified value. Using the same approach as with an averaged model the following definition of an admissible reference is proposed.

Definition 1: $z_0 = Fx_0$ is called an admissible reference for system (10) and (2) where u is constant, if there exists $\rho_0 \in \mathbb{R}^p$, $0 \le \rho_{0i} \le 1$ such that constraint (12):

$$0 = \left(J(\rho_0) - \tilde{R}(\rho_0) \right) z_0 + g(\rho_0) u, \tag{12}$$

is satisfied.

2) Lyapunov Function:

Definition 2: A function H is a Lyapunov function for the system represented by (1) or (10) and (2) in x_0 if:

- $H(x) > 0$ except for x_0 where $H(x_0) = 0$,
- H is radially unbounded,
- for any x, a control ρ can be chosen such that $\dot{H}(x) < 0$.

If such a control law is applied, then x will converge asymptotically toward x_0. The following result [7] states how a Lyapunov function can be determined for the case where the power converter is directly connected to the load.

Theorem 3: Considering the system represented by (10) and (2), it is always possible to find a boolean state feedback $\rho(x)$ such that the function defined by $H_p(x) = E(x - x_0) = \frac{1}{2}(x - x_0)^T F(x - x_0)$, where x_0 is an admissible reference according to definition 1, is a Lyapunov function for the resulting closed–loop system.

Proof: Since there is no jump, H_p is positive, continuous and null only for $x = x_0$. Moreover, the time derivative of H_p depends on the value of the control ρ:

$$\dot{H}_p = -(z - z_0)^T \tilde{R}(\rho)(z - z_0) + \sum_1^p (z - z_0)^T ((J_i - R_i) z_0 + g_i u)(\rho_i - \rho_{0i}) \tag{13}$$

Since $\tilde{R}(\rho)$ is a nonnegative matrix, the first term of this expression is never positive, and since $0 \le \rho_{0i} \le 1$, the second term can be made negative by choosing each ρ_i according to the sign of $(z - z_0)^T ((J_i - R_i) z_0 + g_i u)$. ∎

Remark 4: Developing further (13) by making use of (9) and of (11), one can identify a term which may depend on ρ and one which is independent of ρ:

$$\dot{H}_p = -(z - z_0)^T R(\rho)(z - z_0) - (z - z_0)^T g_l \tilde{R}_L g_l^T (z - z_0) + \sum_1^p (z - z_0)^T ((J_i - R_i) z_0 + g_i u)(\rho_i - \rho_{0i}) \tag{14}$$

$$= D_\rho - (w - w_0)^2 \tilde{R}_L.$$

B. The Power Converter Connected to the Load Using a Transmission Line

1) Admissible Reference: An equilibrium point for the line is defined by:

$$\frac{\partial Q}{\partial t} = \frac{\partial \varphi}{\partial t} = 0, \tag{15}$$

which, due to (7), implies that $(V_0(q), I_0(q))$ is the solution of:

$$\begin{pmatrix} \frac{\partial V_0}{\partial q} \\ \frac{\partial I_0}{\partial q} \end{pmatrix} = \begin{pmatrix} 0 & -R_l \\ -G_l & 0 \end{pmatrix} \begin{pmatrix} V_0(q) \\ I_0(q) \end{pmatrix}, \tag{16}$$

Moreover, the solution of (16) has to respect the boundary conditions:

$$\begin{aligned} V_0(0) &= V_{l0} \\ I_0(0) &= I_{l0} \\ V_0(1) &= R_L I_0(1). \end{aligned} \tag{17}$$

Thus, at equilibrium, (v_0, w_0) is equal either to (I_{l0}, V_{l0}), for the situation represented by figure 1(a), or by (V_{l0}, I_{l0}) for the situation represented by 1(b). In this way, the equilibrium of the power converter part is defined as the solution (z_0, ρ_0) of:

$$\begin{cases} 0 = (J(\rho_0) - R(\rho_0)) z_0 + g(\rho_0) u + g_l v_0 \\ w_0 = -g_l^T z_0 \end{cases} \tag{18}$$

Then, the admissible reference for the case when the power converter is connected to the resistive load through a transmission line is formulated like in the case without line:

Definition 5: The triple (z_0, V_0, I_0) is an admissible reference for the system formed by the power converter connected to a resistive load through a lossy transmission line if there exists $\rho_0 \in \mathbb{R}^p$, $0 \le \rho_{0i} \le 1$, such that constraints (16)–(18) are satisfied.

