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STATE CONVERGENCE OF DISSIPATIVE NONLINEAR SYSTEMS GIVEN BOUNDED-ENERGY INPUT SIGNAL*

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Abstract. In this paper, we show the state convergence of a class of dissipative nonlinear systems given bounded-energy input functions $u : \mathbb{R}_+ \to \mathbb{R}^m$, where the energy function is related to the dissipation inequality (or the supply rate).

Key words. state convergence, asymptotic stability, robust stability, nonlinear systems

AMS subject classifications. 34A34, 34D05, 34D10, 34E10, 93C10, 93D05, 93D20

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1. Introduction.

Consider system **P** described by

$$\dot{x} = f(x,u), \qquad x(0) = x_0 \tag{1.1}$$

$$y = h(x), \tag{1.2}$$

where the state x, the input u, and the output y are functions of $t \geq 0$ such that $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^l$. We assume that $f \in \mathcal{C}(\mathbb{R}^n \times \mathbb{R}^m, \mathbb{R}^n)$ with $f(0,0) = 0$ and $h \in \mathcal{C}(\mathbb{R}^n, \mathbb{R}^l)$ with $h(0) = 0$.

Sontag in [13] has studied the converging-input converging-state property in nonlinear systems. It is shown in [13] that, under some Lipschitz-type condition on f and under the global asymptotic stability (GAS) property of the origin when $u = 0$, if for a given converging input u and an initial state x_0 there exists a unique solution $x(t)$ of (1.1) defined for all $t \geq 0$ and x is bounded, then $x(t) \to 0$ as $t \to \infty$. Ryan in [10] generalizes the result to an L^p input signal. Under the global asymptotic stability of the origin, supposing that $f(\cdot, 0)$ is locally Lipschitz and assuming that for all compact sets $K \in \mathbb{R}^n$ there exists $c > 0$ such that

$$\|f(\xi,v) - f(\xi,0)\| \leq c\|v\| \qquad \forall (\xi,v) \in K \times \mathbb{R}^m, \tag{1.3}$$

it is shown in [10] that if for a given L^p input u and initial state x_0 there exists a solution of the state trajectory x defined for all $t \geq 0$ and x is bounded, then $x(t) \to 0$ as $t \to \infty$.

In both articles, the analysis rests on finding the ω-limit set of the bounded state trajectory x given a certain input function u. It is shown in [10] and [13] that the ω-limit set in both problems is $\{0\}$.

Using an infinite-dimensional version of the La Salle invariance principle, which also relies on the concept of ω-limit set theory, we can generalize the result of [10] by allowing **P** as in (1.1) to satisfy the following: for every $a \in \mathbb{R}^n$ there exists $c_1, c_2 > 0$ such that

$$\|f(a,v) - f(a,0)\| \leq c_1 + c_2\|v\| \qquad \forall v \in \mathbb{R}^m, \tag{1.4}$$

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and it is dissipative with respect to the supply rate $s(y,u) = \|u\|^p - k\|y\|^p$, $k > 0$. Note that the dissipativity assumption is not posited in [10, 13].

A recent development in [7] offers an alternative approach to the study of state convergence given bounded energy input via integral input-to-state stability. The result relies on the stronger assumption than (1.4); viz., for every compact set $K \subset \mathbb{R}^n$ there exists $c_1, c_2 > 0$ such that

$$\|f(a,v) - f(a,0)\| \leq c_1 + c_2\|v\| \qquad \forall (a,v) \in K \times \mathbb{R}^m.$$

In section 3, we show the convergence of the state of nonlinear systems given an L^p input signal where $p \in [1,\infty)$. Assuming that f satisfies the above condition, it is shown that if $\mathbf{P}$ is dissipative with respect to supply rate $s(y,u) = \|u\|^p - k\|y\|^p$, $k > 0$, and with a proper storage function H (the definitions of dissipativity and properness are given in section 2), and is zero-state detectable, then for any L^p input u, the state trajectories x of (1.1) are defined on $\mathbb{R}_+$ and x converges to zero. This result implies also that the systems satisfying these conditions are L^p-stable. In section 4, we generalize this result for $\mathbf{P}$ which can have a nonproper storage function and is dissipative with the supply rate $s(y,u) = \sigma(\|u\|) - \gamma(\|y\|)$, where σ, γ belong to a class of continuous and increasing functions.

The result of this paper reveals an additional property that an L^p-stable system can have. The standard properties of an L^p-stable system are as follows:

1. If the input is zero and the system satisfies a detectability condition, then the state converges to zero (see, for example, [11]).
2. Using a Barbălat-type argument, it can be shown that if the storage function is proper, then the output signal y converges to zero for any L^p input signal (see [14]).

However, these standard stability results do not describe the convergence of the state trajectory x to zero for any L^p input.

For nonlinear systems analysis, the result is useful for robustness analysis of nonlinear controllers or nonlinear state observers when the closed-loop system is disturbed by a bounded-energy signal. For example, it can be used to show that the estimator state converges to the actual plant state despite the existence of a bounded-energy disturbance signal.

An earlier version of this paper is presented in [4], where the condition on f is stronger. In this paper, we extend the result in [4] by assuming a weaker condition on f, allowing a different type of supply rate and allowing nonproper storage functions. The technique for showing the state convergence has also been used in Jayawardhana and Weiss [5, 6] to show the convergence of the state of strictly output passive systems given any L^2 input. The paper [5] proposes an LTI (linear time-invariant) controller for passive nonlinear plants to solve the disturbance rejection problem, where the disturbance signal can be decomposed into a component generated by an exosystem and an L^2 signal.

For a stable LTI system, the state convergence given any L^p input signal with $p \in [1,\infty)$ can be shown easily. Indeed, suppose that $\mathbf{P}$ is described by

$$\dot{x} = Ax + Bu, \tag{1.5}$$

$$y = Cx + Du, \tag{1.6}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^l$. Stability of $\mathbf{P}$ implies that A is Hurwitz. Thus, $u \in L^p$ implies that $x \in L^p$. From (1.5) we also have $\dot{x} \in L^p$. Thus, using a generalized Barbălat's lemma (see, for example, [9]) $x, \dot{x} \in L^p \Rightarrow \lim_{t\to\infty} x(t) = 0$.

