

University of Groningen

## The ac-Josephson effect above the gap frequency

de Lange, G.; Kuipers, J. J.; Klapwijk, T. M.; Panhuyzen, R. A.; van de Stadt, H.; de Graauw, M. W. M.

*Published in:*  
Journal of Applied Physics

*DOI:*  
[10.1063/1.357919](https://doi.org/10.1063/1.357919)

**IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.**

*Document Version*  
Publisher's PDF, also known as Version of record

*Publication date:*  
1994

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

de Lange, G., Kuipers, J. J., Klapwijk, T. M., Panhuyzen, R. A., van de Stadt, H., & de Graauw, M. W. M. (1994). The ac-Josephson effect above the gap frequency. *Journal of Applied Physics*, 76(12), 8016-8021. <https://doi.org/10.1063/1.357919>

### Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

### Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

*Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.*

## The ac-Josephson effect above the gap frequency

G. de Lange, J. J. Kuipers, T. M. Klapwijk, R. A. Panhuyzen, H. van de Stadt, and M. W. M. de Graauw

Citation: *Journal of Applied Physics* **76**, 8016 (1994); doi: 10.1063/1.357919

View online: <https://doi.org/10.1063/1.357919>

View Table of Contents: <http://aip.scitation.org/toc/jap/76/12>

Published by the *American Institute of Physics*

---

---

**AIP** | Journal of  
Applied Physics

SPECIAL TOPICS



# The ac-Josephson effect above the gap frequency

G. de Lange,<sup>a)</sup> J. J. Kuipers, and T. M. Klapwijk

Department of Applied Physics and Materials Science Center, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands

R. A. Panhuyzen, H. van de Stadt, and M. W. M. de Graauw

Space Research Organisation of the Netherlands, Landleven 12, 9747 AD Groningen, The Netherlands

(Received 9 May 1994; accepted for publication 26 August 1994)

The rf-power dependence of the ac-Josephson steps is measured at 720 GHz, using small area Nb tunnel junctions. This frequency is well above the gap frequency of Nb. The junction is placed in a waveguide, and connected to a superconducting stripline, which effectively tunes out the junction capacitance and facilitates the coupling of the radiation to the junction. We observe three Josephson steps, and the first step crosses the zero current axis over a considerable range of rf-power. This indicates the possible application of THz Josephson steps in voltage standards. The data are compared to the theory and we find clear evidence for the predicted intrinsic roll-off of the Josephson current amplitude above the gap frequency. © 1994 American Institute of Physics.

## I. INTRODUCTION

The occurrence of current steps at voltages equal to  $V_{dc} = N\hbar\omega/2e$  (where  $N$  is an integer and  $e$  is the electron charge) in tunnel junctions driven with a radiation of frequency  $\omega$ , was already predicted in the original paper on the Josephson effect<sup>1</sup> and soon after measured by Shapiro.<sup>2</sup> Due to the high precision with which frequency can be set, these so-called ac-Josephson or Shapiro steps now serve as the voltage standard. To get a voltage of order 1 V, voltage standards have to consist of large series arrays of junctions which contain up to several thousands of junctions, which are irradiated with a frequency of approximately 100 GHz. For frequencies below 100 GHz it is known that the steps can cross the zero dc-current axis and this allows the occurrence of quantized voltages across the junction, without any bias leads connected. Levinsen<sup>3</sup> pointed out that this is of potential interest to voltage standards, since the only voltages which can appear across the junction are multiples of  $\hbar\omega/2e$ . Furthermore, the removal of the bias leads to a substantial decrease of the influence of noise. The original idea of Levinsen was further worked out by Kautz,<sup>4-6</sup> who calculated the parameters necessary for zero crossing Josephson steps and the stability of these steps. Niemeyer *et al.*<sup>7</sup> actually used this concept to make a voltage standard operating around 100 GHz.

Since the quantized voltage scales linearly with frequency, operation at higher frequencies can reduce the number of junctions in a series array. Several papers have therefore made a theoretical investigation of the possibilities of the inverse ac-Josephson effect at THz frequencies. In the early work of Kautz,<sup>4</sup> the Resistively Shunted Junction model (RSJ-model), developed by Stewart and McCumber,<sup>8,9</sup> is used, and with this approximate model it is predicted that the inverse ac-Josephson effect can only be observed up to the gap frequency ( $\omega = 2\Delta/\hbar$ , with  $2\Delta$  the superconducting energy gap). In the more recent work of

Danchi *et al.*,<sup>10</sup> Mikolajczak *et al.*,<sup>11</sup> and Frank and Meyer,<sup>12</sup> the complete Werthamer-model<sup>13</sup> is used, and this model predicts zero crossing Josephson steps up to twice the gap frequency. The main difference between the RSJ-model and the Werthamer-model is the inclusion of the frequency dependence and reactive components of both the quasiparticle and pair currents in the tunnel junction in the Werthamer-model.

