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Defect branes

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Abstract

We discuss some general properties of “defect branes”, i.e. branes of co-dimension two, in (toroidally compactified) IIA/IIB string theory. In particular, we give a full classification of the supersymmetric defect branes in dimensions $3 \leq D \leq 10$ as well as their higher-dimensional string and M-theory origin as branes and a set of “generalized” Kaluza–Klein monopoles. We point out a relation between the generalized Kaluza–Klein monopole solutions and a particular type of mixed-symmetry tensors. These mixed-symmetry tensors can be defined at the linearized level as duals of the supergravity potentials that describe propagating degrees of freedom. It is noted that the number of supersymmetric defect branes is always twice the number of corresponding central charges in the supersymmetry algebra.

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1. Introduction

Branes are a fundamental ingredient of string theory. Prime examples of their many applications are the calculation of the entropy of certain black holes [1] and the AdS/CFT correspondence [2]. The properties of branes crucially depend on two quantities: the scaling of the brane tension with the string coupling constant g_s in the string frame and the number T of transverse directions. The first quantity can be characterized by a number α such that

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$$\text{Tension} \sim (g_s)^\alpha. \quad (1)$$

It turns out that α is a non-positive number.¹ Branes with $\alpha = 0, -1, -2, \dots$ are called Fundamental, Dirichlet, Solitonic, etc. The second quantity T naturally splits the branes into two classes: the *standard* branes with $T \geq 3$ and the *non-standard* ones with $T = 2, 1, 0$. Only the standard branes are asymptotically flat. The non-standard branes require special attention. For instance, the non-standard branes with $T = 0$ are space-filling branes which can only be defined consistently in combination with an orientifold. The ones with $T = 1$ are domain walls. The potentials coupling to these domain walls are dual to constants such as mass parameters or gauge coupling constants. By T-duality, these domain walls need orientifolds as well [3].

In this paper we wish to focus on non-standard branes with $T = 2$. We call such branes “defect branes” since branes with co-dimension 2, like the D7-brane or 4D cosmic strings, are not asymptotically flat and can have non-trivial deficit angles at spatial infinity. A prime example of a Dirichlet defect brane is the ten-dimensional D7-brane [4] whose solution has been discussed in [5–7]. It is well known that the *single* D7-brane solution has no finite energy [5,6]. To obtain such a finite-energy solution one should construct a *multiple* brane solution which includes orientifolds. In this paper we will only consider *single* defect branes and assume that finite energy solutions can be obtained by applying the same techniques as for the D7-brane.

Defect branes couple to $(D - 2)$ -form potentials. These potentials are dual to the $\dim G - \dim H$ scalars that parametrize the non-linear coset G/H of the corresponding maximal supergravity theory.² It turns out that the number n_P of $(D - 2)$ -form potentials is not equal to the number n_S of coset scalars, i.e. $n_P \neq n_S$, see Table 1. The reason of this is that the $(D - 2)$ -form potentials transform in the adjoint representation of the duality group G . Their $(D - 1)$ -form field strengths are essentially the Hodge duals of the Noether current 1-forms associated to the global invariance under G [8], which transform in the adjoint representation of G . The $\dim G$ Noether currents are constrained by $\dim H$ relations [9] and, therefore, the $(D - 2)$ -form potentials describe as many physical degrees of freedom as the coset scalars. These constraints, however, do not lead to algebraic relations among the potentials themselves and therefore do not play a role in the present discussion.

To determine whether we are dealing with a *supersymmetric* defect brane we will use a criterion that is based on the construction of a gauge-invariant Wess–Zumino (WZ) term that describes the coupling of the defect brane to a given $(D - 2)$ -form potential [10,11]. This WZ term should contain worldvolume fields that precisely fit into a half-supersymmetric vector or tensor multiplet. This supersymmetric brane criterion leads to a full classification of supersymmetric defect branes in dimensions $3 \leq D \leq 10$. It turns out that the number n_D of supersymmetric defect branes in any dimension is less than the number n_P of $(D - 2)$ -form potentials, i.e. $n_D < n_P$. This means that not all potentials correspond to supersymmetric branes, see Table 1. This is different from the standard branes where the number of potentials always equals the number of supersymmetric branes. The number of *all* non-standard branes have been recently derived in dimension higher than five in [12] using the method of [11], and in *all* dimensions in [13] using an approach based on E_{11} [14] and the observation that imaginary roots do not lead to supersymmetric branes [15]. As far as the number n_D of defect branes is concerned, we will give yet

¹ We do not consider instantons here. They will be shortly discussed in the conclusion section.

² We note that it is non-trivial to use these potentials to describe *multiple* defect branes. For instance, it is not clear how to express the branch-cuts of the holomorphic axion–dilaton solution in terms of properties of the corresponding dual potentials. We thank Jelle Hartong for a discussion on this point.

Table 1

Comparison between the number $n_P = \dim G$ of $(D - 2)$ -form potentials, the number $n_D = \dim G - \text{rank } G$ of supersymmetric defect branes and the number $n_S = \dim G - \dim H$ of coset scalars for the coset spaces G/H of maximal supergravity in $3 \leq D \leq 10$ dimensions. The derivation of the expression for n_D may be found in Section 2.

D	G/H	n_P	n_D	n_S
IIA	–	–	–	–
IIB	$SL(2, \mathbb{R})/SO(2)$	3	2	2
9	$SL(2, \mathbb{R})/SO(2) \times \mathbb{R}^+$	4	2	3
8	$SL(3, \mathbb{R})/SO(3) \times SL(2, \mathbb{R})/SO(2)$	11	8	7
7	$SL(5, \mathbb{R})/SO(5)$	24	20	14
6	$SO(5, 5)/SO(5) \times SO(5)$	45	40	25
5	$E_6/Sp(8)$	78	72	42
4	$E_7/SU(8)$	133	126	70
3	$E_8/SO(16)$	248	240	128

another derivation of this number using a different method, see Section 2. The final result can be found in Table 1. This Table also shows that in $D < 10$ the number n_D of supersymmetric defect branes is not equal to the number n_S of coset scalars, i.e. $n_D \neq n_S$. It is just a coincidence that these two numbers are the same in ten dimensions.

The lower-dimensional branes with $\alpha = 0, -1, -2, -3$ can all be seen to arise as dimensional reductions of branes and a set of generalized KK monopoles in ten dimensions [16,17]. The generalized KK monopoles can be schematically represented by the introduction of mixed-symmetry fields in ten dimensions, provided that one applies a restricted dimensional reduction rule when counting the branes in the lower dimension: given a mixed-symmetry field $A_{m,n}$ with $m > n$, indicating a Young tableaux consisting of a column of length m and a column of length n , one requires that the n indices have to be internal and parallel to n of the m indices [16,17].³ Here we generalize this result, and we determine all the ten-dimensional mixed-symmetry fields that are required to generate all the defect branes for any value of α using the restricted reduction rule. We also derive the eleven-dimensional origin of these fields.

Remarkably, all the solutions corresponding to the generalized KK monopoles that we introduce here were already determined in [18], and as we will show the restricted reduction rule automatically translates into the dictionary used in [18] to classify these solutions. The mixed-symmetry fields we introduce can all be seen as generalized duals [19,20] of the graviton and the other potentials in the ten- or eleven-dimensional theory.⁴ This means that at least at the linearized level one can impose a duality relation, which can be used to predict the behavior of the fields in the various solutions. By explicitly writing down some of the explicit solutions of [18], we will show that this predicted behavior is indeed correct.

The organization of this paper is as follows. In Section 2 we derive the expression for the number n_D of supersymmetric defect branes given in Table 1. In Section 3 we give the string and M-theory origin of these defect branes in terms of branes and a set of “generalized” Kaluza–Klein (KK) monopoles. Furthermore, we discuss the relation between the generalized KK monopoles and mixed-symmetry fields of a certain type. In Section 4 we show how these mixed-symmetry fields classify all defect brane solutions. As an example we give the string

³ This rule naturally generalizes to the case of fields with more than two sets of antisymmetric indices corresponding to a Young tableaux with more than 2 columns [17].

