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Imaging neutral hydrogen on large scales during the Epoch of Reionization with LOFAR

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ABSTRACT

The first generation of redshifted 21 cm detection experiments, carried out with arrays like Low Frequency Array (LOFAR), Murchison Widefield Array (MWA) and Giant Metrewave Telescope (GMRT), will have a very low signal-to-noise ratio (S/N) per resolution element ($\lesssim 0.2$). In addition, whereas the variance of the cosmological signal decreases on scales larger than the typical size of ionization bubbles, the variance of the formidable galactic foregrounds increases, making it hard to disentangle the two on such large scales. The poor sensitivity on small scales, on the one hand, and the foregrounds effect on large scales, on the other hand, make direct imaging of the Epoch of Reionization of the Universe very difficult, and detection of the signal therefore is expected to be statistical. Despite these hurdles, in this paper we argue that for many reionization scenarios low-resolution images could be obtained from the expected data. This is because at the later stages of the process one still finds very large pockets of neutral regions in the intergalactic medium, reflecting the clustering of the large-scale structure, which stays strong up to scales of $\approx 120 h^{-1}$ comoving Mpc ($\approx 1^\circ$). The coherence of the emission on those scales allows us to reach sufficient S/N ($\gtrsim 3$) so as to obtain reionization 21 cm images. Such images will be extremely valuable for answering many cosmological questions but above all they will be a very powerful tool to test our control of the systematics in the data. The existence of this typical scale ($\approx 120 h^{-1}$ comoving Mpc) also argues for designing future EoR experiments, e.g. with Square Kilometre Array, with a field of view of at least 4° .

Key words: methods: statistical – cosmology: observations – cosmology: theory – diffuse radiation – large-scale structure of Universe – radio lines: general.

1 INTRODUCTION

During the Epoch of Reionization (EoR), gas in the Universe reionized after having been neutral for about 500 Myr during the so-called Dark Ages. The EoR is thought to be caused by the first

radiating sources, and its study is crucial to our understanding of the physics of these sources and how they influenced the formation of later generations of astrophysical objects. Current observational constraints indicate that the EoR occurred at $6.5 \lesssim z \lesssim 12$, as inferred from Sloan Digital Sky Survey high-redshift quasar spectra (Fan et al. 2003, 2006), *Wilkinson Microwave Anisotropy Probe* (WMAP; Page et al. 2007), South Pole Telescope (SPT; Zahn et al. 2011) and intergalactic medium (IGM) temperature

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measurements (Theuns et al. 2002; Bolton et al. 2010). In addition, recent *Hubble Space Telescope* observations of the *Hubble Ultra Deep Field* taken with the new Wide Field Camera 3 (WFC3) found a large sample of Lyman break galaxies at $7 \lesssim z \lesssim 10$ (see e.g. Bouwens et al. 2010; Bunker et al. 2010; Oesch et al. 2010). These authors found that these galaxies do not produce enough ionizing photons to account for the Universe's full reionization by redshift 6 and concluded that an additional source of ionizing photons is required (Bouwens et al. 2011). Furthermore, measurements of the number of ionizing photons per baryon from Lyman α forest spectra at $z \approx 6$ yields a low number of such photons (~ 3), that is, the reionization process is photon starved (Bolton & Haehnelt 2007; Calverley et al. 2011). When combined, these measurements lend themselves to the notion that reionization is a slow and drawn out process.

Our current best hope to study this epoch in detail lies in observations of the redshifted $H\text{I}$ 21cm emission line (see e.g. Madau, Meiksin & Rees 1997; Shaver et al. 1999; Furlanetto, Oh & Briggs 2006; Pritchard & Loeb 2011). To date, a number of experiments are planning to measure the EoR with the redshifted 21 cm line (e.g. LOFAR,¹ GMRT,² MWA,³ 21CMA,⁴ PAPER⁵). These experiments seek statistical detection of the cosmological 21cm signal, with the most widely studied such statistics being the rms and the power spectrum of the brightness temperature and their evolution with time (e.g. Morales & Hewitt 2004; Barkana & Loeb 2005; Bowman, Morales & Hewitt 2006; McQuinn et al. 2006; Pritchard & Furlanetto 2007; Jelić et al. 2008; Pritchard & Loeb 2008; Harker et al. 2009a, 2010). Though recently, Datta et al. (2012) have shown that one can image large bubbles of ionization around very high redshift powerful quasars with LOFAR. In particular, Jelić et al. (2008), Harker et al. (2010) and more recently Chapman et al. (2012) showed that despite the low signal-to-noise ratio (S/N), prominent foregrounds and instrumental response, the 21 cm rms and power spectrum can be extracted from the data collected with the LOFAR. Similar studies have been carried out for the MWA case (Geil et al. 2008; Geil, Gaensler & Wyithe 2011).

The current generation of telescopes are designed to detect the EoR statistically, rather than image it, for a number of reasons. On small scales the noise level per resolution element is relatively high. For example, at 150 MHz LOFAR will have a 56 mK system noise per resolution element (≈ 3 arcmin) after 600 h of observations with a 1 MHz bandwidth, corresponding to a S/N of ~ 0.2 at these scales. On large scales there are two issues. The first one has to do with the typical sizes of ionized and neutral regions at each redshift, which limits the maximum scale at which smoothing of the data remains useful. It can also be shown that during the dark ages (no ionization bubbles) smoothing on very large scales does not help so much due to the low level of contrast and to the fact that beyond 1° the power due to cosmological fluctuations drops very quickly (see e.g. Santos, Cooray & Knox 2005; Jelić et al. 2008). The other, and potentially more severe, issue is that of the foregrounds which dominate the measurement on all scales and become even more prominent on large scales with power increasing as $\theta^{2.5-3}$ (Tegmark et al. 2000; Giardino et al. 2002; Santos et al. 2005; Jelić et al. 2008, 2010; Bernardi et al. 2009, 2010). This means that on

large scales the influence of these foregrounds will be harder to filter out. The common wisdom is that scales beyond half a degree will remain inaccessible even after a number of years of observations.

In this paper we argue that, despite the hurdles we have listed, imaging of the EoR on very large scales is possible with the current generation of telescopes. In presenting our case we rely on two arguments. First, inspection of large-scale EoR simulations ($\gtrsim 200 h^{-1}$ comoving Mpc) shows that towards the end of the reionization process one can still find sufficiently large neutral patches so as to allow imaging of this process after smoothing on sufficiently large scales. For example, a neutral patch towards the end of reionization with a scale of roughly $120 h^{-1}$ comoving Mpc (1°) would have a S/N per smoothing cell, after 600 h of observations, of about 4 since it would have about 20×20 independent resolution elements ($1^\circ/3 \text{ arcmin} = 20$). Notice that this argument would not work early in the reionization process since the variation in the 21 cm intensity in a given field, measured by radio interferometers, will be driven by the cosmological density fluctuation field, δ , which is relatively small – unless the spin temperature itself exhibits fluctuations above and below the CMB temperature (Pritchard & Furlanetto 2007; Pritchard & Loeb 2008, 2010; Baek et al. 2010; Thomas & Zaroubi 2011). Towards the later stages of the reionization process, however, the variations in the intensity are driven by the difference between neutral and ionized regions. The $120 h^{-1}$ comoving Mpc scale is driven mainly by the clustering scale of the Universe's large-scale structure, which the ionization sources, independent of their nature, tend to follow.