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2. Preliminaries and notation. Let $\mathbb{R}_+ = [0,\infty)$. For a finite-dimensional vector x, we use the norm $\|x\| = \left(\sum_n |x_n|^2\right)^{\frac{1}{2}}$. For any finite-dimensional vector space $\mathcal{V}$ endowed with a norm $\|\cdot\|$, the space $L^p(\mathbb{R}_+,\mathcal{V})$, $p \in [1,\infty)$, consists of all the measurable functions $f : \mathbb{R}_+ \to \mathcal{V}$ such that $\int_0^\infty \|f(t)\|^p \mathrm{d}t < \infty$. The L^p-norm of a function $f \in L^p(\mathbb{R}_+,\mathcal{V})$ is given by $\|f\|_{L^p} = \left(\int_0^\infty \|f(t)\|^p \mathrm{d}t\right)^{\frac{1}{p}}$. For $f \in L^p(\mathbb{R}_+,\mathcal{V})$ and $T > 0$, we denote by f_T the truncation of f to $[0,T]$. The space $L^p_{loc}(\mathbb{R}_+,\mathcal{V})$ consists of all the measurable functions $f : \mathbb{R}_+ \to \mathcal{V}$ such that $f_T \in L^p(\mathbb{R}_+,\mathcal{V})$ for all $T > 0$. The space $\mathcal{W}^{1,p}(\mathbb{R}_+,\mathcal{V})$ consists of all the functions $f : \mathbb{R}_+ \to \mathcal{V}$ such that $f, \frac{\mathrm{d}f}{\mathrm{d}t} \in L^p(\mathbb{R}_+,\mathcal{V})$ (where $\frac{\mathrm{d}f}{\mathrm{d}t}$ is understood in the sense of distributions).

The space $\mathcal{C}(\mathbb{R}^l,\mathbb{R}^p)$ (respectively, $\mathcal{C}^1(\mathbb{R}^l,\mathbb{R}^p)$) consists of all the continuous (respectively, continuously differentiable) functions $f : \mathbb{R}^l \to \mathbb{R}^p$. For any $\epsilon \geq 0$, we denote $\mathbf{B}_\epsilon = \{x \in \mathbb{R}^n \mid \|x\| \leq \epsilon\}$.

The set $\mathcal{K}$ consists of all the continuous functions $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\alpha(0) = 0$, and it is strictly increasing. The function $H : \mathbb{R}^n \to \mathbb{R}_+$ is called *proper* if for all $c \geq 0$, the set $\{x \mid H(x) \leq c\}$ is compact or, equivalently, $H(x) \to \infty$ whenever $\|x\| \to \infty$.

We consider system $\mathbf{P}$ described by (1.1), (1.2) satisfying the assumptions stated thereafter. Let us recall a result on the existence of the solution of the differential equation (1.1) (see also Hale [1, Theorem I.5.1] for details).

DEFINITION 2.1. *A solution of* (1.1) *with a measurable input u on an interval $\mathcal{I}$ containing* 0 *is an absolutely continuous function $x : \mathcal{I} \to \mathbb{R}^n$ such that*

$$x(t) - x(0) = \int_0^t f(x(\tau), u(\tau))\mathrm{d}\tau$$

for almost every $t \in \mathcal{I}$.

THEOREM 2.2. *Assume that $u : \mathbb{R}_+ \to \mathbb{R}^m$ is measurable, $f \in \mathcal{C}(\mathbb{R}^n \times \mathbb{R}^m, \mathbb{R}^n)$, and the following Carathéodory condition holds:*

(A1) *For every $a \in \mathbb{R}^n$, there exists a locally integrable function $\beta : \mathbb{R}_+ \to \mathbb{R}_+$ such that*

$$\|f(a, u(t))\| \leq \beta(t)$$

for almost every $t \in \mathbb{R}_+$.

Then for every $x_0 \in \mathbb{R}^n$ there exists $I(x_0) > 0$ and a solution x of (1.1) *with input u on the maximal interval of existence* $[0, I(x_0))$.

Moreover, the solution x of (1.1) *is unique (on $[0, I(x_0))$) if f also satisfies the following condition:*

(A2) *For every compact set $K \subset \mathbb{R}^n$, there exists a locally integrable function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ such that*

$$\|f(a, u(t)) - f(b, u(t))\| \leq \alpha(t)\|a - b\|$$

for almost every $t \in \mathbb{R}_+$ and for all $a, b \in K$.

We need this result in the following sections when dealing with an L^p input signal. If $I(x_0) < \infty$ is as in Theorem 2.2, then it is called *the finite escape time*, i.e., $\lim_{t\to I(x_0)} \|x(t)\| \to \infty$ (see, for example, [1, Theorem I.5.2]).

Let $\mathcal{X}$ be a metric space with distance μ. A set $G \subset \mathcal{X}$ is *relatively compact* (or *precompact*) if the closure of G is compact. Let $z : \mathbb{R}_+ \to \mathcal{X}$. A point $\xi \in \mathcal{X}$ is said to be an *ω-limit point of z* if there exists a sequence (t_n) in $\mathbb{R}_+$ such that $t_n \to \infty$ and $z(t_n) \to \xi$. The set of all the ω-limit points of z is denoted by $\Omega(z)$.

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A map $\pi : \mathbb{R}_+ \times \mathcal{X} \to \mathcal{X}$ is said to be a *semiflow on* $\mathcal{X}$ if π is continuous, $\pi(0, x_0) = x_0$ for all $x_0 \in \mathcal{X}$, and

$$\pi(s + t, x_0) = \pi(s, \pi(t, x_0)) \qquad \forall s, t \in \mathbb{R}_+, \quad \forall x_0 \in \mathcal{X}.$$

A nonempty set $G \subset \mathcal{X}$ is π-*invariant* if $\pi(t, G) = G$ for all $t \in \mathbb{R}_+$.