2) Lyapunov Function: Like in section III-A.2, a suitable Lyapunov function can be formulated for the entire system based on energy considerations.

Theorem 6: For the system including a power converter, a transmission line and a resistive load, it is always possible to find a boolean state feedback $\rho(x)$ such that the function defined by $H(x) = H_p(x) + H_l(Q, \varphi)$, with $H_l(Q, \varphi) = E_l(Q - Q_0, \varphi - \varphi_0)$, is a Lyapunov function for the resulting closed–loop system, where (x_0, Q_0, φ_0) correspond to an admissible reference according to definition 5, (z_0, V_0, I_0).

Proof: Consider first the term $H_p(x)$. Then, from (1), the computation of the time derivative of this term leads to:

$$
\begin{aligned}
\dot{H}_p &= (x - x_0)^T F \dot{x} \\
&= (z - z_0)^T [(J(\rho) - R(\rho)) z + g(\rho) u + g_l v] \\
&= -(z - z_0)^T R(\rho)(z - z_0) + (z - z_0)^T g_l (v - v_0) \\
&\quad + (z - z_0)^T \sum_1^p [(J_i - R_i) z_0 + g_i u] (\rho_i - \rho_{i0}) \\
&= D_\rho - (w - w_0)(v - v_0).
\end{aligned}
\tag{19}
$$

Second, using also (5) – (8), the expression of the time derivative of $H_l(Q, \varphi)$ is given by:

$$
\begin{aligned}
\dot{H}_l &= -\int_0^1 \left[\frac{\partial I}{\partial q}(V - V_0) + G_l V(V - V_0) \right] dq \\
&\quad -\int_0^1 \left[\frac{\partial V}{\partial q}(I - I_0) + R_l I(I - I_0) \right] dq \\
&= -\int_0^1 \frac{\partial (I - I_0)}{\partial q}(V - V_0) \, dq \\
&\quad -\int_0^1 \frac{\partial (V - V_0)}{\partial q}(I - I_0) \, dq \\
&\quad -\int_0^1 (V - V_0)\left(\frac{\partial I_0}{\partial q} + G_l V \right) dq \\
&\quad -\int_0^1 (I - I_0)\left(\frac{\partial V_0}{\partial q} + R_l I \right) dq \\
&= (w - w_0)(v - v_0) - (V(1,t) - V_0(1))^2 \big/ R_L \\
&\quad -\int_0^1 \left[(V - V_0)^2 G_l + (I - I_0)^2 R_l \right] dq
\end{aligned}
\tag{20}
$$

Thus, the global time derivative is given by:

$$
\begin{aligned}
\dot{H} &= D_\rho - (V(t,1) - V_0)^2 \big/ R_L \\
&\quad -\int_0^1 \left[(V - V_0)^2 G_l + (I - I_0)^2 R_l \right] dq \\
&= D_\rho + D_l,
\end{aligned}
\tag{21}
$$

where the term D_l (which is independent of ρ) is the dissipated power due to the resistance and conductance of the line and to the resistive load. Similar to theorem 3, ρ can be chosen such that $D_\rho < 0$, and, thus, the same choice for ρ can be used to make $\dot{H}$ negative. ∎

Remark 7: Let $\dot{H}_p \big|_{\text{lineless}}$ be the evaluation of the derivative of H_p in the case where the power converter is
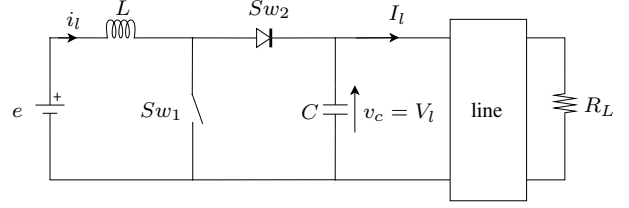