Secondly, the current state-of-the-art foreground fitting methods, such as Wp smoothing (Harker et al. 2010) or Independent Component Analysis (Chapman et al. 2012), do a very good job even on large scales, rendering them accessible for EoR analysis. The availability of such techniques together with the existence of the large-scale neutral patches towards the end of reionization makes it possible to image the EoR from 21 cm data on large scales.

This paper is organized as follows. In Section 2 we describe the cosmological signal and its basic equation. In Section 3 we introduce the very large-scale simulations needed to demonstrate our argument. In these simulations we also include the influence of the telescope response, noise and foregrounds. In Section 4 it is shown that, based on these large-scale simulations, imaging of the EoR on large scales is indeed possible with current instruments. We also argue that the key to successful imaging on large scales is the ability to remove the foreground signal with sufficient accuracy on scales larger than 1° . The paper concludes with a summary and outlook (Section 6).

2 COSMOLOGICAL 21 CM SIGNAL

In radio astronomy, where the Rayleigh–Jeans law is usually applicable, the radiation intensity, $I(\nu)$, is expressed in terms of the brightness temperature, so that

$$I(\nu) = \frac{2\nu^2}{c^2} k_B T_b, \quad (1)$$

where ν is the radiation frequency, c is the speed of light and k_B is Boltzmann's constant (Rybicki & Lightman 1986). This in turn can only be detected differentially as a deviation from the cosmic microwave background (CMB) temperature, T_{CMB} . The predicted differential brightness temperature $\delta T_b \equiv T_b - T_{\text{CMB}}$, which reflects the fact that the only meaningful brightness temperature measurement, insofar as the IGM is concerned, is when it deviates from T_{CMB} . Derivation of δT_b yields (Field 1958, 1959; Madau et al.

¹ Low Frequency Array (<http://www.lofar.org/>).

² Giant Metrewave Telescope (<http://www.gmrt.ncra.tifr.res.in/>)

³ Murchison Widefield Array (<http://www.haystack.mit.edu/ast/arrays/mwa/>)

⁴ 21 Centimetre Array (<http://web.phys.cmu.edu/~past/>)

⁵ Precision Array to Probe the EoR (<http://astro.berkeley.edu/~dbacker/eor/>)

1997; Ciardi & Madau 2003)

$$\delta T_b \approx 28\text{mK} (1 + \delta) x_{\text{HI}} \left(1 - \frac{T_{\text{CMB}}}{T_{\text{spin}}}\right) \left(\frac{dv_r/dr}{H(z)} + 1\right)^{-1} \times \left(\frac{\Omega_b h^2}{0.023}\right) \sqrt{\left(\frac{1+z}{10}\right) \left(\frac{0.15}{\Omega_m h^2}\right)}, \quad (2)$$

where h is the Hubble constant in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$, δ is the mass density contrast, v_r is the line-of-sight velocity component, x_{HI} is the neutral fraction, Ω_m and Ω_b are the mass and baryon densities in units of the critical density, and $H(z)$ is the Hubble parameter. Note that the three quantities, δ , v_r , x_{HI} and T_s , are all functions of 3D position.

Equation (2) shows that the differential brightness temperature is composed of a mixture of cosmology-dependent and astrophysics-dependent terms. The equation is clearly complex yet at the same time information-rich. This is simply because at different stages in the evolution of reionization δT_b is dominated by different contributions. For example, at certain redshift ranges no significant ionization has taken place, i.e. $x_{\text{HI}} \approx 1$ everywhere, yet there is enough heating to render $T_{\text{spin}} \gg T_{\text{CMB}}$. In such a case, the brightness temperature is proportional to the density fluctuations making its measurement an excellent probe of cosmology. However, at low redshifts ($z \lesssim 9$) a significant fraction of the Universe is expected to be ionized and the measurement is dominated by the contrast between the neutral and ionized regions, hence, probing the astrophysical source of ionization (see e.g. Iliev et al. 2008; Thomas et al. 2009; Thomas & Zaroubi 2011). During the stage at which reionization occurs it is safe to assume that $T_{\text{spin}} \gg T_{\text{CMB}}$ (Pritchard & Loeb 2008, 2010).

Since interferometers do not measure the mean δT_b but rather are sensitive to its fluctuations, it is easy to see that early in the reionization process, when the ionized regions are quite negligible, the rms of the measurement will be driven by δ , the cosmological density contrast fluctuations (assuming $T_{\text{spin}} \gg T_{\text{CMB}}$). Later in the reionization process, however, when the typical size of the ionized regions becomes larger than the interferometer's resolution scale, the measured signal is driven by x_{HI} , which increases the contrast and as a result the ability to observe the signal.

In this paper we ignore the issue of calibration errors. This issue might become important for the real data (Datta, Bowman & Carilli 2010), but so far the indication is that the calibration will be achieved with very high accuracy (Yatawatta et al. 2009; Kazemi et al. 2011) as shown recently by the observation of two LOFAR fields (Yatawatta and Labropoulos, private communication).

3 DATA SIMULATIONS

Since here we argue that instruments such as LOFAR will enable imaging the EoR on large scales, it is important to test the influence of the noise, instrument response, foreground extraction and the distribution of the 21 cm signal power on various scales as a function of redshift. Hence one needs to explore scales well in excess of $\approx 120 h^{-1}$ comoving Mpc, namely, about 1° on the sky. This is the natural scale of the large-scale structure and is marked in the cosmological power spectrum by the turn over from $P(k) \propto k^n$ to $P(k) \propto k^{n-4}$, where the primordial power-law index $n \approx 1$. This scale is equal to the comoving horizon size at the era of equality between matter and radiation.

Therefore, as a first step, we have to create very large scale cosmological reionization simulations, from which a δT_b signal cube is produced (signal as a function of frequency). We then add

to this cube a realistic model of foregrounds, expected instrumental response and (system) noise.

3.1 Large-scale 21 cm simulations

Full radiative transfer simulations on very large scales – in excess of $200 h^{-1}$ comoving Mpc – are not available. Hence, recourse must be had to semi-analytical methods. Here we make use of two sets of simulations. The first simulations are produced using the publicly available package 21cmFAST (Mesinger & Furlanetto 2007; Zahn et al. 2007; Mesinger, Furlanetto & Cen 2010) which uses a semi-analytical approach to produce very large scale simulations of the reionization process. See also Santos et al. (2010) who use an alternative method to create fast large-scale reionization and redshifted 21 cm simulations, called SIMFAST21.

21cmFAST uses perturbation theory, the excursion set formalism and analytic prescriptions to generate evolved 3D realizations of the density, ionization, peculiar velocity and spin temperature fields, which it then combines to compute the 21-cm brightness temperature. The method has been thoroughly tested against more accurate reionization codes (Mesinger et al. 2010).