PROPOSITION 2.3. *Let $\pi : \mathbb{R}_+ \times \mathcal{X} \to \mathcal{X}$ be a semiflow on a metric space $\mathcal{X}$. Let $x_0 \in \mathcal{X}$ and denote $z(t) = \pi(t, x_0)$. If $z(\mathbb{R}_+)$ is relatively compact, then $\Omega(z)$ is nonempty, compact, and π-invariant, and*

$$\lim_{t\to\infty} \mu(z(t), \Omega(z)) = 0. \tag{2.1}$$

The proof is a straightforward extension from the result for finite-dimensional systems, where $\mathcal{X} \subset \mathbb{R}^n$ (see, for example, the result of La Salle [8] or Logemann and Ryan [9]). Several extensions of the La Salle invariance principle to the infinite-dimensional systems can also be found in Hale [2] and Slemrod [12]. This result will be used for an infinite-dimensional system in sections 3 and 4. The proof is given below to make the paper self-contained. We mention that $\Omega(z)$ is also connected.

Proof. Since $z(\mathbb{R}_+)$ is relatively compact, $\Omega(z)$ is nonempty and compact.

To prove π-invariance, take $\xi \in \Omega(z)$ so that there exists a sequence (t_n) in $\mathbb{R}_+$ such that $t_n \to \infty$ and $z(t_n) \to \xi$. Take $t > 0$; then

$$\pi(t, \xi) = \lim_{n\to\infty} \pi(t, z(t_n)) = \lim_{n\to\infty} \pi(t + t_n, x_0) \in \Omega(z)$$

so that $\pi(t, \Omega(z)) \subset \Omega(z)$. To prove the opposite inclusion, take $\eta \in \Omega(z)$ so that $\eta = \lim_{n\to\infty} z(\tau_n)$ for some sequence (τ_n) with $\tau_n \to \infty$. The sequence $\pi(\tau_n - t, x_0)$ (defined for n large enough so that $\tau_n - t > 0$), being contained in a compact set, has a convergent subsequence $\pi(\theta_n, x_0)$, where (θ_n) is a subsequence of $(\tau_n - t)$. If we put $\xi = \lim_{n\to\infty} \pi(\theta_n, x_0)$, then $\pi(t, \xi) = \eta$.

To prove (2.1), assume that (2.1) is false. Then there exists a sequence $(t_n) \in \mathbb{R}_+$ such that $t_n \to \infty$ and $\mu(z(t_n), \Omega(z)) \geq \epsilon > 0$ for all n. This is a contradiction since for a subsequence (θ_n) of (t_n), we have $z(\theta_n) \to \xi \in \Omega(z)$. ◻

3. State convergence for dissipative systems given any L^p input. We need a few more definitions for this section. System **P** is called *dissipative with respect to supply rate* $s(y, u)$ if there exists a storage function $H \in \mathcal{C}^1(\mathbb{R}^n, \mathbb{R}_+)$ such that

$$\frac{\partial H(x)}{\partial x} f(x, u) \leq s(y, u). \tag{3.1}$$

See also Willems [16] for a description of dissipative systems.

P is said to be *zero-state detectable* if the following is true: if $u(t) = 0$ and x is a unique solution of (1.1) on $\mathbb{R}_+$ such that $y(t) = 0$ for all $t \geq 0$, then $\lim_{t\to\infty} x(t) = 0$ (see also [3, Definition 10.7.3]).

Suppose that $p \in [1, \infty)$. We need the following additional assumption on f in (1.1):

(B1) For every $a \in \mathbb{R}^n$, there exist constants $c_1, c_2 > 0$ such that

$$\|f(a, v)\| \leq c_1 + c_2 \|v\|^p \qquad \forall v \in \mathbb{R}^m. \tag{3.2}$$

Remark 3.1. For any $p \in [1, \infty)$, it can be shown that assumption (B1) is satisfied for affine passive nonlinear systems **P** described by

$$\dot{x} = \tilde{f}(x) + g(x)u, \tag{3.3}$$

$$y = h(x), \tag{3.4}$$

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where $\tilde{f} \in \mathcal{C}(\mathbb{R}^n, \mathbb{R}^n)$, $f(0) = 0$, $g \in \mathcal{C}(\mathbb{R}^n, \mathbb{R}^{n\times m})$, $g(0)$ has rank m, and h is as in (1.2). This class of systems includes also the port-controlled Hamiltonian systems [11].

For any $\tau \geq 0$, we denote by $\mathbf{S}_\tau^*$ the left-shift operator by τ, acting on $X = L^p(\mathbb{R}_+, \mathbb{R}^m)$. Suppose that $d_0 \in L^p(\mathbb{R}_+, \mathbb{R}^m)$ and $d_t = \mathbf{S}_t^* d_0$; it follows that $d_t \in L^p(\mathbb{R}_+, \mathbb{R}^m)$ for all $t \geq 0$, and the following equation holds for almost every $t \geq 0$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\|d_t\|_{L^p}^p = \frac{\mathrm{d}}{\mathrm{d}t}\int_t^\infty ||d_0(\xi)||^p \mathrm{d}\xi = -||d_0(t)||^p. \tag{3.5}$$

THEOREM 3.2. *Suppose that* $p \in [1, \infty)$*, and let system* $\mathbf{P}$ *defined by* (1.1), (1.2) *be zero-state detectable and satisfy* (B1). *Assume that* $\mathbf{P}$ *has a storage function* H *such that it is dissipative with respect to the supply rate* $s(y, u) = \|u\|^p - k\|y\|^p$, $k > 0$. *Suppose that* $H(x) > 0$ *for* $x \neq 0$, $H(0) = 0$, *and* H *is proper.*

Then for every initial condition $x_0 \in \mathbb{R}^n$ *and for every* $u \in L^p(\mathbb{R}_+, \mathbb{R}^m)$*, the state trajectories* x *of* (1.1) *exist on* $\mathbb{R}_+$ *and* $x(t) \to 0$ *as* $t \to \infty$.

Proof. Let $x_0 \in \mathbb{R}^n$ and $u \in L^p(\mathbb{R}_+, \mathbb{R}^m)$.