Fig. 2. The Boost converter with a transmission line

directly connected to the load. Then, from (14) and (21), it follows that:

$$
\dot{H} = \dot{H}_p \big|_{\text{lineless}} + (w - w_0)^2 \tilde{R}_L + D_l.
\tag{22}
$$

Since the term $(w - w_0)^2 \tilde{R}_L$ is nonnegative it is not true that any switching rule for ρ such that $\dot{H}_p|_{\text{lineless}} \le 0$ automatically ensures that $\dot{H} \le 0$; neither does the converse hold. See section III-C for a further discussion of possible switching rules which keep $\dot{H} \le 0$.

C. Example - Boost Converter

Figure 2 represents a simplified circuit of the well known boost power converter. Under normal operating conditions, the diode is conducting when the controlled physical switch is open ($\rho = 1$) and blocked when the controlled physical switch is closed ($\rho = 0$).

The state vector $x = (x_l, x_c)^T$ is composed of the flux linkage in the inductance and the charge in the capacitor. The co-state vector $z = (i_l, v_c)^T$ is composed of the current in the inductance and voltage on the capacitor. The matrices corresponding to (1), (4) and (10) are:

$$
J(\rho) = \begin{pmatrix} 0 & -\rho \\ \rho & 0 \end{pmatrix} \quad R(\rho) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}
\tag{23a}
$$

$$
g(\rho) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad g_l = \begin{pmatrix} 0 \\ -1 \end{pmatrix}
\tag{23b}
$$

$$
F = \begin{pmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{C} \end{pmatrix} \quad \tilde{R}(\rho) = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{1}{R_L} \end{pmatrix}
\tag{23c}
$$

The state equation is:

$$
\begin{pmatrix} \dot{x}_l \\ \dot{x}_c \end{pmatrix} = \begin{pmatrix} 0 & -\frac{\rho}{C} \\ \frac{\rho}{L} & 0 \end{pmatrix} \begin{pmatrix} x_l \\ x_c \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e + \begin{pmatrix} 0 \\ -1 \end{pmatrix} I_l
\tag{24}
$$

The admissible reference is defined by:

$$
(v_{c0}, i_{l0}) = \left(\frac{e}{\rho_0}, \frac{I_{l0}}{\rho_0} \right),
\tag{25a}
$$

$$
\begin{pmatrix} V_0(1) \\ I_0(1) \end{pmatrix} = \exp \begin{pmatrix} 0 & -G_l \\ -R_l & 0 \end{pmatrix} \begin{pmatrix} V_{l0} \\ I_{l0} \end{pmatrix},
\tag{25b}
$$

$$
V_0(1) = R_L I_0(1).
\tag{25c}
$$

The proposed Lyapunov function is:

$$
\begin{aligned}
H &= \frac{1}{2} \frac{(x_l - x_{l0})^2}{L} + \frac{1}{2} \frac{(x_c - x_{c0})^2}{C} \\
&\quad + \frac{1}{2} \int_0^1 \left[\frac{(Q - Q_0)^2}{C_l} + \frac{(\varphi - \varphi_0)^2}{L_l} \right] dq
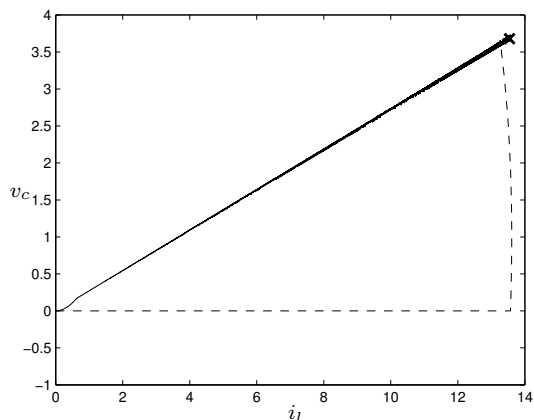\end{aligned}
\tag{26}
$$