⁴ The actual duality relations are not strictly ten-dimensional covariant since we assume the presence of a number of isometry directions, see also footnote 13.

and M-theory monopole solutions that give rise to all the $D = 8$ defect branes. We also show in Section 5 how the linearized duality relations between these mixed-symmetry fields and the propagating forms determine the behavior of the fields in the various solutions. This is compared with the explicit known results in all cases. In Section 6 we explain why the number n_D of supersymmetric defect branes is, for each dimension D , equal to twice the number n_Z of corresponding central charges in the supersymmetry algebra. In the final section we give our conclusions.

2. Supersymmetric defect branes

At first sight one might think that the number n_D of supersymmetric defect branes is equal to the number n_P of dual $(D - 2)$ -form potentials. However, this is not the case. A prime example is ten-dimensional IIB string theory where the 8-forms are in the $\mathbf{3}$ of $SL(2, \mathbb{R})$ and we only have a supersymmetric D7-brane and its S-dual, i.e. $n_D = 2$ [10]. The reason why we only have two supersymmetric seven-branes can be seen as follows: using an $SO(2, 1)$ notation the WZ terms for the three candidate seven-branes can be written in a duality-covariant way schematically as follows:

$$WZ_i \sim A_{8,i} + \overline{\mathcal{F}}_2 \Gamma_i A_6 + \dots, \quad i = +, -, 3, \tag{2}$$

where we have used lightcone notation to label the $SO(2, 1)$ gamma matrices Γ_i . Here \mathcal{F}_2 is a 2-component spinor⁵ of $SO(2, 1)$ whose components are the worldvolume curvatures of the Born–Infeld vector and its S-dual. Similarly, the target-space potentials A_6 are a spinor (doublet) of $SO(2, 1)$ whose components are the NS–NS and RR 6-form potentials. In general the above expression (2) for the WZ term contains *two* worldvolume vectors which do not fit into a single vector multiplet. Therefore we need that, for a given value of the index i the gamma matrices Γ_i act as a projection operator that projects out one of the two worldvolume vectors in the expression (2). It turns out that this is the case for $i = +$ and $i = -$ but not for $i = 3$. This explains why there is no supersymmetric solitonic ($\alpha = -2$) seven-brane in ten dimensions.

We now consider the counting of supersymmetric defect branes in $D < 10$ dimensions. We first decompose the adjoint of the U-duality group G under the direct product of the T-duality group $T = SO(10 - D, 10 - D)$ and a scaling symmetry \mathbb{R}^+ of the D -dimensional string coupling constant. We find that for each dimension $D \geq 5$ this adjoint representation decomposes into a Dirichlet, i.e. $\alpha = -1$, spinor of T-duality with real components, a Solitonic adjoint plus singlet of T-duality and a charge-conjugate spinor of T-duality with $\alpha = -3$:

$$Adj|_U = \text{spinor}_{\alpha=-1} + (Adj|_T + \text{singlet})_{\alpha=-2} + (\text{conj. spinor})_{\alpha=-3}. \tag{3}$$

In four and three dimensions this decomposition is modified, and one gets

$$Adj|_{E_7} = \text{singlet}_{\alpha=0} + \text{spinor}_{\alpha=-1} + (Adj|_T + \text{singlet})_{\alpha=-2} + (\text{conj. spinor})_{\alpha=-3} + \text{singlet}_{\alpha=-4} \tag{4}$$

in four dimensions and

$$Adj|_{E_8} = \text{vector}_{\alpha=0} + \text{spinor}_{\alpha=-1} + (Adj|_T + \text{singlet})_{\alpha=-2} + (\text{conj. spinor})_{\alpha=-3} + \text{vector}_{\alpha=-4} \tag{5}$$

in three dimensions.

⁵ We do not write explicitly the spinor indices here.

Table 2
Defect branes in different dimensions.

D	U repr.	$\alpha = 0$	$\alpha = -1$	$\alpha = -2$	$\alpha = -3$	$\alpha = -4$
IIB	$2 \subset \mathbf{3}$		1	–	1	
9	$2 \subset \mathbf{3}_3$		1	–	1	
8	$6 \subset (\mathbf{8}, \mathbf{1})$ $2 \subset (\mathbf{1}, \mathbf{3})$		$(\mathbf{2}, \mathbf{1})$	$2 \subset (\mathbf{3}, \mathbf{1})$ $2 \subset (\mathbf{1}, \mathbf{3})$	$(\mathbf{2}, \mathbf{1})$	
7	$20 \subset \mathbf{24}$		$\bar{\mathbf{4}}$	$12 \subset \mathbf{15}$	$\mathbf{4}$	
6	$40 \subset \mathbf{45}$		$\mathbf{8}_V$	$24 \subset \mathbf{28}$	$\mathbf{8}_V$	
5	$72 \subset \mathbf{78}$		$\mathbf{16}$	$40 \subset \mathbf{45}$	$\bar{\mathbf{16}}$	
4	$126 \subset \mathbf{133}$	1	32	$60 \subset \mathbf{66}$	32	1
3	$240 \subset \mathbf{248}$	14	64	$84 \subset \mathbf{91}$	64	14

The non-standard branes with $\alpha = -1$, $\alpha = -2$ and $\alpha = -3$ have been classified in [11,21] and [17] respectively. By looking at Eq. (3), this implies the classification of all defect branes in any dimension above four. Moreover, the $\alpha = -4$ branes in four and three dimensions can easily be obtained by the S-duality⁶ properties of the defect branes. Starting with a defect brane whose tension scales as $(g_s)^\alpha$ and using the fact that under S-duality the string-frame metric occurring in the Nambu–Goto action transforms as $(g'_{\mu\nu})_S = e^{-8\phi/(D-2)}(g_{\mu\nu})_S$ one finds that under S-duality the value of α changes as

$$\alpha' = -\alpha - 4. \quad (6)$$

This means that under D -dimensional S-duality the solitonic defect branes are mapped to each other while the Dirichlet and Fundamental defect branes are mapped to defect branes with $\alpha = -3$ and $\alpha = -4$, respectively. Using the fact that the number of fundamental branes is well known this implies that the number of $\alpha = -4$ branes is known as well.

Applying our supersymmetric brane criterion we find that all Dirichlet defect branes are supersymmetric (there are no non-supersymmetric Dirichlet branes within the spinor representation) and the same applies to the charge-conjugate spinor of defect branes with $\alpha = -3$ [17]. On the other hand, from [21] we know that not every component of the soliton representations corresponds to a supersymmetric brane: rank T solitons out of the Adj_T solitons as well as the singlet soliton are *not* supersymmetric. Using the fact that $\text{rank } G = \text{rank } T + 1$ we therefore conclude that the number n_D of supersymmetric defect branes, in each dimension $D \geq 5$, is given by

$$n_D = \dim G - \text{rank } G, \quad (7)$$

in agreement with the statement under Table 1. The analysis for $D = 3, 4$ is the same as the $D \geq 5$ cases because all the fundamental defect branes within the singlet ($D = 4$) and the vector ($D = 3$) representations of T-duality are supersymmetric, and consequently by S-duality the $\alpha = -4$ branes are supersymmetric too, leading again to Eq. (7).

Summarizing, we find that in any dimension $5 \leq D \leq 10$ we have a chiral T-duality spinor of Dirichlet defect branes, a set of solitonic defect branes that transforms as an antisymmetric 2-tensor under T-duality and a charge-conjugate T-duality spinor of $\alpha = -3$ branes, see Table 2. In $D = 8$ dimensions the solitonic defect branes split into two parts: one part that transforms as

⁶ We are referring here to the D -dimensional S-duality in which the D -dimensional string coupling constant (the exponential of the D -dimensional dilaton) is inverted.