Using assumption (B1), we have that for each fixed $a \in \mathbb{R}^n$, there exist constants $c_1, c_2 > 0$ such that (3.2) holds. By denoting $\beta(t) = c_1 + c_2\|u(t)\|^p$, and since $u \in L^p(\mathbb{R}_+, \mathbb{R}^m)$, β is locally integrable and satisfies condition (A1) in Theorem 2.2 for state equation (1.1).

Then using β as above and using initial value $x_0 \in \mathbb{R}^n$, it follows from Theorem 2.2 that there exists $I(x_0) > 0$ and a solution of (1.1) with input $u \in L^p(\mathbb{R}_+, \mathbb{R}^m)$ on $\mathcal{I} = [0, I(x_0))$. In particular, x is absolutely continuous as a function of t on $\mathcal{I}$.

We define an infinite-dimensional signal generator for the signal u. This signal generator has the state space $X = L^p(\mathbb{R}_+, \mathbb{R}^m)$ and the evolution of its state is governed by the operator semigroup $(\mathbf{S}_\tau^*)_{\tau\geq 0}$. Thus, the state of the signal generator at time t is $d_t = \mathbf{S}_t^* d_0$, where $d_0 \in X$ is the initial state. The generator of this semigroup is $\mathcal{A} = \frac{\mathrm{d}}{\mathrm{d}\xi}$ with domain $\mathcal{D}(\mathcal{A}) = \mathcal{W}^{1,p}(\mathbb{R}_+, \mathbb{R}^m)$. The observation operator of this signal generator is $\mathcal{C}$, defined for $\phi \in \mathcal{D}(\mathcal{A})$ by $\mathcal{C}\phi = \phi(0)$. It can be checked that $\mathcal{C}$ is admissible in the sense of Weiss [15]. We need the Lebesgue extension of $\mathcal{C}$, denoted by $\mathcal{C}_L$, defined by

$$\mathcal{C}_L\phi = \lim_{\epsilon\to 0} \mathcal{C}\frac{1}{\epsilon}\int_0^\epsilon \mathbf{S}_t^*\phi \ \mathrm{d}t = \lim_{\epsilon\to 0}\frac{1}{\epsilon}\int_0^\epsilon \phi(\xi)\ \mathrm{d}\xi,$$

with $\mathcal{D}(\mathcal{C}_L)$ being the set of all $\phi \in L^p(\mathbb{R}_+, \mathbb{R}^m)$ for which the above limit exists. We refer to [15] for more information on the concept of Lebesgue extension. The output function of the signal generator is $u(t) = \mathcal{C}_L d_t$, which is defined for almost every $t \geq 0$. It turns out that $u = d_0$ (the generated signal is the initial state).

We define an extended system $\mathbf{L}$ by connecting $\mathbf{P}$ to the generator for d_0. Then we have

$$\dot{x}(t) = f\left(x(t), u(t)\right), \tag{3.6}$$

$$d_t = \mathbf{S}_t^* d_0, \tag{3.7}$$

$$u(t) = \mathcal{C}_L d_t, \tag{3.8}$$

$$y(t) = h(x(t)). \tag{3.9}$$

Let $z(t) = \left[\begin{smallmatrix} x(t) \\ d_t \end{smallmatrix}\right]$ denote the state at time t of the above system so that $z(t) \in Z = \mathbb{R}^n \times X$.

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Consider the storage function $V : Z \to \mathbb{R}_+$ defined for $\left[\begin{smallmatrix} x \\ d \end{smallmatrix}\right]$ by $V(z) = H(x) + 2\|d\|_{L^p}^p$. It can be shown that the function $t \mapsto V(z(t))$ is absolutely continuous. Since H is continuously differentiable and the solution x of (3.6) is absolutely continuous as a function of t defined in $\mathcal{I}$, it follows that $H(x(t))$ is absolutely continuous on $\mathcal{I}$. From (3.5), and since $d_0 \in L^p(\mathbb{R}_+, \mathbb{R}^m)$, it follows that $t \mapsto \frac{\mathrm{d}}{\mathrm{d}t}\|d_t\|_{L^p}^p \in L^1(\mathbb{R}_+, \mathbb{R}^m)$. This implies that $\|d_t\|_{L^p}^p$ is absolutely continuous on $\mathbb{R}_+$.

Using (3.1), (3.5), (3.6)–(3.8), we obtain that, for almost every $t \in \mathcal{I}$,

$$\begin{aligned} \dot{V} &= \frac{\partial H(x)}{\partial x} f\left(x, u(t)\right) - 2\|u(t)\|^p \\ &\leq -k\|y(t)\|^p + \|u(t)\|^p - 2\|u(t)\|^p, \\ &\leq -k\|y(t)\|^p - \|u(t)\|^p. \end{aligned} \tag{3.10}$$

Let us prove that $\mathcal{I} = \mathbb{R}_+$. If the maximal interval of definition $I(x_0)$ is finite, $\lim_{t \to I(x_0)} \|x(t)\| = \infty$. Since V is absolutely continuous as a function of t and bounded from below, (3.10) implies that $V(z(t))$ is bounded and nonincreasing for all $t \in \mathcal{I}$. In particular, the state $x(t)$ never leaves the compact set $\{\xi \in \mathbb{R}^n \mid H(\xi) \leq V(z(0))\}$ for all $t \in \mathcal{I}$. Hence, we conclude that $\mathcal{I} = \mathbb{R}_+$ and $x(t)$ is bounded for all $t \geq 0$.

Since V is bounded from below and $V(z(t))$ is nonincreasing for all $t \in \mathbb{R}_+$ (i.e., monotonic decreasing function), $V(z(t))$ has a limit $c \in \mathbb{R}$ as $t \to \infty$.

We will prove the relative compactness of $z(\mathbb{R}_+)$. It has been shown that $x(t)$ is bounded for all $t \in \mathbb{R}_+$; hence $x(\mathbb{R}_+)$ is relatively compact in $\mathbb{R}^n$. Since $\lim_{t\to\infty} \|d_t\|_{L^p} = 0$, it implies that $\{d_t \mid t \geq 0\}$ is relatively compact in $L^p(\mathbb{R}_+, \mathbb{R}^m)$. Therefore $z(\mathbb{R}_+)$ is relatively compact in $\mathbb{R}^n \times X$.