Due to the difficulty of both getting rf-power in the sub-THz frequency range and coupling this power into the tunnel junctions, most of the previous work published on the ac-Josephson effect is done at frequencies below 100 GHz. The actual experimental observation of the inverse ac-Josephson effect is reported only at highest frequencies of 100 GHz.<sup>7</sup> In the experimental work of Danchi *et al.* on Sn tunnel junctions at 604 GHz (Refs. 14 and 15) (which is to our knowledge the highest frequency at which the ac-Josephson effect in tunnel junctions is investigated to date) the rf-power dependence of the ac-Josephson step amplitudes is studied, but the inverse ac-Josephson effect is not observed. Due to the rather large scatter in their data, there is also no clear difference between a comparison of the measurements with the RSJ-model or the Werthamer model.

In this paper we report on the measurement of the ac-Josephson effect at 720 GHz, measured with small Nb tunnel junctions placed in a waveguide mount. The applied frequency is well above the 670 GHz gap frequency of the used junction. In order to improve the coupling of the high frequency radiation the junction is connected to an integrated tuning structure, which effectively tunes out the junction capacitance. We observe three Josephson steps, and the first step clearly crosses the zero current axis. In Sec. II the ac-Josephson effect and the possible occurrence of zero crossing steps is briefly described. The sample parameters, including the integrated tuning structure, and the experimental techniques are described in Sec. III. Results of the measured  $I$ - $V$  curves with and without radiation, and the ac-Josephson step amplitudes are described in Sec. IV. The comparison of the measured data with the Werthamer and the RSJ model is described in Sec. V. We end the paper with a brief summary and discussion of this work.

<sup>a)</sup>Present address: Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA 02139.

## II. THE ac-JOSEPHSON EFFECT

The tunnel current in a superconducting tunnel junction driven with radiation of frequency  $\omega$ , is a combination of the phase-independent (quasiparticle) current  $I_{qp}$  and the phase-dependent (Cooper pair) current  $I_P$ . Expressions for  $I_{qp}$  and  $I_P$  as a function of time and the applied dc-bias voltage  $V_0$  and rf-voltage  $V_\omega \cos \omega t$  are:<sup>13</sup>

$$I_{qp}(V_0, V_\omega, t) = \text{Im} \sum_{n,m=-\infty}^{\infty} J_n(\alpha) J_{n+m}(\alpha) e^{+im\omega t} \times j_1(V_0 + n\hbar\omega/e), \quad (1)$$

$$I_P(V_0, V_\omega, t) = \text{Im} \sum_{n,m=-\infty}^{\infty} J_n(\alpha) J_n(\alpha) e^{-i(n+m)\omega t - i\omega_0 t + i\varphi_0} \times j_2(V_0 + n\hbar\omega/e) \quad (2)$$

with  $\alpha = eV_\omega/\hbar\omega$ ,  $\omega_0 = 2eV_0/\hbar$ ,  $V_0$  the dc-bias voltage,  $\varphi_0$  the phase difference between the two superconducting electrodes and  $J_n$  the  $n$ th order Bessel-function.  $j_1(V_0; \omega)$  and  $j_2(V_0; \omega)$  are the frequency and the bias voltage-dependent quasiparticle and pair current, which both have a real and imaginary part.<sup>13</sup> The ac-voltage across the junction will in general be proportional to the square root of the incident radiation power  $P_{rf}$  and hence  $\alpha \sim \sqrt{P_{rf}}$ .

For a time domain solution of the currents in a circuit containing a Josephson device, the junction capacitance and the internal impedance of the (equivalent) radiation source have to be taken into account. Due to the highly nonlinear character of both  $I_P(t)$  and  $I_{qp}(t)$  there are no general analytical solutions for the currents in a circuit containing a Josephson junction, and computer simulations have to be used. But in this paper we restrict ourselves to the dc components, and to the quantized bias voltages at the  $N$ th Josephson step, where we define  $V_N = N\hbar\omega/2e$ . Expressions for the dc-components of Eqs. (1) and (2) at the  $N$ th step are

$$I_{qp}^{dc}(V_N) = \sum_{n=-\infty}^{\infty} J_n^2(\alpha) \text{Im} \left( j_1 \left( \left( n + \frac{1}{2}N \right) \hbar\omega/e \right) \right), \quad (3)$$

$$I_P^{dc}(V_N) = \sum_{n=-\infty}^{\infty} J_n(\alpha) J_{N-n}(\alpha) (\text{Re}(j_2(n, N, \omega)) \sin \varphi_0 + \text{Im}(j_2(n, N, \omega)) \cos \varphi_0), \quad (4)$$

where  $j_2(n, N, \omega) = j_2((n - \frac{1}{2}N)\hbar\omega/e)$ .  $\text{Im}(j_1(V_0, 0))$  equals the quasiparticle  $I$ - $V$  curve of the tunnel junction without radiation, and therefore can be directly measured. The  $\text{Re}[j_2(n, N, \omega)]$  component of Eq. (4) is the Josephson current amplitude. It includes the zero voltage supercurrent, and the frequency dependence of this term shows a singularity at the gap frequency (the Riedel-peak). It can be shown<sup>16</sup> that the summation of the  $\text{Im}(j_2)$ -part of the pair current (the  $\cos \varphi_0$ -term) equals zero and this term can be omitted in the calculation of the rf-induced Josephson steps.