Table 3

The string-theory origin of all the potentials that couple to defect branes in all dimensions $D \geq 3$: antisymmetric tensors (coupling to branes), and mixed-symmetry fields (coupling to (generalized) KK monopoles). We have not indicated that in $D = 3$ also the IIA/IIB pp-wave, represented by the metric, contributes to the defect 0-branes.

$\alpha = 0$	$\alpha = -1$		$\alpha = -2$	$\alpha = -3$		$\alpha = -4$
	IIA	IIB		IIA	IIB	
B_2	C_1	C_2	D_6	$E_{8,1}$	E_8	$F_{8,6}$
	C_3	C_4	$D_{7,1}$	$E_{8,3}$	$E_{8,2}$	$F_{8,7,1}$
	C_5	C_6	$D_{8,2}$	$E_{8,5}$	$E_{8,4}$	
	C_7	C_8		$E_{8,7}$	$E_{8,6}$	

a positive-dual 2-tensor under the $SO(2, 2)$ T-duality and one part that transforms as a negative-dual 2-tensor. The positive-dual defect branes have a worldvolume vector multiplet and they transform under U-duality into the defect branes with $\alpha = -1$ and $\alpha = -3$ which have worldvolume vector multiplets as well. The negative-dual defect branes have a worldvolume self-dual tensor multiplet and they transform under U-duality into each other. On top of all these defect branes we have in $D = 4$ dimensions a singlet Fundamental, or $\alpha = 0$, defect brane and in $D = 3$ dimensions a T-duality vector of Fundamental defect branes. These are the usual fundamental string and 0-branes which indeed become defect branes in $D = 4$ and $D = 3$ dimensions respectively. Finally, the $\alpha = -4$ branes corresponding to the S-duals of the $\alpha = 0$ branes. This analysis coincides with the one recently given in [12] for $D \geq 6$ and in [13] for $D \geq 3$.

3. String and M-theory origin

In this section we wish to consider the string and M-theory origin of the defect branes of the previous Section, see Table 2.

The string-theory origin of the Fundamental defect branes is the IIA/IIB Fundamental string supplied with the fundamental wrapping rule

$$F \begin{cases} \text{wrapped} \rightarrow \text{doubled}, \\ \text{unwrapped} \rightarrow \text{undoubled}. \end{cases} \tag{8}$$

This means that the IIA/IIB fundamental string, upon applying the wrapping rule (8) leads to the numbers of fundamental defect branes given in Table 2. We can represent the Fundamental string by the NS–NS 2-form field B_2 it couples to, see Table 3.

Similarly, the string-theory origin of the Dirichlet defect branes given in Table 2 are the IIA and IIB Dirichlet branes supplied with the Dirichlet wrapping rule

$$D \begin{cases} \text{wrapped} \rightarrow \text{undoubled}, \\ \text{unwrapped} \rightarrow \text{undoubled}. \end{cases} \tag{9}$$

The string-theory origin of the solitonic defect branes is the IIA/IIB NS5-brane together with the solitonic wrapping rule [17]

$$S \begin{cases} \text{wrapped} \rightarrow \text{undoubled}, \\ \text{unwrapped} \rightarrow \text{doubled}. \end{cases} \tag{10}$$

To realize the fundamental wrapping rule (8) one needs the pp-wave. No additional objects (other than the ten-dimensional D-branes themselves) are needed to realize the Dirichlet wrapping rule (9). To realize the solitonic wrapping rule (10) the ten-dimensional Kaluza–Klein

(KK) monopole is needed, but that is not enough: one also needs the so-called *generalized* KK monopoles. These are extended objects which have, in addition to worldvolume and transverse directions, *isometry* directions of various kinds with inequivalent properties. The standard KK monopole only has one isometry direction. We find that the string-theory origin of the solitonic defect branes is given by the NS5-brane, the standard KK monopole and one generalized KK monopole with two isometry directions. One can associate mixed-symmetry fields to generalized KK monopoles, and, in particular, as far as the solitonic branes are concerned, the standard KK monopole is associated with the field $D_{7,1}$ and the generalized KK monopole with two isometries with the field $D_{8,2}$ provided that the restricted reduction rule of [16] is applied. For later convenience we give this reduction rule below for a mixed-symmetry field A_{m,n_1,n_2} corresponding to a Young tableaux with 3 columns.

3.1. Restricted reduction rule

For a mixed-symmetry field A_{m,n_1,n_2} to yield, upon toroidal reduction, a potential corresponding to a supersymmetric brane, we require that the n_2 indices are internal and along directions parallel to n_2 of the n_1 indices and n_2 of the m indices, and that the remaining $n_1 - n_2$ indices in the second set are also internal and along directions parallel to $n_1 - n_2$ of the m indices.

To summarize, all solitonic defect branes in any dimensions are generated by the fields

$$D_6, \quad D_{7,1}, \quad D_{8,2} \tag{11}$$

using the restricted reduction rule formulated above. The field D_6 is the dual of B_2 , the field $D_{7,1}$ can be seen as a dual graviton at the linearized level and similarly $D_{8,2}$ is an exotic dual of B_2 at the linearized level.

Using the same reasoning, the string-theory origin of the $\alpha = -3$ defect branes are given by the following branes and generalized KK monopoles:

$$E_{8,n}, \quad n = 0, \dots, 7, \tag{12}$$

where n is even in the IIB case and odd in the IIA case [17]. The eight-form potential E_8 (corresponding to $n = 0$) couples to the S-dual of the D7-brane. The other fields are all exotic duals of the RR fields $C_n, n = 1, \dots, 7$, and correspond to generalized KK monopole solutions. In the same way as the D-branes, upon using the Dirichlet wrapping rule (9), build up a chiral spinor representation of the T-duality group, the S-dual of the D7-brane, upon using the exceptional wrapping rule

$$E \quad \begin{cases} \text{wrapped} \rightarrow \text{doubled}, \\ \text{unwrapped} \rightarrow \text{doubled}, \end{cases} \tag{13}$$

builds up the *charge-conjugate* spinor representation of the same T-duality group. This exceptional wrapping rule is realized through the generalized monopoles given in (12), using the restricted reduction rule given above.

Finally, there is no conventional brane origin and corresponding brane wrapping rule of the $\alpha = -4$ branes. All these branes follow from the reduction of generalized KK monopoles. This is to be expected since the only available $\alpha = -4$ brane in string theory is the S-dual of the D9 brane. However, this is a space-filling brane that upon reduction cannot give rise to a defect brane. We find that we need two $\alpha = -4$ generalized monopoles in ten dimensions, that can be associated to the mixed-symmetry fields

$$F_{8,6}, \quad F_{8,7,1}. \tag{14}$$

One can easily see that $F_{8,6}$ gives an $\alpha = -4$ singlet 1-brane in four dimensions, while using the restricted reduction rule in three dimensions one gets (here we denote with i the internal indices)

$$\begin{aligned} F_{8,6} &\rightarrow F_{1i_1\dots i_7, i_1\dots i_6} \quad (7), \\ F_{8,7,1} &\rightarrow F_{1i_1\dots i_7, i_1\dots i_7, i_1} \quad (7) \end{aligned} \tag{15}$$

adding up to a total of 14 0-branes, in agreement with Table 2. A new feature is that one of the monopoles is described by a mixed-symmetry field $F_{8,7,1}$ corresponding to a Young tableaux with *three* columns. This corresponds to a generalized KK monopole with $6 + 1$ inequivalent isometry directions.⁷ The field $F_{8,6}$ can be seen as an exotic dual of B_2 , while $F_{8,7,1}$ is an exotic dual of the graviton. The complete result, including all the fields that after restricted dimensional reduction give rise to the defect branes, is summarized in Table 3.