We produce three-dimensional $400 h^{-1}$ comoving Mpc simulation boxes of the brightness temperature from redshift 12 down to redshift 6. The resolution of the simulation box is $1 h^{-1}$ comoving Mpc. The simulations assume the standard *WMAP* cosmological parameters (Spergel et al. 2007). The simulation box is binned with a 400^3 grid. The output of the code includes the spin temperature, the ionization fraction, the kinetic temperature and peculiar velocity field. These outputs are used by 21cmFAST to create a brightness temperature box. For more details please see Mesinger et al. (2010) and Mesinger & Furlanetto (2007). We then use the method developed by Thomas et al. (2009) to create an observational box of the 21 cm signal spanning the frequency range of 115–200 MHz. A slice through the signal cube along the frequency direction is shown in Fig. 1. In this simulation the ionization process reaches its mid-point at $z \approx 10$. We will present a number of plots in the remainder of the paper at $z = 9$ where the neutral fraction is 0.2.

The second set of simulations we use to test the imaging possibility is based on the EoR simulations program called BEARS (Thomas & Zaroubi 2008; Thomas et al. 2009). This scheme includes the physics of reionization in more detail relative to 21cmFAST but needs cosmological simulations in order to create the reionization history, which makes it more time consuming than 21cmFAST. BEARS assumes a spherically symmetric ionization region around each ionizing source but can easily allow for a wide range of sources with very different spectral energy distributions (SEDs). Here we use a dark matter simulation of 512^3 particles in a cube with comoving side length of $200 h^{-1}$ comoving Mpc with *WMAP* standard parameters. The sides thus have twice the length of the simulations shown in Thomas et al. (2009) and used in our previous work on LOFAR EoR signal extraction (Harker et al. 2009a,b). This leads to a minimum resolved halo mass of around $3 \times 10^{10} h^{-1} M_\odot$. Dark matter haloes are populated with sources whose properties depend on some assumed model. For this paper we explore both the ‘quasar-type’ and the stellar source models of Thomas & Zaroubi (2008), Thomas et al. (2009) and Zaroubi et al. (2007). The topology and morphology of reionization are different in the two source models. We might expect quasar reionization to allow an easier detection than stellar reionization, since the regions where the sources are found are larger and more highly clustered, producing larger fluctuations in the signal. This is used here as a check on whether a

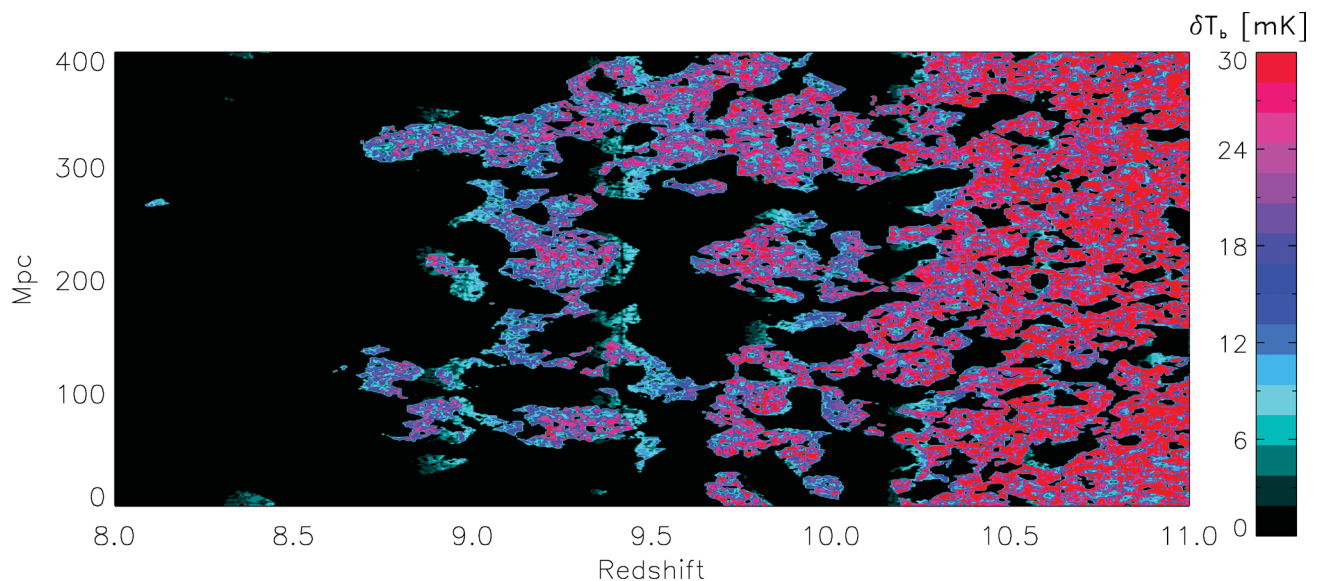


Figure 1. A slice of δT_b along the frequency direction for the $400 h^{-1}$ comoving Mpc simulation obtained with 21cmFAST. The redshift at which the box is half ionized is 10. We will show results from redshift 9 in this paper where the neutral fraction is 0.2. The colour scale is shown to the right of the figure.

completely different and more detailed simulation scheme yields similar conclusions to the ones based on 21cmFAST simulations.

Unfortunately, a full and detailed radiative transfer simulation on such a scale is not publicly available at this stage and we expect our conclusions to vary somewhat with higher resolution simulations. Still, the main results are expected to remain valid, as is indeed seen in such a simulation that is currently being analysed by Iliiev & Mellema (private communication).

3.2 Instrumental response

In radio interferometry the measured spatial correlation of the electric field between two interferometric elements (stations) i and k at a given time, t , is called the visibility and is given by (Taylor, Carilli & Perley 1999; Thompson, Moran & Swenson 2001)

$$V_v^{j,k}(u, v; t) = \int I_v^{j,k}(l, m; t) e^{2\pi i(ul+vm)} dl dm, \quad (3)$$

where $I_v^{j,k}$ is the observed intensity at frequency ν observed by correlating stations j and k and $i = \sqrt{-1}$. The coordinates l and m are the projections (direction cosines) of the source in terms of the baseline.⁶ The size of the station gives the resolution, i.e. minimum uv cell size, at which the uv -plane is covered. From this equation it is evident that the observed visibility is basically the Fourier transform of the intensity measured at the coordinates u and v (uv -plane). Following Harker et al. (2010), we define a sampling function,

$$S_v(u, v) \equiv \sum_{\forall k \in \text{pixel}(u,v)} \int \delta^D(u' - u_k, v' - v_k; t, v) dt du' dv' \quad (4)$$

which gives how a distribution of interferometer baselines sample Fourier space during the time of observation. Here δ^D is the Dirac delta function, and k is a uv -track of a baseline.

Obviously, equation (3) indicates that the sampling function depends on frequency as well as on the number of baselines and their distribution, i.e. how the visibilities are distributed in the uv -plane.

⁶ We ignore the effect of the Earth's curvature, the so-called w-projection.

Here we assume that the uv coverage is the same at all frequencies. In practice, a uniform uv coverage at all frequencies could be achieved by ignoring all the uv points that are only partially covered within the frequency range of interest. This would require discarding some of the data – in the case of LOFAR one loses approximately 20 per cent of the data. This has two effects: an increase in the level of noise and a reduction in the resolution at high frequencies. However, the assumption here is that the uv coverage of the telescope is quite dense and complete. In practice, all the baselines data, including from the long ones, will be used to model and remove compact sources.