Fig. 3. The state evolution, from the origin to the reference point, $(i_{l0} = 13.54\text{A}, v_{c0} = 3.68\text{V})$ indicated by the **x**. The solid line represents the (co-)state trajectory when a maximum descent strategy is used, while the dashed line represents the (co-)state trajectory when a minimum switching strategy is used

And its derivative:

$$\dot{H} = [(v_c - v_{c0})\, i_{l0} - (i_l - i_{l0})\, v_{c0}]\,(\rho - \rho_0) + D_l \quad (27)$$

In the simulation, normalized values have been used ($e = 1$V, $R_L = 1\Omega$, $L = 1$H, $C = 1$F). The line has been modeled using a ladder representation with ten cells. The numerical values of the storage elements used in the cell model are 0.005H for the inductance and 0.01F for the capacitor, such that $L_l \approx 0.05$H/m and $C_l \approx 0.1$F/m. The numerical values of the dissipative elements used in the cell model are 0.01Ω for the resistance and 0.01S for the conductance, such that $R_l \approx 0.1\Omega$/m and $G_l \approx 0.1$S/m. First the output voltage is specified $V_0\,(1) = 3.33$V. Then, from (25), $I_0\,(1) = 3.33$A, $(V_{l0}, I_{l0}) \approx (3.68\text{V}, 3.68\text{A})$, $v_{c0} \approx 3.68$V, $\rho_0 \approx 0.27$ and $i_{l0} \approx 13.54$A.

The simulations were obtained using two control strategies: maximum descent and minimum switching, with the origin used each time as the initial value for the state vector. In figure 3 the (co-)state evolution for the maximum descent control strategy is represented by the solid line. This strategy ensures that the derivative of the Lyapunov function is always negative by keeping negative the term $[(v_c - v_{c0})\, i_{l0} - (i_l - i_{l0})\, v_{c0}]\,(\rho - \rho_0)$. Such a strategy results in a sliding motion on the hyperplane described by the equality with zero of the previous expression. In figure 4(a) is presented the time evolution of the load voltage drop when such a strategy is applied. In figure 3 the (co-)state evolution for a minimum switching control strategy is represented by the dashed line. This strategy takes the decision of changing mode only when the Lyapunov function derivative is becoming zero. Figure 4(b) shows the time evolution of the load voltage drop when such a strategy is applied. It can be noticed that, even though there is overshoot, the system converges faster than when the maximum descent strategy is used.
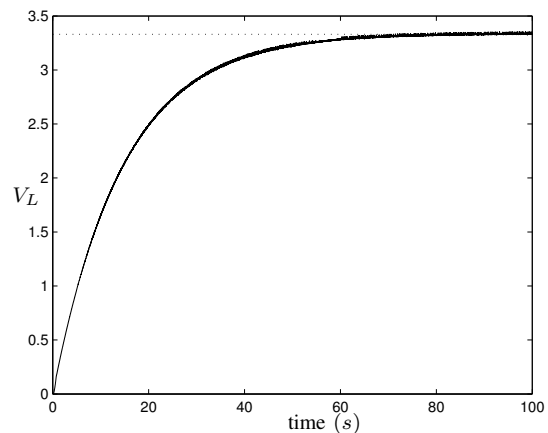
(a) Maximum descent strategy
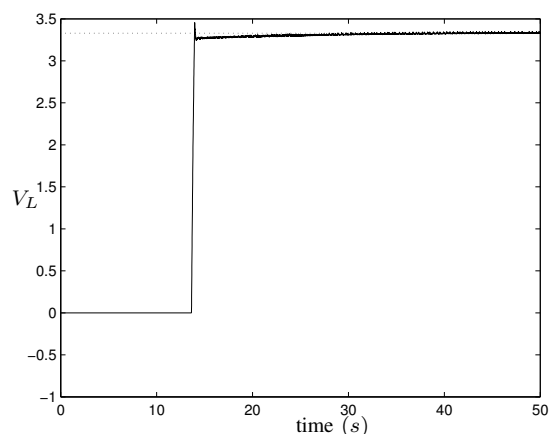
(b) Minimum switching strategy

Fig. 4. The time evolution of the load voltage drop, $V\,(t, 1)$, first when a maximum descent strategy is employed and second when a minimum switching strategy is used. The dotted lines represent the admissible reference point, i.e. $V_0\,(1) = 3.33$V.