The final proof is to use Proposition 2.3 by showing that $\Omega(z) = \{0\}$. Let π denote the semiflow of (3.6)–(3.7) so that $z(t) = \pi(t, z_0)$, $z_0 = z(0)$. Using Proposition 2.3 and the relative compactness of $z(\mathbb{R}_+)$, $\Omega(z)$ is nonempty, compact, and π-invariant.

For any $\xi \in \Omega(z)$, there is a sequence $(t_n) \in \mathbb{R}_+$ such that $t_n \to \infty$ and $z(t_n) \to \xi$. By continuity of $V(z)$, $V(\xi) = \lim_{n\to\infty} V(z(t_n)) = c$. Therefore, $V(\xi) = c$ on $\Omega(z)$. Using (3.10), this implies that on $\Omega(z)$, $u = 0$ and $y = 0$ along such trajectories.

By the assumptions of the theorem, the system described by (3.6) is zero-state detectable; i.e., $u(t) = 0$ and $y(t) = 0$ for all $t \geq 0$ implies that $x(t) \to 0$ as $t \to \infty$. Also, if $u(t) = 0$ for all $t \in \mathbb{R}_+$, then $d_0 = 0$ so that $d_t = 0$ for all $t \in \mathbb{R}_+$. Hence, in the π-invariant set $\Omega(z)$, $V(\xi) = V(0) = 0$ for all $\xi \in \Omega(z)$. Since $V(\xi) > 0$ for all $\xi \neq 0$, we obtain that $\Omega(z) = \{0\}$. In particular, using (2.1) it follows that $x(t) \to 0$ as $t \to \infty$.

The above argument is true for any initial conditions $x_0 \in \mathbb{R}^n$ and for any $u \in L^p(\mathbb{R}_+, \mathbb{R}^m)$. ∎

Remark 3.3. Note that Theorem 3.2 assumes that the range of H is $\mathbb{R}_+$ and $H(x) > 0$ for $x \neq 0$. It is given in this form for clarity of presentation but it is not restrictive. It suffices to assume that the range of H is bounded from below and $H(0)$ is the unique global minimum of H.

Remark 3.4. For system **P** satisfying all the assumptions in Theorem 3.2, it has been shown that for every $u \in L^p(\mathbb{R}_+, \mathbb{R}^m)$ and for every initial state $x_0 \in \mathbb{R}^n$, the state trajectories x of (1.1) exist on $\mathbb{R}_+$ and are bounded. It is easy to see from (3.1) that $y(t)$ is also in $L^p(\mathbb{R}_+, \mathbb{R}^m)$ and satisfies $\|y_T\|_{L^p}^p \leq \frac{1}{k}\|u_T\|_{L^p}^p + \frac{1}{k}H(x_0)$. Therefore, system **P** as in Theorem 3.2 is L^p-stable and has a finite L^p-gain $\leq \frac{1}{k}$ (see van der Schaft [11] for details).

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Note that for system **P** satisfying the assumptions in Theorem 3.2, the state convergence result may not be obtained by using Barbălat's lemma. We also cannot apply the state convergence result from [10, Theorem 4.2] to **P** since the dissipativity condition is not assumed there.

In Theorem 3.2, the state trajectories x of **P** are not necessarily unique given an initial state x_0 and an L^p input u. Despite the fact that there can be several solutions, it has been shown in the proof of the theorem that all trajectories are bounded and converge to zero.

The state trajectories x are unique if f assumes the following additional local Lipschitz-type condition:

(B2) For every compact set $K \subset \mathbb{R}^n$, there exist constants $c_3, c_4 > 0$ such that

$$\|f(a,v) - f(b,v)\| \leq (c_3 + c_4\|v\|^p)\|a-b\| \tag{3.11}$$

for all $v \in \mathbb{R}^m$, $a, b \in K$.

THEOREM 3.5. *Suppose that* $p \in [1,\infty)$*, and let system* **P** *defined by* (1.1), (1.2) *be zero-state detectable and satisfy* (B1) *and* (B2)*. Assume that* **P** *has a storage function* H *such that it is dissipative with respect to supply rate* $s(y,u) = \|u\|^p - k\|y\|^p$, $k > 0$. *Suppose that* $H(x) > 0$ *for* $x \neq 0$, $H(0) = 0$, *and* H *is proper.*

Then for every initial condition $x_0 \in \mathbb{R}^n$ *and for every* $u \in L^p(\mathbb{R}_+, \mathbb{R}^m)$*, there exists a unique state trajectory* x *of* (1.1) *on* $\mathbb{R}_+$ *and* $x(t) \to 0$ *as* $t \to \infty$*.*

Proof. Let $x_0 \in \mathbb{R}^n$ and $u \in L^p(\mathbb{R}_+, \mathbb{R}^m)$.

Using assumption (B1), we have that for each fixed $a \in \mathbb{R}^n$ there exist constants $c_1, c_2 > 0$ such that (3.2) holds. By denoting $\beta(t) = c_1 + c_2\|u(t)\|^p$, and since $u \in L^p(\mathbb{R}_+, \mathbb{R}^m)$, β is locally integrable and satisfies condition (A1) in Theorem 2.2 for state equation (1.1).

Using the assumption (B2), we have that for each fixed $a \in \mathbb{R}^n$ there exist constants $c_3, c_4 > 0$ such that (3.11) holds. By denoting $\alpha(t) = c_3 + c_4\|u(t)\|^p$, and since $u \in L^p(\mathbb{R}_+, \mathbb{R}^m)$, α is locally integrable and satisfies condition (A2) in Theorem 2.2 for state equation (1.1).

Then using β, α as above, using initial value $x_0 \in \mathbb{R}^n$, and using the measurable input $u \in L^p(\mathbb{R}_+, \mathbb{R}^m)$, it follows from Theorem 2.2 that there exists $I(x_0) > 0$ and a unique solution of (1.1) with input $u \in L^p(\mathbb{R}_+, \mathbb{R}^m)$ on $\mathcal{I} = [0, I(x_0))$.