The condition for the occurrence of the inverse ac-Josephson effect is

$$I_{qp}^{dc} + I_P^{dc} < 0. \quad (5)$$

Danchi *et al.*<sup>10</sup> and Frank and Meyer<sup>12</sup> calculate the possibility to fulfill Eq. (5), using time domain simulations and the expressions for the tunnel currents given in Eqs. (1) and (2). They find that zero crossing steps can occur at least up to  $\omega = 4\Delta/\hbar$  and over a considerable range of  $\alpha$  ( $\sim \sqrt{P_{rf}}$ ). Kautz<sup>4</sup> previously analyzed the possibility of zero crossing steps within the RSJ-model and found possible zero crossing steps up to  $\omega = 2\Delta/\hbar$ . The RSJ-model simplifies the Werthamer solution by assuming that the quasiparticle current is independent of the bias voltage and equal to  $I_N$ , the normal state (or subgap) current at a given bias voltage, and by the omission of the frequency-dependent behavior of  $j_2(\omega)$  (i.e., the supercurrent amplitude shows no Riedel peak and no roll-off above  $2\Delta$ ).

### A. Noise and stability

The ac-Josephson is a consequence of the phase locking of the supercurrent in the tunnel junction to the externally applied radiation field. This phase locking can be lost due to external disturbances, like (thermal) noise or electromagnetic pulses. The condition for the inverse ac-Josephson effect [Eq. (5)] would be most easily fulfilled if the phase difference  $\varphi_0$  could reach the value  $-\pi/2$ , but due to the influence of thermal and shot noise, the experimental phase limit will be always smaller than  $-\pi/2$ . Noise cannot only trigger a premature jump out of a phase locked state, but can also give rise to the rounding of ac-Josephson steps.<sup>17,18</sup> Detailed experiments on the reduction of the zero voltage supercurrent due to thermal noise and the rounding effects of the noise on higher order ac-Josephson steps are performed by Danchi *et al.*<sup>15</sup>

For a practical voltage standard, the Josephson step should not only fulfill the condition given in Eq. (5), but the zero crossing should also be stable under the influence of small external perturbations. The conditions for the stability of the zero crossing steps are discussed by Kautz<sup>5</sup> (using the RSJ-model) and by Frank and Meyer<sup>12</sup> (using the Werthamer-model).

## III. EXPERIMENTAL TECHNIQUES

The main interest in our work is the development of high sensitivity quasiparticle heterodyne receivers for (sub-) THz frequencies, and in order to employ the potential quantum-limited sensitivity of the quasiparticle mixer, it is necessary to have an efficient coupling of the high frequency signals into the junction. The main problem in achieving this is the (unavoidable and relatively high) geometric capacitance of the tunnel junction, which can cause a large reflection of the incident submillimeter power. We therefore developed techniques to fabricate small area ( $\approx 1 \mu\text{m}^2$ ), high current density junctions which have a capacitance of  $\approx 50-60$  fF and a normal state resistance of order  $30 \Omega$ .<sup>19</sup> The junction size is at the limit of what can be fabricated with optical lithography, but the junction capacitance is still relatively high. For a further improvement of the rf-power coupling the junction is connected to an integrated superconducting stripline, which has an inductive impedance at the desired frequency and ef-

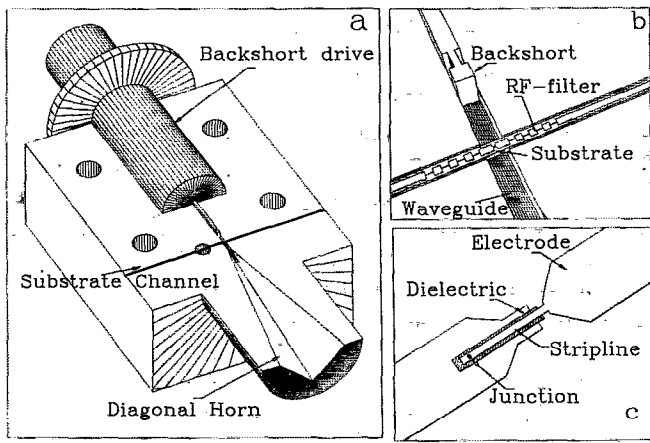


FIG. 1. (a) Geometry of the mixerblock. (b) Detail of the mixerblock showing the backshort tuner and the substrate channel with the substrate supporting the junction and the rf-filter. (c) Geometry of the integrated tuning structure.

fectively tunes out the junction capacitance. The geometry of the integrated tuning structure is shown in Fig. 1(c). A detailed discussion on the behavior of the integrated tuning structure is given in Ref. 20. We only note here that due to the loss of the stripline, the junction capacitance cannot be tuned out completely, resulting in a small remaining capacitive impedance for the tuned junction capacitance.