In the case of LOFAR, to simulate our data in the uv -plane we perform a two-dimensional Fourier transform on the image of the foregrounds and signal at each frequency, and multiply by a mask (the uv coverage) which is unity at grid points in Fourier space (uv cells) where $S(u, v) > 0$, and is zero elsewhere.

3.3 Noise

Assuming the noise in the measurement follows a Gaussian distribution for each component, the uncertainty in the measurement of the visibility at a given uv -plane pixel is inverse proportional to \sqrt{S} . The noise realization in the uv -plane is created at each uv point by drawing from a complex Gaussian field with an rms proportional to $1/\sqrt{S}$ for all the cells within the mask down to a uv distance that is equivalent to 4 arcmin resolution. The realization in the uv -plane is done so as to fulfil the reality condition. At scales smaller than this we truncate the sampling. The truncation is performed in order to avoid the noise normalization being controlled by the very low number of samplings at the edge of our uv sampling, namely, at the limit in which the Poisson-noise can not be approximated by white Gaussian noise. The noise realization in the image plane can then be obtained by inverse Fourier transform of the uv -plane noise. The overall normalization of the level of noise is chosen so that the noise images have an rms in the image plane of 56 mK on an image using 1 MHz bandwidth at 150 MHz at the resolution limit. This is the rms expected from LOFAR after 600 h of observation of one EoR window with one synthesized beam. The noise level

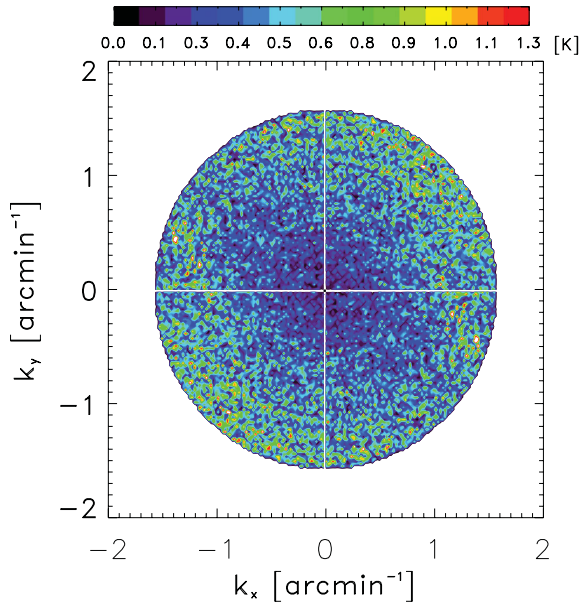


Figure 2. A noise realization in the uv -plane for 600 h of observation. The absolute value of the noise is drawn here and shown in K (and not mK). The rms of the noise in the image plane is about 56 mK. In order to convert from Jy to mK we assume that $k = 2\pi/\theta$. The truncation in the map is made at a radius corresponding to 4 arcmin.

depends on the system temperature which is assumed to be $T_{\text{sys}} = 140 + 60 \times (\nu/300 \text{ MHz})^{-2.55} \text{ K}$ (Jelić et al. 2008). A much more detailed account of the calculation of noise levels and the effects of instrumental corruption for the LOFAR EoR project may be found in Labropoulos et al. (2009).

Fig. 2 shows a noise realization used in the simulations. The circle beyond which there is a cut in the uv data corresponds to 4 arcmin resolution at 150 MHz. The distribution of the LOFAR stations was chosen to ensure a constant noise at each scale for a measurement of the power spectrum. This is why the noise level per pixel increases as a function of uv radius. Remember that the experiment was optimized for a power spectrum measurement and not for imaging.

3.4 Foregrounds and extraction

As mentioned earlier, a very important ingredient to consider here is the accuracy of extracting the foregrounds, especially on large scales. This is because the foreground’s power increases with scale and there is no guarantee that the extraction algorithm, which fits the foregrounds along the frequency direction at small spatial scales, will not leave any large-scale residuals.

For the foregrounds we use the simulations of Jelić et al. (2008, 2010). These incorporate contributions from Galactic diffuse synchrotron and free-free emission, and supernova remnants. They also include unresolved extragalactic foregrounds from radio galaxies and radio clusters. We assume, however, that point sources bright enough to be distinguished from the background, either within the field of view or outside it, have been removed well below the noise level from the data. Observations of foregrounds at 150 MHz at low latitude (Bernardi et al. 2009, 2010) indicate that these simulations describe the properties of the diffuse foregrounds well.

To test the foregrounds’ influence, especially with the amount of noise in the data, we apply an extraction algorithm on mock data. The mock data include the simulated cosmological signal, the

instrument response, noise and foregrounds. As mentioned earlier, calibration errors and other systematics are not taken into account in this simulation.

In order to extract the foregrounds, we assume that they have no small-scale features along the frequency direction. Given what we know about the physical origin of the galactic and extragalactic foregrounds, this assumption is quite reasonable (Petrovic & Oh 2011). For the extraction we use the W_p algorithm which is a non-parametric method that is very suitable for fitting the spectrally smooth foregrounds in EoR data sets. The method was developed for general cases by Mächler (1995), and has been used by Harker et al. (2009b) as an algorithm for fitting EoR foregrounds. Briefly, the method is a penalized maximum likelihood algorithm that is designed to find the maximum likelihood fit for the data but penalizes relative change of curvature. That is so to say, the method finds the best-fitting curve to the data with minimum the smallest possible ruggedness.

The W_p method has shown very good results for fitting the foregrounds both in real space (image plane) and in Fourier space (uv -plane) for up to $100 h^{-1}$ comoving Mpc and its influence on the power spectrum statistic has been tested and shown not to be significant up to the scales of the simulation (Harker et al. 2010). Here, however, we would like to test it on much larger scales than considered previously. In this paper, application of this method is done using uv -plane fitting which has shown slightly better results than image-plane fitting (Harker et al. 2010).

One should note that using predetermined functions, e.g. polynomials, to fit the data might introduce systematics due to over- or under-fitting of the foregrounds. Hence, the use of more advanced non-parametric techniques is essential in this case (see e.g. Harker et al. 2009a; Chapman et al. 2012).

4 RESULTS

We plot, in Fig. 3, the standard deviation of the smoothed cosmological signal (solid lines) and the smoothed noise field (dashed lines) as a function of redshift. The smoothing is done with a Gaussian kernel of 5, 10, 15, 20 and 25 arcmin. It is clear that for smoothing scales $\gtrsim 15$ arcmin there is a redshift range in which the signal becomes larger or comparable to the noise. Obviously, if the cosmological signal is coherent on scales larger than the smoothing scales, these structures will show up as EoR features in the 21 cm maps.

The fact that the EoR signal catches up with the noise at large smoothing scales is driven by the existence of very large ionized and neutral patches. Regardless of their nature, the sources that drive the ionization bubbles follow the large-scale structure which has a natural scale of $120 h^{-1}$ comoving Mpc. This sets the largest scale up to which one can still see coherent structures of neutral and ionized regions.