The rest of the proof is similar to the proof of Theorem 3.2 to show that $\mathcal{I} = \mathbb{R}_+$, and the unique state trajectory x converges to zero. The arguments are valid for every initial condition $x_0 \in \mathbb{R}^n$ and $u \in L^p(\mathbb{R}_+, \mathbb{R}^m)$. ☐

4. Some generalizations. In this section, we discuss several generalizations of the state convergence result from the previous section. We extend the problem by supposing that the supply rate is given by $s(y,u) = \sigma(\|u\|) - \gamma(\|y\|)$ with arbitrary $\sigma, \gamma \in \mathcal{K}$, the input function u satisfies $\int_0^\infty \sigma(\|u(\tau)\|)\,\mathrm{d}\tau < \infty$, and the storage function H is not proper.

Let $\sigma \in \mathcal{K}$, and denote by $\mathcal{U}_\sigma(\mathbb{R}_+, \mathbb{R}^m)$ the space of measurable functions $g : \mathbb{R}_+ \to \mathbb{R}^m$ such that $\int_0^\infty \sigma(\|g(\tau)\|)\,\mathrm{d}\tau < \infty$. It is clear that the space $\mathcal{U}_\sigma(\mathbb{R}_+, \mathbb{R}^m)$ is not necessarily a metric space.

THEOREM 4.1. *Assume that* $\sigma, \gamma \in \mathcal{K}$*. Let system* **P** *defined by* (1.1), (1.2) *be zero-state detectable. Assume that there exists a storage function* H *such that* **P** *is dissipative with respect to supply rate* $s(y,u) = \sigma(\|u\|) - \gamma(\|y\|)$*. Suppose that* $H(x) > 0$ *for* $x \neq 0$ *and* $H(0) = 0$*.*

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If for a given initial condition $x_0 \in \mathbb{R}^n$ *and for a given measurable input* $u \in \mathcal{U}_\sigma \cap L^p(\mathbb{R}_+, \mathbb{R}^m)$, $p \in [1, \infty)$, *the state trajectories* x *of* (1.1) *exist on* $\mathbb{R}_+$ *and remain in a compact set* $K \subset \mathbb{R}^n$, *then* $x(t) \to 0$ *as* $t \to \infty$.

Proof. Let us define an infinite-dimensional signal generator for the signal u. Denote $X = \mathcal{U}_\sigma \cap L^p(\mathbb{R}_+, \mathbb{R}^m)$, which is a metric space with distance $\mu(g_1, g_2) = \|g_1 - g_2\|_{L^p}$ for any $g_1, g_2 \in X$. We use X as the state space of the signal generator for u. The evolution of its state is governed by the operator semigroup $(\mathbf{S}^*_\tau)_{\tau \geq 0}$ acting on X. Thus, the state of the signal generator at time t is $d_t = \mathbf{S}^*_t d_0$, where $d_0 \in X$ is the initial state. It can be checked that if $d_0 \in X$, then $d_t \in X$.

We need the observation operator $\mathcal{C}_L$ defined by

$$\mathcal{C}_L \phi = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \int_0^\epsilon \phi(\xi)\ \mathrm{d}\xi,$$

with $\mathcal{D}(\mathcal{C}_L)$ being the set of all $\phi \in X$ for which the above limit exists. The output function of the signal generator is $u(t) = \mathcal{C}_L d_t$, which is defined for almost every $t \geq 0$. It turns out that $u = d_0$.

In the following, we define an extended system $\mathbf{L}$ by connecting $\mathbf{P}$ to the generator for u:

$$\dot{x}(t) = f\left(x(t), u(t)\right), \tag{4.1}$$

$$d_t = \mathbf{S}^*_t d_0, \tag{4.2}$$

$$u(t) = \mathcal{C}_L d_t, \tag{4.3}$$

$$y(t) = h(x(t)). \tag{4.4}$$

Let $z(t) = \left[\begin{smallmatrix} x(t) \\ d_t \end{smallmatrix}\right]$ denote the state at time t of the above system so that $z(t) \in Z = \mathbb{R}^n \times X$. Since it is assumed that the state trajectory x never leaves the compact set $K \subset \mathbb{R}^n$ and that $\lim_{t\to\infty} \int_0^\infty \sigma(\|d_t(\tau)\|)\,\mathrm{d}\tau = 0$, the state trajectory z is defined for all $t \geq 0$, and the set $\{z(t) \,|\, t \in \mathbb{R}_+\}$ is relatively compact.

For $\left[\begin{smallmatrix} x \\ d \end{smallmatrix}\right] \in Z$, define the function $V : \left[\begin{smallmatrix} x \\ d \end{smallmatrix}\right] \mapsto \mathbb{R}_+$ by

$$V(z) = H(x) + 2\int_0^\infty \sigma(\|d(\tau)\|)\,\mathrm{d}\tau.$$

Since H is continuous and the solution x of (4.1) is absolutely continuous as a function of t defined on $\mathbb{R}_+$, it follows that $t \mapsto H(x(t))$ is absolutely continuous on $\mathbb{R}_+$. Since $d_0 \in \mathcal{U}_\sigma(\mathbb{R}_+, \mathbb{R}^m)$, the function

$$t \mapsto 2\frac{\mathrm{d}}{\mathrm{d}t}\int_0^\infty \sigma(\|d_t(\tau)\|)\,\mathrm{d}\tau = 2\frac{\mathrm{d}}{\mathrm{d}t}\int_t^\infty \sigma(\|d_0(\tau)\|)\,\mathrm{d}\tau = -2\sigma(\|d_0(t)\|) \tag{4.5}$$

belongs to $L^1(\mathbb{R}_+)$. This implies that the function $t \mapsto 2\int_0^\infty \sigma(\|d_t(\tau)\|)\,\mathrm{d}\tau$ is absolutely continuous on $\mathbb{R}_+$. Hence the function $t \mapsto V(z(t))$ is absolutely continuous on $\mathbb{R}_+$.

