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QUADRUPOLE BOSON STRUCTURE FORM FACTORS FOR PROTON SCATTERING FROM THE IBA MODEL AND ELECTRON SCATTERING

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Inelastic proton scattering from ^{110}Pd has been measured at $E_p = 30.7$ MeV. Angular distributions for three low-lying 2^+ states are satisfactorily reproduced by employing reduced matrix elements from the IBA model and form factors evaluated from two quadrupole transition charge densities (determined from electron scattering) folded with a nucleon–nucleon interaction. The relative phase between these two transition charge densities could be determined because of coupled channel effects in inelastic proton scattering. This leads to boson structure form factors for proton scattering resembling a Woods–Saxon first derivative for the d-boson non-conserving part and a Woods–Saxon second derivative with a reduced radius for the d-boson conserving part.

The success of the interacting boson approximation model [1] (IBA) in describing the "static" properties of low-lying collective states of even–even medium and heavy-mass nuclei, has been demonstrated by a large number of recent studies. For example, Van Isacker and Puddu [2] have shown that the IBA model can reproduce many properties of Pd and Ru isotopes, such as level sequences, B(E2) values, magnetic moments and two-neutron separation energies.

More recently attempts have been made to use information from the IBA model to describe the "dynamic" properties of nuclei [3], such as transition densities for inelastic excitations induced by external

probes, e.g. electrons [4,5]. In refs. [4,5] transition charge densities for the first two 2^+ states in an even–even nucleus were determined from electron scattering experiments. These were further used to calculate, by means of the IBA model, the transition charge densities for other 2^+ states. A similar attempt to use the IBA model to reproduce proton inelastic scattering data, was reported by Pignanelli et al. [6]. In these calculations only the reduced matrix elements (rme), i.e. numbers that contain the nuclear structure information, were obtained from the IBA model. Shapes of boson structure form factors (here form factor refers to the radial part of the transition potential for proton scattering) which yield good fits to the data in a coupled channel (CC) analysis were, however, assumed. In a different approach, Saha et al. [7] used the IBA mod-

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el to calculate M_N and M_Z, the neutron and proton transition strengths for the excitation of the first 2^+ states of Pd isotopes. After some normalizations, equivalent to the introduction of effective charges, these values were found to account for π^+ and π^- inelastic scattering cross sections.

For Pd isotopes the ground state (gs) charge distribution and transition charge densities for the excitation of low-lying 2^+ states have been determined in a model-independent way in a recent electron scattering experiment [5] at NIKHEF. In this paper we present elastic and inelastic proton scattering data from ^{110}Pd which we fit with CC analyses in which rme obtained from the IBA model are used in a more involved way. In particular, making use of two transition charge densities deduced from electron scattering for the lowest two 2^+ states [5] folded with a nucleon–nucleon (N–N) interaction, we calculate, using the IBA predicted rme's, all the form factors involved in a 0^+ (g.s.), 2_1^+ (0.374 MeV), 2_2^+ (0.814 MeV), and 2_4^+ (1.470 MeV) CC calculation.

A self-supporting ^{110}Pd target of approximately 1 mg/cm^2 thickness was bombarded with a 30.7 MeV analysed beam of protons from the KVI cyclotron. Scattered protons were detected using the QMG/2 magnetic spectrograph [8] and its detection system [9] with an overall energy resolution of 18 keV. The differential cross sections for the 2_1^+, 2_2^+ and 2_4^+ states of ^{110}Pd which are the first three 2^+ states predicted in the IBA calculations of ref. [2], are presented in fig. 1. The 2_3^+ state which can not be described in IBA is assumed [2] to be an intruder state.

In version two of the IBA model, which distinguishes between neutron and proton bosons, the quadrupole transition between two levels can be expressed [1] with the generalized rme's:

$$Q_{\pi,\nu} = \alpha_{\pi,\nu} A_{\pi,\nu} + \beta_{\pi,\nu} B_{\pi,\nu} , \qquad (1)$$

where

$$A_{\pi,\nu} = \langle J_f \| (d^\dagger \tilde{s} + s^\dagger \tilde{d})_{\pi,\nu} \| J_i \rangle ,$$

$$B_{\pi,\nu} = \langle J_f \| (d^\dagger \tilde{d})_{\pi,\nu} \| J_i \rangle$$

are the d-boson non-conserving and d-boson conserving rme's, respectively.