IV. THE NONLINEAR CASE

As noticed before, the power converter model (10) is equally valid for *nonlinear* storage elements (capacitors and inductors), in which case the energy function $E\,(x)$ is *not* anymore a quadratic function of the state as in (3), or, equivalently, the relation between the state variables x and the co-energy variables z is *not* anymore a linear relation as in (2). Indeed, for nonlinear storage elements the relation between state and co-energy variables is given by

$$z = \frac{\partial E}{\partial x}\,(x) \quad (28)$$

Note that the resistive elements are still considered to be *linear*, corresponding to the matrix $\tilde{R}$ in (10).

For the stabilizing switching control in the nonlinear case once more the case *without* transmission line is first analyzed. Theorem 3 extends to the nonlinear case as follows.

Theorem 8: Consider the system (10), with z being given by the nonlinear relation (28). Let $z_0 = \frac{\partial E}{\partial x}\,(x_0)$ be an

admissible reference as in Definition 1. Furthermore assume that the energy function E is *convex*, that is, its Hessian matrix is everywhere positive definite. Then it is possible to find a Boolean state feedback $\rho(x)$ such that the function

$$H_p(x) := E(x) - (x - x_0)^T \frac{\partial E}{\partial x}(x_0) - E(x_0) \quad (29)$$

is a Lyapunov function for the equilibrium x_0 of the resulting closed-loop system.

Proof: By the fact that

$$\frac{\partial H_p}{\partial x}(x_0) = \frac{\partial E}{\partial x}(x_0) - \frac{\partial E}{\partial x}(x_0) = 0 \quad (30)$$

it follows that x_0 is a critical point for H_p. Trivially $H_p(x_0) = 0$, while convexity of H_p follows from convexity of E. Thus $H_p(x) > 0$ for all $x \neq x_0$.

The time-derivative $\dot{H}_p$ is now given by

$$\dot{H}_p = \left(\frac{\partial E}{\partial x}(x) - \frac{\partial E}{\partial x}(x_0)\right)\dot{x} = (z - z_0)^T \dot{x} \quad (31)$$

which leads to the same formula (13) as in the proof of Theorem 3. Hence the same conclusion as in Theorem 3 follows. ∎

Remark 9: Notice that the above definition of H_p reduces in the *linear* case to the definition of H_p in Theorem 3.

The extension of Theorem 6 to the case of power converters with nonlinear capacitors and inductors proceeds along the same lines. Indeed, as in Theorem 6, the candidate Lyapunov function H is given by:

$$H(x, Q, \varphi) = H_p(x) + H_l(Q, \varphi) \quad (32)$$

with H_p the nonlinear candidate Lyapunov function for the power converter (as defined in (29)), and H_l the candidate Lyapunov function for the transmission line as defined before in Theorem 6. The proof of Theorem 6 now directly extends to the nonlinear case. The same shifted energy function as in (29) has been recently employed in [16].

V. CONCLUSIONS

A switching stabilizing control law has been presented that brings the system to an admissible set-point in two situations: first, the power converter and the load are connected via a lossy transmission line and, second, the power converter has nonlinear energy storing elements. To achieve the objective in the first case, a Lyapunov function has been deduced as the sum of the candidate Lyapunov functions for the power converter part and for the transmission line. This has been applied to the boost converter, with two strategies outlined: the maximum descent strategy, where the derivative of the Lyapunov function is minimized, and the minimum commutation strategy, where the commutation decision is taken only when the derivative of the Lyapunov function becomes equal to zero. Finally, to achieve the second objective, the construction of a Lyapunov function based on a shifted version of the energy has been extended to the nonlinear case. Future work will be concerned with extending the method to systems that involve nonlinear resistors, in which case the model (1) is no longer valid.

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