Since $120 h^{-1}$ comoving Mpc is the natural scale of the large-scale structure, one can ask why such smoothing does not facilitate imaging of the EoR at all redshifts? The answer is simply that without the ionized regions the contrast within the map is driven by the underlying density. This of course assumes that one can ignore the fluctuations due to T_{spin} , which a good assumption in the redshift range of 6–11.5 that LOFAR will probe, but not a good assumption at much higher redshifts. That is to say, if the brightness temperature fluctuations were driven by the cosmological density alone, then it would be more difficult, with LOFAR, to image the signal even on large scales.

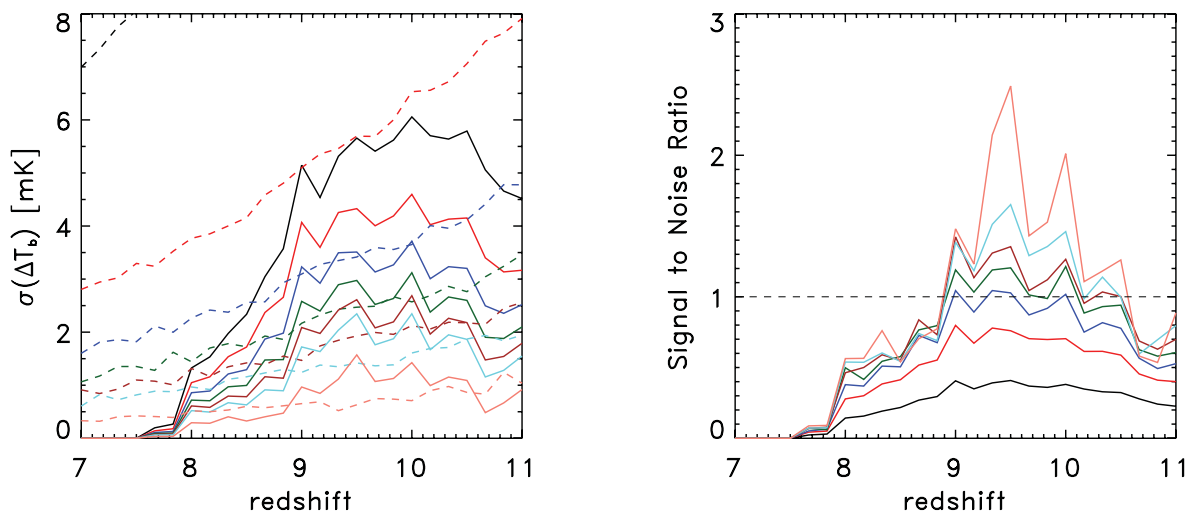


Figure 3. The left-hand panel shows the standard deviation of the signal (solid lines) and the noise (dashed lines) as a function of redshift for the $400 h^{-1}$ comoving Mpc simulation noise and signal. Here, we assume 600 h of observation. The black, red, blue, dark green, brown, cyan and pink solid lines are for instrument resolution (≈ 3 arcmin), 5, 10, 15, 20, 25 and 30 arcmin resolutions, respectively. The right-hand panel shows the S/N for the same cases. Notice that with 15 arcmin resolution and above there is a redshift range in which the signal rms exceeds the noise rms. This high S/N region is typically centred around the redshift at which the IGM is 50 per cent ionized. It is also worth noting that although the rms of the signal decreases with the smoothing scale, the decrease in the noise is even larger.

Fig. 4 shows original and reconstructed maps of the simulated EoR signal in a $2^\circ 5' \times 2^\circ 5'$ field of view at redshift 9. We note here that this is about a quarter of the LOFAR field of view assuming a single beam. We compare here the 20 arcmin Gaussian smoothed map shown in Panel B with the following cases: Panel C, with a noisy signal assuming 600 h of integration with LOFAR but without including foreground effects; Panel D, noisy signal assuming 2400 h of integration of the same field (half the noise level), still without the inclusion of the foreground effects; Panel E, the same map as in C, i.e. with noise added assuming 600 h of integration, but with inclusion of the foregrounds and their extraction with the W_p fitting procedure (Harker et al. 2009a). The smoothing is done with a 20 arcmin Gaussian kernel. After smoothing the signal rms in map A is 2.6 mK. The noise level after 600 h of integration in maps C and D is 2 mK and 2.2 mK (0.9 mK of which are due to foregrounds residual), respectively. The noise levels after 2400 h of integration and 20 arcmin smoothing in maps E and F are 1.1 mK and 1.3 mK (0.7 mK of which are due to foreground residuals), respectively. It is clear that after 600 h of observation, one has the ability, albeit a limited one, to map the EoR signal, and the contour map is dominated by the noise. However, Panel F shows that after 2400 h of observation the noise influence drops significantly at this smoothing scale, and a more reliable map can be seen. This remains true even after the inclusion of the foreground effect, that is, when traces of the foreground extraction are present on large scales.

A visual inspection shows a clear similarity between Panel B and Panels C–F in Fig. 4. To quantify this similarity we use two different methods. The first method is the Pearson cross-correlation coefficient, ρ , calculated with the formula:

$$\rho = \frac{\sum_i (x_i - \langle x \rangle)(y_i - \langle y \rangle)}{\sqrt{\sum_i (x_i - \langle x \rangle)^2} \sqrt{\sum_i (y_i - \langle y \rangle)^2}}, \quad (5)$$

where x_i and y_i are the value of the pixel, i , in the two maps. The results of this calculation are shown in Table 1. The table shows the correlation coefficient between the maps C–F and the noise- and foreground-free map B from Fig. 4. The existence of foregrounds and their extraction in maps D and F clearly reduces the correlation.

Also the higher noise in maps C and D results in smaller correlation coefficient. Still, the correlation coefficients shown in the table are very high in all cases.

The other method we use to quantify the correlation between the maps is to inspect their phase information. This is done by Fourier transforming each map and then checking whether the Fourier space phases of the maps C–F correspond to those of the original map B. If this were done to all the points in the Fourier space, then one would obtain no correlation between the phases. This is because the amplitudes of most points in the Fourier transform of maps C–F is dominated by numerical noise and contain no useful information. In Fig. 5 we show a log–log plot of the rank-ordered Fourier coefficient amplitudes of the five images. The solid black line is the one for the image shown in Panel B of Fig. 4, whereas the others are for the rest of the 20 arcmin smoothed images. Each line is normalized such that its maximum amplitude is one. All the lines show the same typical behaviour where the amplitude of the first few hundred pixels is high but then it drops exponentially to slowly varying values (almost flat) which is typical white noise behaviour. Note that the number of significant coefficients is larger in maps C–F than in map B because the former contain a contribution from the (correlated) system noise. The flatness of this part of the plots indicates that they are dominated by white noise. We would like to emphasize that this is not the system noise contribution which typically has a much larger amplitude and is not white (see Fig. 2). Therefore, in order to compare the phases of the various images, we only take into account the pixels that have values larger than 10^{-4} times the maximum amplitude.