The $\alpha_{\pi,\nu}$ and $\beta_{\pi,\nu}$ are the boson coupling constants that are assumed to be identical for all transitions of equal multipolarity in one nucleus. A straightforward
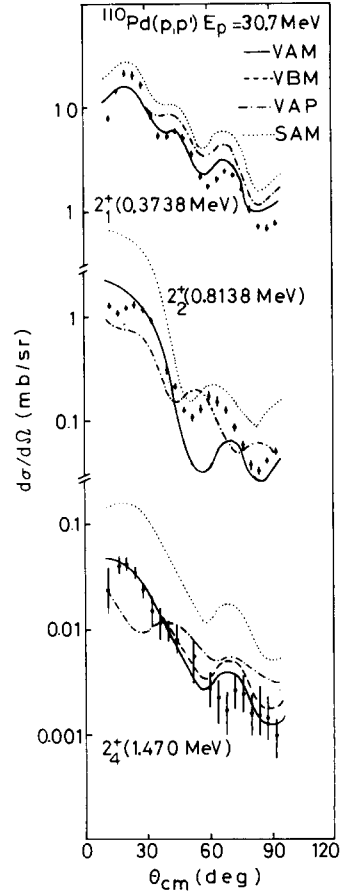

Fig. 1. Differential cross sections for inelastic proton scattering to the 2_1^+, 2_2^+ and 2_4^+ states of ^{110}Pd compared with CC calculations in which the transition potentials were obtained from rme's from the IBA model and quadrupole charge transition densities obtained from electron scattering to the lowest two 2^+ states. The meaning of the different symbols is explained in the text. Where dashed lines are missing these coincide with the full curves.

generalization [3] of the above operators to "dynamic" properties for quadrupole transition densities can be made by replacing the boson coupling constants with boson structure functions $\alpha_{\pi,\nu}(r)$ and $\beta_{\pi,\nu}(r)$ which represent the transition densities for the transitions between s- and d-bosons and two d-bosons, respectively:

$$\rho_{\pi,\nu}(r) = \alpha_{\pi,\nu}(r) A_{\pi,\nu} + \chi_{\pi,\nu} \beta_{\pi,\nu}(r) B_{\pi,\nu} , \qquad (2)$$

where $\chi_{\pi,\nu}$ can be set equal to one or to $(\beta_{\pi,\nu}/\alpha_{\pi,\nu})$ as will be discussed later. The associated quadrupole moment is

$$Q_{\pi,\nu} = \int \rho_{\pi,\nu}(r)\, r^4 \, dr \, . \tag{3}$$

The overall electric transition strength M_Z induced by the electromagnetic field between levels whose wavefunctions have been calculated in a truncated basis space, is defined after the introduction of the effective nucleon charges, e_π and e_ν, which account for the truncation of the basis space:

$$M_Z = e_\pi Q_\pi + e_\nu Q_\nu \, . \tag{4}$$

M_Z is linked to the reduced transition probability by: $B_{em}(EL, J_i \to J_f) = |M_Z|^2/(2J_i + 1)$. The effective electric transition density associated with this is

$$\rho(r) = e_\pi \rho_\pi(r) + e_\nu \rho_\nu(r) \, , \tag{5}$$

where ρ_π and ρ_ν are given by eq. (2).

For Pd isotopes the rme's in eqs. (1), (2) have been calculated in ref. [2]. As nucleon effective charges and boson coupling constants we can use the empirical values ($e_\pi = e_\nu = 1$) determined in ref. [2] [in such a case our analyses will be labeled (V)], or the nucleon effective charges ($e_\pi = 1.35$ and $e_\nu = 0.49$) of Saha et al. [7] [analyses (S)] who used the boson coupling constants calculated by Scholten [10] in a generalized seniority scheme. A check of the two sets of nucleon effective charges and boson coupling constants can be made through the comparison of evaluated B_{em}(E2)'s

(by means of eqs. (1)–(4)) and the experimental B_{em}(E2) values [11] (see table 1) for all the transitions examined here. We note that those calculated with the S procedure (column 8) show in general bad agreement; in particular the B(E2)-value from the 2_2^+ level to the gs is overestimated.

