# Finite range effects in pionic atoms

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# FINITE RANGE EFFECTS IN PIONIC ATOMS

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Finite range effects are shown to be important in the derivation of the $\pi$-nucleus optical potential.

Strong interaction shifts in $\pi$-mesic atoms have been succesfully analyzed [1] with the optical model of Ericson and Eriscon [2]. In a purely phenomenological context the finite range of the $\pi$-nucleon interaction may be ignored and its effects included in the effective parameters. However, any attempt to *derive* the optical parameters from the $\pi$-N interaction must also take into account its finite range, particularly in view of the near cancellation of the s-wave scattering lengths, $2\alpha_3+\alpha_1 = 0$.

Furthermore contrary to what is commonly stated, the ranges of the $\pi$-N interaction are *not* significantly smaller than internucleon distances. The contributions to the $\pi$-N interaction have been studied in detail by Hamilton and coworkers [3–5]. They are: (i) The exchange of an s-wave, $T = 0$ $\pi\pi$ pair ($\sigma$), (ii) the exchange of a p-wave, $T = 1$ $\pi\pi$ pair ($\rho$), (iii) the long range Born term (nucleon exchange) and (iv) the crossed physical cut term (mainly isobar exchange). In addition to these there is the Born pole, present however only in the $P_{11}$ wave, and a very short range interaction about little is known (the core). The Born pole and N-exchange have the longest range ($1/m_\pi \sim 1.4$ fm) [5]. Next come the $\sigma$-exchange ($1/2m_\pi \sim 0.7$ fm) and the $\rho$-exchange ($1/5.4m_\pi \sim 0.3$ fm); finally the $N^*$ exchange and the core part ($1/10m_\pi \sim 0.1$ fm). The nucleon and the nucleon isobar $N^*$ exchanges contribute only to the p-waves.

A calculation along the lines of ref. [5] of the strengths and ranges of the low-energy potentials associated with these terms will be presented elsewhere [6]. Here instead we present results of a phenomanological treatment of the effects of the finite range of the s-wave $\pi$-N interaction on level shifts in pionic atoms. We write the first order optical potentials as convolution integrals of the nuclear density $\rho(\boldsymbol{r}_0)$ with the appropriate projections of the $t$-matrix

$$V_0(r) = A\int \tfrac{1}{3}\,[2t_3(\boldsymbol{r}-\boldsymbol{r}_0)+t_1(\boldsymbol{r}-\boldsymbol{r}_0)]\;\rho(\boldsymbol{r}_0)\,\mathrm{d}^3\boldsymbol{r}_0$$
$$V_1(r) = (N-Z)\int \tfrac{1}{3}\,[t_3(\boldsymbol{r}-\boldsymbol{r}_0)-t_1(\boldsymbol{r}-\boldsymbol{r}_0)]\,\rho(\boldsymbol{r}_0)\,\mathrm{d}^3\boldsymbol{r}_0\,. \tag{1}$$

where $t_3$ and $t_1$ are the s-wave isospin $\frac{3}{2}$ and $\frac{1}{2}$ projections of the $\pi$N transition matrix. The subscript on $V$ refers to the isoscalar (0) and isovector (1) potentials respectively. For zero-range

$$t_i(\boldsymbol{r}-\boldsymbol{r}_0) = -\frac{4\pi}{2m}\,\alpha_i\,\delta(\boldsymbol{r}-\boldsymbol{r}_0)\,, \quad i = 1.3 \tag{2}$$

the optical s-wave potentials assume the usual Eriscon [1,2] form

$$V_0^{(\mathrm{E})}(r) = -\frac{4\pi}{2m}\frac{A}{3}\,(2\alpha_3+\alpha_1)\,\rho(r)$$
$$V_1^{(\mathrm{E})}(r) = -\frac{4\pi}{2m}\,\frac{N-Z}{3}\,(\alpha_3-\alpha_1)\,\rho(r) \tag{3}$$

where $m$ denotes the reduced $\pi$N mass and $\alpha_3, \alpha_1$ the s-wave scattering lengths.