One may also consider the M-theory origin of the defect branes. It turns out that all the fields in Table 3 have their origin in the eleven-dimensional fields⁸

$$A_3, \quad A_6, \quad A_{8,1}, \quad A_{9,3}, \quad A_{9,6}, \quad A_{9,8,1}. \tag{16}$$

They correspond to two branes (the M2- and M5-brane), the standard M-theory monopole and three generalized KK monopoles one of which has *two* inequivalent isometry directions as we will describe in the next section. The fields A_6 , $A_{9,3}$ and $A_{9,6}$ are duals and exotic duals of the 3-form potential A_3 , while $A_{8,1}$ and $A_{9,8,1}$ are duals of the graviton.

4. Mixed-symmetry fields and monopole solutions

In this section we show how the mixed-symmetry fields, together with the restricted reduction rule, are in one to one correspondence with the classification of generalized KK monopole solutions of [18].⁹ We are going to use the following notation: an extended object of D -dimensional string theory with mass proportional to g_s^α , T transverse dimensions, p spacelike worldvolume dimensions and I_1, I_2, \dots inequivalent isometry directions,¹⁰ with $T + p + \sum_i I_i = D - 1$, will be denoted by $(T, p, I_1, I_2, \dots)_\alpha$. We will omit by convention all the entries to the right of the last non-vanishing I_i . Thus, standard Dp -branes ($I_i = 0$) are denoted by $(T, p)_{-1}$, the standard KK monopole in D dimensions is denoted by $(3, D - 5, 1)_{-2}$, etc. For M-theory objects we will omit the subindex α .

The association between the $(p + 1)$ -form potentials $B_2, C_1, \dots, C_8, D_6, E_8$ and p -branes is well established. Mixed-symmetry potentials are associated to generalized KK monopoles as follows: the symmetry of the potential $A_{m,n}$ is that of a Young tableau with two columns, one with m rows and one with n rows,¹¹ and it corresponds to the generalized KK monopole

$$\begin{aligned} A_{m,n} &\leftrightarrow (D - m, m - n - 1, n)_\alpha, \quad \text{or} \\ (T, p, I)_\alpha &\leftrightarrow A_{D-T, I}, \end{aligned} \tag{17}$$

⁷ In which sense they are inequivalent, will be discussed later.

⁸ We have not indicated the M-theory pp-wave which is represented by the metric.

⁹ The relation between mixed-symmetry fields and generalized KK monopole solutions has also been recently pointed out in [13].

¹⁰ In this work we will not have to consider more than two inequivalent sets of isometries, but to account for all domain-wall and space-filling branes, one has to consider more.

¹¹ An antisymmetric potential is denoted by $A_{m,0} = A_m$.

where the value of α can be read off from Table 2. This rule can be extended to include monopoles with two inequivalent isometry directions as follows

$$\begin{aligned} A_{m,n_1,n_2} &\leftrightarrow (D - m, m - n_1 - 1, n_1 - n_2, n_2)_\alpha, \quad \text{or} \\ (T, p, I_1, I_2)_\alpha &\leftrightarrow A_{D-T, I_1+I_2, I_2}. \end{aligned} \quad (18)$$

From now on, for simplicity, we will denote the correspondence between the mixed-symmetry fields and the solutions with an equality, i.e. $A_{m,n} = (D - m, m - n - 1, n)_\alpha$. In this notation, the string-theory origin of the solitonic defect branes mentioned in the previous section (the NS5-brane, the standard KK monopole and one generalized KK monopole) reads

$$D_6 = (4, 5)_{-2}, \quad D_{7,1} = (3, 5, 1)_{-2}, \quad D_{8,2} = (2, 5, 2)_{-2}, \quad (19)$$

the string-theory origin of the $\alpha = -3$ defect branes reads

$$E_{8,n} = (2, 7 - n, n)_{-3}, \quad n = 0, \dots, 7, \quad (20)$$

and the string-theory origin of the $\alpha = -4$ defect branes reads

$$F_{8,6} = (2, 1, 6)_{-4}, \quad F_{8,7,1} = (2, 0, 6, 1)_{-4}. \quad (21)$$

Finally, the M-theory origin of the defect branes reads

$$\begin{aligned} A_3 &= (6, 4), \quad A_6 = (5, 5), \quad A_{8,1} = (3, 6, 1), \\ A_{9,3} &= (1, 5, 3), \quad A_{9,6} = (2, 2, 6), \quad A_{9,8,1} = (2, 0, 7, 1). \end{aligned} \quad (22)$$

One advantage of this notation is that it makes it easy to write the mass of a toroidally compactified 10-dimensional monopole solution $(T, p, I_1, I_2)_\alpha$, which is given by ($\ell_s = 1$)

$$M_{(T,p,I_1,I_2)_\alpha} = R_1 \cdots R_p (R_{p+1} \cdots R_{p+I_1})^2 (R_{p+I_1+1} \cdots R_{p+I_1+I_2})^3 (g_s)^\alpha, \quad (23)$$

while for an 11-dimensional monopole it is given by ($\ell_{\text{Planck}}^{(11)}/2\pi = 1$)

$$M_{(T,p,I_1,I_2)} = R_1 \cdots R_p (R_{p+1} \cdots R_{p+I_1})^2 (R_{p+I_1+1} \cdots R_{p+I_1+I_2})^3. \quad (24)$$

Here the R are the compactification radii in the spacelike worldvolume and two isometry directions. It is this different dependence on the compactification radii that makes the isometry directions inequivalent. For instance, the mass of the $F_{8,7,1} = (2, 0, 6, 1)_{-4}$ generalized KK monopole is given by

$$M_{(2,0,6,1)_{-4}} = (R_1 \cdots R_6)^2 (R_7)^3 (g_s)^{-4}, \quad (25)$$

where $1, \dots, 7$ indicate the $6 + 1$ isometry directions. Similarly, the mass of the $A_{9,8,1} = (2, 0, 7, 1)$ generalized KK monopole is given by

$$M_{(2,0,7,1)} = (R_1 \cdots R_7)^2 (R_8)^3, \quad (26)$$

where $1, \dots, 8$ refer to the $7 + 1$ isometry directions.

This identification is based on the consistency between the restricted reduction rules of the potentials and the dimensional reduction of the objects. One can reduce a monopole solution given by (T, p, I_1, I_2) in four different ways: over a transverse (T), worldvolume (p) or one of the two inequivalent isometry directions (I_1, I_2). This leads to brane solutions as soon as one has reduced over all isometry directions. The branes corresponding to such solutions couple to a number of potentials. In order to obtain the same number of potentials following from the

Table 4

This table indicates the string-theory origin of the eight $D = 8$ half-supersymmetric defect brane solutions. The common factor V_5 in the expression for the mass is given by $V_5 = R_1 \cdots R_5$. We have set $\ell_s = 1$. The 8, 9 directions are the two transverse directions. The free index $i = 6, 7$ indicates two defect brane solutions. In the IIA case, the D_6 and $D_{8,2}$ solutions lead to the two tensor defect branes, while in the IIB case they arise from the $D_{7,1}$ solution.

IIA	Monopole	$M = \text{mass}/V_5$
$\alpha = -1$	$C_7 = (3, 6)_{-1}$	$M = R_i (g_s)^{-1}$
$\alpha = -2$	$D_6 = (4, 5)_{-2}$	$M = (g_s)^{-2}$
	$D_{7,1} = (3, 5, 1)_{-2}$	$M = (R_i)^2 (g_s)^{-2}$
	$D_{8,2} = (2, 5, 2)_{-2}$	$M = (R_6 R_7)^2 (g_s)^{-2}$
$\alpha = -3$	$E_{8,1} = (2, 6, 1)_{-3}$	$M = R_i (R_{i+1})^2 (g_s)^{-3}$
IIB	Monopole	$M = \text{mass}/V_5$
$\alpha = -1$	$C_8 = (2, 7)_{-1}$	$M = R_6 R_7 (g_s)^{-1}$
	$C_6 = (4, 5)_{-1}$	$M = (g_s)^{-1}$
$\alpha = -2$	$D_6 = (4, 5)_{-2}$	$M = (g_s)^{-2}$
	$D_{7,1} = (3, 5, 1)_{-2}$	$M = (R_i)^2 (g_s)^{-2}$
	$D_{8,2} = (2, 5, 2)_{-2}$	$M = (R_6 R_7)^2 (g_s)^{-2}$
$\alpha = -3$	$E_8 = (2, 7)_{-3}$	$M = R_6 R_7 (g_s)^{-3}$
	$E_{8,2} = (2, 5, 2)_{-3}$	$M = (R_6 R_7)^2 (g_s)^{-3}$

reduction of the mixed-symmetry fields one must use the restricted reduction rule formulated in Section 3.