Using (4.1)–(4.5), we obtain that, for almost every $t \in \mathbb{R}_+$,

$$\begin{aligned}\dot{V}(z(t)) &= \frac{\partial H(x)}{\partial x} f\left(x, u(t)\right) - 2\sigma(\|u(t)\|) \\ &\leq -\gamma(\|y(t)\|) + \sigma(\|u(t)\|) - 2\sigma(\|u(t)\|) \\ &\leq -\gamma(\|y(t)\|) - \sigma(\|u(t)\|). \end{aligned} \tag{4.6}$$

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Let π denote the semiflow of (4.1)–(4.4) so that $z(t) = \pi(t, z_0)$, $z_0 = z(0)$. Using Proposition 2.3 and the relative compactness of $z(\mathbb{R}_+)$, $\Omega(z)$ is nonempty, compact, and π-invariant.

Similar to the last arguments in the proof of Theorem 3.2, using the zero-state detectability of $\mathbf{P}$, using (4.6), and using the fact that $V(\xi) > 0$ for all $\xi \neq 0$, we obtain that $\Omega(z) = \{0\}$. Hence, using (2.1) it follows that $x(t) \to 0$ as $t \to \infty$. ∎

For a measurable function $u : \mathbb{R}_+ \to \mathbb{R}^m$ and $\sigma \in \mathcal{K}$, one often defines the energy of u by the energy functional $E_\sigma : u \mapsto \int_0^\infty \sigma(\|u(\tau)\|)\,\mathrm{d}\tau$. In this situation, Theorem 4.1 shows that, given a bounded-energy input u and given an initial state x_0, the boundedness of the state trajectories x and the dissipativity condition imply that x converge to zero. The theorem neither imposes any condition on f nor assumes the properness of H.

Under additional assumptions on $\mathbf{P}$ which are similar to the ones found in Theorems 3.2 and 3.5, we can have a stronger result of Theorem 4.1 which ensures the existence of state trajectories x on $\mathbb{R}_+$ and the convergence of x to zero for every bounded-energy input and for every initial state.

THEOREM 4.2. *Assume that $\sigma, \gamma \in \mathcal{K}$. Let system $\mathbf{P}$ defined by* (1.1)*,* (1.2) *be zero-state detectable and satisfy the following:*

(C1) *For every $a \in \mathbb{R}^n$, there exist constants $c_1, c_2 > 0$ such that*

$$\|f(a,v)\| \le c_1 + c_2\sigma(\|v\|) \qquad \forall v \in \mathbb{R}^m. \tag{4.7}$$

Assume that there exists a storage function H such that $\mathbf{P}$ is dissipative with respect to supply rate $s(y,u) = \sigma(\|u\|) - \gamma(\|y\|)$. Suppose that H is proper, $H(x) > 0$ for $x \neq 0$, and $H(0) = 0$.

Then for every $x_0 \in \mathbb{R}^n$ and for every $u \in \mathcal{U}_\sigma \cap L^p(\mathbb{R}_+, \mathbb{R}^m)$, $p \in [1, \infty)$, the state trajectories x of (1.1) *exist on $\mathbb{R}_+$ and $x(t) \to 0$ as $t \to \infty$.*

Moreover, the state trajectory x of (1.1) *on $\mathbb{R}_+$ is unique for every $x_0 \in \mathbb{R}^n$ and for every $u \in \mathcal{U}_\sigma \cap L^p(\mathbb{R}_+, \mathbb{R}^m)$, $p \in [1, \infty)$, if f additionally satisfies the following:*

(C2) *For every compact set $K \subset \mathbb{R}^n$, there exist constants $c_3, c_4 > 0$ such that*

$$\|f(a,v) - f(b,v)\| \le (c_3 + c_4\sigma(\|v\|))\|a - b\| \tag{4.8}$$

for all $v \in \mathbb{R}^m$, $a, b \in K$.

Proof. Let $x_0 \in \mathbb{R}^n$ and $u \in \mathcal{U}_\sigma \cap L^p(\mathbb{R}_+, \mathbb{R}^m)$.

Using (C1), we have that for each fixed $a \in \mathbb{R}^n$, there exist constants $c_1, c_2 > 0$ such that (4.7) holds. By denoting $\beta(t) = c_1 + c_2\sigma(\|u(t)\|)$, and since $u \in \mathcal{U}_\sigma(\mathbb{R}_+, \mathbb{R}^m)$, β is locally integrable and satisfies condition (A1) in Theorem 2.2 for state equation (1.1).

Then using β as above, using initial value $x_0 \in \mathbb{R}^n$, and using the measurable input $u \in \mathcal{U}_\sigma(\mathbb{R}_+, \mathbb{R}^m)$, it follows from Theorem 2.2 that there exists $I(x_0) > 0$ and a solution of (1.1) with input $u \in \mathcal{U}_\sigma(\mathbb{R}_+, \mathbb{R}^m)$ on $\mathcal{I} = [0, I(x_0))$.

We will prove that $\mathcal{I} = \mathbb{R}_+$ and x is bounded. Using the dissipativity of $\mathbf{P}$, we have that

$$\dot{H} = \frac{\partial H(x)}{\partial x} f(x, u(t)) \le -\gamma(\|y(t)\|) + \sigma(\|u(t)\|) \le \sigma(\|u(t)\|)$$

holds for almost every $t \in \mathcal{I}$. Integrating the above inequality from 0 to $T \in \mathcal{I}$, we obtain

$$H(x(T)) \le H(x(0)) + \int_0^T \sigma(\|u(\tau)\|)\,\mathrm{d}\tau \le H(x_0) + \int_0^\infty \sigma(\|u(\tau)\|)\,\mathrm{d}\tau.$$

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Using the fact that $u \in \mathcal{U}_\sigma(\mathbb{R}_+, \mathbb{R}^m)$ and using the properness of H, the inequality implies that $x(t)$ never leaves the compact set

$$K = \left\{ \xi \in \mathbb{R}^n \mid H(\xi) \le H(x_0) + \int_0^\infty \sigma(\|u(\tau)\|)\,\mathrm{d}\tau \right\}$$

for all $t \in \mathcal{I}$. Hence, we conclude that $\mathcal{I} = \mathbb{R}_+$, and $x(t)$ remains in a compact set K for all $t \ge 0$.