For finite range

$$t_i(\boldsymbol{r}-\boldsymbol{r}_0) = -\frac{4\pi}{2m}\,\alpha_i\,f(\boldsymbol{r}-\boldsymbol{r}_0)\,, \quad i = 1,3 \tag{4}$$

the s-wave optical potentials take on the form

$$V_0^{(\mathrm{FR})}(r) = -\frac{4\pi}{2m}\,\frac{A}{3}\,[2\alpha_3 u_3(r)+\alpha_1 u_1(r)] \tag{5a}$$

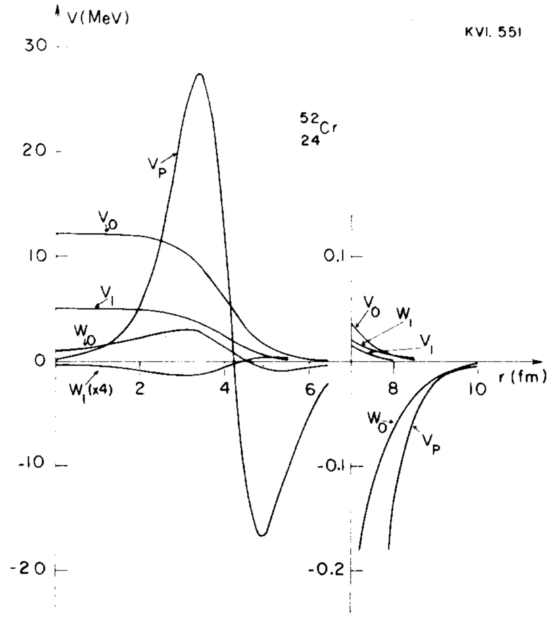


Fig. 1. The correction potentials $W_0$ and $W_1$ compared with the real part of Ericson's standard potentials ($V_0$, $V_1$ and $V_P = V_{\text{p-wave}}$) in the local representation. The strength parameters of the Ericson set [1] are given in table 1. The correction potentials shown here were generated with $\alpha_3 = -0.088\,\mu_\pi^{-1}$, $R_3 = 0.3$ fm, $\alpha_1 = 0.171\,\mu_\pi^{-1}$, $R_1 = 0.7$ fm.

$$V_1^{(\mathrm{FR})}(r) = -\frac{4\pi}{2m}\,\frac{N-Z}{3}\,[\alpha_3 u_3(r) - \alpha_1 u_1(r)] \tag{5b}$$

where the functions $u_3(r)$ and $u_1(r)$ are given by

$$u_i(r) = \int f_i(r - r_0)\,\rho(r_0)\,\mathrm{d}^3 r_0\,, \quad i = 1,3\,. \tag{6}$$

The correction to the first order optical potential introduced by the finite range of the s-wave $\pi$N interaction is

$$W_0(r) = V_0^{(\mathrm{FR})}(r) - V_0^{(\mathrm{E})}(r)\,,$$
$$W_1(r) = V_1^{(\mathrm{FR})}(r) - V_1^{(\mathrm{E})}(r)\,. \tag{7}$$

To estimate the importance of the s-wave finite range, we have calculated level shifts and widths in mesic atoms by solving the Klein–Gordon equation with and without the correction term (7) added to the Ericson standard set of parameters [1]. We have assumed Yukawa-type $t$-matrices with ranges $R_3$ and $R_1$

$$f_i(R) = \frac{1}{R_i^3}\,\frac{\exp(-R/R_i)}{(R/R_i)} \tag{8}$$

and scattering lengths [7] $\alpha_3 = -0.088\,\mu_\pi^{-1}$, $\alpha_1 = 0.171\,\mu_\pi^{-1}$.