Table I gives an overview of the characteristic parameters of the junction used in this work. For a comparison with previous work within the RSJ-model, we have added the Stewart-McCumber parameter  $\beta$  and the junction plasma frequency  $\omega^p$ . Since the integrated tuning structure effectively reduces the capacitance, values for  $\beta$  and  $\omega^p$  are given both based on the junction capacitance only and on the effective junction capacitance, resulting from the tuning.

The junctions of the particular batch used in this work are of a somewhat reduced quality with respect to the subgap leakage current and the 4.2 K gap voltage. The 2.55 mV gap voltage of the junction at 4.2 K is significantly lower than the 2.8–2.9 mV gap voltage of “standard” high quality Nb junctions. At 2.2 K the gap voltage increases to 2.7 mV. From the analysis of the behavior of these junctions as heterodyne mixers,<sup>21</sup> we have found that due to the reduced quality of the Nb-layers, the removal of heat generated close to the junction (by dc- and rf-dissipation) is not optimal. The ambient junction temperature is therefore dc-bias and rf-power

TABLE I. Parameters of the junction used in this work.  $I_c^{\text{theo}}$  and  $I_c^{(0)}$  are the theoretical and measured maximum zero voltage Josephson current.  $C$ ,  $\beta (=2eI_c R_N^2 C/\hbar)$  and  $\omega^p$  are the junction capacitance, the Stewart-McCumber parameter and the junction plasma frequency, respectively. The index  $T$  or  $j$  indicates whether the parameter is taken with or without the effect of the integrated tuning structure included. The normalized frequency  $\Omega = \omega(\hbar/2eI_c R)$  equals 2.3.

$A = 1 \mu\text{m}^2$	$R_N = 16 \Omega$	$J_c = 1.5 \times 10^4 \text{ A/cm}^2$	$V_g = 2.7 \text{ mV}$
$I_c^{(0)} = 90 \mu\text{A}$	$I_c^{\text{theo}} = 180 \mu\text{A}$	$C_j = 55 \text{ fF}$	$C_T = 5 \text{ fF}$
$\beta = 8$	$\beta_T = 0.7$	$\omega_j^p = 1.7 \text{ THz}$	$\omega_T^p = 0.5 \text{ THz}$

dependent. This complicates the analysis of the data, especially at bias voltages above the gap voltage and at high rf-power levels.

The junctions and the integrated tuning structure are fabricated, together with a band-stop rf-filter, on a 200  $\mu\text{m}$  thick fused quartz substrate. After fabrication the quartz is polished down to 45  $\mu\text{m}$  and diced in widths of 90  $\mu\text{m}$ . The substrates are glued in the substrate channel, across the waveguide of the mixer block. The mixer block consists of a full height waveguide with dimensions 300 $\times$ 150  $\mu\text{m}$  and a substrate channel of dimensions 100 $\times$ 100  $\mu\text{m}$  (perpendicular to the waveguide). One end of the waveguide is closed by an adjustable backshort tuner and the other end contains a transition to a diagonal horn. The geometry of the mixerblock is shown in Figs. 1(a) and 1(b). The radiation is coupled to the horn by a polyethylene lens. A magnetic field can be applied to the junction by a coil of 10,000 turns of Nb wire, placed in front of the mixer block. The mixer block is placed on the cold plate of a liquid He dewar and the signals enter the dewar via a 1-mm-thick TPX-window and a 190  $\mu\text{m}$  quartz heat filter. The dc-bias of the junction can be either a voltage or a current bias, and is provided by a home-made bias supply. This measurement setup is the same as the one we use to perform heterodyne mixing measurements.<sup>21,22</sup>

The high frequency source we use consists of a combination of a Thomson carcinotron with a frequency of 345 to 375 GHz and a Radiometer Physics Schottky diode frequency doubler. The power level of the incident radiation is varied by using a set of two polarizers, of which one can be turned by means of a stepper motor controlled unit. The power level is monitored by a Golay cell (coupled via a beam splitter). The setup is computer-controlled and the data are obtained by changing the power level and subsequently measuring the  $I$ - $V$  curve of the junction. The typical time interval between the measurements at different power levels is 1 minute. The maximum power level is limited by the maximum power of the rf-source and is of order of several tens of  $\mu\text{W}$ .