For probes which are also sensitive for neutrons one can also define an effective neutron transition matrix element M_N similar to the effective proton transition matrix element M_Z. In the Pd case where the core is self-conjugate, M_N is given by [7]

$$M_N = e_\pi Q_\nu + e_\nu Q_\pi \, . \tag{6}$$

Analogous to eqs. (4) and (5), one can then define effective proton and neutron transition densities. In the case of proton scattering, the transition potential ΔU_{tr} is obtained from these effective transition densities by folding with a p–p and p–n interactions, respectively. Thus:

$$\Delta U_{tr} = e_\pi \int \rho_\pi(r)\, v_{p-p}(|r - r'|)\, d^3r'$$
$$+ e_\nu \int \rho_\nu(r)\, v_{p-p}(|r - r'|)\, d^3r'$$
$$+ e_\nu \int \rho_\pi(r)\, v_{p-n}(|r - r'|)\, d^3r'$$
$$+ e_\pi \int \rho_\nu(r)\, v_{p-n}(|r - r'|)\, d^3r' \, , \tag{7}$$

Table 1

B(E2) values (in e^2 fm^4 between the levels investigated in this paper. The sign in front of each calculated B(E2) value is that of the corresponding matrix element M (eq. (4)).

$J_i \to J_f$	VAP [a]	SAM [a]	VAM [a]	VBM [a]	EXP [b]	IBA [c]	Saha [d]
$0_1^+ \to 2_1^+$	−8770	−8870	−8770	−8770	9100 ± 600	−9100	−9446
$0_1^+ \to 2_2^+$	−149	+149	+149	+149	128 ± 11	+143	+506
$0_1^+ \to 2_4^+$	+1.8	+11	+15	+18		+18	+9
$2_1^+ \to 2_1^+$	−2360	−1078	−1015	−1047	1037	−1111	−763
$2_1^+ \to 2_2^+$	−1030	−1571	−1725	−1740	1800 ± 300	−1790	−2007
$2_1^+ \to 2_4^+$	+26	−5	−1.1	−0.05		+0.2	−20
$2_2^+ \to 2_2^+$	+716	+548	+531	+564		+592	+533
$2_2^+ \to 2_4^+$	+626	+309	+308	+289		+304	+227
$2_4^+ \to 2_4^+$	−2280	−769	−670	−782		−835	−435

[a] The symbols VAP, SAM, VAM, and VBM refer to the (p,p′) analyses described in the text.
[b] Experimental B_{em}(E2) values from ref. [11].
[c] Calculated B_{em}(E2) values (ref. [2]), normalized arbitrarily to reproduce the experimental B(E2, $0_1^+ \to 2_1^+$) of ref. [11].
[d] Calculated B_{em}(E2) values from parameters in ref. [7].

where $\rho_\pi(r)$ and $\rho_\nu(r)$ are given by eq. (2).

In the present analysis, as in those of refs. [4,5], only two experimental charge transition densities are well determined. To reduce the number of the unknown quantities it is useful, in eq. (2), to assume $\alpha_\pi(r) = \alpha_\nu(r) = \alpha(r)$ and $\beta_\pi(r) = \beta_\nu(r) = \beta(r)$. This seems reasonable because of the expected strong polarization of the self-conjugate proton and neutron core. However, the above hypothesis does not completely restrict the analysis to an IBA-1 calculation since the effective charges and the probe interaction with the protons and neutrons of the target still remain different.