As an example we consider the string and M-theory origin of the eight $D = 8$ defect brane solutions, see Table 2. We have indicated the IIA and IIB string-theory origin of these eight solutions in Table 4. Assuming that we reduce over the $i = 6, 7$ directions the three eight-dimensional dilatons are given by g_s, R_6 and R_7 , where R_6 and R_7 are the radii in the 6 and 7 directions. The IIA origin of the remaining 4 axions is given by g_{67}, B_{67}, C_6 and C_7 where $C_{i\mu}$ is the RR vector. Similarly, the IIB origin of the same axions is given by g_{67}, B_{67}, C_{67} and C_0 where C_0 is the IIB axion. The two transverse directions of the defect brane are 8 and 9.

The IIA/IIB string-theory origin of all eight $D = 8$ supersymmetric defect branes are given in Table 4. Note that each object has a different mass. All these objects and corresponding solutions are known in the literature. For instance, in the IIA case, the $(3, 6)_{-1}$ object is the D6-brane. This object gives rise to two defect branes depending on whether we take $i = 6$ or $i = 7$ along the worldvolume directions of the D6-brane. $(4, 5)_{-2}$ is the NS5A-brane and $(3, 5, 1)_{-2}$ is the standard KK5A monopole. $(2, 5, 2)_{-2}$ is a generalized KK monopole whose M-theory origin is another generalized KK monopole: $A_{9,3} = (2, 5, 3)$. The corresponding explicit solution of the latter can be found in Eq. (3.9) of [18].¹² Finally, $(2, 6, 1)_{-3}$ is the reduction of the $(3, 6, 1)$ standard M-theory monopole solution over one of its transverse directions and corresponds to the $p = 6$ case of Eq. (1.1) of [18]. Together, the M-theory origin of all the IIA solutions is given by the $(5, 5)$ M5-brane solution, the $(3, 6, 1)$ standard M-theory KK monopole and the $(2, 5, 3)$ generalized KK monopole solution.

¹² Some of the generalized monopoles have been constructed using E_{11} techniques [22].

In the IIB case the two $\alpha = -1$ objects are the D5-brane and the D7-brane. The three $\alpha = -2$ objects are the same as in the IIA case. Finally, the two $\alpha = -3$ objects are the S-dual of the D7-brane and the $(2, 5, 2)_{-3}$ generalized KK monopole.

The masses of all the objects of Table 4 transform into each other under the T-duality rules

$$R \rightarrow 1/R, \quad g_s \rightarrow g_s/R, \quad (27)$$

in agreement with the T-duality representations given in Table 2. Note that the mass of the solutions is not left invariant under this T-duality. The mass multiplets form representations of the SO(3) subgroup of the SO(2, 2) T-duality group. Note that under the S-duality rules

$$g_s \rightarrow 1/g_s, \quad R \rightarrow R/(g_s)^{1/2} \quad (28)$$

the masses do not transform in agreement with the S-duality rule (6). This is because the S-duality underlying (6) refers to the *eight-dimensional* dilaton whereas the S-duality rules (28) refer to the *ten-dimensional* dilaton.

5. Duality relations and explicit solutions

In this section we want to show that the linearized duality relations that the mixed-symmetry fields satisfy can be used to deduce the behavior of the fields of the corresponding solution. We consider as a first example the ten-dimensional field B_2 together with all its generalized duals D_6 , $D_{8,2}$ and $F_{8,6}$. For each of these fields, there is a (T, p, I) solution in which the field can be considered to be electric, that is with non-zero components along the $p + 1$ worldvolume directions and along the isometry directions. We now want to show that using linearized duality relations each of these solutions becomes a solution in which only the B_2 field occurs. The duality relation reveals in each case the particular form that the B_2 field takes.

We denote the T transverse directions with ω^a , with $a = 1, \dots, T$, the worldvolume direction with $y^\mu = (t, y^1, \dots, y^p)$ and the isometry directions with z^m , with $m = 1, \dots, I$. In all cases the fields only depend on the transverse directions ω . We start considering the B_2 solution. This is the solution $(8, 1)_0$, and simply corresponds to an electric field $B_{\mu\nu}(\omega)$. We next consider the D_6 solution $(4, 5)_{-2}$. This corresponds to a non-vanishing $D_{\mu_1 \dots \mu_6}(\omega)$. Using the duality relation

$$\partial_a D_{\mu_1 \dots \mu_6} \sim \epsilon_{a\mu_1 \dots \mu_6}{}^{b_1 b_2 b_3} \partial_{b_1} B_{b_2 b_3} \quad (29)$$

we see that this corresponds to a B_2 field along the four transverse directions $B_{a_1 a_2}(\omega)$.

We next consider the field $D_{8,2}$. The solution $(2, 5, 2)_{-2}$ has two isometries, and corresponds to turning on the components $D_{\mu_1 \dots \mu_6 mn, mn}(\omega)$. Dualizing we get¹³

$$\partial_a \partial_b D_{\mu_1 \dots \mu_6 mn, mn} \sim \epsilon_{a\mu_1 \dots \mu_6 mn}{}^c \partial_b \partial_c B_{mn} \quad (30)$$

which means that the solution can be seen as a solution in which one turns on B_2 along the isometry directions, $B_{mn}(\omega)$. The linearized duality relation is at second order in derivatives because the field has mixed-symmetry with two sets of antisymmetric indices [19,20].

¹³ Note that the duality relations involving mixed-symmetry fields we use in this section are not truly ten-dimensional ones. They are only applied to solutions that exhibit a number of isometries. Effectively, this means that, after reduction over the isometry directions, we apply standard lower-dimensional duality relations between forms. We thank Axel Kleinschmidt for a discussion on this point.

We finally consider the field $F_{8,6}$ corresponding to the solution $(2, 1, 6)_{-4}$ with six isometries. This solution is carried by the electric mixed-symmetry field $F_{\mu_1\mu_2m_1\dots m_6,m_1\dots m_6}(\omega)$ which can be dualized as follows:

$$\partial_a \partial_b F_{\mu_1\mu_2m_1\dots m_6,m_1\dots m_6} \sim \epsilon_{a\mu_1\mu_2m_1\dots m_6}{}^c \epsilon_{bm_1\dots m_6}{}^{dv_1v_2} \partial_c \partial_d B_{v_1v_2}. \quad (31)$$

This corresponds to a solution with $B_{\mu\nu}(\omega)$ non-vanishing, exactly as in the first case, but now, since there are isometry directions, this B_2 is not electric.

The same reasoning can be applied to the solutions $(2, 7 - n, n)_{-3}$ corresponding to the fields $E_{8,n}$. These solutions have n isometries. The field $E_{\mu_1\dots\mu_{8-n}m_1\dots m_n,m_1\dots m_n}(\omega)$ is non-vanishing, and can be dualized according to

$$\begin{aligned} \partial_a \partial_b E_{\mu_1\dots\mu_{8-n}m_1\dots m_n,m_1\dots m_n} &\sim \epsilon_{a\mu_1\dots\mu_{8-n}m_1\dots m_n}{}^c \partial_b \partial_c C_{m_1\dots m_n} \\ &\sim \epsilon_{a\mu_1\dots\mu_{8-n}m_1\dots m_n}{}^c \epsilon_{bm_1\dots m_n}{}^{v_1\dots v_{8-n}d} \partial_c \partial_d C_{v_1\dots v_{8-n}}, \end{aligned} \quad (32)$$

corresponding to a solution with the RR field C_n along the isometry directions (or a dual RR field C_{8-n} along the worldvolume directions).