Next, we plot the phases of the pixels with relatively high amplitudes ($\geq 10^{-4}$ of the maximum amplitude). Each of the four panels of Fig. 6 plots the phases of the reconstructed images (maps C–F in Fig. 4) versus the phases of the original map (map B in Fig. 4). These plots are presented as density plots. A high correlation shows as high concentration of points (high contour values) along the diagonal. In all the panels the correlation between the phases is obvious. The best correlation is clearly obtained in the lower-left panel because map E has the lowest noise and is without foregrounds. The

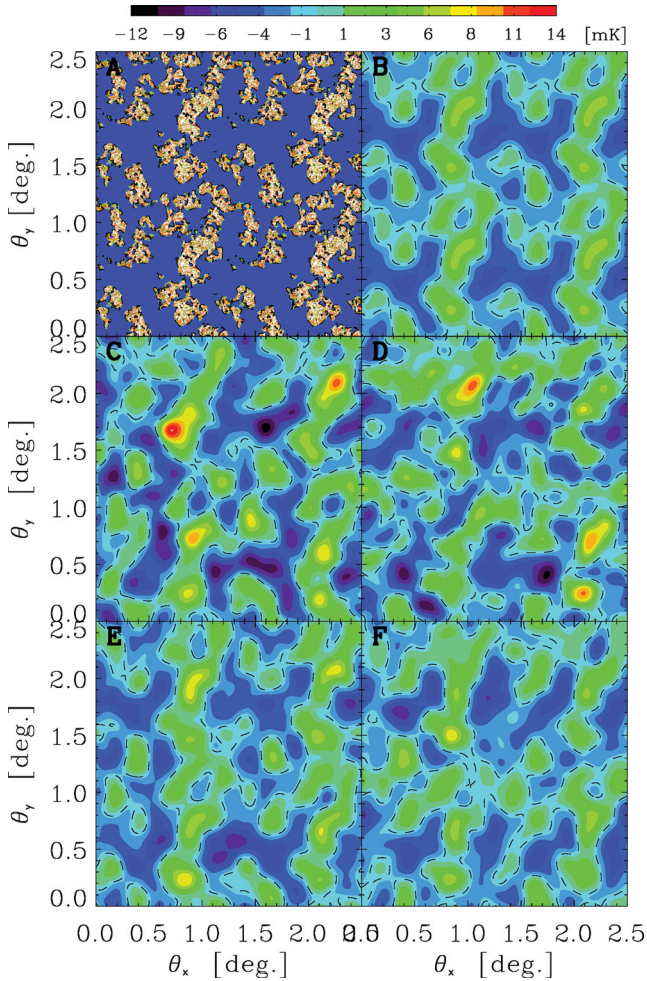


Figure 4. EoR maps at redshift 9, with a $\langle x_{\text{HI}} \rangle = 0.2$ field of view of 2.5×2.5 (note that the LOFAR field of view will be around $5^\circ \times 5^\circ$). Panel A shows the original simulated EoR map at full resolution. The map shown in panel B is smoothed with a 20 arcmin (standard deviation) Gaussian kernel. Panel C shows the same map as in B but with noise added to it assuming 600 h of observation with LOFAR. Panel D is the same as C, i.e. with 600 h of observation noise, but here the foregrounds were added and then extracted with the W_p fitting procedure (Harker et al. 2009b). Panel E is the same as C but with half the noise level of the map in panel B (2400 h of observation). Panel F is the same but with 2400 h of observation noise and foregrounds that were added and then extracted with the W_p fitting procedure. The contour levels are colour coded as shown in the colour table at the top of the figure.

Table 1. Spearman correlation coefficients between Map B and the other maps.

| Map | C | D | E | F |
|--------|------|------|------|------|
| ρ | 0.77 | 0.68 | 0.93 | 0.80 |

worst correlation, though still a very clear correlation, is obtained in the upper-right panel because map D has high noise and still has some residuals from the subtraction of the foregrounds.

We repeated the same procedure on the $200 h^{-1}$ comoving Mpc BEARS simulations and get very similar results. To cover the same angular size as the previous simulation we tile the BEARS simulation box to reach $400 h^{-1}$ comoving Mpc. The result of this simulation is shown in Fig. 7, where the left-hand panel shows the original 20 arcmin smoothed simulation assuming 2400 h of observation

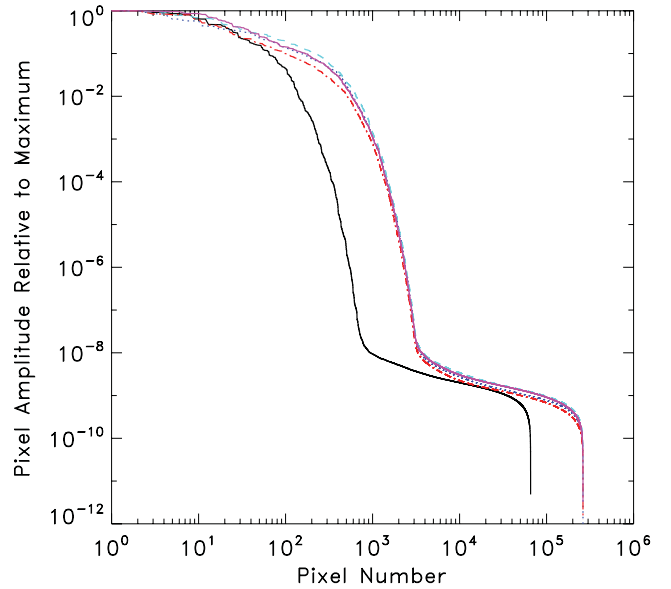


Figure 5. The rank-ordered Fourier space amplitude for each of the five images, B–F, shown in Fig. 4. All curves are normalized with respect to their maximum amplitudes. The solid black line is the plot for map B whereas the other curves show the Fourier space amplitudes of image C (blue dotted line), D (cyan dashed line), E (red dot–dashed line) and F (magenta solid line). All uv -maps are dominated by the highest few hundred pixels. The rest of the Fourier space pixels are noise dominated as demonstrated by the sudden drop in the amplitudes and their almost flat slope thereafter.

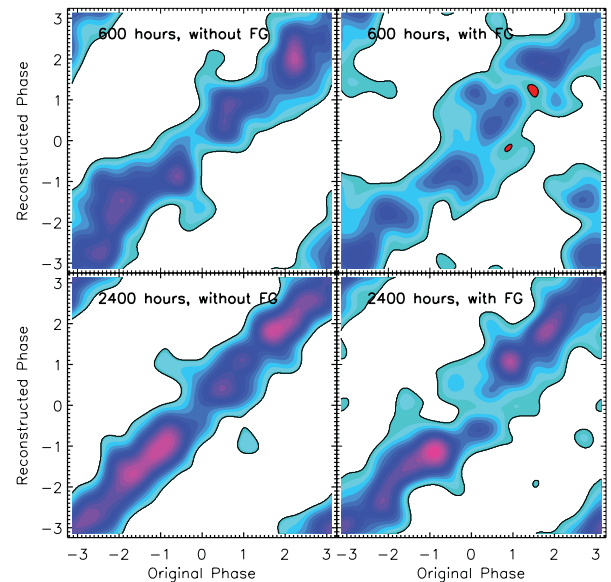


Figure 6. The phases of the reconstructed images (maps C–F in Fig. 4) versus the phases of the original map (map B in Fig. 4). The plots are shown as density plots where the density represents the number of point per unit area, hence, highly correlated maps should show as high-density contours at the diagonal. The map shows the highest 67 per cent of the density PDF. The high correlations at the upper-left side and lower-right side of the figures simply reflect the periodicity of the phases.