An important feature of $W_0$ is its similarity to the potential due to a p-wave interaction. This can be easily understood in terms of a momentum expansion of the $\pi$N scattering amplitude

$$F(\boldsymbol{k},\boldsymbol{k}') = (\alpha_0' + \alpha_1' \boldsymbol{t}\cdot\boldsymbol{\tau}) + (\beta_0 + \beta_1 \boldsymbol{t}\cdot\boldsymbol{\tau})k^2 + (\gamma_0 + \gamma_1 \boldsymbol{t}\cdot\boldsymbol{\tau})k^2 \cos(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{k}}') \tag{9}$$

where the s-wave finite range gives $\beta_0, \beta_1 \neq 0$. In fig. 1 we compare the correction potentials $W_0$ and $W_1$ with the real parts of the local and gradient terms corresponding to the Ericson standard set.

It is convenient to parametrize the finite range shifts in terms of one of the ranges $R_3$ and of the difference $\Delta R = R_1 - R_3$. Because of the near cancellation of the isobar scattering lengths $\frac{1}{3}(2\alpha_3 + \alpha_1)$ the finite-range effect is very sensitive to $\Delta R$. If $2\alpha_3 + \alpha_1 = 0$, the zero-range isoscalar potential (3) vanishes but not the finite range potential (5). On table 1 we show 2p shifts in $^{52}$Cr for typical values of the ranges $R_3$ and $R_1$. For $\Delta R \approx 0.4$ fm, a value consistent with theoretical estimates [6], the level shift is 20–30% of the total shift. This results holds for all 2p, 3d and

Table 1

s-wave finite-range corrections to the 2p levels shift in $^{52}_{24}$Cr. The shift corresponding to the Ericson standard set is $-2.561$ keV. The parameters of the Ericson set are $b_0 = -0.030\,\mu_\pi^{-1}$, $b_1 = -0.080\,\mu_\pi^{-1}$, $c_0 = 0.24\,\mu_\pi^{-3}$ ($\xi = 1$), Im $B_0 = 0.040\,\mu_\pi^{-4}$, Im $C_0 = 0.14\,\mu_\pi^{-6}$. The last row corresponds to the theoretical estimate [6].

| S-wave ranges(fm) | | | Finite-range-correction (keV) | | |
|---|---|---|---|---|---|
| $R_1$ | $R_3$ | $\Delta R$ | isoscalar | isovector | isoscalar + isovector |
| 0.4 | 0.1 | 0.3 | −0.379 | +0.032 | −0.346 |
| 0.7 | 0.1 | 0.6 | −1.153 | +0.088 | −1.057 |
| 1.0 | 0.1 | 0.9 | −2.216 | +0.162 | −2.032 |
| 0.7 | 0.4 | 0.3 | −0.740 | +0.103 | −0.632 |
| 1.0 | 0.4 | 0.6 | −1.778 | +0.177 | −1.583 |
| 1.0 | 0.7 | 0.3 | −0.962 | +0.206 | −0.746 |
| 0.7 | 0.3 | 0.4 | −0.926 | +0.096 | −0.824 |

4f shifts considered in ref. [1]. The effect on 1s shifts is, instead, negligible.

One important consequence of the s-wave finite range correction is that the p-wave optical parameters needed to fit mesic atoms are reduced to ≈80% of the previous values. This result is relevant in the study of the Pauli quenching of the p-wave $\pi$-nucleus interaction [8].

Although the finite range s-wave correction is similar to the p-wave contribution to the optical $\pi$-nucleus potential it differs sufficiently from it to render mesic atoms a unique tool in the study of the ranges of the elementary $\pi$N interaction. For instance the 2p level shift corresponding to $R_3 = 0.3$ fm, $R_1 = 0.7$ fm is 28% of the total shift in $^{40}$Ca but ~50% in $^{59}$Co. Thus a measurement of the 2p level shifts from $^{27}$Al to $^{59}$Co with a precision of < 10% could detect small differences from the Ericson standard set and thus allow a determination of the ranges.

Finally we remark that the finite range of the $\pi$N interaction will also influence the second and higher order terms. This problem has been studied for the p-wave interaction by Eisenberg et al. [9].

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