#### IV. EXPERIMENTAL RESULTS

Results of measured  $I$ - $V$  curves at an ambient temperature of 2.2 K are shown in Fig. 2. The solid and dashed lines in Fig. 2 are the  $I$ - $V$  curves measured with and without radiation, and with a magnetic field applied. The magnetic field is adjusted to a value such that the effective cross-section of the junction contains one flux quantum, which in effect suppresses the ac- and dc-Josephson currents. The pumped curve clearly shows the first photon assisted tunneling step, which extends from 2.7 to  $-0.2$  mV.

The thick solid line in Fig. 2 gives a typical  $I$ - $V$  curve of an irradiated junction without magnetic field. The current steps at 1.5 and 3 mV are the first and second ac-Josephson step from the 720 GHz radiation ( $\hbar\omega/2e = 1.5$  mV). The Josephson step at 1.5 mV is very sharp and shows no tilt or rounding effects due to noise. The second step is slightly rounded but not tilted and becomes sharp at higher rf-power levels. The third step shows the same behavior as the second step. It can be seen in Fig. 2 that the first step clearly crosses

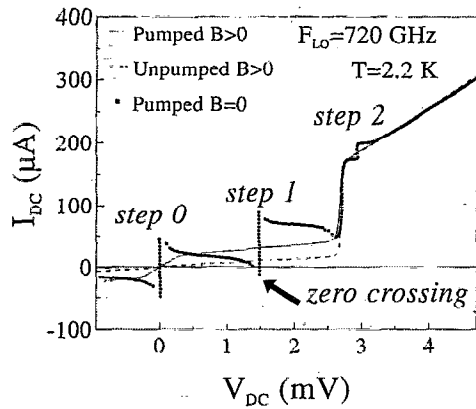


FIG. 2. Unpumped (dashed line) and pumped  $I$ - $V$  curve (solid line) at 720 GHz and 2.2 K of Nb tunnel junction with a magnetic field applied. Pumped  $I$ - $V$  curve without magnetic field applied (thick solid line). The zero crossing of the first Josephson step is clearly observed.

the zero dc-current axes. The zero crossing probably can be even more pronounced in a junction with lower leakage current.

The data are measured with the bias supply in the voltage mode, and at the unstable voltage regions below and above the first Josephson step the bias circuit oscillates. The displayed  $I$ - $V$  curve in this region is the average of the relaxation oscillations.<sup>23</sup>

$I$ - $V$  curves as shown in Fig. 2 are measured at 15 different power levels  $P_{rf}$ , and from these curves the parameters  $I_{min}^n$  (the minimum current of step  $n$ ),  $I_{max}^n$  (the maximum current of step  $n$ ) and  $\Delta I^n = I_{max}^n - I_{min}^n$  (the step amplitude of the  $n$ th step) are obtained. The results of the measured  $\Delta I^n$  are shown in Figs. 3(a) and 3(b) as a function of the square root of the measured power, together with calculated values based on Eq. (4) (discussed below). The measured data on the step-heights of the first, second, and third step all show a gradual change with changing rf-power. This indicates that the noise level and the applied magnetic flux stay constant or change gradually during the measurement, and are not influenced by the switching of the bias voltage between the subsequent measurements.

The zero voltage Josephson step shows more scatter at low values of  $\alpha$ . This is consistent with the observation made by Danchi *et al.*<sup>15</sup> on the effects of thermal and shot noise on the zero voltage step height.

## V. ANALYSIS OF DATA

For the analysis of the data with the Werthamer theory it is necessary to know the frequency dependence of  $\text{Re}(j_1)$ , but this function cannot be deduced directly from the measured data. The only measurable tunnel current is the  $\text{Im}(j_1)$  component of Eq. (3), which corresponds to the dc quasiparticle tunnel curve without radiation and with a magnetic field applied (the dashed line in Fig. 2). The Kramers-Kronig transform of this dc  $I$ - $V$  curve gives the reactive quasiparticle current component  $\text{Re}(j_1)$  and this curve shows a singularity at the gap voltage (the "Werthamer-peak", similar to the Riedel-peak in the pair response function).

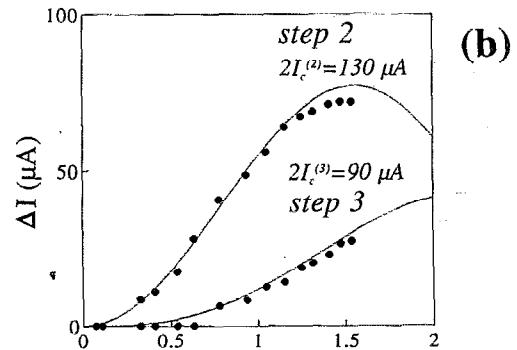
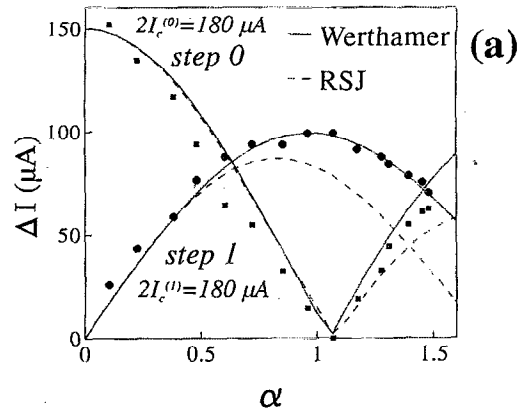