In the following our (p,p′) analyses that make use of eqs. (5) and (7) will be labeled (A). When applied to the experimental results from (e,e′) scattering to deduce $\alpha(r)$ and $\beta(r)$, the (A) analyses utilize only the IBA-rme numbers in eq. (2) and do not use the quantitative information given by the boson coupling constants α and β in eq. (1) since no restriction is placed on the quadrupole moments of the deduced boson structure functions $\alpha(r)$ and $\beta(r)$; i.e. $\chi_{\pi,\nu}$ is set equal to 1 in eq. (2). The ratio of the quadrupole moments of the boson structure functions $\alpha(r)$ and $\beta(r)$ can, however, be constrained to the ratio of the above boson coupling constants $\alpha_{\pi,\nu}$ and $\beta_{\pi,\nu}$ in eq. (1) as deduced from either the analysis of Van Isacker and Puddu [2] or of Saha et al. [7]. This is implemented by taking $\chi_{\pi,\nu} = \beta_{\pi,\nu}/\alpha_{\pi,\nu}$ in eq. (2). In this case we have (B) analyses. The influence on the boson structure functions resulting from the use of (A) or (B) analyses is moderate (see fig. 2a). In particular $\beta(r)$ flips its sign and changes its absolute value to compensate the differences in β_ν and β_π. This change in sign takes place because χ_ν has a different sign from χ_π in the analyses of refs. [2,7]. All the results shown in fig. 2a were obtained using effective charges of Van Isacker and Puddu [2], but similar results have been obtained using those of Saha et al. [7]. In general we expect no big differences in the CC calculations from the use of procedures (A) or (B) since the transition densities produced by these procedures will be about the same.

An uncertainty in the use of eqs. (5) and (7) is connected to the choice of the relative sign of the experimental transition charge densities that cannot be determined experimentally from (e,e′) data. As shown in fig. 2a, this choice influences noticeably the shape of
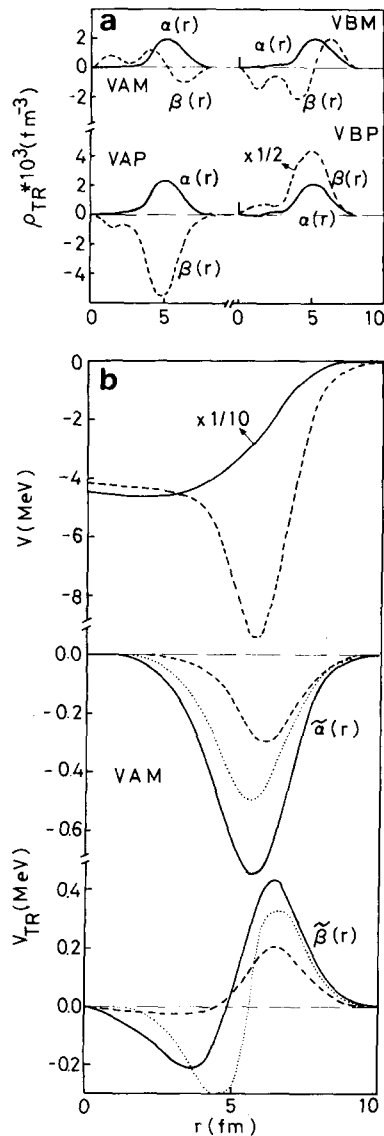

Fig. 2. (a) Quadrupole boson structure functions in ^{110}Pd as extracted from the experimental $0_1^+ \rightarrow 2_1^+$ and $0_1^+ \rightarrow 2_2^+$ charge transition densities of ref. [5] and effective charges rme's and boson coupling constants from ref. [2]. (b) Diagonal potentials and quadrupole boson transition form factors for the ^{110}Pd(p,p′) deduced from the VAM curves of the upper part, folded with the JLM potential. Full and dashed lines refer to the real and imaginary parts of the diagonal and transition potentials, respectively. Dotted curves for $\tilde{\alpha}(r)$ and $\tilde{\beta}(r)$ refer to Woods–Saxon first- and second-derivative form factors, respectively.

$\beta(r)$ which changes from an approximate Woods–
Saxon second derivative shape when $\rho_{2_1}(r)$ and $\rho_{2_2}(r)$
from (e,e') are taken with opposite sign [(M) analyses],
to an almost Woods–Saxon first-derivative shape when
they are assumed to have the same sign [(P) analyses].
In the following three symbols (such as VAM, VAP
etc.) will be used together to indicate the various
choices and procedures used to perform the calcula-
tions.