We now consider the purely gravitational solutions. The KK monopole solution is $(3, 5, 1)_{-2}$. The corresponding ten-dimensional field is $D_{7,1}$, and turning on the component $D_{\mu_1\dots\mu_6m,m}(x)$ the linearized duality relation becomes

$$\partial_a \partial_b D_{\mu_1\dots\mu_6m,m} \sim \epsilon_{a\mu_1\dots\mu_6m}{}^{cd} \partial_c \partial_b h_{dm}, \quad (33)$$

corresponding to a linearized graviton fluctuation of the form

$$h_{am}(x). \quad (34)$$

This is the well-known KK monopole solution.

The other (generalized) KK monopole solution is $(2, 0, 6, 1)_{-4}$ where now there are two sets of isometries: a six-plet and a singlet isometry direction. The corresponding field is $F_{8,7,1}$ which has non-vanishing components $F_{\mu m_1\dots m_7,m_1\dots m_7,m_1}$. This corresponds to a linearized graviton given by the duality relation

$$\partial_a \partial_b \partial_c F_{\mu m_1\dots m_7,m_1\dots m_7,m_1} \sim \epsilon_{a\mu m_1\dots m_7}{}^d \epsilon_{bm_1\dots m_7}{}^{ve} \partial_d \partial_e \partial_c h_{vm_1}, \quad (35)$$

which corresponds to a linearized graviton fluctuation

$$h_{\mu m}(x) \quad (36)$$

where m is the singlet isometry direction, and thus this solution is a pp-wave.

Finally, we consider the eleven-dimensional solutions. Repeating the analysis just done for the B_2 field and its duals in ten dimensions, one can deduce that the solution $A_{9,3} = (2, 5, 3)$ corresponds to the field A_3 along the isometry directions, while the solution $A_{9,6} = (2, 2, 6)$ corresponds to A_3 along the worldvolume directions. The gravitational solutions $A_{8,1} = (3, 6, 1)$ and $A_{9,8,1} = (2, 0, 7, 1)$ are exactly as in the ten-dimensional case.

These results can be tested by looking into the explicit supergravity solutions given in [18], which we reproduce here for the sake of completeness. Recently, some of these solutions have been rederived in [13] by performing U-duality transformations on known solutions in the E_{11} framework.

Let us start with the 10-dimensional (string-theory) fields in Table 3: $B_2 = (8, 1)_0$ is the Fundamental (IIA/IIB) string, $C_{p+1} = (9 - p, p)_{-1}$ are the Dirichlet p -branes, $D_6 = (4, 5)_{-2}$ is the

(IIA/IIB) NS5 brane, $D_{7,1} = (3, 6, 1)_{-2}$ is the standard (IIA/IIB) KK monopole. The explicit form of all these solutions is well known.

The solution corresponding to $D_{8,2} = (2, 5, 2)_{-2}$ is, in the string frame¹⁴

$$\begin{aligned}
 ds_s^2 &= dt^2 - d\bar{y}_5^2 - H d\omega d\bar{\omega} - \frac{H}{\mathcal{H}\bar{\mathcal{H}}} d\bar{z}_2^2, \\
 e^\phi &= \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}}\right)^{\frac{1}{2}}, \\
 B_{(6)ty^1\dots y^5} &= \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}}\right)^{-1}, \quad B_{(2)z^1 z^2} = -\frac{A}{\mathcal{H}\bar{\mathcal{H}}},
 \end{aligned} \tag{37}$$

which is Eq. (3.1) of [18]. Observe that, as anticipated, B_2 only has non-vanishing components in the two isometric directions. The solutions corresponding to the $E_{8,n} = (2, 7 - n, n)_{-3}$ are, with $7 - n = p$ and in the string frame, given by

$$\begin{aligned}
 ds_s^2 &= \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}}\right)^{-1/2} [dt^2 - d\bar{y}_p^2 - H d\omega d\bar{\omega}] - \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}}\right)^{1/2} d\bar{z}_{7-p}^2, \\
 e^\phi &= \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}}\right)^{\frac{3-p}{4}}, \\
 C_{(p+1)ty^1\dots y^p} &= (-1)^{\lfloor \frac{p+1}{2} \rfloor} \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}}\right)^{-1}, \quad C_{(7-p)z^1\dots z^{7-p}} = -\frac{A}{\mathcal{H}\bar{\mathcal{H}}},
 \end{aligned} \tag{38}$$

which is Eq. (1.1) of [18]. As anticipated, this corresponds to the field C_{8-n} along the worldvolume directions or the dual field C_n along the isometry directions. The solution corresponding to the $F_{8,6} = (2, 1, 6)_{-4}$ is

$$\begin{aligned}
 ds_s^2 &= \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}}\right)^{-1} [dt^2 - dy^2 - H d\omega d\bar{\omega}] - d\bar{z}_6^2, \\
 e^\phi &= \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}}\right)^{-\frac{1}{2}}, \\
 B_{(2)ty} &= -\left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}}\right)^{-1}, \quad B_{(6)z^1\dots z^6} = \frac{A}{\mathcal{H}\bar{\mathcal{H}}},
 \end{aligned} \tag{39}$$

which is Eq. (3.2) of [18] and as anticipated corresponds to the field B_2 along the worldvolume directions. The purely gravitational solution corresponding to $F_{8,7,1} = (2, 0, 6, 1)_{-4}$ is

$$ds^2 = -2 dt dy - \frac{H}{\mathcal{H}\bar{\mathcal{H}}} dy^2 - \mathcal{H}\bar{\mathcal{H}} d\omega d\bar{\omega} - d\bar{z}_6^2, \tag{40}$$

which is Eq. (3.11) of [18] and is a pp-wave.

As for the explicit solutions corresponding to the eleven-dimensional fields in Eq. (16), $A_3 = (8, 2)$ and $A_6 = (5, 5)$ are the M2 and M5 branes, $A_{8,1} = (3, 6, 1)$ is the standard KK monopole and all their solutions are well known. $A_{9,3} = (2, 5, 3)$ is given by

¹⁴ In all these defect brane solutions function $\mathcal{H} = \mathcal{H}(\omega) = A + iH$ is a complex, holomorphic (multivalued) function of ω .

Table 5

This table indicates the R -representations of the p -form central charges and the corresponding standard supersymmetric p -branes of $7 \leq D \leq 10$ maximal supergravity. A prime indicates that the worldvolume multiplet is not a vector but a tensor multiplet. The pp-wave corresponds to the translation generator.