Therefore, Theorem 4.1 can be used to conclude the first claim of the theorem. The arguments are valid for every initial condition $x_0 \in \mathbb{R}^n$ and for every $u \in \mathcal{U}_\sigma(\mathbb{R}_+, \mathbb{R}^m)$.

The second claim follows once we establish that the local solution x of (1.1) on $\mathcal{I}$ is unique. Using assumption (C2), we have that for each fixed $a \in \mathbb{R}^n$ there exist constants $c_3, c_4 > 0$ such that (4.8) holds. By denoting $\alpha(t) = c_3 + c_4\sigma(\|u(t)\|)$, and since $u \in \mathcal{U}_\sigma(\mathbb{R}_+, \mathbb{R}^m)$, α is locally integrable and satisfies condition (A2) in Theorem 2.2 for state equation (1.1). Then using β, α, using initial value $x_0 \in \mathbb{R}^n$, and using the measurable input $u \in \mathcal{U}_\sigma(\mathbb{R}_+, \mathbb{R}^m)$, it follows from Theorem 2.2 that there exists $I(x_0) > 0$ and a unique solution of (1.1) with input $u \in \mathcal{U}_\sigma(\mathbb{R}_+, \mathbb{R}^m)$ on $\mathcal{I} = [0, I(x_0))$. □

5. Example.

Let system $\mathbf{P}$ be described by

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = f(x,u) \tag{5.1}$$

$$= \begin{bmatrix} -x_1(1 + 1/(u^p+1) + u^p) - x_2^{p-1} + u \\ x_1^{p-1} \end{bmatrix}, \tag{5.2}$$

$$y = x_1, \tag{5.3}$$

where $p \in \{2n \mid n \in \mathbb{N}\}$. It can be checked that for each compact set $K \subset \mathbb{R}^2$, there exists $b_1, b_2, b_3 > 0$ such that $\|f(a,v) - f(a,0)\| \le b_1 + b_2\|v\| + b_3\|v\|^p$ for all $(a, v) \in K \times \mathbb{R}$. Hence, it does not satisfy the Lipschitz condition in [10].

It is easy to check that for every $a \in \mathbb{R}^2$, there exist $c_1, c_2 > 0$ such that $\|f(a,v)\| \le c_1 + c_2\|v\|^p$ for all $v \in \mathbb{R}$; i.e., (B1) holds. Also, for every compact set $K \in \mathbb{R}^2$, there exists $c_3, c_4 > 0$ such that

$$\|f(a,v) - f(b,v)\| \le (c_3 + c_4\|v\|^p)\|a - b\|$$

holds for all $v \in \mathbb{R}$ and $a, b \in K$; i.e., (B2) holds.

Using a proper storage function $H(x) = x_1^p + x_2^p$, it can be checked that by using Young's inequality,

$$\begin{aligned} \dot{H} &= -px_1^p(1 + 1/(u^p+1) + u^p) + px_1^{p-1}u \\ &\le -px_1^p + (p-1)x_1^p + u^p = -\|y\|^p + \|u\|^p. \end{aligned}$$

This shows that $\mathbf{P}$ is dissipative with respect to supply rate $s(y,u) = \|u\|^p - \|y\|^p$. System $\mathbf{P}$ is also zero-state detectable. Based on this information, the application of Barbălat's lemma can show only that $\lim_{t\to\infty} \|y(t)\| = 0$ (see, for example, Teel [14]). It follows from Theorem 3.2 that for any L^p input u, there exists a unique solution $x(t)$ defined for all $t \ge 0$ and $\lim_{t\to\infty} x(t) = 0$.

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REFERENCES

[1] J. K. Hale, *Ordinary Differential Equations*, Robert E. Krieger Publishing Company, Malabar, FL, 1980.

[2] J. K. Hale, *Dynamical systems and stability*, J. Math. Anal. Appl., 26 (1969), pp. 39–59.

[3] A. Isidori, *Nonlinear Control Systems* II, Springer-Verlag, London, 1999.

[4] B. Jayawardhana, *Remarks on the state convergence of nonlinear systems given any L^p input*, in Proceedings of the 45th IEEE Conference on Decision and Control, San Diego, CA, 2006, pp. 995–1000.

[5] B. Jayawardhana and G. Weiss, *State convergence of passive nonlinear systems with an L^2 input*, IEEE Trans. Automat. Control, 54 (2009), pp. 1723–1727.

[6] B. Jayawardhana and G. Weiss, *Convergence of the state of a passive nonlinear plant with an L^2 input*, in Proceedings of the European Control Conference, Kos, Greece, European Union Control Association, 2007.

[7] B. Jayawardhana, A. R. Teel, and E. P. Ryan, *iISS gain of dissipative systems*, in Proceedings of the 46th IEEE Conference on Decision and Control, New Orleans, LA, 2007, pp. 3835–3840.

[8] J. P. La Salle, *The Stability of Dynamical Systems* (with an Appendix by Z. Artstein), CBMS Reg. Conf. Ser. Appl. Math., 25, SIAM, Philadelphia, 1976.

[9] H. Logemann and E. P. Ryan, *Asymptotic behavior of nonlinear systems*, Amer. Math. Monthly, 111 (2004), pp. 864–889.

[10] E. P. Ryan, *Remarks on the L^p-input converging-state property*, IEEE Trans. Automat. Control, 50 (2005), pp. 1051–1052.

[11] A. J. van der Schaft, *L_2-Gain and Passivity Techniques in Nonlinear Control*, Springer-Verlag, London, 2000.

[12] M. Slemrod, *Asymptotic behavior of a class of abstract dynamical systems*, J. Differential Equations, 7, pp. 584–600.

[13] E. D. Sontag, *A remark on the converging-input converging-state property*, IEEE Trans. Automat. Control, 48 (2003), pp. 313–314.

[14] A. R. Teel, *Asymptotic convergence from L_p stability*, IEEE Trans. Automat. Control, 44 (1999), pp. 2169–2170.

[15] G. Weiss, *Admissible observation operators for linear semigroups*, Israel J. Math., 65 (1989), pp. 17–43.

[16] J. C. Willems, *Dissipative dynamical systems, Part* I: *General theory*, Arch. Rational Mech. Anal., 45 (1972), pp. 321–351.

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