with LOFAR. The right-hand panel shows the extracted image after adding noise and foregrounds also smoothed with 20 arcmin Gaussian. The two maps are clearly very similar with a correlation coefficient of ≈ 0.61 . The correlation coefficient here is lower

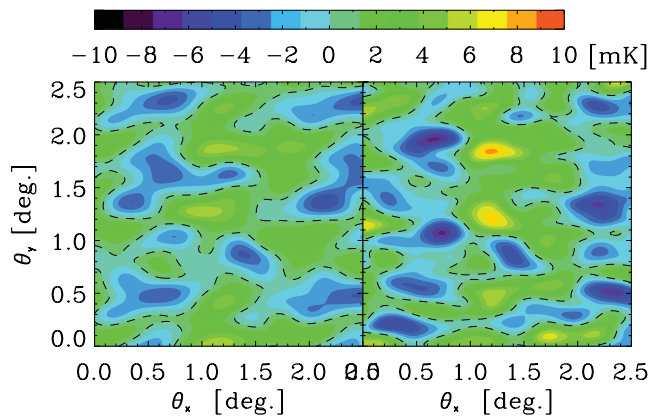


Figure 7. EoR maps at redshift 7 produced by the BEARS algorithm, with a $\langle x_{\text{HI}} \rangle = 0.5$ field of view of 2.5×2.5 . The left-hand panel shows the original 20 arcmin smoothed simulation assuming 2400 h of observation with LOFAR. The right-hand panel shows the extracted image after adding noise and foregrounds also smoothed with 20 arcmin Gaussian. The two maps are clearly very similar with a correlation coefficient of ≈ 0.61 .

than the same comparison done with the previous simulation (between panels B and F in Fig. 4) due the relatively small size of the simulation box where the number of large-scales modes is smaller.

This conclusion is insensitive to the type of source we assume to power reionization, i.e. thermal or power law. This is reassuring and indicates that this effect is driven more by the very large scale structure than by the details of the reionization process. It should be emphasized here that with higher resolution the increased number of low-mass sources might slightly change the picture, but not in a drastic way, as already seen in very large scale high-resolution simulations (Iliev & Mellema, private communication).

5 ENHANCING THE LARGE-SCALE IMAGING CAPABILITIES OF LOFAR

Clearly, the best imaging quality is obtained when one assumes a very large amount of observing time (2400 h) focusing on one

single field (Fig. 4). Obviously, this is a vast amount of telescope time that is hard to accumulate especially on open time telescopes, such as LOFAR. Hence, in what follows we show that a relatively inexpensive modification of the LOFAR telescope can significantly enhance its sensitivity on large scales.

Increasing the S/N of the data on scales $\gtrsim 30$ arcmin, corresponding to a comoving scales of $\gtrsim 60h^{-1}$ comoving Mpc at high redshifts, will allow imaging the EoR, especially the last phases of reionization ($\langle x_{\text{HI}} \rangle \lesssim 0.5$) within a very reasonable amount of observational time. The quality of imaging on such angular scales for a 2 m wavelength depends on the number of baselines with length of $\lesssim 200$ m, and hence on the number of stations within the very central area, i.e. around and within the so-called superterp of LOFAR. For clarity, high band antenna (HBA) station here means a collection of 24 HBA tiles with each tile having 4×4 antennas (dipoles). The superterp, which in Dutch means ‘super mound’, is shown in the left-hand panel of Fig. 8 and is a circular area with a radius of about 150 m (baselines up to 300 m). In this area there are currently about 12 HBA stations of LOFAR that operate in the frequency range of 115–230 MHz. Below we consider the effect of increasing the number of HBA stations in the superterp as a possible way to cut the observation time to reach the desired sensitivity for imaging. The right-hand panel of Fig. 8 shows a possible configuration of 24 stations (pink filled circles) in the superterp – the current 12 stations (red filled circles) and 12 new stations (pink filled circles). This would increase the number of baselines within the superterp and the area immediately surrounding it by a factor of 4. The S/N for the same time of observation would therefore be enhanced by a factor of, at least, 2, i.e. reducing the integration time by a factor of 4 as well. An addition of HBA stations would not be too costly in relative terms since most of the infra structure needed for rolling out the added station will be relatively small. Fig. 9 shows the improvement in the uv coverage for the case of a 24 HBA stations in the superterp (right-hand panel) relative to that of the current 12 HBA stations (left-hand panel). Such an enhancement, which is relatively easy and cheap to obtain, would increase the sensitivity of LOFAR on these scales by a factor of 2, thus boosting the LOFAR imaging capability at large scales. Such an enhancement would

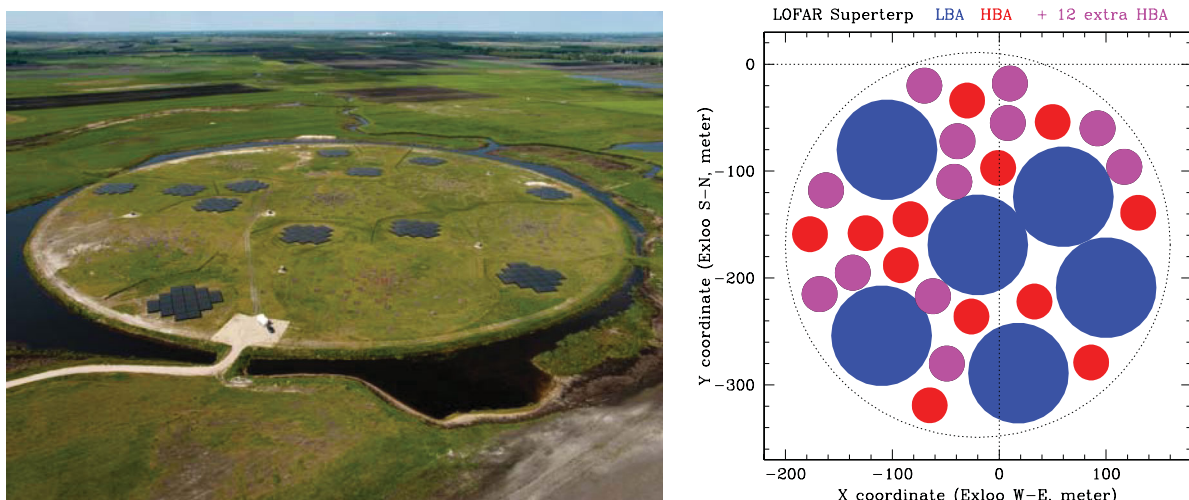


Figure 8. The very central area of the LOFAR core, the ‘superterp’. The left-hand panel shows a picture of the superterp that was taken in the spring of 2011 (credit for the picture goes to ‘TopFoto, Assen’). The right-hand side shows a sketch of the layout of the stations in the superterp. The large blue filled circles indicate the locations of the existing LOFAR 6 low band antenna (LBA) stations in the superterp. The red filled circles show the locations of the existing 12 HBA stations, whereas the pink filled circles show a possible configuration for an additional 12 HBA stations which would double the collecting area. The sizes of the stations are to scale. The addition of 12 HBA stations would significantly increase the sensitivity of LOFAR on large scales.