D	R	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
IIA	1	1 D0	1 F1 –	1 D2		1 D4 D6	1 S'5 + KK5
IIB	SO(2)		2 F1 + D1		1 D3		1 ⁺ + 2 ⁺ KK'5 + (D5 + S5)
9	SO(2)	1 + 2 F0 + (F0 + D0)	2 F1 + D1	1 D2	1 D3	1 + 2 KK4 + (D4 + S4) S'5 + (D5 + S5)	
8	U(2)	2 × 3 2 × (2F0 + D0)	3 F1 + 2D1	2 × 1 2 × D2	1 + 3 KK3 + (2D3 + S3)	3⁺ + 3⁻ (D4 + 2S4) + (D4 + 2S4)	
7	Sp(4)	10 6F0 + 4D0	5 F1 + 4D1	1 + 5 KK2 + (4D2 + S2)	10 4D3 + 6S3		

$$ds^2 = \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}} \right)^{-1/3} [dt^2 - d\bar{y}_5^2 - H d\omega d\bar{\omega}] - \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}} \right)^{2/3} d\bar{z}_3^2,$$

$$A_{(6)ty^1\dots y^5} = - \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}} \right)^{-1}, \quad A_{(3)z^1z^2z^3} = - \frac{A}{\mathcal{H}\bar{\mathcal{H}}}, \tag{41}$$

which is Eq. (3.9) of [18] and as anticipated has A_3 along the isometry directions, while $A_{9,6} = (2, 2, 6)$ is given in

$$ds^2 = \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}} \right)^{-2/3} [dt^2 - d\bar{y}_2^2 - H d\omega d\bar{\omega}] - \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}} \right)^{1/3} d\bar{z}_6^2,$$

$$A_{(3)ty^1y^2} = - \left(\frac{H}{\mathcal{H}\bar{\mathcal{H}}} \right)^{-1}, \quad A_{(6)z^1\dots z^6} = \frac{A}{\mathcal{H}\bar{\mathcal{H}}}, \tag{42}$$

which is Eq. (3.8) of [18] and as anticipated has A_3 along the worldvolume directions. Finally, $A_{9,8,1} = (2, 0, 7, 1)$ is given by the purely gravitational solution

$$ds^2 = -2 dt dy - \frac{H}{\mathcal{H}\bar{\mathcal{H}}} dy^2 - \mathcal{H}\bar{\mathcal{H}} d\omega d\bar{\omega} - d\bar{z}_7^2, \tag{43}$$

which is Eq. (3.7) of [18] and again corresponds to a pp-wave.

6. Central charges

It is well known that there is a 1–1 correspondence between standard branes and the central charges in the supersymmetry algebra in type II string and M-theory. The standard branes, for $3 \leq D \leq 10$ dimensions have a universal behavior with respect to T-duality. For each dimension they are given by a singlet and vector of Fundamental branes, a chiral spinor of D-branes and antisymmetric tensors of solitonic branes. On top of this we have in each dimension a pp-wave and a $(3, D - 5, 1)_{-2}$ standard KK monopole. The pp-wave is represented by the translation generator whereas all other branes are represented by the most general central charges in the supersymmetry algebra. This is summarized in Tables 5 and 6, that indicate the R -representations of the p -form central charges and the corresponding standard supersymmetric p -branes of $3 \leq D \leq 10$

Table 6

This table indicates the R -representations of the p -form central charges and the corresponding standard supersymmetric p -branes of $3 \leq D \leq 6$ maximal supergravity. The pp-wave corresponds to the translation generator.

D	R	$p = 0$	$p = 1$	$p = 2$	$p = 3$
6	$\text{Sp}(4) \times \text{Sp}(4)$	(4, 4) 8F0 + 8D0	(1, 1) (1, 5) (5, 1) KK1 F1 + 4D1 4D1 + S1	(4, 4) 8D2 + 8S2	(10, 1)⁺ (1, 10)⁻
5	$\text{Sp}(8)$	1 + 27 KK0 + 10F0 + 16D0 + S0	27 F1 + 16D1 + 10S1	36	
4	$\text{SU}(8)$	28 + $\overline{28}$ 12F0 + 16D0 16D0 + 12S0	63	36⁺ + $\overline{36}$⁻	
3	$\text{SO}(16)$	120	135		

Table 7

The number n_D of supersymmetric defect branes is twice the number n_Z of corresponding n -form central charges.

D	H	$n = 0$	$n = 1$	$n = 2$	$n = 3$	n_D
IIB	$\text{SO}(2)$				1	2
9	$\text{SO}(2)$				1	2
8	$\text{U}(2)$				3 + 1	6 + 2
7	$\text{Sp}(4)$				10	20
6	$\text{Sp}(4) \times \text{Sp}(4)$				(10, 1)⁺ + (1, 10)⁻	40
5	$\text{Sp}(8)$			36		72
4	$\text{SU}(8)$		63			126
3	$\text{SO}(16)$	120				240

maximal supergravity. In [Tables 5 and 6](#), if applicable, we have indicated the space–time duality of the central charges with a superscript \pm . We also use the following abbreviations in the tables: F (Fundamental), D (D-brane), S (Soliton), and KK (Kaluza–Klein monopole). All branes have worldvolume vector multiplets except for the ones indicated by a prime. Note that in $D = 3$ dimensions there are no standard branes.

One does not expect a similar 1–1 relation to hold between the non-standard branes and the central charges of the supersymmetry algebra. The reason is that these non-standard branes are not asymptotically flat and therefore the standard Poincaré supersymmetry algebra is not realized at spatial infinity. Nevertheless, since we calculated the number n_D of supersymmetric defect branes, it is of interest to compare these numbers with the number n_Z of relevant p -form central charges.¹⁵ These are the 3-form central charges for $D \geq 6$ and the $(D - 3)$ -form central charges for $3 \leq D \leq 5$. We have collected these numbers in [Table 7](#). We observe that there is a universal 2–1 relation between n_D and n_Z , i.e. we find that $n_D = 2n_Z$. The reason that this is the case is due to the universal behavior of the central charges and defect branes. In any dimension the central charges corresponding to defect branes transform in the adjoint representation of the R -symmetry

¹⁵ We do not consider here the charges corresponding to the generalized KK monopoles.

group H , which is the maximal compact subgroup of the U-duality group G , i.e. we always have that $n_Z = \dim H$. On the other hand, we found that the number of supersymmetric defect branes n_D is universally given by $n_D = \dim G - \text{rank } G$. We now use that the U-duality groups of all maximal supergravity theories are of split-form and therefore we have that $\dim H = P$ and $\dim G - \text{rank } G = 2P$ where P is the number of positive roots. This indeed implies that $n_D = 2n_Z$. The reason that we find this 1–2 relation between defect branes and central charges is most likely related to the fact that under S-duality the central charges remain invariant whereas the defect brane solutions are mapped to S-dual defect-brane solutions according to the S-duality rule (6).¹⁶

7. Conclusions

In this work we have discussed some basic properties of branes with co-dimension 2, i.e. defect branes. Requiring the existence of a supersymmetric gauge-invariant WZ term we gave a full classification of these branes, see Table 2. Their string and M-theory origin as seven-branes and a set of generalized KK monopoles was determined. These included monopoles with two inequivalent isometry directions. We explained why the number n_D of supersymmetric defect branes does not equal the number n_P of $(D - 2)$ -form potentials or the number n_S of coset scalars and we presented the string and M-theory origin of all defect branes. As an example we gave explicit results for the $D = 8$ case. We observed that the number n_D of supersymmetric defect branes is always twice the number n_Z of central charges in the supersymmetry algebra and we explained why this is the case.

There is a simple alternative way to count the number of supersymmetric defect branes and to verify that for a U-duality group G the number n_D of supersymmetric defect branes is given by $n_D = \dim G - \text{rank } G$. Each basic half-supersymmetric defect brane is carried by an axion–dilaton combination that parametrizes an $\text{SL}(2, \mathbb{R})$ subgroup of the U-duality group. Together with the S-dual defect brane this leads to two branes for each inequivalent embedding of $\text{SL}(2, \mathbb{R})$ into G . For instance, for $G = \text{SL}(n, \mathbb{R})$, which is the case for $D = 7$ and $D = 9$, one has to choose 2 out of the n directions. This leads to $n(n - 1)/2$ inequivalent embeddings and hence $n(n - 1)$ supersymmetric defect branes. On the other hand, for $G = \text{SL}(n, \mathbb{R})$ we have that $\dim G = n^2 - 1$ and $\text{rank } G = n - 1$ so that we indeed verify the expression for n_D given above. The other dimensions proceed in a similar way.