FIG. 3. (a) Measured stepwidth of the zeroth and first Josephson step as a function of  $\alpha$  at 720 GHz, together with the calculated power dependence based on the Werthamer-model (solid line) and the RSJ-model (dashed line). (b) Measured and calculated stepwidth of the second and third Josephson step as a function of  $\alpha$ .

We now estimated  $\text{Re}(j_2)$  by the adaption of the (analytic) zero temperature  $j_2$  from Werthamer<sup>13</sup> by a rounding parameter  $\delta$ , as described by Danchi *et al.*<sup>10</sup> The parameter  $\delta$  is set to a value such that the resulting rounding of the Riedel peak singularity is comparable to the rounding of the calculated reactive quasiparticle response function  $\text{Re}(j_1)$ . The resulting pair response function is shown in Fig. 4. For a comparison Fig. 4 also shows the frequency independent pair response function of the RSJ-model.

From Eq. (4) it follows that in the calculation of the Josephson step amplitude at step  $N$  the only important values of the pair response function are the values of  $\text{Re}(j_1)$  at voltages  $V_N \pm n\hbar\omega/e$ . For low values of  $\alpha$  ( $< 2$ ) a summation of Eq. (4) over  $n = -2, -1, 0, 1, 2$  is sufficient to calculate the actual step amplitude and the open and filled squares in Fig. 4 indicate the values of  $\text{Re}(j_2)$  which give the main contribution in the calculation of the zeroth and first Josephson step. From Fig. 4 it follows that the most significant difference between the RSJ-model and the Werthamer-model can be expected in the amplitude of the first Josephson step, where the  $n=1$  term of the summation probes the intrinsic roll-off of the Josephson current amplitude above the gap frequency.

Results of the calculated step-height dependence using Eq. (4) and the deduced pair response function are shown as the solid lines in Figs. 3(a) and 3(b). The maximum step height is obtained by assuming that the limiting values of the

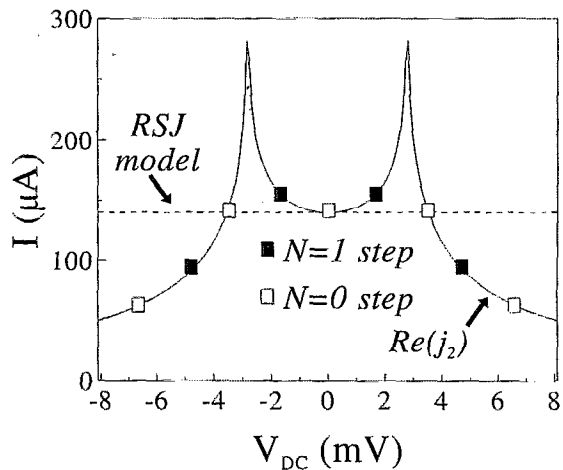


FIG. 4. Deduced frequency dependence of the Josephson current amplitude. The dashed line is the frequency independent Josephson current amplitude used in the RSJ-model. The open and filled blocks indicate the important values of the pair response function in the calculation of the zeroth and first Josephson step amplitude.

phase  $\varphi_0$  in Eq. (4) are  $-\pi/2$  and  $\pi/2$ . As a further assumption we take  $j_2(V_0, \omega)$  scaled to the maximum pair current  $I_c^{(0)}$  at  $V=0$  ( $90 \mu\text{A}$ ), which is approximately half of the theoretical value  $I_c^{\text{theo}} = \pi V_{\text{gap}}/4R_N$ . The lower value of the