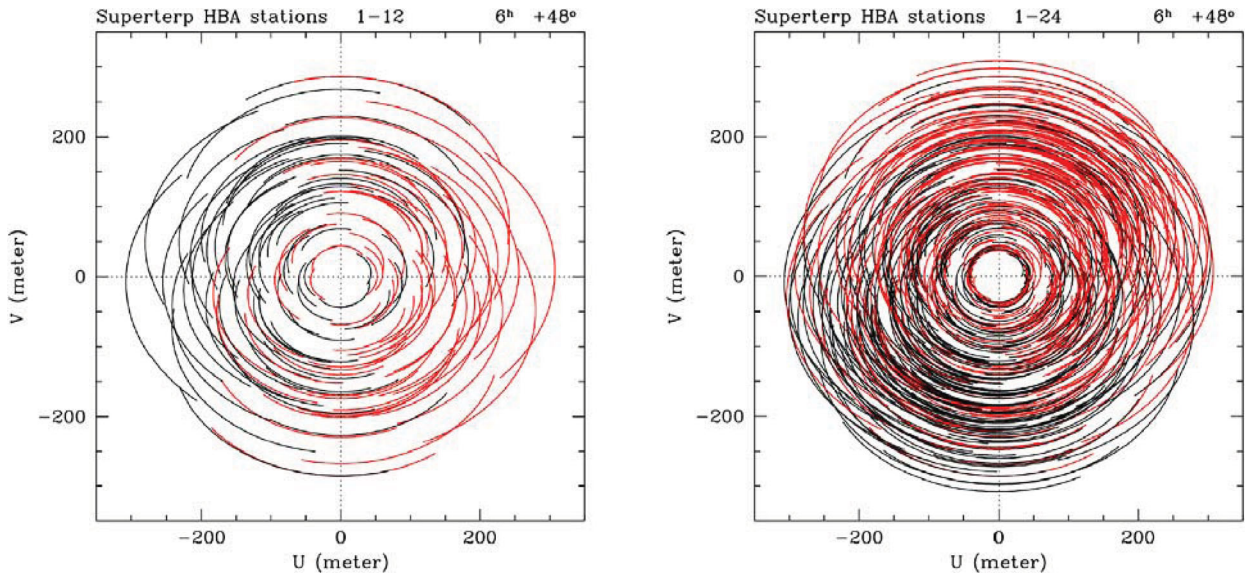


Figure 9. The uv -plane covered by the current HBA stations after 6 h of observation at a declination of $+48^\circ$. The left-hand panel shows the uv -plane obtained with the current superterp HBA stations whereas the right-hand panel shows the uv -plane with the proposed additional 12 HBA stations.

enable the LOFAR-EoR project to image the reionization process on large scales rather than only detecting it statistically. Another issue that we are studying is the possibility of correlating all the tiles within the superterp, instead of whole stations, which will add many baselines to the measurement. This extension proposal has to be studied in more detail before accurate estimations of its ramifications can be appreciated.

6 CONCLUSIONS AND DISCUSSION

A new generation of low-frequency interferometers has recently come on line [LOFAR, MWA, PAPER, Square Kilometre Array (SKA)]. Exploring the EoR is a major science driver for these telescopes. The current common wisdom in the field is that these telescopes will detect the EoR statistically but that they will not be able to image it, due to their poor sensitivity. The low sensitivity affects both small scales and large scales. The influence of the noise on small scales is quite clear; its influence on large scales is indirect and has to do with the ability to fit the foregrounds well along the frequency direction. Since this fitting is done for each image or uv -plane pixel along the frequency direction, the accuracy of the fit will depend sensitively on the noise level in the data. Hence, the low S/N will decrease the quality of the foreground fit, especially on large scale where the foreground power becomes larger. With the projected level of noise per resolution element in the current generation experiments it is no wonder that imaging has been deemed possible only with future instruments such as the SKA (see e.g. Zaroubi 2010).

In this paper we have shown that imaging of the neutral IGM at the later stages of reionization with the current generation of radio interferometers is possible on very large scales ($\gtrsim 0.5$). The existence of very large scale neutral regions towards the later phases of the reionization process and their large contrast with the ionized regions enhances the EoR signal in two ways: first, the large neutral regions increase the amplitude beyond that expected from mere cosmological density fluctuations; secondly, their large size gives a coherent signature over about a hundred resolution elements for

LOFAR, which overcomes the poor S/N in each of them. These two effects make it possible in principle to image the reionization process on large scales.

A simple argument in support of our conclusion could also be cast as follows. The ionizing sources are preferentially located in the high-density regions which typically ionize before the low-density regions. Since the density fluctuation power spectrum peaks at $120 h^{-1}$ comoving Mpc this will be roughly the scale of the ionized and neutral regions at the midpoint of reionization. It is this scale that essentially allows us to image the EoR on large scales with instruments like LOFAR. Notice that this argument holds, almost, regardless of the type of reionization sources.

The second issue we address here is the issue of the influence of the foregrounds on the extracted EoR signal, especially on large scales. We show that for realistic foregrounds and noise models current state-of-the-art extraction techniques do very well even on scales in excess of a degree, which have so far been thought to be inaccessible to the current generation of experiments. This again demonstrates that imaging of the EoR on large scales is possible with LOFAR. We have also shown that a modest enhancement of the capabilities of LOFAR at the central area of the core, known as the superterp, would greatly boost the possibility of imaging the EoR on large scales.

The importance of imaging the EoR with current telescopes cannot be overstated. Astrophysically, imaging would allow addressing a large number of issues that would otherwise be difficult to deal with. For example, it would make it possible to identify spatially where the ionized regions are and hence it would allow targeting these regions with follow up optical and infrared surveys, and iterating on the foregrounds' subtraction (Petrovic & Oh 2010). However, given the issues that face the current experiments, imaging will be of utmost importance in discovering and addressing systematic effects whose existence would otherwise be impossible to realize. In other words, imaging of the EoR would boost our confidence in the reliability of the measured signal and the properties attributed to it. Conversely, if the measured power spectrum indicates the existence of EoR power on very large-scales, then the availability of images on such scales would allow us to determine whether this is a result

of the cosmological signal or of systematic effects that have not yet been brought under control.

In the future, the SKA will provide enough S/N to image the EoR with very high accuracy on scales up to 5° and down to scales of the order of 1 arcmin. This obviously will surpass LOFAR's performance. SKA will also be able to go to much lower frequencies (down to 50 MHz) enabling a direct observation of the Universe's Dark Ages. However, SKA will still take about a decade to become operational whereas LOFAR and the other current instruments are already here, and can make significant scientific discoveries that can be enhanced even further with relatively little extra cost.

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