It is interesting to also consider the electric duals of the defect branes, i.e. instantons. These instantons occur in the same U-duality representations, with the value of α given by the general relation

$$\alpha_{\text{magnetic}} = -\alpha_{\text{electric}} - 2, \quad (44)$$

where in this case α_{magnetic} is the value of α of a given defect brane and α_{electric} is the value of α of the dual instanton. This implies that the values of α for instantons in $D \geq 3$ are $\alpha = 2, 1, 0, -1, -2$. Since under S-duality the value of α transforms according to $\alpha \rightarrow -\alpha$ we see that the instantons are symmetric around the $\alpha = 0$ Fundamental instantons. The Fundamental, Dirichlet and Solitonic instantons can all be understood as the result of extending the corresponding wrapping rule to wrapping over time. For instance, the Fundamental instantons arise as the result of applying the fundamental wrapping rule (8) to the fundamental string, see Table 8. Note that fundamental instantons only arise in $3 \leq D \leq 8$ dimensions.

¹⁶ We thank the referee for raising this point.

Table 8

Upon applying the fundamental wrapping rule (8) one obtains in each dimension the U-duality representations of the Fundamental instantons, cf. Table 2.

Fp-brane	IIA/IIB	9	8	7	6	5	4	3
−1			4	12	24	40	60	84
0		2	4	6	8	10	12	14
1	I/I	1	1	1	1	1	1	1

The main result of our work is that we have associated a mixed-symmetry field to each of the generalized KK monopoles using the general rule (18). All the generalized KK monopoles considered have a single set of isometry directions, with the notable exception of the ten-dimensional solution $(2, 0, 6, 1)_{-4}$ and the eleven-dimensional solution $(2, 0, 7, 1)$, which have two *inequivalent* isometry directions. Such monopoles have a quadratic and cubic dependence of the mass on the radii, see Eqs. (25) and (26).

It turns out that the specific mixed-symmetry fields we found are precisely the ones predicted by E_{11} [14]. Indeed, E_{11} naturally contains fields that are all possible dual descriptions of the supergravity fields, and thus naturally includes the fields in Table 3 and in Eq. (16) [23]. Moreover, selecting out of the various potentials the ones that are associated to supersymmetric branes corresponds to selecting the real roots of E_{11} , and this gives automatically all supersymmetric branes in all dimensions [13]. This is one more application where E_{11} is used to learn about the properties of supergravity.

It is important to distinguish between the status of the mixed-symmetry fields and that of the monopole solutions. The monopole solutions have been given in the literature as solutions of the full non-linear supergravity theory [18,22].¹⁷ On the other hand the mixed-symmetry fields can only be made consistent with supersymmetry at the level of *linearized* supersymmetry. A prime example is the dual graviton field $A_{8,1}$ in $D = 11$ dimensions whose supersymmetry properties have been discussed in [24]. The restricted reduction rule of the mixed-symmetry fields $A_{m,n}$ we found suggests that they couple to a generalized KK monopole via a Wess–Zumino term where the last n indices are taken into the isometry directions and n of the first m indices are taken into the same isometry directions. The remaining $m - n$ indices couple to the worldvolume directions of the monopole in the usual way. It would be interesting to see whether such a gauge-invariant WZ term describing the coupling of the background fields to the monopole can be constructed.

Note added

During the course of this work the paper [13] appeared which has some overlap with this work. In particular, Section 3 of [13] discusses defect brane solutions from an E_{11} -point of view.

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¹⁷ It remains to be seen whether these monopole solutions can be turned into non-singular finite-energy solutions.

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References

- [1] A. Strominger, C. Vafa, Microscopic origin of the Bekenstein–Hawking entropy, *Phys. Lett. B* 379 (1996) 99–104, hep-th/9601029.
- [2] J.M. Maldacena, The large N limit of superconformal field theories and supergravity, *Adv. Theor. Math. Phys.* 2 (1998) 231–252, hep-th/9711200.
- [3] J. Polchinski, E. Witten, Evidence for heterotic–type I string duality, *Nucl. Phys. B* 460 (1996) 525–540, hep-th/9510169.
- [4] J. Polchinski, Dirichlet branes and Ramond–Ramond charges, *Phys. Rev. Lett.* 75 (1995) 4724–4727, hep-th/9510017.
- [5] B.R. Greene, A.D. Shapere, C. Vafa, S.-T. Yau, Stringy cosmic strings and noncompact Calabi–Yau manifolds, *Nucl. Phys. B* 337 (1990) 1.
- [6] G.W. Gibbons, M.B. Green, M.J. Perry, Instantons and seven-branes in type IIB superstring theory, *Phys. Lett. B* 370 (1996) 37–44, hep-th/9511080.
- [7] E.A. Bergshoeff, J. Hartong, T. Ortín, D. Roest, Seven-branes and supersymmetry, *JHEP* 0702 (2007) 003, hep-th/0612072.
- [8] E.A. Bergshoeff, J. Hartong, O. Hohm, M. Hübscher, T. Ortín, Gauge theories, duality relations and the tensor hierarchy, *JHEP* 0904 (2009) 123, arXiv:0901.2054 [hep-th].
- [9] P. Meessen, T. Ortín, An $SL(2, Z)$ multiplet of nine-dimensional type II supergravity theories, *Nucl. Phys. B* 541 (1999) 195, hep-th/9806120;
G. Dall’Agata, K. Lechner, M. Tonin, $D = 10$, $N = IIB$ supergravity: Lorentz-invariant actions and duality, *JHEP* 9807 (1998) 017, hep-th/9806140.
- [10] E.A. Bergshoeff, M. de Roo, S.F. Kerstan, T. Ortín, F. Riccioni, $SL(2, R)$ -invariant IIB brane actions, *JHEP* 0702 (2007) 007, hep-th/0611036.
- [11] E.A. Bergshoeff, F. Riccioni, D-brane Wess–Zumino terms and U-duality, *JHEP* 1011 (2010) 139, arXiv:1009.4657 [hep-th].
- [12] E.A. Bergshoeff, F. Riccioni, The D-brane U-scan, arXiv:1109.1725 [hep-th].
- [13] A. Kleinschmidt, Counting supersymmetric branes, arXiv:1109.2025 [hep-th].
- [14] P.C. West, E_{11} and M theory, *Class. Quant. Grav.* 18 (2001) 4443, hep-th/0104081.
- [15] L. Houart, A. Kleinschmidt, J. Lindman Hornlund, An M-theory solution from null roots in E_{11} , *JHEP* 1101 (2011) 154, arXiv:1101.2816 [hep-th].
- [16] E.A. Bergshoeff, F. Riccioni, Dual doubled geometry, *Phys. Lett. B* 702 (2011) 281–285, arXiv:1106.0212 [hep-th].
- [17] E.A. Bergshoeff, F. Riccioni, Branes and wrapping rules, arXiv:1108.5067 [hep-th].
- [18] E. Lozano-Tellechea, T. Ortín, 7-branes and higher Kaluza–Klein branes, *Nucl. Phys. B* 607 (2001) 213–236, hep-th/0012051.
- [19] T. Curtright, Generalized gauge fields, *Phys. Lett. B* 165 (1985) 304.
- [20] C.M. Hull, Duality in gravity and higher spin gauge fields, *JHEP* 0109 (2001) 027, hep-th/0107149.
- [21] E.A. Bergshoeff, F. Riccioni, String solitons and T-duality, *JHEP* 1105 (2011) 131, arXiv:1102.0934 [hep-th].
- [22] F. Englert, L. Houart, A. Kleinschmidt, H. Nicolai, N. Tabti, An $E(9)$ multiplet of BPS states, *JHEP* 0705 (2007) 065, hep-th/0703285.
- [23] F. Riccioni, P.C. West, Dual fields and $E(11)$, *Phys. Lett. B* 645 (2007) 286, hep-th/0612001.
- [24] E.A. Bergshoeff, M. de Roo, S.F. Kerstan, A. Kleinschmidt, F. Riccioni, Dual gravity and matter, *Gen. Rel. Grav.* 41 (2009) 39–48, arXiv:0803.1963 [hep-th].