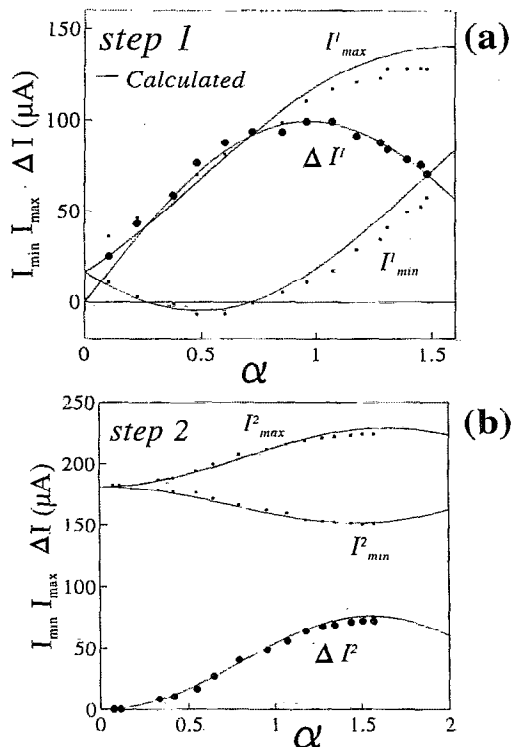


FIG. 5. (a) Measured and calculated maximum and minimum current of the first Josephson step and the step amplitude as a function of  $\alpha$ . Note the 0.3–0.7  $\alpha$ -range of the zero-crossing of the Josephson step. (b) Measured and calculated maximum and minimum current of the second Josephson step and the step amplitude as a function of  $\alpha$ .

measured critical current is caused by the escape rate from the zero voltage state due to noise and perhaps also by the effect of remnant magnetic fields of frozen-in vortices located close to the junction. Instead of lowering  $I_c^{(0)}$ , we could also have decreased the limiting values of  $\varphi_0$ . This reflects better the physical origin of a reduced step amplitude, but gives the same numerical results.

The only remaining free parameter in the comparison of the measured data to the calculated data, is the scaling factor of  $\sqrt{P_{\text{rf}}}$  to the parameter  $\alpha$ . The fit which is obtained with a fixed value of  $180 \mu\text{A}$  for  $2I_c^{(0)}$  shows good agreement with the measured zeroth and first ac-Josephson step (located below the gap), but if we fit to the second and third step (located above the gap) with the same  $2I_c^{(0)}$  value, we find that the calculated step height is far too high. The calculated data for the second and third step do fit to theory if we adjust the value of  $2I_c^{(0)}$  to  $2I_c^{(2)} = 130 \mu\text{A}$  for the second step and  $2I_c^{(3)} = 90 \mu\text{A}$  for the third step. It can be seen in Fig. 3 that the data for the second and third step then follow the Bessel function dependence very well. Possible explanations for this discrepancy will be discussed below.

The data for the zeroth and first step are also compared with the RSJ-model. In the RSJ-model the step height of the  $n$ th step is a simplified form of Eq. (4) and is given by  $J_n(2\alpha)2I_c(0)$ . We find that the simple  $J_n(2\alpha)$  dependence no longer holds. This is illustrated in Fig. 3(a), where the zeroth and first Josephson step for the two models are given, with the same  $I_c^{(0)}$ . In the RSJ-model we used a (slightly lower) scaling factor for  $\alpha$ , such that the measured and calculated minimum of the zeroth step coincide. The most striking differences are the position of the maximum of the first step relative to the minimum of the zeroth step, and the differences in the step height at values of  $\alpha > 1$ . The agreement of the data with the Werthamer model is significantly better than with the RSJ-model.

Figures 5(a) and 5(b) show the calculated and measured power-dependence of  $I_{\text{max}}^n$  and  $I_{\text{min}}^n$  of the first and second Josephson step. The calculated data of the first step shown in Fig. 5(a) were adjusted with a  $7 \mu\text{A}$  offset and then show a very good general agreement with the measured data. The offset is probably caused by the oscillatory behavior of the bias supply. At the  $\alpha$  range from 0.3 to 0.7, the step exhibits the zero crossing feature. This range is comparable to the  $\alpha$ -range calculated in Ref. 10 for the curves with a comparable Riedel-peak smearing and at a frequency of  $4\Delta/\hbar$ . At power levels above  $\alpha = 1$ , the predicted minimum and maximum current values remain somewhat below the calculated values, although the total step width is in good agreement with the calculated value [see Fig. 3(a)].

In Fig. 5(b) the measured and calculated values of the second Josephson step are shown. In the calculation of these data we use the value of  $I_c^{(2)} = 90 \mu\text{A}$ , and without any further adjustment we get a very good fit over the whole power range.

## VI. DISCUSSION AND CONCLUSION

All our measured data are in good agreement with the Werthamer model, if we scale  $I_c^{(N)}$  to an appropriate value for the specific  $N$ th Josephson step. The scaled  $I_c^{(N)}$  is substan-

tially lower than the theoretical value and rapidly decreases for Josephson steps at voltages above the gap voltage. Instead of decreasing  $I_c^{(N)}$ , we can also decrease the phase range of Josephson current to get the same result. Since the phase range is set by the noise in the junction, the decrease of the step amplitudes at higher voltages is in agreement with an increasing noise level due to both shot noise and thermal noise. It is pointed out by Danchi *et al.*<sup>10</sup> that the shot noise of a junction biased above the gap voltage corresponds to an effective "thermal" noise temperature of  $T^{\text{eff}} = eV/2k$ , which in our case corresponds to 15–20 K. This is in qualitative agreement with the rapid decrease of the Josephson step amplitude above the gap voltage. We note here that the sample used in this work has even an elevated noise level due to the heating effects and the subgap current mentioned in Sec. III. Measurements with this sample at 4.2 K do not show the zero crossing Josephson step, indicating the strong dependence of the Josephson step amplitude on the thermal noise level.