The boson structure functions thus obtained from
electron scattering data and also the ^{110}Pd gs charge
density have been employed in the analysis of proton
scattering. In principle, the point densities should first
be calculated from these by deconvoluting out the fi-
nite-charge distribution of the proton, after which the
form factors should be obtained by folding a N–N in-
teraction. However, equivalent results have been ob-
tained by folding the determined transition charge den-
sities with a N–N interaction of a slightly shorter range.
The N–N potential of Jeukenne et al. [12] (JLM) has
been used here with the range suggested by Mellema et
al. [13]. The density dependence of the JLM interac-
tion leads to different shapes of the real and imaginary
potentials (full and dashed lines, respectively, in fig.
2b). As shown in ref. [13], the choice of the local ap-
proximation influences, to a great extent, only the
imaginary parts. Here, as nuclear density, the average
of the densities at the positions of the projectile and
of the target nucleon is taken. The values obtained do
not need any further renormalization and the volume
integrals of the diagonal potentials (shown in the up-
per part of fig. 2b) were found to be very similar to
those used in ref. [6]. The obtained form factors
(summed over the proton and neutron components;
see eq. (7)) are also plotted in fig. 2b. The $\tilde{\alpha}(r)$ real
part closely resembles a Woods–Saxon first derivative
(dotted line in the middle part of fig. 2b), while the
$\tilde{\beta}(r)$ part is similar to a Woods–Saxon second deriva-
tive (dotted line in the lower part of fig. 2b) with a
reduced radius. We conclude, therefore, that the
shapes found here are indeed very near to the phenom-
enological form factors used in ref. [6]. The strengths
of the form factors in fig. 2b depend upon the N–N
interaction range. These become larger or smaller for
shorter (as also for the M3Y interaction [14] or
longer ranges, respectively.

The final results of our CC calculations are re-
ported in fig. 1. When Van Isacker and Puddu's [2]

parameters are used, the 2_1^+ and 2_2^+ calculations are in-
dependent of whether method (A) or (B) is used, and
those for the 2_4^+ are only slightly affected. The 2_1^+
cross section is reproduced well by the (M) method
and reasonably by the (P) method, but the 2_2^+ and the
2_4^+ calculations suggest that choice (P) should be
avoided since it fails in reproducing the experimental
cross section shapes. The (e,e') data for the 2_2^+ cross
section are insensitive to the choice (P) or (M) since
two-step effects are negligibly small. Similar conclu-
sions can be drawn from the use of effective charges
of ref. [7]. A comparison between the (V) and (S)
calculations suggests that the latter overestimate all
cross sections, reflecting their generally larger neutron
transition matrix elements M_N contributions which in-
creases their sensitivity to the p–n effective interac-
tion.

We would like to stress the point that all the curves
in fig. 1 have been obtained without free parameters.
In table 1 the B(E2) values between the four coupled
levels examined here are reported. Those obtained
from our present (p,p') analyses have been determined
from the effective electric transition strengths, i.e.
from formulas 4, 3 and 2, using the effective charges
and boson structure functions employed in the vari-
ous analyses. We also include the B_{em} values calcu-
lated in ref. [2] and those calculated using the param-
eters of Saha et al. [7], and the existing experimental
B_{em} values [11]. We note that all VAM and VBM cal-
culations, which reproduce the data better, also repro-
duce all the known experimental B_{em}(E2)-values of
ref. [11]. Even the SAM calculations which yield bad
fits to the (p,p') data because of their higher M_N val-
ues, reproduce the B(E2)-values reasonably well. Only
the VAP results which reproduce the B(E2, $0_1^+ \rightarrow 2_1^+$)
and B(E2, $0_1^+ \rightarrow 2_2^+$) values as they should, fail badly in
reproducing the other known experimental B(E2)-val-
ues as well as the experimental (p,p') data.

In summary, we have shown that quadrupole exci-
tations in proton scattering can be satisfactorily repro-
duced from rme's evaluated from the IBA model and
form factors given by an IBA-weighted sum of two
parts evaluated from two quadrupole transition charge
densities deduced from (e,e') scattering folded with a
N–N interaction. The d-boson non-conserving form
factor is similar in shape to a Woods–Saxon first de-
rivative, and the d-boson conserving one comes out
similar to a second derivative with a smaller radius. At

the moment the only remaining uncertainty in the calculations results from the use of neutron—boson structure functions equal to those of protons. To eliminate such uncertainty the (e, e') scattering to four 2^+ states should be measured.

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