In conclusion, we have measured the ac-Josephson effect at 720 GHz and observed for the first time the inverse ac-Josephson effect at frequencies above the gap frequency. Voltage standards operating at much higher frequencies than used thus far therefore seem to be possible. The measured data are significantly in better agreement with the Werthamer-model than with the RSJ-model and therefore show in a very direct way the frequency dependence of the pair response function.

#### ACKNOWLEDGMENTS

We gratefully acknowledge the excellent work done by C.E. Honingh and H.H.A. Schaeffer in developing the microwave setup used in these experiments. We also thank Van Duc Nguyen and H. Golstein for technical support in the

preparation of these experiments. This work is financially supported by the European Space Agency under Contract No. 7898/88/NL/PB(SC) and the Stichting voor Technische Wetenschappen and the Stichting voor Fundamenteel Onderzoek der Materie.

- <sup>1</sup>B. D. Josephson, *Phys. Lett.* **1**, 251 (1962).
- <sup>2</sup>S. Shapiro, *Phys. Rev. Lett.* **11**, 80 (1963).
- <sup>3</sup>M. T. Levinsen, R. Y. Chiao, M. J. Feldman, and B. A. Tucker, *Appl. Phys. Lett.* **31**, 776 (1977).
- <sup>4</sup>R. L. Kautz, *Appl. Phys. Lett.* **36**, 386 (1980).
- <sup>5</sup>R. L. Kautz, *J. Appl. Phys.* **52**, 3528 (1981).
- <sup>6</sup>R. L. Kautz, *J. Appl. Phys.* **52**, 6241 (1981).
- <sup>7</sup>J. Niemeyer, J. H. Hinken, and R. L. Kautz, *Appl. Phys. Lett.* **45**, 479 (1984).
- <sup>8</sup>W. C. Stewart, *Appl. Phys. Lett.* **12**, 452 (1968).
- <sup>9</sup>D. E. McCumber, *J. Appl. Phys.* **39**, 3113 (1968).
- <sup>10</sup>W. C. Danchi, W. J. Golightly, and E. C. Sutton, *J. Appl. Phys.* **65**, 2772 (1989).
- <sup>11</sup>B. Mikolajczak, H. G. Meyer, and P. Seidel, *J. Appl. Phys.* **66**, 735 (1989).
- <sup>12</sup>B. Frank and H. G. Meyer, *J. Appl. Phys.* **72**, 2973 (1992).
- <sup>13</sup>N. R. Werthamer, *Phys. Rev.* **147**, 255 (1966).
- <sup>14</sup>W. C. Danchi, F. Habbal, and M. Tinkham, *Appl. Phys. Lett.* **41**, 883 (1982).
- <sup>15</sup>W. C. Danchi, J. Bindslev Hansen, M. Octavio, F. Habbal, and M. Tinkham, *Phys. Rev. B* **30**, 2503 (1984).
- <sup>16</sup>R. E. Harris, *Phys. Rev. B* **10**, 84 (1974).
- <sup>17</sup>M. J. Stephen, *Phys. Rev.* **182**, 531 (1969).
- <sup>18</sup>P. A. Lee, *J. Appl. Phys.* **42**, 325 (1971).
- <sup>19</sup>M. M. T. M. Dierichs, R. A. Panhuyzen, C. E. Honingh, M. J. de Boer, and T. M. Klapwijk, *Appl. Phys. Lett.* **62**, 774 (1993).
- <sup>20</sup>G. de Lange, C. E. Honingh, J. J. Kuipers, H. H. A. Schaeffer, R. A. Panhuyzen, T. M. Klapwijk, H. van de Stadt, and M. W. M. de Grauw (unpublished).
- <sup>21</sup>G. de Lange, C. E. Honingh, J. J. Kuipers, H. H. A. Schaeffer, R. A. Panhuyzen, T. M. Klapwijk, H. van de Stadt, and M. W. M. de Grauw, *Appl. Phys. Lett.* **64**, 3039 (1994).
- <sup>22</sup>G. de Lange, C. E. Honingh, M. M. T. M. Dierichs, H. H. A. Schaeffer, R. A. Panhuyzen, T. M. Klapwijk, H. van de Stadt, and M. W. M. de Grauw, *IEEE Trans. Appl. Supercond.* **3**, 2613 (1993).
- <sup>23</sup>K. K. Likharev, *Dynamics of Josephson Junctions and Circuits* (Gordon and Breach, New York, 1986).